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## Promoting Risk Taking in Mathematics Classrooms: The importance of Creating a Safe Learning Environment

Sashi Sharma

University of Waikato, New Zealand

**Abstract:** Students beliefs and attitudes towards risk taking can impact on their mathematics learning and performance. However, at present, risk is not established in the field of mathematics education. The challenge for mathematics teachers in developing their students' risk taking dispositions is to choose appropriate activities and tools that match this concept and the learning needs of the students. This paper describes some research-based ideas for promoting risk taking behaviours in a mathematics classroom. It presents interactional pedagogical strategies from a design collaborative research conducted at one secondary school. As part of the learning activities, students critically evaluated statistical investigations undertaken by others. Students had to take risks: ask critical questions, construct statistical arguments and respond to others' arguments in face of criticisms. The findings are interpreted in relation to recent writing about students' abilities to take risks in mathematics classrooms. The final section considers the issues arising out of the paper and offers suggestions for meeting these challenges.

*Keywords:* mathematics education, risk taking, collaborative research, secondary school students, critical statistical literacy, implications for research.

*Do the one thing you think you cannot do. Fail at it. Try again. Do better the second time. The only people who never tumble are those who never mount the high wire. This is your moment. Own it.*

~Oprah Winfrey

Risk abounds in our everyday life and workplace. Risk can delight, annoy and engage us. As the above quote suggests people need to be aware of the importance of risk taking. Notions of risk appear in various messages that we encounter such as when receiving forecasts of medical, financial or environmental risks from media research reports and public officials. In such situations, we have to make decisions in the presence of conflicting goals and constraints (Schlottmann & Wilkening, 2011; Shapira, Nattinger, & McHorney, 2001). In recent times, risk taking and risk-aversion has drawn increased attention of educators and researchers for a range of reasons (Anthony & Walshaw, 2007; Fesser, Martignon, Engel & Kuntze, 2010; Martignon & Kurz-Milcke, 2006; Till, 2014). These include recognition of its importance for solving differences and reaching consensus, as well as its central role in thinking and making informed decisions (Gaissmaier & Gigerenzer, 2008; Galesic & Garcia-Retamero, 2010; Martignon & Krauss, 2009; Schlottmann & Wilkening, 2011). Indeed, to learn and grow people must take risks, but most people will not take risks in an emotionally unsafe environment.

Moreover, there has been significant shifts in the way teaching and learning of mathematics is conceptualised internationally. The American policy document, Principles and Standards for School Mathematics states that if students are to learn to “construct mathematical arguments and respond to others” arguments, then creating an environment that fosters these kinds of activities is essential (National Council of Teachers of Mathematics, 2000, p. 18). This reform teaching aligns with vision promoted by NCTM's Professional Teaching Standards (National Governor's Association for Best Practices and Council of Chief State School Officers, 2010) that encourages

students to communicate mathematical ideas and nurtures intellectual risk-taking by students. However, reasoning at complex cognitive levels and risk taking through mathematical discourse is not something many students are able to achieve easily without adult mediation (Anthony & Walshaw, 2007; Cobb & McClain, 2004; French, 2009; Hunter, 2010).

While there has been ample discussion of students at risk (Clark, 2001; Franco, Sztajn, & Ortigão, 2007; Lubienski, 2007; Winsor, 2007), mathematics education at risk, and nations at risk (Center for the Study of Mathematics Curriculum, 2005; Nasir, Hand, & Taylor, 2008; Wagner, 2008), risk assessment (Gigerenzer, 2002; Gigerenzer, Gaissmaier, Kurz-Milcke, Schwartz, & Woloshin, 2009; Martignon & Krauss, 2009), there has been limited discussion of students' willingness to take risks in mathematics classrooms where they are asked to solve open-ended tasks (Lubienski, 2007; Sullivan, Mousley, & Zevenbergen, 2006). Individuals who have different perceptions of risk are more or less likely to do well in these environments (Clifford, 1991; Hills, Stroup, & Wilensky, 2005). It follows that learning environments must take into account individual student's and teachers' risk-taking or risk-averse behaviours. There is very little research on students' mind-sets and motivation for risk tolerance in mathematics education. In 2010, for the first time, risk was included in a session of the ICOTS, as a topic for instruction in school in connection with statistics education (Fesser et al., 2010).

This paper describes some research-based ideas for promoting risk taking behaviours in a mathematics classroom. It presents interactional pedagogical strategies from a design collaborative research (Sharma et al., 2011) conducted at one secondary school. As part of the learning activities, students critically evaluated statistical investigations undertaken by others (Ministry of Education, 2007). Students had to take risks: ask critical questions, construct statistical arguments and respond to others' arguments. Teachers used a range of strategies to initiate, sustain, direct, shift, and conclude mathematical discussions.

The first section reviews literature on risk taking and provides the theoretical framework taken in this paper. Theoretical standpoint of this paper is drawn from a sociocultural perspective which supports a view of mathematical teaching and learning as inherently social and founded on active participation in communicative reasoning processes. The section explains the importance of mathematical communication and questioning in teaching and learning mathematics. The second section will draw upon Sharma et al (2011) study to explain pedagogical strategies used in the study to promote risk taking. Episodes are provided to illustrate the demand for the teacher to engage in a number of pedagogical actions in order to maintain student participation and engagement in the discourse community. The final section outlines some implications and offers suggestions for teaching and further inquiry into risk literacy.

## **Literature Review**

### **Research on Risk Taking**

Understanding how people think about risk and their attitudes towards risk taking is important for educators and policy makers (Xie, Wang & Xu, 2003). It is widely assumed that people differ considerably in their motivation and attitudes towards risks, ranging from cautiousness to risk-seeking and even pleasure in risk-taking (Burrill, 1998; Rohrmann, 2005). However, there is no convincing evidence from these studies that this is a general trait (Hills, Stroup, & Wilensky, 2005; Rohrmann, 2005). According to Rohrmann, risk attitudes are multi-dimensional and that risk orientations are not consistent across domains and the motivation for accepting risks depend on the context. There is very little research on students' mind-sets towards risk taking in mathematics education. To address this issue, Atkins, Leder, O'Halloran, Pollard and Taylor (1991) investigated the tendency of students to take risks in mathematics examination (in multi-choice format). Atkins, et al. (1991, p. 297) defined risk taking as "the preparedness of a student to attempt to answer a

question when not certain of the result”.

Specifically, data from the Australian Mathematics Competition were analysed to measure risk taking by groups of secondary students by gender, school year and achievement level. An example of a problem from their study is given below.

*Mary wanted to divide a certain number by 4 to get an answer. However, she used the calculator incorrectly and multiplied by 4 instead and got 60. The correct answer would be*  
 (A) 3.75    (B) 15    (C) 4    (D) 12    (E) 240

The statistics for this question were as follows:

(A) 34%    (B) 42%    (C) 5%    (D) 7%    (E) 4%    No response 6%

Atkins et al. (1991) explained that the incorrect responses ©, (D) and (E) were more likely to be due to risk taking techniques whereas the high response for the alternative (B) was probably due to participants who believed that they had answered correctly but had not read the question carefully.

The examination paper consisted of 30 multi-choice questions. The first 10 questions were worth 3 points, the next 10 were worth 4 points and the last 10 were worth 5 points. The range of possible scores was from 0 (for 30 incorrect responses) to 150 (for 30 correct responses). The authors measured risk by finding the average value of penalty marks which ranged from 0 to 30 for an individual student.

The measures showed relatively high values for the lower level grades decreasing each year to grade 12. Males consistently obtained higher z values than females except for grade 10. The researchers concluded that among high school students, the highest risk taking behaviour takes place among year 7 students and steadily decrease until year 12. Findings regarding gender differences resonate with the findings of Fesser et al. (2010). Data from Fesser et al. sample exhibited interesting gender differences, girls tended to be more risk averse than boys in a typical gamble and a ludo game situation. However, Fesser et al. also reported that younger students lacked effective tools for assessing and communicating risk which is inconsistent with the findings of Atkins et al. Fesser et al. findings motivate the development of research programmes for implementing risk assessment and risk communication as a mathematics topic in school curricula. Moreover, Fesser et al. argued that an early familiarisation with risk as a topic in school may strengthen intuitions on risk and convey competencies for sound decision making in risky situations.

It appears that children can be taught to be more risk seeking in their activities. Dehaene, Izard, Spelke and Pica (2008) contributed to this picture through their examination of whether the move from risk aversion (logarithmic) to risk taking (linear mapping) was a result of formal schooling or a natural process of brain maturation. To study this, they undertook some number-mapping exercises with the Mundurucu, an Amazonian group with little access to education or other instruments that may affect their perception of numbers (such as maps and rulers). The researchers concluded that the education of children in mathematics tended to increase their risk tolerance through changing the way they see numbers. The conclusions conform the findings of Fesser et al. (2010).

Streimatter (1997) claims that in order for people to understand who they are, they must experiment and take risks. She further adds that one appropriate place for young people to take risks as they explore who they are is in the mathematics classroom. Streimatter argues that “without taking academic risks, asking or answering questions in the classroom a large part of students’ lives may be excluded from their conscious or subconscious deliberations during this period of identity” (p. 18). Indeed, students who are active participants in their own education tend to be higher achievers. It must be noted that although asking and answering questions in class may seem rather trivial

compared to risk taking behaviours, for some students, these behaviours involve substantial risk tolerance.

Streimatter (1997) explored the dynamics of a girls-only class in a public co-educational middle school. Her classroom observations and interviews with girls revealed that girls were more likely to ask and answer questions about subject matter in the mathematics class than they were in their other classes which were coeducational. The girls also said that the girls-only setting enhanced their ability to learn mathematics and their views of themselves as mathematicians.

Indeed in numerous areas, including financial decisions, health/safety, and recreation females are found to be more risk-averse than males (Weber et al., 2002). These gender differences in everyday decisions may also carry over to educational performance in mathematics. For example, a meta-analysis performed by Hyde et al., (1990) (cited in Hills, Stroup, & Wilensky, 2005), assessing the performance of over 3,000,000 students, suggests that females outperform males in most grades, although, this gender gap has been shrinking. However, they also noted a significantly lower performance of females in mathematical problem solving at the high school level.

From the above discussion, it appears that gender influences students' risk seeking behaviour. If gender issues are prevalent in risk taking behaviour in-inquiry based and student-centered opportunities may be inappropriately biased towards a specific gender. As well, if there is a variation in risk-seeking or risk-averse behaviour across students, then proposals to introduce risk-intensive curriculum must be appropriately guided to meet students at appropriate comfort levels.

According to a number of writers (Ames & Archer, 1988; Burrill, 1998); Elliot, 1991; Hills, Stroup, & Wilensky, 2005; Meyer & Turner, 2002; Stipek, Salmon, Givvin, Kazemi, Saxe & MacGyvers, 1998) tolerance for risk is directly related to students' perceptions about their own goals (performance or mastery). Elliot (1999) states that performance goals focus on the demonstration of competence relative to others, whereas mastery goals focus on the development of competence. Elliot adds that performance goals are hypothesised to be linked to a negative set of processes and outcomes, for example, withdrawal of effort in the face of failure, decreased task enjoyment, surface processing of study materials. On the other hand, mastery goals are hypothesised to be linked to a positive set of processes and outcomes, for example, persistence in the face of failure, enhanced task enjoyment and deep processing of study materials.

Ames and Archer (1988) claimed that individuals who see learning as the ultimate goal and who are less worried about risks of assessment are more likely to pursue challenging material and thereby engage the risk and confusion associated with learning. However, when success is normatively defined, then both self worth and the perception of the material are at stake. Hence, in a given activity, students and teachers may perceive performance or mastery goals differently, and this may affect their willingness to participate or engage in 'risky' learning situations.

The above claims are consistent with the findings of Dweck (2000) and Burrill (1998) who state that how teachers approach a curriculum may be a consequence of their perception of associated risks. Dweck explained that students who are more likely to avoid risk taking have a performance orientation, meaning they seek social affirmation rather than understanding of the content. Dweck claims that teachers may inadvertently encourage such responses by affirming easy successes and by failing to affirm effort. According to the Burrill's (1998) review of the Third International Mathematics and Science Study, 61% of lesson goals among United States teachers focused on skills, 22% focused on thinking, and 6% were on test preparation. Burrill adds that though many aspects are undoubtedly involved in these decisions, the above evidence would suggest that students and teachers perceptions of risk associated with performance and the complexities associated with thinking are likely to be significant factors. If students are unwilling to tackle challenging content with its associated risks of confusion and failure because of external performance evaluations, then these performance evaluations inhibit learning advocated by current

reforms (NCTM, 2000). With or without these performance evaluations, students and teachers are likely to show natural variation in their willingness in challenging situations

Hills, Stroup and Wilensky (2005) explain that student-centered learning implies perceived risks by both teachers and students. By nature open-ended problems have multiple entry and exit points, require high levels of cognitive demand and may have fuzzy criteria for correctness (Ferguson, 2009; Sullivan & Mornane, 2014). It follows that people who have different perceptions of risk (Hills, Stroup & Wilensky, 2005; Rohrmann, 2005) are more or less likely to do well in these environments. Appropriate educational environments must therefore take into account students and teachers risk-taking or risk-averse attitudes.

Using a risk-eliciting investment game, a survey of preferences, and a data-blind evaluation of participation in class discussion, Hills, Stroup, and Wilensky (2005) investigated the hypothesis that there are risk-seeking and risk-averse pre-service teachers and that this risk is conserved across activities. They found strong correlations between risky behavior, risk preference, and willingness to engage in both in class discussions and open-ended problems in general in mathematics and science. Approximately 65% of pre-service teachers only engaged in discussions when directly asked a question and these teachers also strongly preferred non-constructivist type of mathematics and engaged in 'safe' behavior in the investment game. Risk-seeking students in the game were more likely to engage in discussion and preferred open-ended and confusing activities. Unlike Streimatter (1997) and Atkins et al. (1991) studies, gender was not associated with students' risk preference except males were more prone to seek open-ended science activities.

Meyer and Turner (2002) investigated academic risk taking behaviour of students in an upper elementary mathematics classroom during several project-based activities. The researchers used two surveys, one that assessed academic risk taking and the extent of students response to failure and the other assessed individual goals, self-efficacy and strategy use. In addition, they videotaped classroom lessons and interviewed students before and after the learning goals. In contrast, the risk avoiders note more negative emotions, limited social support and very different goals and strategies for their project. project to understand how the students' beliefs were related to students' actions. They found that the risk takers approached the project's learning opportunities with positive feelings and in ways that met their goals.

### **Reforms in mathematics education**

Over the last two decades, mathematics education reform has called for a shift in approach to the teaching and learning of mathematics in order to meet the needs of a knowledge society. A major aspect of this reform is the move from traditional teacher-centred classrooms focusing on content, to classrooms where classroom discourse and conceptual development are facilitated (Cobb, Boufi, McClain, & Whitenack, 1997; Fraivillig, Murphy & Fuson, 1999; Franke, Kazemi & Battey, 2007; Hunter, 2010; Mercer & Sams, 2006; Silver & Smith, 1996; Stein, 2001). The reforms have recommended that teachers emphasise focus on processes and seeking solutions rather than following a set of solution path (Begg, 2009; French, 2009; Hunter & Anthony, 2011; Stein, 2001). The reforms suggest giving students opportunities to engage in mathematical conversations, incorporating students inadequate solutions into teaching and giving feedback rather than grades on assignments. Dalton (1990) adds that risk taking involves guessing, sharing ideas with others that might involve criticisms or failure or defending ideas. Teaching approaches that foster these learning goals also engender a willingness to engage in challenging tasks and to take risks (Begg, 2009; Sullivan & Mornane, 2014).

The above recommendations resonate with principles of socio-cultural theories combined with elements of constructivist theory which provide a useful model of how students learn mathematics. Constructivist theory in its various forms, is based on a generally agreed principle that learners actively construct ways of knowing as they strive to reconcile present experiences with

already existing knowledge (Confrey & Kazak, 2006; Lerman, 2006; von Glasersfeld, 1993). Students are no longer viewed as passive absorbers of mathematical knowledge conveyed by adults; rather they are considered to construct their own meanings actively by reformulating the new information or restructuring their prior knowledge through reflection (Cobb, 1994).

Another notion of socio-cultural theory derives its origins from the work of socio-cultural theorists such as Vygotsky (1978) and Lave and Wenger (1991) who suggest that learning should be thought of more as the product of a social process and less as an individual activity. There is strong emphasis on social interactions, language, experience, collaborative learning environments, catering for cultural diversity, and contexts for learning in the learning process rather than cognitive ability only. Children learn through social interaction, by talking, explaining, listening, and actively exploring concepts with their peers in whole-class and small-group situations. Through the process of verbalising, including asking/responding to questions, children learn to make connections between concrete and abstract thought (Perso, 2003). Hence, the development of mathematical understanding requires learning contexts in which children can discuss and reflect on their construction of knowledge (Cobb et al, 1997; Franke et al, 2007; Hunter & Anthony, 2011; Mercer & Sams, 2006; McFeetors & Mason, 2006).

The sociocultural perspective has led to an increased attention to mathematical communication and questioning in curriculum documents and materials produced for teachers (Anthony & Walshaw, 2007; Lampert & Cobb, 2003, Lerman, 2006; Ministry of Education, 2007; National Council of Teachers of Mathematics [NCTM], 2000). However, this can only happen if teachers create social norms in their classrooms that give students the confidence to ask questions, discuss with others and to listen and respond actively to the ideas of others (Cobb & McClain, 2004; Franke et al., 2007; Hunter, 2010).

Studies on reform teaching offer contradictory results. A study in England by Jo Boaler (2008) used a fixed average (low income) student socioeconomic status (SES) in two schools while examining varied teaching approaches. When Boaler compared students' performance between schools, her results pointed to reform teaching as beneficial to low SES students. In a study in the United States by Lubienski (2007) all the students learned math by a reform approach but the SES of the students varied. Lubienski's analysis of data from students of differing SES in the same classroom showed that reform was problematic for the low SES students. Lubienski suggested that these differences between the teacher's intentions and students' perceptions were manifestation of cultural confusion

Stipek, Salmon, Givvin, Kazemi, Saxe and MacGyvers (1998) claim that students willingness to take risks can be seen in students reactions to having difficulty in regular classroom contexts. Students may not seek help because of the risk of being perceived as being dumb in asking questions, they may give up, persist on ineffective strategies instead of seeking assistance. Indeed, without explicit discussion of the structure of the discourse-how it works, its norms and rules some students may not be able to take risks and participate fully in the rich conversations.

Recent research has shown that intervention can assist low SES students to learn about the value of classroom mathematical discourse. Hunter (2010) investigated discourse patterns within two year 5 classes of mostly Pasifika and Maori children attending a low SES school. The teachers used explicit strategies to develop and maintain student participation in the discourse community because these were not familiar to the children. Hunter (2010) argues that children from minority groups need to be encouraged by their teachers to participate in mathematics discussion and taught how to do this.

### **Sharma et al (2011) study**

The following inter-related research questions guided the study:

1. How can we develop a classroom culture where students learn to make and support statistical arguments based on data in response to a question of interest to them?
2. What learning activities and tools can be used in the classroom to develop students' statistical critical thinking skills?

A design research approach (Cobb, 2000) with its progressive cycles of testing and revision was used for this study. Design research generally involves cycles of three phases: a preparation and a design phase, a teaching experiment phase, and a retrospective analysis phase. Teachers were key stakeholders in this collaborative research project consistent with Kieran, Krainer and Shaughnessy (2013).

#### **Preparation for the teaching experiment**

This phase consisted of literature review and the first attempt at reformulating a Hypothetical Learning Trajectory (HTL). Then, the research team proposed a sequence of ideas, skills, knowledge and attitudes that they hoped students would construct as they participated in activities. The team planned activities to help move students along a path towards the desired learning goals. As part of the activities, students evaluated statistical investigations or activities undertaken by others including data collection methods, choice of measures and validity of findings (Ministry of Education, 2007).

#### **Teaching Experiment**

The teaching tool place in regular classrooms and as part of mathematics teaching. The teaching activities were spread over up to two weeks to suit the school schedule. The research team was involved in designing, teaching, observing and evaluating sequences of activities. There were two cycles of teaching experiments. Their goal was to improve the design by checking and revising conjectures about the trajectory of learning for both the classroom community and the individual students.

#### **Data Collection**

During the teaching experiment, the data set consisted of video-recordings of classroom sessions conducted during the research, copies of all the students' written work, audio recorded mini-interviews conducted with students, and set of field notes of the classroom sessions. Semi-structured interviews were also conducted with a selected number of students from each class while the design experiment was in progress. These interviews were scheduled after class sessions and focus on students' interpretation of classroom events with a particular emphasis on the identities they were developing as consumers of statistics. Each teacher-researcher kept a logbook of specific events that took place during the data collection period. The team was engaged in conscious reflection and evaluation of situations as they unfolded.

#### **Data Analysis**

The research team performed a retrospective analysis after each lesson to reflect on and redirect the learning trajectory. In addition the team performed analysis of the HTL after an entire teaching experiment has been completed. During this phase the team developed specific instructional theory to help future instruction. The continually changing knowledge of the research team created continual change in the hypothetical learning sequence.

### **Results and Discussion**

A number of learning activities were trialled in our study. In this section, music survey activity which intended to capitalise on students' interest in music, is used to discuss the results.



### Activity – Music Survey

The purpose of this activity is to introduce students to some critical questions that will help them evaluate surveys undertaken by others. The activity provides opportunities to students to express their opinions and concerns both verbally and in writing.

Start by introducing the context of the activity by asking questions such as:

How many of you like music? What type of music do you like the most? How much do you spend on music?

Then present the scenario below. Students can read the music survey scenario individually and make a note of any difficult words. These words can be posted on the whiteboard. Students could be encouraged in pairs to represent the words through interpretative drawings and labels.

### Survey shows Recorded Music Appeals to Teens

A marketing research company was asked to determine how much money teenagers (age 13 – 19) spend on recorded music (cassette tapes). The company randomly selected 80 malls located around the country. A researcher stood in a central location in the mall and asked passers-by who appeared to be the appropriate age to fill out a questionnaire. A total of 2 050 questionnaires were completed by teenagers. On the basis of this survey, the research company reported that the average teenager in this country spends \$1155 each year on recorded music.

Ask students to answer the questions below. Having students working in small groups enables discussion of their opinions.

1. Who carried out the survey?
2. What was the purpose of the survey?
3. How was the survey done? Why do you think this method was used?
4. What was the target population?
5. What are the main findings of the survey?
6. How do you think the average was calculated? Is the average a good estimate of the spending of all teenagers? Why?
7. Does the data support the headline? Justify your answer.
8. Comment on two concerns you have about the survey.

Encourage students to write down their response for each question. You may want to have groups present their decisions to the class.

When we used the music survey activity, concerns raised by students about the sample selection process used in the study were:

*They only asked passers-by in the mall. As Mr T says, this might bias the results because teenagers outside the mall were not asked.*

*How did they calculate the average? Did they get rid of odd numbers [outliers]? Some teenagers spend more money than others.*

*They should have done the survey at more than 80 malls if they wanted an average based on teenagers throughout the country.*

The last student quote above reinforces to us that students can struggle with thinking of the sample size in relation to the size of the country, rather than in relation to the representativeness of the sample. However, for the above scenario, theirs is a valid concern because hardly any information had been given regarding the target population.

Like the activity above, most of our classroom activities included group and whole class discussion of the data. Typically, we used a small group activity (2-4 members) in which the students worked on tasks together and then reported back to the whole class. The groups were organised to include students with a range of mathematical, statistical, and language abilities because we believed that varying insights would enhance overall understandings.

During group work learning opportunities arose for students from collaborative dialogue and resulting of differing points of view. Working in groups also provided less confident or less able students with opportunities to explain, question, agree and disagree and test their thinking in a less threatening context before engaging in class discussions. Our findings are consistent with the views of Hunter (2010) and Begg (2009) who illustrated the importance of small group discussion as a means for students to rehearse their explanations, justifications and analysis of their solution strategies as the students prepared for questioning and challenge from the whole class.

The teachers ensured that students understood and adhered to effective group problem solving practices, including listening, writing, answering, questioning, and critically evaluating information. The teacher in the following transcript explains their expectations:

*When working in the group, first of all each one of you has to say what your concerns are. Then I want you to come up with a group response. You should not only say bias, headline and questions, you need to clearly explain your reasons. Remember - at the end of your group discussion, everyone in the group needs to be able to explain why you chose that response. Also, anyone in the group can be asked to present to the whole class.*

During reporting back the whole group was required to stand with the reporter and share the responsibility for explaining their reasoning and responding to any questions from the class. These norms encouraged the students to work together, communicate, and be responsible for the learning of everyone in the group. The teachers took time to remind students of effective group practices (e.g., ways of agreeing and disagreeing and how to present to the class):

*The group is not finished until everyone in the group can explain and defend their answer. When we come together as a class to share our ideas, I will simply ask any one student to report on why they agree or disagree with a particular statement.*

Anthony and Walshaw's (2009) research provides evidence that learners enjoy learning in communities of mathematical enquiry as they gain a variety of strategies that can be used to solve problems and they feel their belonging to a group that allows them to engage with their own sense-making and the sense-making of others. Our study shows that working in groups provides learners with more than just opportunities to engage with mathematics, it also provides learners with opportunities to develop other competencies identified in the New Zealand curriculum document (Ministry of Education, 2007). For example, students become aware that working with others will support their learning and understanding and the community provides a safe environment where they can take risks asking questions and defending ideas.

Students in our classes were of different language and statistical abilities and activities were designed to ensure students were interacting with each other in order to improve their statistical communication. Support was provided for both reading and writing in statistics. Supports included assisting with vocabulary acquisition, such as using pre-reading and further reading strategies such as shared reading and scanning techniques. Writing support included the use of writing frames, cloze activities and composing responses individually and in groups. The student-student interactions presented various demands on students' literacy skills, as indicated in the following student quote:

*Because usually, like in normal maths, we don't use literacy ... like we use addition, subtraction but we actually need some kind of literacy for the things we do in statistics.*

Questioning was one of the key teaching strategies used in our study. We wanted both the students and the teachers posing questions that would support student learning. The teachers posed prompting and probing questions that diagnosed and extended student thinking - questions that elicited student ideas and encouraged them to explain and justify their contributions in respectful ways (Cobb & McClain, 2004; Franke, Kazemi & Battey, 2007). Students needed to formulate and pose critical questions in ways that assisted them to evaluate statistical statements and reports.

Our students came up with critical statistical questions, as reflected in the following:

*The simplest question I want to ask is how they got the information. How were the teenagers chosen? Now that we have talked about random samples in statistics ... did they use random sample?*

*I want to know how many boys and how many girls took part in the research. To show comparison, they need same number of boys and girls.*

It seems that among the students interviewed classroom culture was an important factor in how they responded to asking questions. Some students commented that because the teacher explained that students could ask different questions they were more prepared to ask questions as they felt less pressure in terms of making mistakes in front of their peers. Some students even indicated that they were willing to risk making mistakes in front of their peers.

When asked what helped them to try harder in maths each interviewed student reported that the teacher was the most influential factor. One student explained why this relationship was so critical to their learning

*I have a good relationship with my teacher, I am not scared to ask him questions. I don't ask other teachers because I am afraid they might get angry*

Another theme that emerged was that understanding what was being asked was important when beginning the lesson. Students specifically pointed out that at the initial stage key terms were explained and connections made to their prior knowledge. This an important insight to incorporate into the recommended pedagogies.

The classroom discourse was important for statistical literacy. Most of our classroom activities included group and whole class discussion of the data. This typically involved a small group activity in which the students worked on problems together and then reported back to the whole class. The two teachers took time to remind the students how to work in groups (e.g. how to agree and disagree and how to present to the class). Our results show that students can be taught how to question and challenge in respectful ways as part of classroom discourse.

Students found group work useful:

*When you are working alone you just get one point of view and when you are working in a group you get different perspectives of other ideas ... how other people are thinking, learning in class*

*Oh ... just because when we work alone we might get it right, we might get it wrong, but if we work in a group we'll get more ideas. We will be able to discuss it with the group.*

Context is an important component of statistical literacy. Our findings show that students need exposure to both familiar and unfamiliar contexts. Engagement with context helps students develop higher order thinking skills. However, our results show that some contextual knowledge may be a barrier for some students. Teachers were able to address this in two ways. The first was to start from familiar contexts before moving to unfamiliar contexts. The other was to use contexts of interest to the students.

Our study shows that students concerns about revealing ignorance by asking questions might be overcome in a classroom context that focuses attention on learning rather than just getting right answers. The findings align with the conclusions of Burrill (1998) and Elliott(1999). According to Elliott, learning goals focus on the development of competence or task mastery whereas performance goals focus on the demonstration of competence relative to others. In our study a context, teachers encouraged students to ask questions about media reports, they were concerned with processes such as persistence in the face of failure, deep processing of tasks, risk taking. In additions, classmates were prohibited from putting down a student who was having difficulty posing an appropriate statistical question. The teachers were aware that students who have been shamed by the teacher or another student will not will not take risk and engage in challenging tasks. Students in the class commented:

*I not bothered about being wrong. Even if you have something wrong he will listen to you and ask you to pose a better question.*

*He always reminds us not to put down our classmates.*

*Sometimes I don't have the right question and answer. It is okay to make mistakes. Mr ... says we learn from our mistakes.*

Overall the interviewed students were willing and some cases eager to evaluate statistical tasks. They were aware of the subtleties in the classroom that affected their learning in mathematics and the factors that either assisted or constrained their ability to take risks.

### **Implications for Teaching and Research**

We find that this work is still in its infancy and there are numerous limitations. As the research questions suggest, the original study was focused on developing statistical literacy. The study was not designed to give us insight into risk taking. Thus, results from these studies are tentative. However, these limitations provide an opportunity for future research. The work that has been done provides a road map for future studies.

We envision statistical literacy going beyond calculations. It is more than the ability to do calculations and read tables and graphs. Students should be able to interpret and critically evaluate statistical information and data related arguments. Additionally, they should be able to take risks and communicate their understanding and opinions to others. This has potential consequences in how the teaching of statistical literacy might be altered for greater effectiveness. For example, ample class time should be spent on discussion and reflection rather than presentation of information.

We believe that the nature of the learning environment and classroom culture are major contributors to success for students, and teachers need to put a high priority on building a classroom climate that positively engages all students. Students need to understand the importance of sharing their opinions in order to advance their statistical ideas. It would be valuable for teachers to help students reflect on the purposes of explaining and justifying their thinking to others This is consistent with the latest New Zealand curriculum document that promotes the ideals of having confident, critical and active learners of mathematics (Ministry of Education, 2007).

The ability to interpret and critically evaluate reports that contain statistical elements is paramount in our information laden society. Teachers need to give students some basic foundations for critiquing and evaluating statistically based information that they encounter in daily life. We assume that students can be taught these reasoning skills through using media articles as a springboard into learning about how to evaluate these reports. Consequently they will become familiar with a list of worry questions and apply them to real life examples without prompting, consistent with Gal (2004).

It was clear that specific actions by the teachers to encourage and affirm risk taking along with information about the processes of explaining their thinking in writing and verbally to other class members and the importance of active listening was helpful. Indeed as students move to statistics study at more senior levels an orientation to risk taking and a growth mindset will be useful asset to them.

Teachers have to be prepared in the initial stages for students to be negative towards challenging tasks and be prepared to scaffold their learning in a safe environment in order to become competent mathematical practitioners. Like Anthony and Walshaw (2009) we found that students find it difficult to provide mathematical explanations, to figure out what questions to ask and often lack the confidence to speak in large groups.

Views about statistics teaching and learning have shifted considerably in New Zealand and internationally over recent decades, and it is important for teachers to be kept informed about changes in the ways that mathematical and statistical processes and thinking are being emphasised (Anthony & Walshaw, 2007). It would be useful if schools were to make a point of highlighting the importance of risk taking in mathematics and other learning areas in their interactions with teachers and families.

Not only does this help with mathematics but learning the process of critical thinking and risk taking will expand to other curriculum subjects and wider society. Our study was a long term teaching experiment involving a team of teachers and researchers and it only begins to suggest ways to promote risk taking. More research is needed on the effectiveness of particular sequences of activities, the use of different technological tools and the teachers role in shifting the students towards desired behaviour. Other forms of classroom based research can be used to study and attempt to answer questions about optimal ways of promoting risk taking in mathematics teaching. Studies that give insight into the challenges faced by teachers and learners in inquiry-based classroom become important. Teachers can reflect on and develop their practices. It would seem that ongoing research is valuable to inform teacher practice.

The results have implications for teacher education. Many discussions around presentation of mathematical concepts to students are focussed on avoiding the complexities. Yet it may be that coming to grips with the complexities is exactly what students need. It seems important that teacher educators include at least some consideration of risk taking in their presentations to prospective and practising teachers.

Ideally, it would have been useful to make links between students' classroom experiences with what happened outside of the mathematics classrooms. This could have been achieved by doing observations in other classrooms and by conversing with other teachers. This could have enabled the research team to gain insight into whether the students were transferring the critical skills from the realms of statistics to other learning areas as advocated by Watson (2006).

Comprehensive theoretical work that articulates how motivation and cognition interact within mathematics classroom contexts is needed if understanding risk taking is to move forward. Although our research has borrowed ideas from socio-cultural learning theory that appeared to support our research questions, this framework provided only some of the pieces to the theoretical puzzle that we are trying to investigate here. We need theories that will help us better understand how risk taking is intertwined with motivation and cognition within the context of classroom learning.

Research discussed in this paper indicates that females are found to be more risk-averse than males (Hills, Stroup, & Wilensky, 2005; Weber et al., 2002). These gender differences in everyday and educational settings may also carry over to performance in mathematics classrooms. However, gender gaps in students willingness to take risks associated with higher-level problem solving has not yet been investigated. Additionally, we failed to ask how teachers and students defined risk and how they felt about risk taking in mathematics learning. Such issues could be addressed in future

research.

The participants in my study were a fairly small non-random sample from one school. Thus, the findings, in particular the number of students who took risks may or may not generalize to the population of secondary school students as a whole in New Zealand. There is a need for more research with larger, more random samples with different backgrounds to determine how common these behaviours are in the general population.

Furthermore, as mentioned earlier, the findings reported in this paper were part of a larger study (Sharma et al. 2011) which focused on developing critical statistical literacy. Since there had been few studies that focused on risk taking internationally, it was not clear when this study was conducted that the questions discussed in this paper would be as rich and interesting as they were. Now that the risk dimension described in this paper has been identified as possible areas of concern, there is a need for more qualitative and quantitative research focused on a deeper understanding of students' thinking about risk and their risk taking behaviours.

### Concluding Thoughts

It is hoped that the findings reported in this paper will generate more interest in research with respect to risk taking, risk perception and risk communication in mathematics education. It will be interesting to explore gender and cultural differences that may impact on students' risk taking behaviour. There is also a need to focus on documenting the challenges and difficulties that researchers face in the process of conducting international studies and how cultural factors can influence researcher activities and research results. Indeed we need to look for new ideas and develop more collaborative and cross cultural research between practitioners and researchers in the future if we are to improve outcomes for all our students. Research methods developed in one cultural setting may not be appropriate in another cultural context (Cao, Forgasz, & Bishop, 2007). Teachers, curriculum developers and researchers need to work together to find better ways to help all students take risks in mathematics classrooms.

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