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Abstract: In the New Zealand school statistics curriculum, year 12 students (aged 16-17) are required to solve problems that involve interpreting risk and relative risk within a range of meaningful contexts. In a small exploratory study we investigate the risk conceptions of four year 13 students who performed at the excellence level in their year 12 externally-assessed examination on this topic. Through questionnaires and interviews we investigate the ways in which these students perceive and express risks associated with a variety of everyday activities and also how they compare the risks of several adverse outcomes. We also explore the strategies they use when confronted with varied representations of risk such as visual, verbal and numerical. We will report on insights gained about these students' reasoning with different risk representations, on how they interpret, evaluate and communicate risk.

Keywords: secondary school students; heuristics; risk perceptions; risk representations; risk interpretations; relative risk.

Introduction

Statistical information is prolific in the media, yet many of us are misled, or have difficulty in interpreting and challenging statements made. In order to be an educated citizen in the twenty-first century, "(S)tatistical literacy is becoming fundamental for living in a full democracy" (Biggeri & Zuliani, 1999, p. 2). An important facet of statistical literacy is the ability to reason and to make judgments when an element of uncertainty is involved. We tend to rely on our gut feelings, combined with personal experiences or beliefs in order to make such decisions, and often use shortcuts such as the availability and representative heuristics (Kahneman, 2011). However, this approach may fail us when circumstances require more of an analytic assessment. Examples of such situations include the choice of a particular medical plan, the decision to invest in stocks or shares, assessing tsunami risk and measuring the effects of changes to scheduling of transport systems. Although sometimes trivial, the consequences of such decisions can be significant. Thus an understanding of risk is important in order to be an educated citizen in the 21st century.

The topic of risk was introduced to the New Zealand school statistics curriculum in 2012. The result of this is that year 12 students (aged 16-17) are now required to solve problems that involve calculating and interpreting risk and relative risk within a range of meaningful contexts. In addition to concepts such as risk and relative risk, the topic involves probability distributions, relative frequencies, two-way tables and probability trees. Students need to be able to demonstrate knowledge of probability concepts and terms, and to communicate using appropriate representations. This small exploratory study, involving four students, is a first attempt to explore how these students now perceive and express risks associated with every day activities, how they compare risks of several adverse outcomes and what strategies they use when confronted with varied representations of risk such as verbal, visual and numerical.

Background Literature

What is Risk?

The Oxford English Dictionary definition of the word 'risk' is "*(Exposure to) the possibility of loss, injury, or other adverse or unwelcome circumstance; a chance or situation involving such a possibility*". The everyday definition of risk will depend on the context in which it is being used. One common way of defining risk is as an uncertainty based on historical observations (Gigerenzer, 2002; Johnson, 2004) with other definitions including a hazard, or a potential adversity or threat that requires exposure and uncertainty (Slovic & Weber, 2002). As a concept, risk can refer to a probability or to a consequence or to the product of probability and consequence. For example, to answer the question "what is the risk of getting cancer if I eat processed meat frequently?" one might respond with a numerical value such as 0.01 or 1%. On the other hand, to answer the question "what is the risk of defaulting on my mortgage payments?" the response may be that my home will be repossessed by the bank. Gardner (2008) noted that risk can be perceived as the product of the probability of an event occurring and the value placed on its consequence. A study carried out by Sadique, Devlin, Edmunds, and Parkin (2013), explored decisions made by mothers who were given a hypothetical situation and asked whether or not they would vaccinate their child against a disease. The severity of the health effects associated with the disease was explained as was the severity of the adverse side-effects due to the vaccine, together with the probabilities of their occurrence. Their findings suggested that the information regarding consequence, in this case the severity of the health effects associated with the disease and the severity of the adverse side-effects due to the vaccination, influenced the mothers' decision to vaccinate or not, while "the probability of these events occurring was not a significant predictor" (p. 1). The very fact that the term risk encompasses multiple definitions and concepts results in many of us having trouble in both interpreting and communicating risk (Slovic & Weber, 2002).

Communication of Risk

It is a well-established fact that our perception of risk is influenced by the way in which that risk is presented (Gigerenzer & Edwards, 2003; Tversky & Kahneman, 1981; Utts, 2005). Since the provider of risk information is at liberty to choose how to convey the information, they are free to select the method that best serves their interest. An example might be a drug company quoting the benefit of their treatment in terms of relative risk instead of absolute risk in order to make results more compelling. Thus, owing to the many and varied ways in which risk can be communicated, we are easily manipulated. In a risk-literate society, citizens should have the ability to interpret risk in its many forms in order to make informed life-decisions (Gal, 2005). It is therefore desirable for learners to experience risk information in several formats, and to appreciate that the provider of information may present information in a manner that suits their own agenda, and not necessarily that of the consumer.

Verbal communication. Verbal representations allow for flexibility in risk communication and, as such, have the potential to capture consumers' emotions and intuitions in a way that numerical and graphical representations cannot. While this may enhance risk comprehension in some instances, there is also the potential for confusion and misunderstanding, particularly when the communicator of the risk information has an agenda to push. In particular, the manner in which information is framed, the *verbal frame*, can affect the way in which that information is perceived and acted upon. For example, when persuading patients to undertake a risky treatment option, evidence suggests that presenting probabilistic or risk information in a positive frame is more effective than presenting it in a negative frame (Edwards, Elwyn, Covey, Mathews, & Pill, 2001). However, when considering how to communicate the risks associated with taking part in a screening programme it appears that loss framing, where the potential losses from refusing to participate in the programme are described, has a greater impact on participation rates than gain framing, where potential gains from agreeing to participate in the screening programme are described (Gigerenzer & Edwards, 2003).

Visual communication. Visual displays of information, whether static or animated, have the potential to capture attention, summarise data effectively and to reveal potentially undetected patterns such as part-to-whole relationships (Eppler & Aeschimann, 2009). However, they are not immune to manipulation by risk information providers. When considering the design of a visual risk representation, it is important to consider the audience and to ensure that important details are highlighted, and superfluous details are avoided. Effective visual representations need to be designed carefully so that misinterpretation is minimised. Often, simpler graphs are more effective than more complicated graphs (Zikmund-Fisher, Fagerlin, & Ubel, 2010).

Numerical communication. Since a common way of representing and communicating risk is in numerical format, consumers of such information require adequate numeracy skills (Fagerlin, Zikmund-Fisher, Ubel, Janlovic, Derry, & Smith, 2007). However, even with sufficient proficiency, the way in which risks are expressed mathematically has an influence on their interpretation (Hoffrage, Lindsey, Hertwig, & Gigerenzer, 2000). Evidence suggests that expressing risks with natural frequencies lends itself to easier interpretation than proportions, probabilities or percentages since this is how statistical information has been presented historically, and the human mind is well-adapted to information in this form (Gigerenzer, Gaissmeier, Kurz-Milcke, Schwartz, & Woloshin, 2007; Martignon, 2014; Peters, Hibbard, Slovic, & Dieckmann, 2007).

Perception of Risk, Heuristics and Biases

People tend to perceive and react to risk in one of two ways; either as *risk as analysis*, or as *risk as feelings* (Slovic & Peters, 2006). While risk as analysis refers to a logical and rational treatment of risk information, risk as feelings refers to a reflexive reaction which may be based on no more than a hunch or a gut feeling. When judging or interpreting risk as feelings, we are susceptible to appealing shortcuts or heuristics which, although reliable in some instances, may also lead us astray. The availability heuristic is a shortcut we use that allows us to make a judgment based on how easy it is to recall a particular situation (Kahneman, 2011). For example, we tend to think that deaths due to causes such as murder, airplane accidents and shark attacks are more common than deaths due to less memorable causes such as flu, heart disease and diabetes due in part to the well-documented tendency for the media to devote more column-inches to relatively rare events than to conditions or behaviours that carry higher risks (Aronson, 2006; Harrabin, Coote, & Allen, 2003; Thirlaway & Heggs, 2010). This phenomenon results in the availability heuristic leading us astray. However, the availability heuristic may also serve us well. For example, if we find ourselves in a problematic position, we may recall unpleasant consequences occurring in a similar situation, and act to better protect ourselves. Another common shortcut is the control heuristic. Thompson, Armstrong, and Thomas (1998) suggest that people often adopt the control heuristic when assessing their personal influence over the occurrence of an outcome, even in chance situations. The control heuristic is comprised of both the intention to achieve an outcome and the perceived connection between one's action and the outcome. When we perceive a connection between action and outcome, the notion of personal control will be high. As with the availability heuristic, the control heuristic can lead to accurate judgments in situations such as those where we do in fact have control. However, in situations where we have little or no control, the control heuristic can give rise to erroneous decisions.

There are also several biases to which we are susceptible when receiving and interpreting risk information. According to Weinstein (1989, p. 1232), the optimism bias has the result that "people regard themselves as more likely than others to experience financial success, career advancement, and long life". This bias results in the belief that one's own risk is lower than the risk faced by other individuals sharing the same behaviour. Other examples of the optimism bias in action include judging our driving skills as higher than average, and our chance of being involved in traffic accidents being lower than for others (Svenson, Fischhoff, & MacGregor, 1985). The bias of risk denial arises when evidence is considered as reliable and informative only if it is consistent with one's beliefs, otherwise it is dismissed (Everitt, 1999). Another bias, the anchoring bias, relates to our inclination to rely heavily on one, often irrelevant, piece of information when making judgments, often evaluating new information against this anchor (Anderson & Iltis, 2008; Kahneman, 2011).

How Risk Should Be Taught

It seems indisputable that an understanding of risk is crucial in order for us to participate in the modern world. Thus, "training young students in the perception of risk has become fundamental in modern society", (Martignon, 2014, p. 157). According to Gal (2005), risk literacy is a branch of probability literacy that is closely related to statistical literacy. He describes five main knowledge classes and some dispositions which he proposes form the basic foundation of probability literacy. The knowledge elements comprise (1) big ideas (such as variation, randomness, and independence), (2) figuring probabilities, (3) language, (4) context, and (5) critical questions. Dispositional elements include critical stance, beliefs and attitudes, and personal sentiments such as risk aversion. Probability instruction in school has a tendency to be based on probability axioms and mathematical calculations (Greer & Mukhopadhyay, 2005; Jones, 2005). However, for the vast majority of us, the situations that necessitate us to draw on probability knowledge will be those requiring judgments or interpretations, not calculations. Gal (2005) has proposed some critical questions that consumers should use when interpreting statistical and probability claims. Based on these, and on what others consider that consumers need to ask about probability, and risk in particular (Gigerenzer et al., 2007; Utts, 2005), the following critical risk questions are proposed (see Table 1).

Big idea	*Details*
The risk of what?	• Baseline information • Risk communication (verbal, visual, numerical format)
What is the frame?	• Time frame • Verbal frame
To whom does the risk apply?	• Does it apply to me?

Table 1. Proposed critical questions for risk literacy

Risk in the New Zealand Mathematics and Statistics Curriculum

In 2012 the New Zealand Mathematics and Statistics Curriculum introduced a National Certificate of Educational Achievement (NCEA) standard at Level 2 (penultimate school year for most students) which in part relates to interpreting risk and relative risk. This standard, *Apply probability methods in solving problems*, requires students to exhibit proficiency in selecting and using methods, demonstrate knowledge of probability concepts and terms and to communicate using appropriate representations. The problems that are used are set in either real-life or statistical contexts, with probability methods including risk, relative risk, probability distributions, relative frequencies, two-way tables and probability trees. A typical risk-based question would describe a study with a real-life context such as, for example, wishing to investigate the prevalence of a disease in individuals with differing characteristics. It may be of interest to establish whether one group of individuals is more susceptible to the disease than another group, or that treatment A is more effective in combatting the disease than treatment B. Information may be given in terms of frequency data, or in a two-way table. Students are expected to be able to calculate probabilities, conditional probabilities, absolute risks and relative risks and possibly to critique media statements based on findings from the study in question. Although interpreting risk formed a small part of this standard, it was of interest to determine to what extent students' experiences from the previous year remained with them as they were confronted with new risk situations and representations.

Methods

This study was exploratory in nature, with the purpose of developing a deeper awareness of the issues associated with learning about risk in the school curriculum. The participants in this study were four Year 13 (aged 16 and 17) students attending a single-sex girls' secondary school in New Zealand. The participants were selected randomly from a larger group of 20 students from the same school, all of whom achieved well. The second author collected the data which comprised questionnaire responses and interviews. The students first completed a questionnaire designed to elicit their understanding of risk in a variety of ways. Six of the nine questions in the questionnaire were from the work of Dargahi-Noubary & Growney (1998), one question was adapted from Iman (1994), one from Fagerlin et al. (2007), and one from a previous Year 12 national examination. The questions required the students to describe, calculate, compare and interpret risk in several contexts and, where relevant, were asked to explain their answers. Once the students had completed the questionnaires, two follow-up interviews were carried out with the students working in pairs. The purpose of the first interview was to gain more insight into student responses in the questionnaires, while the aim of the second interview was to explore their reasoning strategies when presented with varied, and sometimes unfamiliar, risk representations. The interviews were video-recorded and transcribed for analysis. A thematic qualitative data analysis (Braun & Clarke, 2006) was conducted on the student questionnaire responses and student interviews.

Analysis

Since similar themes were obtained from several questions and tasks given to the students, attention will focus on only five of the nine questions in the questionnaire, and on three of the four tasks that formed the second interview. In sections 4.1 to 4.3, we focus on Questions 1, 2, 3 and 6 of the questionnaire. In section 4.4 we present findings from the second interview where students were given four risk representation tasks. Three of the four tasks will be focused on, and at this stage Question 9 from the questionnaire will also be discussed.

Description/Definition of Risk

The first question (Dargahi-Noubary & Growney, 1998) required the students to state what they knew about the topic of risk based on their general knowledge, school work, or from what they had heard in the media. While not specifically asked to do so, three of the four students (Students A, B and C) provided their definition of risk. All of these students defined risk as the likelihood of an event occurring, or words to that effect. Two of these students stated that the event in question was undesirable, that is they associated risk with a negative outcome. Student B's response was "Risk is how likely something is to occur, usually in regards to a negative outcome", whereas Student C responded "Risk is how likely something bad is going to happen". Student A stated that there were "good risks and bad risks" while Student D answered the question by mentioning how she thought the media portrayed risk, and by stating that her school experiences taught her to consider "the consequences of our actions and risk which comes with the choices we make." This student is thinking on a broader level than the other three by reflecting on how risk is generally portrayed in society.

In Question 3 the students were asked to choose their preferred definition of the term risk from a list of four provided in Table 2.

Which of these definitions of risk do you prefer? Explain.

(a) A probability with a negative connotation

(b) A type of probability that has no theoretical model

(c) An uncertainty that has a numerical value

(d) Other – (explain)

Table 2. Risk definitions provided to students in Question 3 (Dargahi-Noubary & Growney, 1998, p. 45)

Given their responses to Question 1, it was not surprising that students B and C chose option (a). Despite her response to Question 1, Student A also selected option (a). Student D chose option (d) with the accompanying explanation that risk could refer to events with positive or negative outcomes. She also stated that the media tend to use the term risk when discussing negative outcomes.

We conjecture that there may be several reasons to explain why risk is associated with negative outcomes. One reason may be that the students cannot envisage the complete *sample space* of outcomes associated with an event under consideration. Confounding this issue is the fact that everyday *language* may interfere with risk perception so that even if students are able to imagine the complete sample space of outcomes, they may use different language to describe the likelihood of each outcome. In addition, the *availability heuristic* may be operating in such a way to influence the students' perception risk, a fact that Student D was aware of. As an example, consider testing the effectiveness of a new drug. A naïve consideration of all possible outcomes would be to state that the drug either worked, or did not work. However, not only would it be of interest to establish whether or not the drug worked, but also if the drug caused any side effects. Therefore, a complete list of outcomes may be: (1) the drug works and there are no side effects, (2) the drug works and there are side effects, (3) the drug doesn't work and there are no side effects, and (4) the drug doesn't work and there are side effects. When asked to assess the riskiness of the four outcomes, it may be that the *language* depends on the outcome. For example, the word 'chance' may be used to describe the likelihood of outcome (1) which describes a positive response, while the word 'risk' is used for the other outcomes which all include at least one negative response.

Ranking Risks

The second question, which was not part of the Dargahi-Noubary and Growney (1998) questionnaire, required the students to rank a list of ten causes of death according to their risk, with a rank of 1 being allocated to the event with the highest risk, and a rank of 10 to the event with the lowest risk. Eight of the causes of death were due to injury, either deliberate or accidental, with the remaining two (smoking and stroke) being disease-related. This question aimed to provide some insight into the students' perceptions of the comparative risks of several events and was based on similar questions adopted by others, for example Iman (1994) and Jones (2012). Students were then asked what background information they would need in order to estimate the risks associated with each event. The rankings from the students are provided in Table 3, with the mean rank being the average awarded across all four students. The final column in Table 3 gives the actual ranking of the deaths due to injury provided by the National Safety Council (National Safety Council, 2006), an organisation that collates data on such events for the United States, with the relative rankings of deaths due to smoking-related illnesses and strokes obtained from the Centers for Disease Control and Prevention website (www.cdc.gov).

Cause of death	Student				*Mean rank*	*NSC/CDC rank*
	A	*B*	*C*	*D*		
Falling	7	10	10	3	9	5
Firearm assault	9	1	6	9	7	6
Motorcycle accident	8	4	1	8	4	8
Motor vehicle accident	3	5	7	7	5	3
Pedestrian accident	2	8	3	4	3	7
Stroke	6	9	4	5	6	2
Smoking	4	7	9	6	8	1
Suicide	1	3	2	1	1	4
Natural forces	10	6	8	10	10	10
Drowning	5	2	5	2	2	9

Table 3. Students' and NSC/CDC rankings for Question 2

There were several interesting aspects to the student responses to Question 2. Firstly, students tended to rank *suicide* very high despite its actual ranking of 4. Secondly, while *smoking* and *stroke* ranked highest according to the data provided by the CDC, students placed both of these causes of death as low (8, 9) or moderately low (4-6). In addition, two students ranked *drowning* as their second highest cause of death, while the data provided by the NSC resulted in a rank of 9. Finally, two students ascribed the lowest possible rank of 10 to *falling*, while its relative ranking is 5. Possible explanations for these results include the use of the availability heuristic, a lack of awareness of critical questions, and the notion of conditioning on an event.

Availability Heuristic. The role that the *availability heuristic* plays is the possibility that the students are relying on media reports and subsequent conversations with their peer groups and/or family to make conclusions about the incidence of suicide, drowning and deaths from smoking or strokes. This study took place around the time when the media had reported on several high-profile celebrity suicides, thereby perhaps distorting the perception of the prevalence of suicide. In Student D's words, "as you grow older you are more familiar with death… from old age or suicide kind of because you hear a lot about it, but when it comes to being hit on a motorcycle you are like oh, I haven't heard that for ages". Cases of drowning also receive heightened media-attention, largely due to the fact that in New Zealand water-based activities such as fishing, swimming, surfing and sailing are popular leisure pursuits and the media therefore have a responsibility to raise an awareness of the dangers associated with these activities. Thus students may perceive drowning to be more common than it actually is. On the other hand, news reports about deaths due to strokes and smoking are few and far between. Given the tendency of the media to report prolifically on relatively rare events, this may have distorted the students' perceptions of the regularity with which some of these events occur.

Critical questions. It appears that these students did not consider the questions that should be asked, such as the critical questions outlined in Table 1, when ranking the events in Table 2. Taking *falling* as an example, it appears that the students may be considering that their own risk and that of their peers, of dying from a fall is very low and that that is the basis for the low ranking. They do not appear to consider the time frame over which the events are considered. If they had taken the time frame into account, then perhaps they would have recognised that on a population level the risk of dying from a fall is not insignificant due to the higher prevalence in the elderly. The same may be said for their comparatively low rankings for deaths from smoking and strokes, since they perhaps don't see these events as being relevant to themselves or to their peer groups. However, even though the students did not appear to reflect on the critical questions when ranking the events in Table 2, they appeared have an awareness of some of the questions when they were asked what information would be required in order to provide numerical estimates. Responses included a requirement to have information in the form of frequencies, for example Student B stated: "How often each of the events occurs" which suggests that she was considering the risks at a population level rather than at a local level. Student B also asked for information as to whom the risk applies, for example "How often does the person smoke?" and "Motor cycle accident – was the person wearing a helmet? What speed were they going?" thereby demonstrating an awareness of *sample space* and its relevance when required to make a judgment.

Conditioning on the event. Follow-up interviews with the students uncovered an interesting feature to one student's interpretation of this task. Student B explained that she allocated a rank of 3 to suicide and explained "I wasn't sure whether thinking … if you're going to commit suicide what is your chance of dying from it, it's pretty high. Like if you're already in the act…" and, for motor vehicle accidents "a lot of people survive car accidents, so it was probably lower than suicide" which she ranked as 5. Student B appears to be using conditioning in her reasoning by considering the risk of dying *given* each event is underway, and therefore ranking them according to the likelihood of surviving the event once it has happened. Despite this reasoning, her rankings for suicide and motor vehicle accidents were similar to the NSC data-based rankings. While assessing the relative rankings of the events in Table 1, Student B appears to be integrating knowledge about context and consequence as part of her reasoning strategy, giving her a sense of the size of risk that she might attribute to the events. Her belief is that once engaged in the act of suicide, the chance of survival is very low, while she believes there is a much higher chance of surviving a motor vehicle accident. Her contextual knowledge does not seem to include an awareness that there will be many unsuccessful suicide attempts which is not surprising given that such events are generally undocumented in the media and elsewhere.

Numbers Associated with Risk

Question 6 required the students to describe a risky activity in which they had been involved, and to estimate the amount of risk associated with that activity. They were then asked to consider how their friends might respond to the risk estimates for the same activity and to explain their reasoning (Dargahi-Noubary & Growney, 1998). The activity provided by Student A was that of going on an overnight hiking expedition in the bush, with negative outcomes comprising getting lost or injured, or suffering from hypothermia. She explained that there was the potential for such a hike to be considered a high-risk activity, but that the preparatory groundwork and the experience of the hike leaders meant that the actual risk was fairly low, giving a numerical value of 2%. In her response, Student A exhibits the personal control heuristic by considering that the preparatory actions of group would directly influence the possibility of a hazardous outcome. She then went on to explain that the risk estimates provided by friends who were involved in the same activity would depend on their individual experience levels and their perceptions of the competence of hike leaders.

Student C's example was of riding her unicycle to dance class. She felt that this was a high risk activity since a unicycle is difficult to control and can be very unstable on uneven ground. She gave a numerical value of 70% for her own risk of falling and explained that this was because of the "numerous things that could go wrong". She believed that her peers would have a much higher ("maybe 90%") chance of falling since they were less experienced. Again, there is evidence of the personal control heuristic in Student C's response although she is aware that, despite her level of experience, she is still vulnerable.

While Student A's risk estimate is very low (2%) compared with that of Student C (70%), it is worth mentioning that the negative outcomes associated with the hiking activity may well be perceived as being more serious than those associated with falling off a unicycle, particularly when protected by a cycling helmet. Therefore the severity of the *consequences* may be influencing the students' risk perceptions more than the actual likelihood of adverse events occurring.

Risk Representations

In order to gain some insight into how these four students reasoned about risk when presented with varied, and sometimes unfamiliar, risk representations, further tasks were given. In these tasks, students worked in pairs to understand and reason with risks presented both visually and numerically. The visual representations included both graphical formats with information being displayed as line graphs, and pictorial formats with information being displayed as icons. Numerical representations involved information being presented either in the form of probabilities, natural frequencies, or two-way tables.

Visual risk representations. Graphical information on the risk of invasive breast cancer diagnosis (Dupont & Plummer, 1996) was presented to the students (see Figure 1).

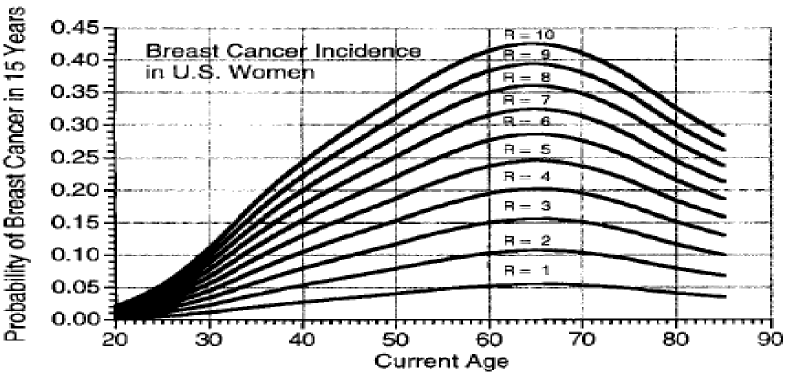


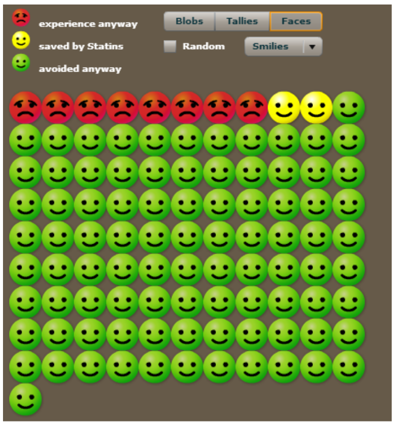
Figure 1. 15 year absolute risk of invasive breast cancer diagnosis by current age and relative risk (Dupont & Plummer, 1996, p. 2196)

This particular format would not have been familiar to the students. However, both pairs of students were able to read information from the graphs, and to think of some risk factors that might explain the different R-lines in the graph. Students A and B suggested that family history and lifestyle would be factors that may influence an individual's breast cancer risk, while Students C and D suggested exposure to UV light. When asked to estimate the probability of someone aged 60 with an R-line value of 4 being diagnosed with invasive breast cancer within a 15 year time period, all four students were able to interpret the graph correctly and provide a sensible answer. Students were then asked why all of the curves appeared to peak at the age of 65. Students A and B were able to make use of their background contextual knowledge with Student A stating: "…and maybe if you've had all those increased risk factors such as family history you'll have got it before then so if you've got all the way to 65 means you've got a good family history." When asked if a corresponding graph for women in New Zealand would look similar to Figure 1, which is for US women, Students A and B noted that the graph would look similar because "we live in very similar lifestyles and have a similar demographic to the US, we're a Western culture and if you went to Africa you'd probably get a different graph". On the other hand, Students C and D thought the graph would be different because in the US there are "different diagnostic techniques or not as many people in New Zealand have scans… so it won't be exactly the same percentage". While each pair of students answered this question differently from one another, their responses were sensible and indicated that they were all using their contextual knowledge about breast cancer, and integrating this into their reasoning.

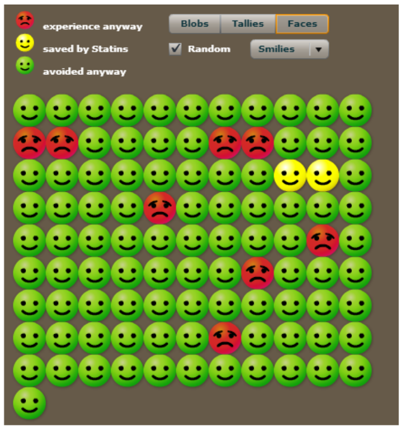
Two different situations, both incorporating pictorial risk information in the form of icons, were also provided (see Figure 2). The information related to the effects of statins (cholesterol-lowering drugs) on the incidence of heart attack/stroke (see www.understandinguncertainty.org website, Spiegelhalter). Scenario 1 used loss framing by focusing on the disadvantages of not taking statins while Scenario 2 used gain framing by describing the positive effects of taking statins. Within each of these scenarios the information was provided visually in two ways; either with grouped icons where like-icons are placed in groups as can be seen in Figures 2(a) and 2(c), or with randomly placed icons where like-icons are placed randomly as can be seen in Figures 2(b) and 2(d).

Scenario 1

Your chance of experiencing a heart attack or stroke in 10 years without Statins is in 100, which is reduced to___in 100 with Statins.



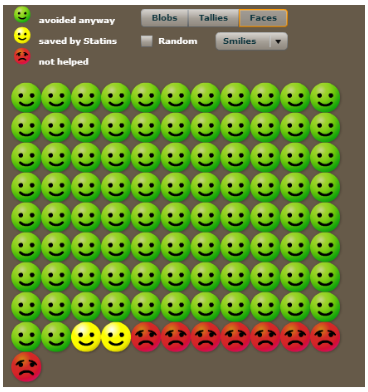
2(a)



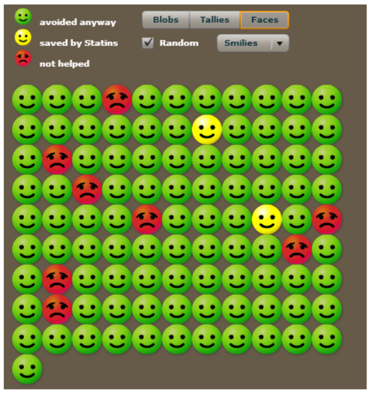
2(b)

Scenario 2

Your chance of avoiding a heart attack or stroke in 10 years without Statins is ___in 100, which is increased to__in 100 with Statins.



2(c)



2(d)

Figure 2. Pictorial representations for risks with loss framing (Scenario 1) and gain framing (Scenario 2) with grouped icons (a, c) or randomly placed icons (b, d) (http://understandinguncertainty.org/node/233)

The students understood the visual representations shown in Figure 2 and did not appear to have any trouble in answering the questions stated within each scenario. When asked for their opinions on the grouped icons versus randomly placed icons, Students A and B expressed a preference for grouped icons with Student A stating that they were "easier to understand conceptually whereas counting them wasn't really that hard but visually looking at it, it was a lot easier when they were grouped". Whether the icons were placed in groups or randomly did not appear to make a difference to Students C and D. The students were then asked if they had a preference for Scenario 1 or for Scenario 2. This questioning was aimed at discovering whether the verbal frame made a difference to the students' reasoning. Again, students C and D had no real preference while Students A and B mentioned that it depended on the numbers. Student A said: "It was interesting because they've both got a difference of two but with the 90s it seems like a smaller gap… so it seems more significant when you're talking about the 8 and the 10, even though they're both just two". Her statement suggests that the effects of the statins are more apparent within the loss frame of Scenario 1 because the difference between 8 and 10 is relatively larger than the corresponding difference between 90 and 92 when the situation is expressed within the gain frame of Scenario 2.

Numerical risk representations. In order to explore these students' reasoning when confronted with numerical risk information, a breast screening and diagnosis problem was presented in two ways; one probabilistically, the other with natural frequencies (Gigerenzer, 2002; see Table 4).

Version (a)	The probability that a woman of age 40 has breast cancer is about 1 per cent. If she has breast cancer, the probability she tests positive on a screening mammogram is 90 per cent. If she does not have breast cancer, the probability that she nevertheless tests positive is 9 per cent. What are the chances that a woman who tests positive actually has breast cancer?
Version (b)	Eight out of every 1000 women have breast cancer. Of these 8 women with breast cancer, 7 will have a positive mammogram. Of the remaining 992 women who don't have breast cancer, some 70 will have a positive mammogram. Imagine a sample of 1000 women who have a positive mammogram. How many of these women actually have breast cancer?

Table 4. Two versions of problem given to students (Gigerenzer, 2002)

Students A and B, who received version (a), immediately drew a probability tree and wrote the correct probabilities along the branches of the tree. As they started to multiply the probabilities along two sets of branches and add them together, they then decided that only the branch reflecting those with breast cancer who tested positive was needed. Student B stated "so isn't it just 90%?" and Student A agreed. Students C and D received version (b), but did not draw a probability tree. Instead, they tackled the problem verbally and concluded that 77 women had a positive mammogram, "and how many of these women actually have breast cancer? So isn't it just seven out of 77?". The students who were given the risk information in probabilistic format appeared to lose sight of the relevant sample space, convincing themselves incorrectly that it was women who have breast cancer. However, the students provided with numerical information gave the impression that they were able to retain information about the relevant sample space whilst discussing the problem, thereby arriving at the correct answer.

In the Question 9 of the questionnaire, which students answered individually, they were given a two-way table describing the outcomes (cured or not cured) of a group of individuals with a serious skin rash who either received treatment A or treatment B. They were asked to calculate the probability of being cured with treatment A, and then to critique a newspaper headline stating that a person with a serious skin rash is twice as likely to be cured if using treatment A as a person using treatment B. In order to answer this question, the students were required to calculate the probability of a person being cured with treatment B, and then to calculate the required relative risk. This question was very familiar to the students as it resembled typical questions asked in their probability standard of the previous year. All of the students answered this question well.

Discussion and Conclusion

This paper explores the strategies used by Year 13 students as they were asked to interpret, rank and reason with risks in a variety of contexts, and were confronted with a variety of risk representations. In this final section we discuss reasoning strategies and risk conceptions displayed by the four students and consider the contribution that the findings may have to existing research in this area. We then discuss the implications the findings have on teaching risk in the classroom. However, we will first comment on the limitations of the study.

Limitations of the Study

This exploratory study was very small, focusing as it did on only four high-achieving students. Within the constraints of the study, many potential areas for investigation were not possible. For example, it was not possible to explore the effects of cognitive limitations and biases and personal characteristics such as cultural background, which researchers have found to be contributory factors in interpreting and misinterpreting risk information (Anderson & Iltis, 2008; Sorensen, Gyrd-Hansen, Kristiansen, Nexoe, & Nielsen, 2008).

Potential Characterisations of Student Conceptions

While acknowledging the limitations of the study, the findings seem to suggest that certain potential characterisations of student conceptions are apparent (see Table 5). These conceptions, based on responses to all questions in the questionnaire and on all tasks in the second interview, will now be discussed further.

Conceptions	Description	Characterisation in study	Student Reasoning
Availability heuristic	Judgement is created on the availability or recall of information in the memory	• Likelihood of risks are given	Led to sensible use of own contextual knowledge to judge reasons for likelihoods
		• Likelihood of risks are estimated	Own contextual knowledge led to errors in judgements
Critical questions to ask when judging risk information	*Risk of what?*		
	• Baseline information	Size of risk depends on the baseline	Not taken into account when judging risk but mentioned when asked how to determine risk
	• Risk communication	Can be verbal, visual or numerical format	Appropriate reasoning seemed to depend on format
	What is the frame?		
	• Time frame	Risk of events depends on the time frame	Not considered when time frame was not given
	• Verbal frame	Information presented in positive/negative, harm/benefit frames influences estimation of risks	• Larger numbers influenced preference for positive frame • Preference for risk expressed as harm or benefit not evident
	To whom does the risk apply? Does it apply to me?	Judgements of risks are influenced by identification of relevant sample space	Relevant sample space not taken into account unless given
Reasoning numerically with risk	• Estimating risk	Sense of size of risk (e.g. 10%)	• Considered context and consequence of risk • Developing a sense of associating numbers appropriately with high and low risk • Personal control ideas tended to lead to a disregard of universal sample space

	• Calculating risk	• Natural frequencies • Proportions	Led to appropriate calculations Led to inappropriate calculations
	• Language of risk	Everyday language is different from probability language. Probability and risk calculations are the same	Some perceived risk as having a negative connotation
Sample space	• Universal sample space • Conditional sample space	All population units Subset of units in population	Tended to disregard when assessing risk Used when thinking about risk applying to me

Table 5. Summary of four students' conceptions and reasoning about risk in the study

The availability heuristic. People naturally make use of their experiences in order to make judgements. The availability heuristic is employed when we recall information from family, peers and the media to analyse risk information (Kahneman, 2011). In this study, the students' use of the availability heuristic was characterised in two ways. Firstly, when faced with several situations from which death could arise, students were asked to rank them in order, from the most to the least risky. The disparity between the students' ranks and those of the NSC and the CDC appear to be due, in part, to the students' reliance on memory which brought to mind examples of moderately common or relatively uncommon causes such as suicide and drowning, and propelled them to the top of their lists. Similarly, common causes of death such as falling or from the effect of smoking, not readily reported in the media, came close to the bottom of their lists. However, when the students were asked to consider the reasons for the different R-lines in Figure 1 (see Section 4.4.1), they were able to draw on their contextual knowledge and provide sensible suggestions. This characterisation of the availability heuristic, when students were told that the risk profiles vary, had a beneficial effect on student reasoning.

Critical questions to ask when judging risk information. The students in the study did not appear to consider the critical questions or to search for background information unless specifically asked how to determine risk. In certain instances, they were aware of notions such as time frame but only if it was provided, such as their reasoning with the visual representation of 15-year breast cancer diagnosis (see Figure 1). In other cases they seemed unaware of the importance of the time frame which meant that they were unable to judge how likely it was to die as the result of a fall or from the effects of smoking. The notion of verbal framing was not investigated thoroughly in the study, although for two students it appeared that framing preference had more to do with numbers than with the wording of the question. With regard to the question "to whom does the risk apply?", students were able to judge risks based on the relevant sample space or conditioning event, but again only if this information was provided. This was evident in their responses to the visual representation of risk in Figure 1, particularly when asked to reflect on the corresponding representation for a different group of women. However, when the sample space was not stated explicitly, for example when ranking the risks in Table 1, the relevant sample space did not appear to be considered. Such findings lend credence to the beliefs of researchers such as Gigerenzer (2002) and Watson (2006) who state that by asking critical questions of risk information and being aware of the ways in which risk can be represented, we may be more informed and not so easily manipulated.

Reasoning numerically with risk. When it came to estimating risk, students appeared to consider both *context* and *consequence*. For example, when describing a risky activity in which they had been involved, their assessment of the size of the risk seemed to be based on the severity of the outcome in conjunction with how likely it would be to experience such an outcome. The student who provided the unicycling example (see Section 4.3) noted that the risk of an adverse event such as falling off was relatively high (70%), but that there were lots of possible things that could go wrong due to the instability of a unicycle and the likelihood of unstable ground. The student who described the hiking scenario gave a much lower estimate (2%), but explained that although the activity in itself could be considered high risk, the preparatory groundwork and experience of the hike leaders would ameliorate this risk substantially. The idea that the severity of the *consequences* may be influencing the students' risk perceptions more than the probability of an undesirable outcome is in keeping with the findings of Sadique, et al. (2013). However, there was also a tendency for students to overestimate the risk of rare but potentially dramatic events such as suicide, which is indicated by the availability heuristic characterisation, and also by the research of Everitt (1999). With regard to calculating risks numerically, the students in the study were more comfortable working with natural frequencies than with probabilistic information such as proportions which is in line with the research (Gigerenzer et al., 2007; Hoffrage et al., 2000; Martignon, 2014; Peters et al., 2007). Another characterisation of reasoning numerically with risk that was evident in the study was the perception of most of the students that risk has a negative connotation. This finding supports Watson's (2006) belief that interpretation and evaluation of the language of risk, when stated in a social context, is crucial. The fact that Gal (2002) includes a language and context component in his probability literacy framework suggests that when teaching risk, the students' perceptions must be considered.

Sample space. The final characterisation of student conception evident in this study is that of sample space. While consideration of sample space underpinned the characterisation of '*Judgement of risks are influenced by identification of relevant sample space*' in Table 4, it is worthy of a mention in its own right. When asked to assess risk by ranking (see Section 4.2) or by calculating having been provided with numerical information in a probabilistic format (see Section 4.4.2), the students tended to be unaware of the sample space of interest. However, when specifically asked to consider risks for a particular group of individuals or units (see Section 4.4.1), or to estimate risks for which they had no relevant numerical information (see Section 4.2.2), they were either able to integrate knowledge of the relevant conditional sample space or to ask for relevant information about particular characteristics to arrive at sensible conclusions. Sample space is well-recognised as a key concept for learning about probability (Batanero & Sanchez, 2005; Jones, Langrall, Thornton, & Mogill, 1997; Savard, 2014). In light of the findings of this small study, it would appear that sample space, as a concept, should also have prominence in the learning of risk.

Curriculum and risk

As evidenced by the findings of this small exploratory study, the four high-achieving students who participated were proficient at calculating risks and relative risks from two-way tables and were able to reason effectively with different risk representations. However, the reasoning strategies employed by these students did not always lead to appropriate judgments or conclusions. The current practice of teaching risk and probability calculations, with little regard to the wider context, results in students being capable of the mathematical manipulations and uncomplicated interpretations of risk statements. However, integrating more context into the situations provided to students, and building in students an awareness of intuitions and potential biases, may result in a deeper conceptual understanding of risk.

Implication for Teaching

Gigerenzer (2002) describes three important steps when it comes to teaching people to reason with risk. The first step, which he calls "Franklin's Law", is to develop the awareness that uncertainty is a given since, according to Benjamin Franklin, "In this world nothing is certain but death and taxes". He recommends demonstrating this through the use of everyday contexts in which uncertainty is present rather than through the use of imaginary situations. The second step, "Beyond ignorance of risks", involves teaching people to know how to use information to estimate risks and to be cognisant of the difficulties that may arise in doing so, such as biased media reporting. The third step, "Communication and Reasoning", includes educating people in the various ways in which risk can be represented, and to develop awareness of the ways in which we can be manipulated by different representations.

With an awareness of the student conceptions that were apparent in this study, we conjecture that attending explicitly to notions such as those outlined in Table 4 may benefit school students such as those who were involved in the study as they learn to interpret and reason with risk in varied situations. Our findings suggest that intuition guided the students' reasoning processes in many of the tasks in this study, particularly when reliable information was not provided. In such instances the students adopted a *risk as feelings* approach. However, when specifically asked for numerical estimates, or for reasons to explain certain factual findings, they took a logical approach, thereby employing *risk as analysis* (c.f. Slovic & Peters, 2006).

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