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Pedagogy of Risk:
Why and How Should We Teach Risk in High School Math Classes?

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Abstract: Risk is everywhere yet the concept of risk is seldom investigated in high school mathematics. After presenting arguments for teaching risk in the context of high school mathematics, the article describes a case study of teaching risk in two grade 11 classes in Canada- an all-boy independent school (23 boys) and a publicly funded religious school (19 girls and 4 boys). The findings suggest that the students possessed intuitive knowledge that risk of an event should be assessed by both its likelihood and its impact. Following and amending pedagogic model of risk (Levinson, R., Kent, P., Pratt, D., Kapadia, R., & Yogui, C., 2012), the study suggests that pedagogy of risk should include five components: 1) knowledge, beliefs, and values, 2) judgment of impact, 3) judgment of probability, 4) representations, and 5) estimation of risk. These components do not necessarily appear in the instruction or students' decision making in chronological order,; furthermore, they influence each other. The implication for mathematics education is that a meaningful instruction about risk should go beyond mathematical representations and reasoning and include other components of the pedagogy of risk. The article also illustrates the importance of reasoning about rational numbers (rates, ratios, and fractions) and their critical interpretation in the pedagogy of risk.

Keywords: probability teaching and learning, risk literacy, teaching and learning of risk, inquiry-based learning, understanding of rational numbers; risk estimation.

Introduction

Risk as a concept permeates all aspects of our society, it appears in every activity we do as humans and it shows in diverse disciplines such as mathematics, physics, engineering, sociology, and psychology. Yet, despite its importance, it is rarely included in school mathematics. In this article, I explore teaching risk in the context of high school mathematics.

There is a consensus amongst experts that most people are unable to adequately interpret and communicate risk (Kahneman, Slovic, and Tversky, 1982; Rothman, Montori, Cherrington, A., & Pigone, 2008). The problem of improving understanding of risk has been addressed in the specific context of public health and financial counselling, yet it has only begun to be explored within educational research (Pratt et al., 2011). Despite a recognized and urgent need for risk education, there is a lack of agreement on its definition. The concept of risk exists at the intersection of many related fields—mathematical, health, statistical, probability, scientific, and financial, among others. In this study, I will situate risk within the fields of statistical and probability literacy as these fields focus on uncertainty and chance, both important elements of risk-based reasoning. Most current approaches to literacy recognize it as more than a minimal subset of content knowledge in a particular field (see, for example, Gal, 2004a). Further, the definition of literacy has been expanded to include "desired beliefs, habits of mind, or attitudes, as well as a general awareness and a critical perspective" (Gal, 2004a, p. 48). Consistent with Gal's (2004a, 2004b, 2005) research on statistical and probability literacy, I define pedagogy of risk to include knowledge (e.g., probability content knowledge) and dispositional elements

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(e.g., beliefs and attitudes about risk) as I examine its place within the secondary mathematics curriculum.

Researchers and policy makers have recognized the need for education about risk (Gigerenzer, 2002; Kolsto, 2001; Levinson, R., Kent, P., Pratt, D., Kapadia, R., & Yogui, C., 2012; Pratt et al., 2011). Pratt et al. (2011) provide examples from the UK curricular documents which call for teachers to teach probability through situations involving risk. In the context of the Ontario secondary school curriculum, the concept of risk can be found in multiple subject areas, including science (e.g., students are expected to learn to analyze risk of introducing particular technology to ecosystems), physical and health education (e.g., risk involved in participation in a physical activity), and family studies (e.g., risk of contamination in food) (Ontario Ministry of Education, 2011). Given that understanding of risk includes a strong quantitative component, the mathematics classroom is an appropriate setting for the exploration of its pedagogy. However, in Ontario, mathematics curriculum documents do not focus on risk. Moreover, throughout the secondary mathematics curriculum, there are limited mentions of risk; these fall within the context of financial mathematics as well as the promotion of students' risk taking which is considered "necessary to become successful problem solvers" (Ontario Ministry of Education, 2005, p. 24). The exploration of the pedagogy of risk has begun only recently. The most comprehensive research in pedagogy of risk was done by the researcher involved in the Institute of Education's TURS Project (Promoting Teachers' Understanding of Risk in Socio-scientific Issues). The research, which involved in-service teachers, problematized risk education as the interplay of mathematical knowledge, beliefs, context, and content knowledge.

Despite calls for teaching risk in the classroom, and despite the explorations by the TURS research group, there remains a lack of research in the mathematics classroom setting and involving students. The purpose of this study is to address the lack of research by exploring the ways in which risk could be taught within the mathematics classroom. Specifically, this study explores the ways that secondary school mathematics instruction can support students' developing understanding of risk, and I focus on the following guiding question, namely how do secondary school students reason and make decisions about risk?

Review of Literature

The Concept of Risk

Risk is a concept that is prevalent in many disciplines and the term 'risk' has been used in many distinct yet connected ways. Hansson (2009) distinguishes between five different definitions of risk: 1) risk as an unwanted event which may or may not occur; 2) the cause of an unwanted event which may or may not occur; 3) the probability of an unwanted event which may or may not occur; 4) the fact that a decision is made under conditions of known probabilities; and 5) the statistical expectation value of unwanted events which may or may not occur.

The third, fourth, and fifth definitions are the most common in mathematics. The third definition aligns with the view that a risk associated with an event is a quantifiable uncertainty (Gigerenzer, 2002), which is equivalent to the likelihood or probability of the event. This definition of risk is suitable when the events have similar consequences, but it becomes problematic if the impact of each event is different. For example, the likelihood of a person catching a cold is relatively large but its impact on the person's life is most likely to be minimal, whereas the likelihood of getting killed in a terrorist attack is relatively small but the impact is immense. In order to account for both likelihood and impact, proper understanding of risk requires the coordination between judgments of probability and impact (Pratt et al., 2011), which corresponds to the fifth definition, the statistical expectation. This coordination can be done informally, but also formally using mathematical representations. Symbolically, the fifth definition of risk can be written as

$$R = \sum_{i=1}^{n} p_i d_i$$

where the overall risk, R, of a hazard, is the sum of the products of the probability (p) and disutility or impact (d) of each event associated with the hazard (Pratt et al., 2011). For example, to assess the overall financial risk of owning a car, one would find the probability of each outcome (e.g., flat tire), multiply those by the financial impact, and then obtain the total sum of all the products. The approach based on the above formula is known as the utility theory of risk (Levinson et al., 2012) and is the standard approach in technical risk analysis (Moller, 2013).

Cultural Perspectives

Utility theory and technical risk analysis are not the only approaches to risk. Technical risk analysis, which is a domain of philosophy, statistics, and economics, has been extended to risk governance which involves actors' understanding and handling of risk (Lidskog & Sundqvist, 2013). However, risk governance is a complex task, particularly in the case of global risks such as terrorism, catastrophic weather due to climate change, financial meltdown, and nuclear accidents such as the radiation leakage due to the Fukushima nuclear disaster. The anticipation of global risks can seldom be determined using methods of science. The less we are able to calculate risk, the more the balance shifts toward the cultural perspectives on risk (Beck, 2009). It follows that assessing risk goes beyond the utility theory. Assessing risk in the vast majority of social situations involves more than individual considerations of maximizing utility; rather, it is a dynamic consensus-making political process involving diverse actors and contexts (Douglas, 1992). Consistent with the cultural perspective on risk, "sociology opposes any kind of reification of risks, in which risks are lifted out of their social context and dealt with as something uninfluenced by the activities, technologies, and instruments that serve to map them" (Lidskog & Sundqvist, 2013, p. 77).

Beyond Utility Theory and Cultural Theories. Reality is "neither reducible to something out there, beyond human action, nor reducible to something in there, to human thoughts and actions" (Lidskog & Sundqkvist, 2013, p. 98). Reification of risk—the belief that risk is void of social context—is problematic. However, the social purification of risk the notion that risk is just the product of social factors (Lidskog & Sundqvist, 2013)—is also problematic. The question, then, is how to reconcile naïve realism and idealism. The third way should not be obtained by the combination of constructivism and realism. Instead, the focus should be on "the dynamic interplay between different factors that make up reality" (Irwin & Michael, 2003; Latour, 1993, 2004, 2005). Latour suggests that one can transcend the perceived dichotomy between utility and cultural theories by focusing on the production of risk:

> Risks are produced by practices, by actors using instruments and technologies. It is therefore misleading as a sociologist to focus on perceptions, opinions and experience. Instead, the focal point for sociology should be to explore how risks are produced, by what means and with what effects. (Lidskog & Sundqvist, 2013, p. 99)

Recently, technical risk analysis has recognized the importance of cultural theory and has treated values (individual or collective) on par with empirical data. For example, the structured decision-making approach (Gregory et al., 2012) takes into account the complexity of environmental decision making by considering uncertain science and multiple stakeholders' values and priorities.

In addition, the precautionary principle (Wiener, 2002) can serve as a mechanism to support decision making when there is a lack of scientific evidence. The precautionary principle states that "[w]hen an activity raises threats of harm to the environment or human health, precautionary measures should be taken even if some cause and effect relationships are not fully established scientifically" (Science and Environmental Health Network, 1998).

Teaching and Learning of Risk. The tension between the utility and cultural theories is reproduced in the teaching and learning of risk. In the following section, I discuss the two predominant approaches to the teaching and learning of risk: deficit theory of risk (utility approach) and pedagogical approach (Levinson et al., 2012).

Deficit Theory of Risk

If we accept the assumption of utility theory that there is such thing as objective or actual risk, then the goal of risk education is to evaluate the learner's perceived risk and, through cognitive adjustment, to align it with actual risk (Levinson et al., 2012; Lidskog & Sundqvist, 2013). The deficit theory has been successful in solving problems that are well defined—in other words, the problems for which there is a unique solution, such as a subset of problems involving Bayesian inference (Gigerenzer, 2002; Kahneman et al., 1982).

Research on students' reasoning suggests the importance of different representations in solving Bayesian problems. For example, Zhu and Gigerenzer (2006) propose that students' abilities to solve Bayesian tasks vary depending on the data being represented in terms of natural frequencies or probability. When the binomial hypothesis problem was presented in terms of probability to fourth, fifth, and sixth grade students, none of the students were able to estimate the Bayesian posterior probability. On the other hand, when the same information was presented as a natural frequency (e.g., rather than represent the probability as 0.07, state that "out of 100 women, 7 have cancer"), reasoning about conditional probability showed a steady increase from fourth to sixth grade, reaching an average level of 19%, 39%, and 53%, respectively, in two studies.

In summary, researchers who apply the deficit theory of risk education begin with the analysis of students' perceptions (heuristics and biases) and then propose an intervention (e.g., the use of natural frequencies) in order to align the perceived risk with the actual risk. (Figure 1)

Critiques of the Deficit Theory of Risk. Simple Bayesian tasks can be approached using the deficit model because, within the context of the problem, there is an actual risk—or, to be more precise, an actual quantifiable risk. However, if the question is reframed as "How often should women over forty get mammograms?" or, even more specifically, "Should a particular person get a mammogram?", it becomes ill-defined, meaning there is neither a clear solution nor a clear method of arriving at a solution. The deficit model is particularly inadequate when dealing with technoscientific situations such as epidemics, climate change, energy policy, or pharmaceutical research, where there is no consensus on how probability or impact should be calculated (Levinson et al., 2012; Lidskog & Sundqvist, 2013).

Even when problems are well defined, it is debatable whether individuals base their decisions simply on cognitive heuristics. In other words, when making risk-based decisions, are we really performing calculations at a varying level of complexity depending on our background in risk analysis? Paul Slovic, a veteran of heuristics research, began to doubt this several decades ago:

> I recall, in the midst of this growing collection of heuristic strategies, wondering how people decided when it was safe to cross a busy street. Certainly they were not calculating probabilities and utilities or their summed products, and the known judgment heuristics did not seem to offer any insight. (Slovic, 2010, p. xx)

He instead proposes that any risk decision includes the affect heuristic, which is a cognitive process in which individuals use feeling (positive or negative) as a guide to evaluating risk.

Deficit theory also does not take into account that risk situations are "constructed by different histories, narratives, and experiences" (Levinson et al., 2012, p. 216). For example, Levinson and Rodd (2009) investigated pre-service teachers' conceptions of risk related to the question of whether malaria is a major risk in travelling to West Africa. A student who had had an experience with malaria downplayed the risk; "what was seen as a major risk by one person was not perceived as a significant risk by another" (Levinson et al., 2012, p. 216).

Another problem with deficit theory is that it places expert knowledge before the knowledge of laypeople, regarding them as "poorly informed in comparison to the 'precise' and 'scientific' analyses of experts" (Beck, 2009, p. 12). Laypeople, however, "have the competence to contribute to discussions and decisions on risks since they concern them much more than scientific facts" (Lidskog & Sundqvist, 2013, p.94). Levinson et al. (2012) assert that "evolving models of interactions between experts and publics point toward a more reflexive expert perception of public concerns and a realization of the importance of public engagement" (pp. 216-217). Beyond public engagement, Gregory et al. (2012) call for the meaningful inclusion of public knowledge into decision-making processes, focusing in particular on local and traditional knowledge characterized by the reliance on experience and observations rather than experimentation, often expressed in more holistic rather than reductionist fashion, and dealing with particular concerns and context-dependent situations.

Consistent with the value of public knowledge and decision making, Levinson et al. (2012) consider personal models in understanding of risk for two reasons:

> learning involves the modification of preexisting personal models in interaction with others, rather than learning being a process of replacing learners "wrong" thinking with models of "right" thinking and (2) it is critical to respect personal models because personal values (as expressions of personal preferences and ethical positions) and social and affective values are inextricable from making decisions. (p. 217)

It follows that risk should be taught and learned in an environment that creates opportunities "to make explicit values, experiences, and representations of those experiences and probabilities that foreground the decision-making process, and where probabilities can be judged in light of, and interact with, expressed values" (Levinson et al., 2012, p. 228). An inquiry-based approach can offer such an environment (Pratt & Yogui, 2010).

Pedagogic Model of Risk

For the purpose of exploring the pedagogy of risk, researchers involved in the Institute of Education's TURS Project (Promoting Teachers' Understanding of Risk in Socio-scientific Issues) developed a computer microworld called Deborah's Dilemma (Levinson et al., 2011; Levinson et al., 2012; Pratt et al., 2011). In Deborah's Dilemma, students were engaged in a narrative involving a fictitious person, Deborah, who suffers from a spinal cord condition. Based on the data about the side effects of a surgery and the consequences of not having the surgery, pairs of math and science teachers had to choose the best possible course of action for Deborah. One of the outcomes of the research program was the development of the pedagogic model of risk (Levinson et al., 2012).

According to this model, probabilistic judgments lead to the estimation of risk but the judgments are informed by values, experiences, personal and social commitments, as well as representations (see Figure 2). This is in contrast with the utility model of risk, where values are separate from the probabilistic judgments and may only play a role in risk management (following an analysis of risk).

Relevant findings from the study have been used throughout this literature review to outline the elements of the pedagogy of risk.

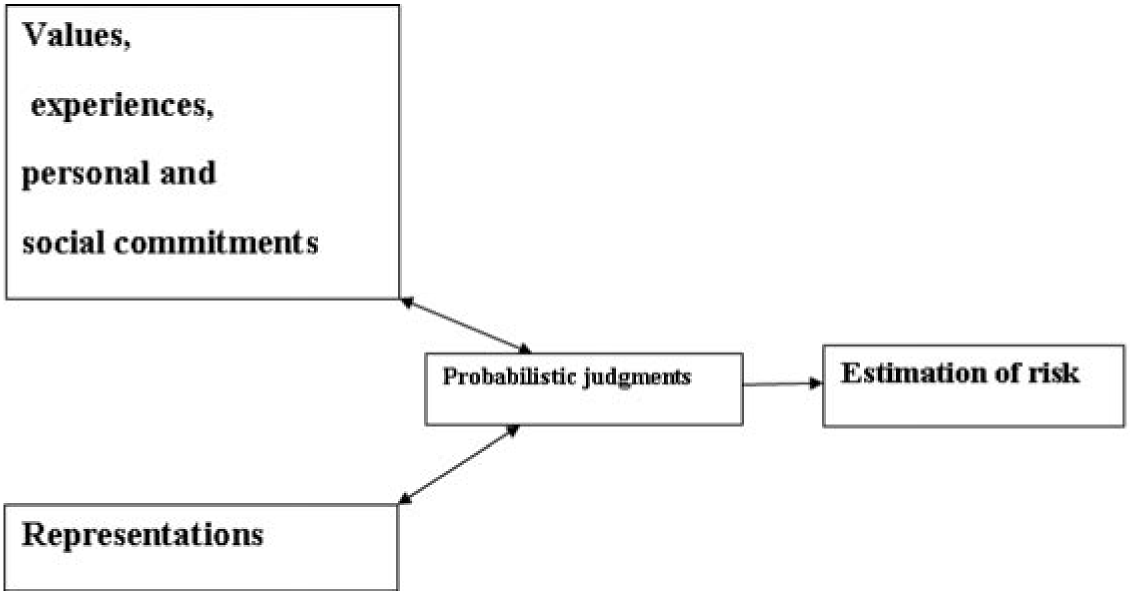

Figure 1. Pedagogic model of risk. Adapted from Levinson, Kent, Pratt, Kapadia, and Yogui (2012).

Elements of the Pedagogic Model of Risk

In this section, I present the existing research that supports and expands upon the Levinson et al. (2012) pedagogic model of risk. This will be followed by research on the estimation of risk, with a particular focus on coordination between probability and impact. Finally, the role of context in the pedagogy of risk will be discussed.

Probabilistic Judgments. In terms of probability knowledge, there are several frameworks that describe probabilistic reasoning. The major frameworks mentioned in the literature are Core Domain of Probability Concepts (Moore, 1990), Probability Thinking Framework (Jones, Langrall, Thornton, & Mogill, 1997; Polaki, Lefoka, & Jones, 2000), and Gal's knowledge elements of probability literacy (Gal, 2004b, 2005). Moore (1990) describes the conditions that students need to satisfy in order to be able to move towards more difficult concepts such as conditional probability. These conditions are: 1) learning to discern the overall pattern of events and not attempt a causal explanation of each outcome; 2) recognizing the stability of long-run frequencies; 3) assigning probabilities to finite sets of outcomes and comparing observed proportions to these probabilities; 4) overcoming the tendency to believe that the regularity described by probability applies to short sequences of random outcomes; and 5) applying an understanding of proportions to construct a math model of probability and develop an understanding of some "basic laws or axioms that include the addition rules for disjoint sets" (Moore, 1990, p. 120).

One of the most comprehensive frameworks describing probabilistic reasoning is provided by Jones et al. (1997). The framework is divided into four constructs: 1) sample space, 2) probability of an event, 3) probability comparisons, and 4) conditional probability. The framework also recognizes four levels of reasoning: 1) subjective, 2) transitional between subjective and naïve quantitative, 3) informal quantitative, and 4) numerical.

Polaki et al. (2000) provide an extension of the Jones et al. (1997) framework. Their framework describes probabilistic thinking across five constructs: 1) sample space, 2) probability of an event, 3) probability comparisons, 4) conditional probability, and 5) independence. The Polaki et al. (2000) framework has been used to explain requirements for understanding compound events in terms of the sample space and the probability of the event constructs (Nilsson, 2007). The Jones et al. (1997) framework, which serves as a basis for the Polaki et al. (2000) framework, was used by Langrall and Mooney (2005) to interpret the grade three students' understanding of probability in Falk and Wilkening's (1998) research.

Representations. One of the biggest challenges for learners of risk is the issue of representations. The claim that the Bayesian problem is simpler when the probability is represented in terms of natural frequencies is made elsewhere (Cosmides & Tooby, 1996; Gigerenzer, 1994; Gigerenzer & Hoffrage, 1995). These researchers offer an evolutionary psychology explanation of why most people find Bayesian tasks represented using natural frequencies easier than others. According to this explanation, humans are "hard wired" to deal with natural frequencies rather than with probabilities.

In the Levinson et al.'s (2012) study, however, the choice of representation seemed to depend on the context. For example, when deciding whether Deborah should have surgery based on data about the success rate, a teacher claimed that, if they were arguing for the surgery, they would represent the rate of failure in terms of probability ("less than a half of seventh"), whereas, if the recommendation was against the surgery, they would represent the data in terms of natural frequency ("four people") because it contains information on actual people impacted by the failure. The idea that natural frequencies carry more weight in representing impact is confirmed by Tim, another teacher in the Levinson et al.'s (2012) study:

> I mean if you're going to say "60 people died from this procedure," is that enough to tempt someone to say "alright I'll give that a go"? Ok that would look bad because 60 people is more impacting on you than one in 1000, one in 10,000. Those big figures will convince you, but I think "60 people died from this last year" convinces you in a different way, even though the figures, you know that's where the one in 50,000 comes from. The way you present your data is very important to an individual. (p. 223)

Levinson et al. (2012) also documented that teachers made mistakes in calculating percentages:

> In paired dialog, it was easy for teachers to miscalculate small percentage values into figures and proportions more commonly used in everyday discourse. Overall, this illustrates a common problem, where people find large numbers and low probabilities difficult to comprehend. It suggests the need to take care in designing materials about risk, possibly highlighting the need to support students and consumers in negotiating and interpreting the ways in which probabilities are represented. (p. 223)

In terms of representations of impact, Pratt et al. (2011) document "fuzzy qualitative descriptors" used by students to roughly calculate impact ("serious," "massive," "bad," "fine," "big") (p. 339). Qualitative representations of impact (outcomes) were similarly documented by the Levinson et al. (2012) study; all three pairs of teachers considered impact in deciding whether Deborah should have the operation using phrases such as "impact on her life," "pain threshold," and "prohibitively dangerous option" to describe impact.

In contrast with qualitative representations of impact, there is a lack of research on what quantification of impact looks like in the classroom. There is, however, empirical evidence that quantification does not come easy. Pratt et al. (2011) state that there was no opportunity for teachers to quantify impact:

> At no point did any of the six teachers attempt to numerically quantify impact and certainty [sic] in designing the task we provided no explicit encouragement to move beyond fuzzy quantifications of how much additional pain might be caused by a lifestyle decision. Perhaps because of the difficulties in quantifying impact, and to some extent of making sense of the odds, we observed no attempts to quantify risk per se in a formal way. (p. 339)

We can conjecture, however, that quantitative reasoning about impact will be consistent with quantitative reasoning described by statistical literacy frameworks. According to these frameworks, understanding of rational numbers, including the understanding of ratio and proportion, is another domain of quantitative knowledge and is a necessary prerequisite for performing at the highest level of statistical understanding (Watson & Callingham, 2003; Watson & Shaughnessy, 2004). According to Watson and Callingham (2003), statistical literacy is a hierarchical construct with proportional reasoning featured at its highest levels (see Table 1). At the highest level, critical mathematical, students are required to use "proportional reasoning associated with ratio and appropriate part-whole interpretations, the ability to use rates in calculating costs" (p. 18).

Proportional reasoning causes difficulty for many middle school (Lamon, 2007) and high school students. For example, Akatugba and Wallace (1999) studied students' proportional reasoning in a physics class and found that students had difficulty performing mathematical operations that were not explicitly stated in the task. The conclusion of the study was that students' understanding of mathematical processes involved in proportional reasoning was inadequate.

Values, Experiences, Personal, and Social Commitments. Students also must be aware that the collection, generation, and interpretation of data are influenced by social factors and are consequently value-laden (Pratt et al., 2011; Watson, 1997). In order to examine the social factors, students must be able to critically examine data (Gal, 2004a; Pratt et al., 2011). In addition, thinking about risk also involves decision making that can function on an individual level, societal level, and within the intersection of these levels (Pratt et al, 2011).

The importance of critical evaluation of data is consistent with the consensus among educators stating that literacy should extend beyond a minimal subset of skills expected for all. According to Gal (2005):

> increasingly the term literacy, when used as part of the description of people's capacity for goal-oriented behavior in a specific domain, suggests a broad cluster not only of factual knowledge and certain formal and informal skills, but also a desired beliefs, habits of mind, or attitudes, as well as a general awareness and a critical perspective. (p. 42)

For the purpose of the critical evaluation of data, Gal (2004a) introduced a list of critical or worry questions which enable individuals to critically evaluate information provided. The critical questions include questions about reliability and the validity of data (Gal, 2004a, 2004b, 2005).

The students' perception of reliability of data is an important critical element. There is evidence that some individuals associate reliability with disinterestedness. For example, Pratt et al. (2011) found that two participants in their study (Linda and Adrian) were suspicious of the surgeon who they thought may be "drumming up the business" (p. 340).

Levinson et al. (2012) found that students "recognize the role of trust and authority in giving meaning to the data" (p. 224). The authors reported that a participant (Linda) found the spine doctor's recommendation reliable because he "knows more about it than the other people, and he's seen more of these people" (Levinson et al., 2012, p. 224). The notion of trust in groups that are involved in measurement is evident in Kolsto's (2001) study. Despite the fact that the students were sceptical of the power company's risk evaluation and the power company was marked as interested, the students accepted the company's claims concerning magnetic field strength from different sources.

Besides the issues of reliability, there is an issue of content validity, which includes the questions of how the statement about data was derived, whether the claims are supported by the data, and whether additional information and interpretations are needed (Gal, 2004b). Kolsto (2001) provides evidence that students are quite vigorous in questioning the source of data but are less likely to question the validity which requires students to analyze the content of the source and evaluate the arguments presented in the document.

Research on affect heuristics also sheds light on the ways that individuals make decisions about risk. Slovic, Finucane, Peters, and MacGregor (2010), influenced by the dual processes theory (Kahneman & Frederick, 2002), suggest that human reasoning about risk consists of two cognitive systems: one is the experiential, intuitive system that helps us make quick assessments about the safety of a situation ("a gut feeling"), the other is the analytic system that helps us evaluate our thinking. Slovic et al. (2010) do not want to fall into a trap of deficit theorists by favouring the analytic system over the experiential. Instead, they use empirical evidence to argue that affect that stems from the experiential system helps us to make decisions quickly in an uncertain and dangerous world:

> We now recognize that the experiential mode of thinking and the analytic mode of thinking are continually active, interacting in what we have characterized as the 'dance of affect and reason' (Finucane et al., 2003). While we may be able to 'do the right thing' without analysis (e.g. dodge a falling object), it is unlikely that we can employ analytic thinking rationally without guidance from affect somewhere along the line. Affect is essential to rational action. (Slovic, Finucane, Peters, & MacGregor, 2010, p. 24)

An example of affect heuristics is the feeling of dread (Fischhoff et al., 1978), which has been shown to be a major predictor of public perception of risk for a wide range of phenomena (Slovic et al., 2010). For example, the feeling of dread towards nuclear power has resulted in the view of nuclear power "as a technology whose risks are uncontrollable, lethal, and potentially catastrophic" (Slovic, Fischhoff, & Lichtenstein, 1982, p. 485).

Estimation of Risk. In contrast to the research on probability, there are no comprehensive frameworks for analyzing students' estimation of risk, particularly the estimation that involves coordination between probability and impact. However, the research of Pratt et al. (2011) suggests that, in order to comprehend the coordination, students must have both algebraic and analytic geometry skills that will enable them to manipulate formulae and graph functional relationship between two variables.

Levinson et al. (2012) state that the participants in their study did not sufficiently coordinate between probability and impact; instead, the majority of discussion was based on personal attitudes of the group of teachers. The authors state that "while there was some discussion of probabilities, these only interacted with the decision on outcomes in a marginal way or provided insufficient background for a decision to be made" (p. 226). In addition, Levinson et al. state that:

> there were relatively few instances where the teachers simultaneously balanced changes in lifestyle against the likelihood of the operation resulting in serious harm. This might have been a problem in the way the data were presented, but it was more likely that personal preferences were driving decisions irrespective of emerging evidence. (p. 228)

The Role of Context and Content Knowledge in Pedagogy of Risk

Pratt et al. (2011) ask whether context may impede students' understanding of risk, drawing on examples from the previous studies which suggest that context may be detrimental to the mathematical understanding of risk. Pratt et al. (2011) conclude that the understanding of risk is closer to statistics than mathematics and that the context is crucial. If we strip away context and reduce the task of assessing risk to the mathematical coordination between likelihood and impact, we can see that the meaning is lost. Also, the numbers (quantitative data) have to be viewed in context. Pratt et al. (2011) also consider the issue of who the decision maker is as an element of context. In other words, students will respond differently depending on whether they are making decisions about themselves or another person.

Data concerning risk and uncertainty are never decontextualized and comprehensive interpretation of data involving risk requires placing data in an appropriate context (Gal, 2005). Therefore, contextual knowledge is essential for risk analysis. For example, the analysis of risk of a nuclear power plant accident requires content knowledge about nuclear power plants. Understanding of risk contains many elements of mathematical reasoning (logical, probabilistic, and proportional reasoning); it also contains many elements of the quantitative reasoning in context (Mayes, Peterson, & Bonilla, 2012). According to the authors:

> Quantitative reasoning in context (QRC) is mathematics and statistics applied in real-life, authentic situations that impact an individual's life as a constructive, concerned, and reflective citizen. QRC problems are context dependent, interdisciplinary, open-ended tasks that require critical thinking and the capacity to communicate a course of action. (p. 10)

Pratt et al. (2011) acknowledge, based on their study of math and science teachers, that reasoning about risk is highly contextualized. They differentiate between the context of the problem and the setting where reasoning takes place, namely reading, inquiry, and pedagogic setting (Monteiro & Ainley, 2003). An example of a reading setting is reading newspaper articles or advertisements, an inquiry setting is one in which an individual engages with data to solve the problem, and a pedagogic setting is the school setting where highly formal and mathematical ways to solve very specific problems are used (Pratt et al., 2011). It can be argued that, when solving problems, students can have different goals within each context.

Pratt et al. (2011) conjecture and show evidence that these three settings differ in the way participants use cognitive resources. They assert that cognitive resources most readily used in the reading setting are affective (emotional) responses followed by the understanding of context. In the inquiry setting, understanding of the problem context takes precedence over mathematical and statistical knowledge, though this knowledge is also important. Finally, the authors conjecture that, in the pedagogic setting, which is intended to teach and assess particular ideas, statistical and mathematical ideas are prioritized, whereas affective resources are less likely to be drawn upon.

One of the prerequisites for understanding of risk is content knowledge (e.g., in order to be familiar with risk of nuclear power plants accidents, we should have relevant knowledge about nuclear power). However, possession of content knowledge does not guarantee that the knowledge will be used in risk assessment. For example, Levinson et al. (2012) state that:

> there were many opportunities in the microworld for the teachers…to make use of relevant scientific knowledge in helping to evaluate risk, but none chose to do so,

reflecting other accounts, where scientific knowledge and information are either marginal or irrelevant to lay decision making. (p. 228)

Teaching Risk in the Classroom

There is a lack of classroom studies of students' understanding of risk; the studies that do exist stress the importance of treating risk-based decision making as a complex enterprise. The importance of the complexity of decision making is highlighted by Monteiro and Ainley (2003) in their study of student teachers who drew on four distinct resources: mathematical knowledge, contextual knowledge, affective responses, and personal experiences. The authors found that "if attention is focused exclusively on one of these sources, then the judgment may be distorted" (as cited in Pratt et al., 2011, p. 338).

In order to create a classroom environment conducive to the complex view of risk, Kolsto (2006) suggests that students need "easy access to an appropriate range of information and viewpoints" (p. 1711). For example, in Kolsto's study of high school student decision making related to electrical power lines, many students were only drawing their conclusions based on research-related information. Kolsto (2006), citing Aikenhead (1985), suggests that if we want students to draw from wider domains (including values), we need to include tasks in which students are confronted with this information.

Summary: Pedagogy of Risk

In summary, a comprehensive pedagogy of risk should embrace complexity of the concept of risk as well as human risk-based decision making. Research supports the claim that classroom instruction needs to be inquiry-based and embrace students' experiences, beliefs, and values (Pratt et al., 2011; Levinson et al., 2012). Based on students' risk-based reasoning, the goal of the pedagogy of risk ought to be the articulation of pedagogical strategies needed for teaching and learning and finding a place for the instruction within the mathematics curriculum. However, in order to articulate the pedagogical strategies, there ought to be more research on how students reason and make decisions about risk in the mathematics classroom setting.

Methods

I applied a qualitative case study approach as I explored the teaching of risk in two 11th grade classrooms that were using an inquiry-based learning approach to pedagogy. This section begins with a justification of my selection of a qualitative case study methodology for conducting research in the classroom, followed by my reasoning for the use of inquiry-based learning. I then outline the selection of the school, teachers, and participants, as well as the chronology of research, including the initial interviews with teachers, initial assessment of students, inquiry-based activities, final assessment, and final interviews with teachers. The section concludes with a detailed description of methods used for data collection and analysis, and also the ethical considerations relevant to my research.

Research Setting and Participants

The first research setting was Dale Academy, an all-boys private secondary school following the International Baccalaureate curriculum. Every student at the Dale Academy had access to many educational resources, including laptop computers and wireless Internet. The reason why this school was chosen was to be able to see how risk pedagogy can be approached in settings in which there is no lack of resources. The 11th grade class was chosen because the International Baccalaureate probability and statistics unit was a good place for teaching and learning about risk. In order to have a greater diversity of participants, the second school was St. Hubertus Secondary School, a co-educational school with no direct access to laptops and no wireless Internet access. I did most of the teaching in the study—the two teachers, Breanna and Clarissa, were there to help me plan the lessons, observe them, and assist me with the logistics and classroom management. Thus, the case study centres on the students and me, whereas the role of the classroom teacher was not explored in the study.

Inquiry-Based Learning Approach

In this section, I describe the inquiry-based learning approach (IL) and explain why it is appropriate for my study. As mentioned in the literature review, students' risk-based reasoning should be rich and complex and involve not only thinking about mathematics and content knowledge but also personal beliefs and experiences (Pratt et al., 2011). IL is consistent with the above statement because, in IL, "students learn content as well as discipline-specific reasoning skills and practices (often in scientific disciplines) by collaboratively engaging in investigations" (Hmelo-Silver, Duncan, & Chinn, 2007, p. 100). Inquiry-based learning involves authentic tasks that enable students to engage with the topic of inquiry. There are different views on what makes a task authentic (Levinson et al., 2012), but for the purpose of this thesis, I follow the distinction made by Murphy et al. (2006) between cultural authenticity and personal authenticity. They assert that cultural authenticity is present when students engage in a common social issue, whereas personal authenticity refers to an issue of importance to an individual student.

I draw upon the 5 E Model (Bybee et al., 2006) in which IL consists of five steps: 1) engage, 2) explore, 3) explain, 4) elaborate, and 5) evaluate. While presented linearly, the five steps do not always proceed chronologically and each of them may contain the teacher's help and guidance. The engagement stage consists of an activity or activities in which students engage in an issue which is either socially or personally authentic. In the exploration stage, students collaborate through an authentic task and begin to clarify their understanding of major concepts. Students are involved in the explanation stage when they construct concepts and processes about which they are exploring and learning. Finally, the elaboration activities challenge students to apply what they have learned to a new situation, and evaluation involves both students' and teachers' assessment of progress for the purposes of informing instruction (Bybee, et al., 2006).

The IL module started with an initial assessment of the student understanding of risk and knowledge of the Fukushima accident in order to engage students in the issue. The initial individual written assessment consisted of two questions setting the context of the issue. The first question was:

> Do you agree or disagree with the following statement? Explain your reasoning.
>
> "There have been around 500 nuclear power plants built since the 1950s. So far there have been only three significant nuclear power plant accidents. This makes nuclear power relatively safe compared to other means of generating power."

The purpose of the initial assessment was to show evidence that students possessed knowledge about impact prior to the instruction and to investigate language the students used when talking about impact. In addition, I wanted to investigate whether there was any evidence for the coordination between likelihood and impact as discussed by Pratt et al. (2011).

The second question consisted of two parts. First, the students were asked to write a paragraph on what they know about the accident. Their responses were then marked on a scale of 0 to 3 corresponding to the content of their answer (as described in the data analysis section as well as the findings). The second part of the question was: "What is your opinion on safety of nuclear power plants and how did the Fukushima incident influence your thinking?" The purpose of the question was to give students more opportunity to consider reasoning about risk in terms of probability and impact as well as to gauge their personal values, experiences, and personal and social commitments. The initial assessment was done during class and took around 30 minutes to complete. Following the initial assessment, there were 3 hours and 45 minutes of instruction (over the three 75-minute periods) on determining the empirical probability of nuclear power plant accidents. The activity started with a 75-minute lecture that defined key terms: probability, theoretical probability, and empirical probability.

Following the introduction, there was a 75-minute group activity (full period) with the following objectives: 1) to critically evaluate the sources of data provided; and 2) to estimate the empirical probability. The activity was specifically designed to contain the "explore," "explain," and "elaborate" elements of the 5 Es. The students were given a worksheet in which they were presented with four websites to use as potential data sources. The websites contained nuclear power plant accident data and could be easily accessed with laptops. The websites were:

•World Nuclear Association (WNA, 2011) – an organization representing the interests of the nuclear energy industry

•Greenpeace (Greenpeace, 2010)– an environmental activist group, giving a comprehensive list of incidents and accidents

•Datablog (Rogers, 2011) – the statistics blog from the *Guardian*

•Ecocentric Blog (Harrel, 2011) – the environmental blog from *Time* magazine

I chose to give students specific websites rather than having them freely explore the Internet because by selecting from a shared set of websites, I was able to gain insight into the reasons why they picked one over the other.

After deciding which website to draw data from, students were instructed to estimate the probability of a nuclear power plant accident. Following the activity, the groups presented their findings to the class; this took approximately 75 minutes (one full period). The group presentations had the potential to contain the "explore," "explain," "elaborate," and "evaluate" elements of the 5 Es.

Following the first activity, the second group activity (also 75 minutes) was completed and involved interpretation of data including likelihood and impact. The objective of the activity was to introduce students to the assessment of the impact, both qualitative and quantitative, and the coordination of the likelihood and impact, and to present them with the idea that the assessment may be value-laden. In terms of the five Es proposed by Bybee et al. (2006), it contained the elements of "explore," "explain," and "elaborate." This was followed by the presentation of data (75 minutes), containing the elements of the last four Es. Throughout the group activities, I was a facilitator assisting in student learning and using direct instruction to clarify certain points—the direct instruction was given to either the groups or the whole class.

Data Collection

Data collection for the first school started in March of 2011, with the four half-hour semi-structured interviews with the teacher in order to prepare for the lessons. These interviews were audio-recorded and helped me plan the activities in terms of choosing an appropriate class, duration of the activities, and the time period in which I could do the data collection. This data informed the IL activities but was not directly used in data analysis. All of the lessons were video-recorded; a camera captured the whole class and each group was video-recorded. In addition, student-written work from the initial assessment questions, handouts, and the construction paper given to each group for the activities was collected. I also kept field notes during the activities. This was challenging since I was also facilitating the lessons, and did not result in a rich and consistent data source. However, after each lesson, I recorded a reflection on each lesson and a short (5 minutes) debriefing with the teacher that was audio-recorded. The debriefing consisted of the teachers' assessment of the lesson, mostly their comments on the student engagement and on the logistics for the next activity. This data was used to make conjectures about the difference in engagement between the two groups. At the culmination of the series of lessons, I had an hour-long semi-structured interview with each teacher about her reflection on the activities as well as on her background as a teacher. These data were used to create a background of the teacher and also to make conjectures about the engagement differences between two groups. Preparation for the data collection at Dale Academy began in March of 2011. The data collection began and ended in May of the same year. Preparation for the data collection at St. Hubertus Secondary School began in early September 2011. The second data collection, at St. Hubertus, began in October and ended in November of 2011 and had the same structure as the first data collection.

Findings and Discussion

The findings suggest that the source of data (the four websites) was chosen based on reliability and the visual presentation of data. The judgment about reliability was based on reputation, sense of disinterestedness, neutrality, caution towards Internet documents, and trust in authority. In both settings, students found the *Guardian* website most trustworthy, being a "reputable newspaper." For many students, a source was reliable if it was seen to be disinterested and neutral. Specifically, for the Dale Academy students, the *Guardian* was considered the website that was not a "stakeholder in the issue" (David, Fahad, Samir, and Sasha) whereas the Greenpeace site was deemed unreliable because it had a clear agenda which was seen as anti-nuclear. Interestingly, the World Nuclear Association (WNA), a pro-nuclear power advocate, was not seen as a stakeholder, possibly because the students were not aware of its agenda since they were never explicitly given any information about it. On the other hand, the students at St. Hubertus were given brief descriptions of the sources and the WNA entry specifically stated that WNA was sponsored by the nuclear industry. The students unanimously found WNA to be unreliable. For example, Chloe, Mina, Larissa, Dana, and Sara found that WNA has "political and financial motives." This is consistent with Kolsto's (2001) study in which the power company was found to be unreliable because it had a financial interest in the issue.

The Dale Academy students were given four choices for the data source—two blogs (the Ecocentric blog associated with *Time* magazine and the *Guardian*-associated blog), the Greenpeace website, and the World Nuclear Association website. Interestingly, the students did not classify the *Guardian*'s website as a blog, while *Time* magazine's website was categorized as a blog—possibly because it has the word "blog" in the title. Some groups dismissed the Ecocentric blog because the blog format was seen as unreliable—Adam, Andrew, Zu-Zhang, and Mario stated that blogs are generally not edited and proofread for content. More bluntly, David, Jared, Luca, and Clint did not trust websites that were not .org or .uk. These concerns about online content are also echoed in the Levinson et al. (2012) study in which two groups were questioning the reliability of information online. For example, a math teacher (Ella) finds a source questionable because it could be from "any old website…could be one person" (p. 221). The students in my study, however, seemed to have a heuristic which they used to differentiate between online sources (e.g., blogs are not reliable) rather than being sceptical towards all information found online.

Levinson et al. (2012) found that students "recognize the role of trust and authority in giving meaning to the data" (p. 224). Consistent with this, two groups in my study found the World Nuclear Association (WNA) most reliable because they are the authority on the issue and they are involved in the production of information: Blair, Federico, Robert, and Cai stated that WNA was "an organization measuring those things"; Jordan, Alex, and Aaron echoed this sentiment by saying that WNA was "the source that records the actual data." This trust in people (authoritative bodies) that are involved in direct observation and measurement is consistent with the findings in Levinson et al. (2012), in which they reported that a participant (Linda) finds the spine doctor's recommendation reliable because he "knows more about it than the other people, and he's seen more of these people" (p. 224).

Gal (2004a, 2004b) lists the questions of validity of data—arguments and data that support claims made within the data source—as an important component of the critical question which assist in the critical evaluation of data and the claims made about the data. As mentioned above, there is a solid body of evidence that the students considered reliability but there is virtually no evidence that the students questioned claims made within the specific data source (the four websites). This is consistent with Kolsto's (2006) finding that pupils evaluated sources of knowledge more than they evaluated the content of the statement.

However, the students did consider the presentation of data within the document. Many students (e.g., Adam from Dale Academy) claimed that they chose the *Guardian*'s website because of the data presentation: the accidents were given a numerical ranking in terms of severity with the addition of the detailed description of the incidents as well as the use of colour to designate severity.

Determining and Interpreting Probability

Probability Estimates. Determination of probability was presented to the students as the calculation of empirical probability. The estimation depended on the group's decision on the data source from the previous step. Findings showed that students used various ways of calculating empirical probability. However, even the groups who used the same data source (e.g., *Guardian*) differed on the data that was included in the calculation. For example, David, Jared, Luca, and Clint used the cut-off value of 5 on the INES scale, bringing the number of nuclear accidents to 6, whereas Daniel's group also used a cut-off value, but 4 instead of 5. These values depended on the students' evaluation and judgment about the "severity" of accidents.

This can be explained by the Levison's et al. (2012) pedagogic model of risk since the estimation of probability was influenced by the students' beliefs on what constitutes a serious accident. However, it also shows that the decisions about impact (interpretation of INES scale) are not separate from the decisions about probability, suggesting that the estimation of probability is also dependent on the estimation of impact, which is not explicitly stated in the Levinson et al. pedagogical model.

Representation of Probability. In the Levinson et al. (2012) study, the teachers' representation of probability (fractions versus natural frequencies) depended on the context. The findings in my study confirm the context-dependency of representations. Some of the groups in my study moved between different representations as they were making sense of the data, and also to defend their claims. For example, Federico, Blair, Cai, and Robert calculated that the probability of a nuclear accident was 0.00016 accidents per day, but they also represented the answer as 3 accidents in 18747 days, or alternatively 1 accident in 6249 days. However, later, when the groups were asked whether this accident showed that nuclear power plants were safe, they said they were. They supported their claim by saying that there was one accident in 17, yet again changing the representation—in this case the representation was used to make the number more approachable, as the interval of 6248 days may not be as tangible as 17 years. This shows that the interpretation of the estimate of risk as being small (on the far right of the Levinson et al. model, Figure 2) has influenced the representation. This is something that seems not to be explicit in the Levinson et al. model, since in the Levinson et al. model the representation of probability influences the estimation of probability, whereas the converse statement is neither documented nor discussed.

Lina, Louvie, Connie, Karl, and Andy also went from the decimal representation (0.55) to one involving frequencies, saying that there is 1 accident every 2 years. The preference for natural numbers has been widely documented by Zhu and Gigerenzer (2006), whose research shows that individuals' probabilistic reasoning is improved if natural frequencies are used instead of probabilistic estimates (fractions and decimals).

Some of the groups in both schools showed elements of being on the highest level of Watson and Callingham's (2003) construct (critical reasoning about rate). However, some students showed a lack of understanding about rational numbers, for instance, incorrect use of percentages amongst one of the St. Hubertus groups. Both groups also made mechanical mistakes in the calculation and communication of the results; there was no mechanism for checking the work amongst the groups.

There was also a lack of contextual understanding of rates. For example, in both cases, there were instances of students calculating the ratio of accidents per nuclear reactor by using the current number of reactors and the total number of accidents (since the 1950s), whereas the more consistent way would be to use the total number of reactors, past and present. Only one group used the notion of reactor years (days) which is the most common way to express operation of nuclear power plants amongst the experts in the field of risk assessment.

Interpretation and Decision Making Based on Probability. Once the students produced the value for the probability (the probability estimate), they were asked whether this indicated that the probability was small, and then, consequently, they were asked to decide on the safety of nuclear power plants. The findings showed that all of the Dale Academy students found the numbers to be small, whereas at St. Hubertus three groups found the numbers to be large and three found them to be small. Interestingly enough, Adam, Andrew, Zu-Zhang, and Mario found 0.076 accidents per reactor to be a small number, whereas Chloe, Mina, Larissa, Dan, and Sara found the same value to be large, arguing that 7.6 accidents per 100 reactors is not acceptable. When asked about the value, Adam said that they also took their personal views into consideration. This confirms Pratt and Levinson's claim that the interpretation depends on students' beliefs and also supports the Levinson et al. (2012) pedagogical model.

The relative frequencies students used for estimating probability had units (accidents/year). (My instruction at Dale Academy did not specifically suggest that students use units, whereas St. Hubertus students were instructed to use them.) The use of units may have given additional context and meaning to the probabilities.

Determining and Interpreting Impact

Qualitative Representation of Impact. There is evidence that students possess pre-existing informal (intuitive) knowledge of impact. From the pedagogic view, this is very encouraging because, in many other domains (such as assessment of probability), there is strong evidence that individuals' intuitions are often erroneous (see, for example, Kahneman, Slovic, & Tversky, 1982). As it was seen from the study, this informal knowledge has a potential to be used in the instruction. The initial assessment in both classrooms shows that students use different language to talk about impact (e.g., "massive" and "dangerous"). Other words used to express impact include: "big," "astronomical," and the students even used the phrase "a barren landscape" to visualize the impact of the Chernobyl accident. This corresponds to what Pratt et al. (2011) label as "fuzzy qualitative descriptors," which students in his study used for the purpose of a rough quantification of impact ("serious," "massive," "bad," "fine," "big") (p. 339).

This is also consistent with the Levinson et al. (2012) study in which the teachers used phrases such as "impact on her life," "pain threshold," and "prohibitively dangerous option" to describe impact. The authors state that there was no opportunity for teachers to quantify impact. They also suggest that a meaningful quantification of impact and probability can only be done in an inquiry-based approach where the students can apply their values, representations, and experiences. This is the reason why the students in my study were given the impact statistic, and why I operationalized impact in terms of accidents and fatalities. Reasoning based on the magnitude of impact leads to reasoning about rational numbers—proportions, rates, and reciprocal values—confirming Watson's thesis that proportional thinking should be in the foreground of research about data. The findings showed that the students were making use of equivalent fractions, rates, and percentages. Sometimes they were correct (Blair's group, described above) but sometimes the use of rational numbers was incorrect. For example, some students were incorrectly using percentages.

Some students did not recognize that the statements were equivalent: 1.72 was the same as 31/18 which is the same as saying that the ratio was 1/18 to 1/31. However, it could be that the students thought that those statements, although mathematically equivalent, conveyed different contextual information. This stresses the differences between mathematical reasoning and quantitative reasoning in context (Mayes, Peterson, & Bonilla, 2012). The study as presented presents the case for the quantitative reasoning in context.

Another quantitative concept that the students were having difficulties with was the concept of reciprocal values. For example, St. Hubertus students understood that 31 fatalities/accident was a greater risk than 18 fatalities/accident. However, they did not understand that 1/31 accidents/fatality was the greater risk than 1/18 accidents/fatality.

Mathematical instruction underplays the importance of units. In the quantitative reasoning in context, however, the units are very important. My study shows that units were seldom used by Dale Academy students, while they were more often used by St. Hubertus students, which may be a consequence of them having been explicitly instructed to use units.

Coordination Between Likelihood and Impact. Unlike the studies of Levinson et al. (2012) and Pratt et al. (2011), the students in my study attempted to quantify both probability and impact. However, once quantified, there was a question of what to do with obtained numerical values. The students used various techniques to coordinate probability and impact—Dale students were instructed to multiply the two numbers, whereas the students at St. Hubertus used the graphing method (as suggested by Pratt et al.). Reflection on the Dale Academy students' use of the utility model (multiplication formula) confirms the observation made by Pratt et al. (2011) that, by reducing decision making to the product formula may have compromised the complexity, rendering the "decision-making process … irrelevant and meaningless" (p. 442). The Dale students' result of the product formula simply confirmed their overwhelming consensus that nuclear power is safe compared to other means of producing energy. The one group that disagreed decided not to use numerical information at all; instead, it used the qualitative descriptor of the impact of nuclear power plant accident as "astronomical."

The St. Hubertus students' use of the graphing method enabled them to visualize the relationship between likelihood and impact, and gave them more diverse ways of coordination. This is consistent with Pratt's call for the offering of other methods of quantification (p. 442). Mathematically speaking, the graphing method can be made equivalent to the product since the area of the rectangle defined by the point and the axes equals the product of the likelihood and impact.

However, in terms of the QRC framework, the graphing gives students an opportunity to locate the risk and then make risk management decisions about how to interpret it. For example, the area of the rectangle corresponding to nuclear power plant accidents—small probability, large impact—can be interpreted as medium risk; however, the students can consider contingency planning in terms of the accident. The graphing method does not necessarily make students make more objective decisions (the St. Hubertus students were still in line with their previous beliefs), however, it preserves the complexity by giving them the opportunity to use the language of risk. Also, it enables students to draw on ethical and other value-laden resources.

Some students did use the quantitative information to locate likelihood and impact on the axes, whereas some students used qualitative information. Another group also used the graph as an opportunity to ask the question about correlation between likelihood and impact—a group hypothesized that the larger the impact the smaller the probability, again leaving room for the richer interpretation.

Another characteristic of the interplay between likelihood and impact is that it does not happen only in the "final stages." The students considered likelihood and impact at various stages. For example, some Dale Academy students considered impact when determining the probability of a nuclear power accident by considering the cut-off value. Similarly, when asked whether 7% probability was large or small, Adam stated that it is small based on impact because the negative impact is smaller than the benefits.

The Role of Context and the Content Knowledge

The role of context. Pratt et al. (2011) discuss whether context may impede students' understanding of risk, drawing on examples from previous studies in which it was shown that context may be detrimental to mathematical understanding. The authors conclude that the understanding of risk is closer to statistics than to mathematics and that the context is crucial. If we strip away context and reduce the task to the mathematical coordination between likelihood and impact, we can see that the meaning is lost. The numbers have to be viewed in context.

The role of content knowledge. Levinson et al. (2012) state that "there were many opportunities in the microworld for the teachers…to make use of relevant scientific knowledge in helping to evaluate risk, but none chose to do so, reflecting other accounts, where scientific knowledge and information are either marginal or irrelevant to lay decision making" (p. 228). In my study, I found otherwise—the students' content knowledge interacted with their knowledge of risk, each at times elucidating the other. The students did seize the opportunity to use content knowledge, as the students seemed to understand that determination of risk and its various components depended on the context. For example, the knowledge about impact depended on students' being able to recall the information about nuclear power plants. However, not only did the content knowledge influence the knowledge about risk—the converse was also true. For example, in order to determine the impact of the nuclear power plant accident, the students at St. Hubertus drew on their knowledge of the content; however, as they were trying to understand impact, they were also making sense of the society.

How the question was formed was also an important part of the context in which students reasoned about risk. If the question had been framed differently, that would have influenced the study. The way I posed the question was whether nuclear power plants were safe relative to other energy sources. Also, the students understood the question to be whether "we" should have nuclear power plants. Some students made personal connections while others did not. This is consistent with the Pratt et al. (2011) study in which the students said that they would react differently depending on whether they were making decisions about somebody else or about themselves.

The role of feelings, beliefs and values. The affective factor is very important in individuals' risk based reasoning. Slovic (2010) discusses the dread factor that creates mental images about the hazards of interest (e.g., nuclear power plant accident). We can infer the feeling of dread in some imagery expressed by the St. Hubertus students, for example, a student talking about the impact of nuclear power plants as "a barren landscape." Similarly, Gregory et al. (2012) have shown that beliefs and values have to be an integral part of risk assessment and that the choice of data and the presentation of data depend on values. This can be seen in my study when Christine's group encountered the table of fatalities and argued about whether it was valid to only consider the accidents that resulted in five immediate fatalities. One of the students had a stern belief that "every death should count," and the other one was more pragmatic. Finally, the student drew on her personal experience, saying that it would matter to her if she was the person or if she knew the person. The students did not draw as much on personal experience as did the students in the Pratt et al. (2011) study. The reason is that the question was framed in terms of the logical statements: Are nuclear power plants safe? This can be compared to the decision statement: Should we have nuclear power plants, or more specifically, should we build more power plants in a certain area? Students did draw on their personal experiences, however. Particularly, one of the Dale Academy students was in the region (Hong Kong) when the Fukushima accident happened and he drew on this experience when making a decision about the safety of nuclear energy. In addition, another student at St. Hubertus stated that she would not like to live next to the nuclear power plant. However, because of how the question was construed, the students did not draw too much on personal experiences. Some students did show empathy (praying for Japan).

The pattern in both case studies was that the students did not seem to shift their beliefs about nuclear power plants (except in the case of one student). There were instances in which the exposition of quantitative data did cause students to question their beliefs. For example, some students were very surprised to find out that the fatalities for coal power plants were higher than those for nuclear plants. This is consistent with Kolsto's (2006) claim that students should be confronted with diverse information and viewpoints. However, students tended to include auxiliary information in order to "salvage" their beliefs.

Implications for Further Research

Some of the existing research suggests gender, race, and socio-economic differences play a role in perceived risk (Finucane et al., 2010), suggesting that white males tend to downplay risks compared to non-white males, white females, and non-white females. Although there is evidence that there are differences in the estimation of risk between the two classes, I am reluctant to make any conclusions based on gender, race, or socio-economic status because not enough data was collected about students' individual histories, identities, and beliefs about the world. There is a potential, however, to conduct research that would explore the interplay between different aspects of students' identity, beliefs, and understanding of risk in the classroom setting.

There also seemed to be gender differences in rhetorical style. Thirteen St. Hubertus students (more than half) expressed uncertainty of their knowledge about the accident (using phrases such as "I think" or "I don't know"), whereas none of the Dale Academy students did so. The expression of uncertainty did not necessarily match with the lack of knowledge. For example, Hiroko, who received the score of 2.5, prefaced her account of the events with, "I don't know much about what happened, but…" St. Hubertus students also expressed their emotions in their description, whereas only one student at Dale Academy did so—the student who had been in Hong Kong had been worried about the accident.

The question of the risk of nuclear power plant accident was framed in terms of a judgment about safety. Another way to frame the question would be in terms of the risk decision, such as whether there should be nuclear power plants. More specifically, we could frame a socially authentic inquiry-based learning instruction placed in the students' own community. For example, students could assess the risks of having a uranium pellet factory in their neighbourhood. Framing the question in terms of decision rather than opinion may give more weight to the project, and it could create an opportunity for students to reflect on the relationship between their values, mathematical and statistical thinking, and the content knowledge. To echo Monteiro and Ainley (2003), the pedagogical setting of the task would resemble the inquiry setting.

The study also showed an importance of representations in reasoning about risk that goes beyond verbal and formal-mathematical representations. For example, my findings show that students were very often using gestures in order to elucidate their arguments. Another potential direction would be to research students' use of gestures and locate it within other research on students' gestures in mathematics. The students were also making sense of the documents containing data, very often choosing data sources that presented data in visually accessible ways. Another potential research direction would be to study visualization of risk data most accessible to students in the classroom setting. As a starting point, the research could apply existing research in the field of data visualization, such as that of Tufte (1997).

Conclusion

This article documents the complexity of the concept of risk and decision making based on risk. It also suggests how risk can be taught in the mathematics classroom. The study contributes to educational research by shedding light on the teaching and learning of risk in the mathematics classroom, whereas there is a lack of research in this area (Pratt et al., 2011). Another major contribution of the research is the identification of the understanding of rational numbers as being crucial to understanding risk.

An important lesson to take from this research is that decision making about risk is an interplay between quantitative reasoning, experiences, values, beliefs, and content knowledge. Restricting the instruction to any of these single components without meaningful consideration of other components will trivialize and reduce the effectiveness of the teaching. Risk is all around us and the pedagogy of risk should play an important role in mathematics education.

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