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Knowledge for Equitable Mathematics Teaching: The Case of Latino ELLs in U.S. Schools

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Abstract: This paper reports the exploration of an aspect of knowledge needed for equitable mathematics teaching. Pedagogical Content Knowledge for Teaching Mathematics to English Language Learners (PCK-MELL) was proposed as a theoretical knowledge construct, a subdomain of MKT, and the construct was investigated through a process of survey instrument development and administration. The survey contained items intended to measure teachers' knowledge of the obstacles encountered by ELLs in math classes, of the resources that ELLs draw upon, and of instructional strategies for teaching ELLs. Analysis of middle school mathematics teachers' responses ($N = 42$) offered insights into how to improve the reliability and measurement validity of this sort of instrument, as well as directions for further theory development.

Key words: English Language Learners; mathematical knowledge for teaching; deficits; affordances

Introduction

Many mathematics teachers worldwide are finding new languages and new cultures in their classrooms. Since the 1970s the number of students who are English Language Learners (ELLs) in United States schools has grown and continues to grow dramatically (U.S. Department of Education, 2012; Payán & Nettles, 2008; Francis et al., 2006; Capps et al., 2005). Mathematics education researchers and others have observed, based upon the lower relative performance of these students on standardized exams of mathematics achievement in comparison with their mainstream (non-ELL) counterparts, that many of these ELLs have infrequently had equitable opportunity to learn mathematics in U.S. schools and have often been underserved by their teachers and schools (Center on Education Policy, 2010; Abedi & Herman, 2010). Furthermore, researchers have noted that using measurements of mathematical achievement to draw attention to disparities between student groups in fact offers little, if any, contribution toward promoting equity in education (Gutiérrez, 2008), and that focusing on achievement gaps may only perpetuate negative stereotypes. Yet, measurements of mathematical achievement have also shown that some teachers and school districts appear to serve their ELLs better than do others.

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For example, consider the following phenomenon that occurred in a large southwestern state in which approximately 16% of all public school students are ELLs². The Samsonville School District and the Wilkins School District (pseudonyms) both had significant percentages of ELL students, 25% and 33% of their approximately 80,000 and 50,000 respective student populations. Samsonville ELLs were approximately 85% Latino students, i.e., students from Spanish-speaking backgrounds, while Wilkins' ELLs were 99% Latinos. Sixty percent of Samsonville students were classified as economically disadvantaged compared to 95% of Wilkins students. Furthermore, less than 40% of Samsonville teachers were Latinos, while in the Wilkins district greater than 70% of teachers were Latinos. Although the characteristics of the students who were ELLs in these two school districts were largely equivalent in terms of language background and ethnicity, over several years Wilkins ELLs performed significantly higher than Samsonville ELLs on the mathematics portion of the state standardized test. To what could this differential performance be attributed?

In trying to explain the above phenomenon, one may ask the obvious question of whether the teachers in the Wilkins district do something differently than do the Samsonville teachers which results in the differential mathematics achievement among ELLs. One may also ask the question of whether there is something that the Wilkins teachers know about teaching mathematics to ELLs that capacitates them to more effectively instruct ELLs and that is less well known among Samsonville teachers. That is, one may seek to explain this phenomenon from the perspective of instructional practice or from the perspective of the teachers' knowledge that informs their practices.

There are also several reasons why it may be valuable to investigate a phenomenon such as this from the later perspective, that of teachers' knowledge. Researchers concerned with the assessment of ELLs have asserted that "with the rapid growth of ELL populations states should place a substantial focus on increasing teacher knowledge of current ELL issues...including pre-service teacher education and continuing teacher education" (Wolf, Herman, & Dietel, 2010, pp. 8–9). Hence, successfully characterizing knowledge that promotes achievement among ELLs would add to theories of mathematics teachers' knowledge. It would also fill a void of content for educator textbooks and professional development materials useful for equipping teachers for the work of teaching mathematics to ELLs (Watson et al., 2005). Ultimately and importantly, it would more fully complete the picture of essential elements that inform equitable mathematics teaching.

The research reported in this paper was done as part of a larger study that attempted to explain the difference in ELL achievement seen in the Samsonville and Wilkins school districts from the viewpoint of teacher qualities. That larger study looked at a number of important teacher and student variables in the hope of identifying the teacher qualities and instructional moves that resulted in higher mathematics achievement for ELLs. One among the many teacher variables to be measured in as a possible predictor of ELLs' achievement was teachers' mathematical knowledge for teaching, MKT (Ball, Thames, & Phelps, 2008). Yet, because of the

² This account regarding the Samsonville and Wilkins school districts arose through conversations between school district administrators and university researchers. Reporting of state standardized test results on the website of the state department of education website between 2002 and 2009 had revealed the higher performance of Wilkins ELLs on the math portion of the state standardized test. Administrators in some districts around the state and nation expressed interest in knowing what Wilkins teachers were "doing" with their ELLs. The study that resulted in part from those conversations was funded by the National Science Foundation (DRL-1055067).

special cultural and linguistic context of that study, the researchers sought an instrument that could capture teachers' MKT that informed their work with students who came to school with languages and prior mathematics learning that might vary from English and from traditional mathematical algorithms and notations used in U.S. schools. However, at the time of the study there were—and still are today—several limitations to doing an investigation of this sort, i.e., a study that can describe and measure the kind of mathematical knowledge needed by teachers of students who are still learning the language of instruction, in this case ELLs. The foremost of these is that, while there exist abundant strategies for teaching mathematics to ELLs, there is a shortage in theory about what effective math teachers of ELLs really need to know. Because of this lack of theory there is also a lack of research tools for investigating this knowledge. For example, most current instruments that measure mathematics teachers' knowledge – and the Learning Mathematics for Teaching (LMT) measures in particular (Hill & Ball, 2004) – do not address ELLs in particular, or aspects related to equity in general, and as a result, may fail to capture many aspects of knowledge for equitable teaching.

This paper narrates an initial attempt to create a research instrument capable of measuring knowledge for teaching mathematics as it is used by teachers in linguistically and culturally diverse classrooms containing large number of ELLs. The work reported here is fundamental in the sense that it was done in hopes of “laying the bearings” for observing MKT in linguistically and culturally diverse contexts and of building the capacity to study this aspect of knowledge for equitable mathematics teaching. Specifically, the goal of this study was to create a survey instrument that could be used to capture the particular kind of knowledge that mathematics teachers like those in the Wilkins district called upon when teaching ELLs. But how could that kind of mathematical knowledge be defined? To what extent could a survey be designed that measures that knowledge reliably? And further, to what extent does it even seem valid to define knowledge for teaching mathematics to ELLs as a special kind of knowledge? Is this a valid construct? Answers to these important questions would determine the usefulness of the survey to be created. Responses to questions like these are given here in the hope that the interested reader may appreciate the nuanced nature of the knowledge being addressed and in hopes that the foundational attempt described in this paper to measure that knowledge may only open the way for even more illuminating work along these lines.

To summarize, if we are to understand the differential performance that some mathematics teachers have with ELLs, like that described above, and to use this understanding to promote equitable access to mathematics instruction for all students, then there is the need to better understand the particular role that teachers' knowledge plays in the context of teaching math to ELLs. Thus, there is an initial need to first develop and test theory related to mathematics teachers' knowledge for teaching ELLs and to then develop research tools capable of observing and describing such knowledge. The ultimate goal of the study reported here was to produce a viable research instrument for measuring teachers' knowledge for teaching mathematics to ELLs and this process was guided by theory in survey and scale development (DeVellis, 2003; Dillman, Smyth, & Christian, 2009; Schuman & Presser, 1996). As a result, this work followed a somewhat linear fashion that commenced with a review of literature with the purpose of defining a theoretical construct, that is, defining what knowledge is needed by mathematics teachers of ELLs. Following this a test blueprint was outlined and original survey items were developed according to the knowledge domains defined in the blueprint. Finally responses were obtained from math teachers for the main purpose of evaluating the reliability and validity of the measure. The sections of this paper that follow elucidate the path that was

taken in trying to capture this knowledge, showing in detail how knowledge for teaching ELLs mathematics was hypothesized based upon the research literature and classroom observations conducted, how the instrument was developed and administered, and finally what was learned about this knowledge as a result of administering the survey.

Conceptualizing Mathematical Knowledge for Teaching ELLs and Developing a Measure

Consulting the Literature: Connecting MKT to ELLs

As an entry into investigating the kind of knowledge needed for equitable mathematics teaching generally, by looking at mathematics teachers of ELLs specifically, this study began by examining the intersection between two existing strands of inquiry: research concerning mathematics teachers' knowledge and research concerning ELLs in the mathematics classroom. Highlights from some of the essential theory and findings that guided this study follow.

An important component of math teachers' knowledge is profound mathematical content knowledge (Ma, 1999, for example) and this kind of deep mathematical knowledge was assumed here to be important for math teachers of ELLs as well. Furthermore, it was assumed that, like all teachers, teachers of ELLs have *knowledge-in-practice* (Schon, 1983), i.e., knowledge that is gained by their practice of teaching ELLs and from their instructional experiences related to the particular linguistic and cultural background that their ELLs bring to the classroom. Shulman (1986) connected content knowledge to this kind of practice-based knowledge by explaining how teachers' knowledge of the content that they teach is shaped by the pedagogy that they practice. To Shulman (1986), *pedagogical content knowledge* (PCK) "includes an understanding of what makes the learning of specific topics easy or difficult" as well as "the most useful forms of representation of those ideas...—in a word, the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). The above utterances concerning teachers' PCK, which have admittedly been reprinted in many a literature review because of their lucidity, are repeated here because of the role that the terms *easy*, *difficult*, and *representations* played in determining a connection between the two strands of research central to the present study and in informing survey development, as is explained in a later paragraph.

Recent research concerning mathematical knowledge for teaching, MKT, has given a robust explication of Shulman's (1986) PCK in the context of mathematics teaching (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008; Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008; Hill, Rowan, & Ball, 2005). These researchers proposed the existence of a system of related knowledge constructs, some more mathematical and some more pedagogical, that together constitute MKT. Furthermore, in their Learning Mathematics for Teaching (LMT) project, they have investigated this knowledge through the development of psychometric instruments, pen-and-paper (survey) tools, for measuring the knowledge in these domains (Hill, Schilling, & Ball, 2004).

Two subdomains of MKT, as theorized by the above researchers, were identified as particularly relevant to this study: knowledge of content and students (KCS) and knowledge of content and teaching (KCT). These researchers define KCS as "knowledge of how students think about, know, or learn this particular content" (Hill, Ball, & Schilling, 2008, p. 375). Furthermore, they define KCT as "mathematical knowledge of the design of instruction, [including] how to choose examples and representations, and how to guide student discussions toward accurate mathematical ideas" (Hill, Ball, Sleep, & Lewis, 2007). Within this model of PCK, the two

domains of KCS and KCT seemed to be of special interest to the present study because they include awareness of students' background and awareness of instructional decisions appropriate to the teaching of specific mathematics topics. The particular linguistic and cultural qualities that ELLs bring to the learning of mathematics, which qualities often vary from those found in non-ELLs, seemed to have potentially the most practical impact on these two domains of teachers' PCK. Knowledge of mathematics and of ELLs and of appropriate instructional decisions that can be made on their behalf seemed central to the kind of knowledge under investigation by this study.

Moschkovich (2002) has investigated ELLs in the mathematics classroom extensively and was among the first mathematics education researchers to begin to make a shift in the perspective taken when studying ELLs. She took a sociocultural view of ELLs as mathematics learners and observed that, historically, much research concerning ELLs had taken a deficit perspective which focuses the research lens on the obstacles, sources of *difficulty*, that ELLs encounter in mathematics classrooms. Because of this lens, such studies, she noted, had failed to observe the resources that ELLs often draw upon to make *easier* the learning and expressing of mathematics. Factors that have been observed to cause difficulty for ELLs in mathematics classes are many, including, for example: language of instruction and limited English proficiency (Cuevas, 1984), word problems and linguistic complexity (Llabre & Cuevas, 1983; Martiniello, 2009), polysemy (Lager, 2006), and whole-class, teacher-centered instruction format (Chang, 2008), to name a few. Conversely, much research since the 1990s has taken note of factors that serve ELLs as resources upon which they can draw to do and express valid mathematics—even if at times using grammatically invalid English, or even no English at all. Such factors include: gesturing (Shein, 2012), first language and bilingualism (Gutiérrez, 2002; Khisty & Morales, 2004; Moschkovich, 2002; Sorto, Mejía Colindres, & Wilson, 2014), non-linguistic mathematical representations (Martiniello, 2009), and prior and cultural knowledge (Gutiérrez, 2002; Gutstein, Lipman, Hernandez, & Reyes, 1997; Henderson & Landesman, 1995). Studies mentioned in this paragraph represent ways in which researchers have investigated the obstacles that ELLs face in mathematics classrooms and have also recently begun to perceive the strengths that many ELLs possess for learning mathematics.

Here a connection can be made between mathematics teachers' knowledge and ELLs. Moschkovich (2002) saw a dichotomy in perspectives taken by mathematics education researchers concerned with ELL issues; they took either a deficit or an affordance perspective. And as has been mentioned above, Shulman (1986) also saw a dichotomy; he saw that teachers' knowledge includes an understanding of the things that make learning *easy* or *difficult*, as mentioned earlier. These two sets of extremes—deficits versus affordances and difficult versus easy—constitute the link found in the present study between the two bodies of research concerning mathematics teachers' knowledge and ELLs. More precisely, the two divergent perspectives concerning ELLs of deficits and affordances, with their different research perspectives, were taken as potentially illuminating the very domains of PCK—KCS and KCT—central to teaching mathematics to ELLs. It seemed that experienced mathematics teachers of ELLs should potentially have pedagogical content knowledge by which they perceive both the *obstacles* (deficits) that their ELLs face and also the *resources* (affordances) that their ELLs draw upon in mathematics classes. These two divergent research perspectives (and their respective results) informed the method used in this study significantly, as will now be seen.

Developing the PCK-MELL Survey

In developing the survey in this study it was hoped that the understanding of knowledge needed by mathematics teachers of ELLs as well as the survey itself could be closely tied to current conceptions of mathematics teachers' knowledge, with a view to possibly investigating the relationship between this construct and related constructs in future studies. As a result, the theoretical basis of the survey was conceived of in connection with the theory behind MKT and the items themselves were written in a similar fashion as the multiple-choice LMT measures (Hill & Ball, 2004), although the measurement purpose and the mathematical topics and contexts were all novel. The process of survey development became a means of exploring this aspect of mathematics teachers' knowledge.

Pedagogical Content Knowledge for Teaching Mathematics to ELLs

Drawing from a review of the literature, a sample of which is given above, a hypothetical framework of pedagogical content knowledge for teaching mathematics to ELLs (PCK-MELL) was first developed (Figure 1).

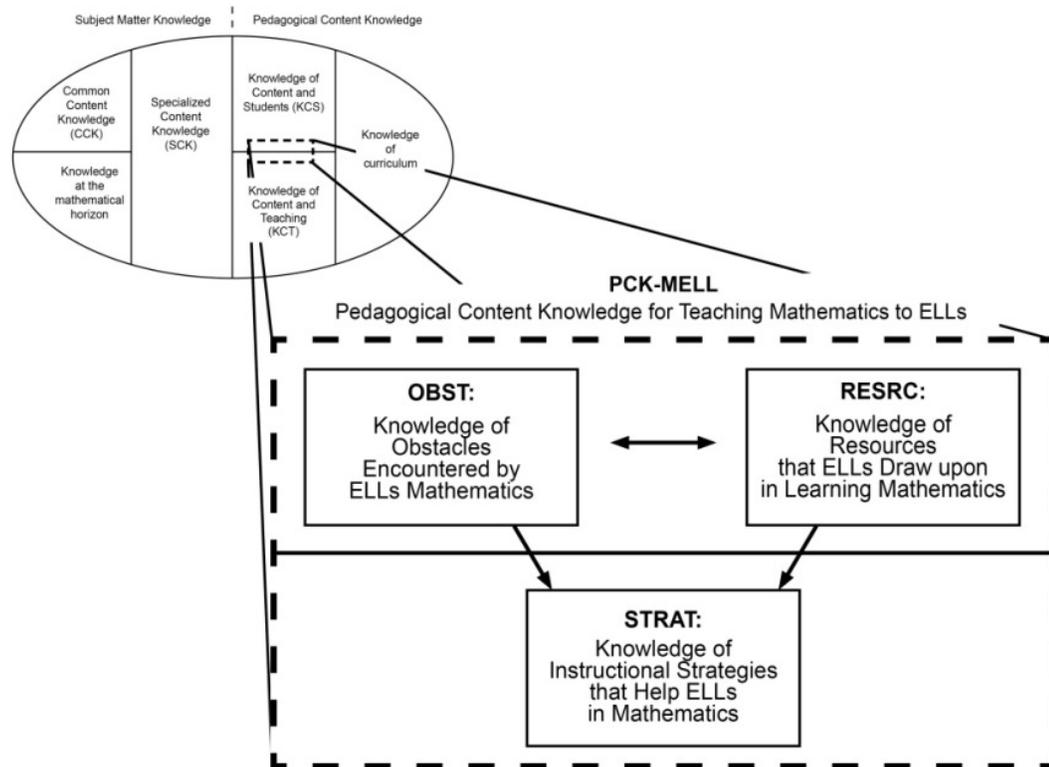


Figure 1. Hypothetical framework of PCK-MELL, proposed here as a subset of MKT (Hill, Ball, & Schilling, 2008).

The framework in Figure 1 makes several structural hypotheses regarding PCK-MELL. Firstly, it identifies the construct as a subset of pedagogical content knowledge and places it squarely within the larger framework of MKT (Hill, Ball, & Schilling, 2008). Secondly, it further embeds PCK-MELL within this framework by identifying it with two specific domains of MKT: knowledge of content and students (KCS) and knowledge of content and teaching (KCT). Thirdly, it posits three subdomains of PCK-MELL: knowledge of obstacles often encountered by

ELLs in math classes (OBST), knowledge of resources that ELLs draw upon when learning math (RESRC), and knowledge of instructional strategies for usage with ELLs in math classes (STRAT). Although not referenced in the brief summary of literature above, knowledge of instructional *strategies* was included because it seemed central to both Shulman's (1986) understanding of PCK and to Ball, Thames, & Phelps' (2008) theorization of MKT (specifically the KCT domain), and because a considerable number of strategies have been posited for teaching ELLs (see for example, Coggins, Kravin, Coates, & Carroll, 2007; Echevarría, Short, & Vogt, 2007). Finally, the arrows between the OBST and RESRC domains and between these and the STRAT domain above suggests a theoretical relationship between these three subdomains: knowledge of obstacles may be related to knowledge of resources, and knowledge in both of these subdomains may inform teachers' knowledge of, and especially selection of, strategies. (It may not be at once clear to the reader why the OBST and RESRC domains were suspected to be related; indeed, the affordance and deficit perspectives of researchers looking at ELL students seem to contradict each other significantly. This suspicion arose through reflection on the conceptual process of writing items in both of these domains and the grounds for this suspicion will be clearly explained in the presentation of the OBST survey item below.)

Furthermore, the following testing framework (Table 1) was crafted to serve as a guide to survey item development. The testing framework again delineates the three proposed knowledge domains of OBST, RESRC, and STRAT and then associates with these domains a number of specific aspects of knowledge within each domain. All of the aspects provided in this framework were derived specifically from research results. That is, in so far as it seemed possible to classify select findings from the literature as either identifying and explaining particular obstacles encountered by ELLs (such as linguistic complexity, Martiniello, 2009), or as describing resources that ELLs draw upon (such as their bilingualism, Moschkovich, 2002), or as enumerating instructional strategies for usage with ELLs (see for example, Coggins, Kravin, Coates, & Carroll, 2007), then those concepts from the literature concerning the mathematics education of ELLs were placed in the framework and hypothesized to be elements of PCK-MELL about which experienced teachers of ELLs should be familiar. At least three assumptions are being made here: first, that the research findings concerning ELLs accurately depict elements of the 'real' situation concerning many ELLs in mathematics classes, second, that the findings are generalizable in contexts beyond those in which the research occurred, and third, that teachers who have PCK-MELL will have gained, possibly through formal training but much more likely through actual teaching experience, an understanding of at least some of those very same elements described in the research literature and identified in the testing framework. In essence the reasoning behind the usage of the testing framework was as follows: if research such as Martiniello's (2009) finds that the linguistic complexity of word problems is an explanative factor in the differential (lower) performance of ELLs in mathematics, then not only is the difficulty caused by linguistic complexity something about which teachers of ELLs *should be aware*, but it even seems probable that experienced teachers of ELLs *will actually be aware* of ways in which linguistically complex word problems can cause difficulty for their students. The survey items were then written for the purpose of capturing this awareness (or lack of awareness). Assumptions like these may be dangerous. Yet, some assumptions had to be made since this effort was an initial attempt at hypothesizing about the very nature and contents of the knowledge domains in question. It was hoped that results from the survey administration would either validate, or indeed invalidate and lead to improvements of, the survey.

Two final notes regarding the testing framework are in order. First, the aspects given in the STRAT domain are merely an adaption of Chval & Chávez’s (2011) synthesis of research-based strategies for instructing ELLs in mathematics. Many specific strategies for teaching ELLs have been posited and their list seemed to be general enough that specific tools such as “word walls”, posters and manipulatives, for example, could be taken as mere instances of the broader aspects of strategic knowledge in the list. Finally and most importantly, it must be made clear that this testing framework is not now and was not believed at the time of the study to be exhaustive. There are, no doubt, many more aspects of the knowledge domains that are not represented here. The items in the testing framework are a best attempt given limited time and resources.

Table 1. *PCK-MELL Testing Framework*

DOMAIN	ASPECTS
OBST Knowledge of Obstacles encountered by ELLs in mathematics classes	<ol style="list-style-type: none"> 1. Limited English proficiency in speaking, reading and writing <hr/> 2. Word problems <ol style="list-style-type: none"> a. Specific words (vocabulary), multiplicity of words (linguistic complexity), shifts of application, polysemy <hr/> 3. Classroom format <ol style="list-style-type: none"> a. High speech formats (direct teaching versus indirect or peer-based methods) <hr/> 4. Assessments <ol style="list-style-type: none"> a. Low performance because of: <ol style="list-style-type: none"> i. Word problems, time limitation, high stakes, cultural-irrelevance
RESRC Knowledge of Resources that ELLs draw upon in learning mathematics	<ol style="list-style-type: none"> 1. Linguistic creativity, mathematics discursive and communicative ability <ol style="list-style-type: none"> a. Fluency in L₁ (i.e., first language) b. Bilingualism c. Usage of gestures, objects, and verbal <i>inventions</i> to convey meaning <hr/> 2. Linguistic and cultural identity <ol style="list-style-type: none"> a. Personal association with cultural icons, people, values, traditions, etc. b. Appreciation of first language <hr/> 3. Prior mathematical knowledge including knowledge of and fluency with alternative or “foreign” mathematical notations and algorithms
STRAT Knowledge of instructional strategies that help ELLs in mathematics	<ol style="list-style-type: none"> 1. Usage of students’ background knowledge—academic, linguistic and cultural—to promote understanding <hr/> 2. Maintenance of classroom environment rich in linguistic and mathematics content <hr/> 3. Emphasis on meanings of words and/or provisions for students’ usage of multiple modes of communication to express mathematics <hr/> 4. Usage of visual supports to—gestures, objects, illustrations—to convey the meanings of classroom conversations <hr/> 5. Connection of mathematical language with multiple forms of mathematical representation <hr/> 6. Available visual display of classroom mathematics concepts, representations and words during instruction <hr/> 7. Rich usage of students’ own mathematical writings and speech with opportunity for them to make revisions

Survey Development

To inform and augment the development of the PCK-MELL theoretical and testing frameworks, more than thirty hours of original classroom observations were also conducted in middle school (and a few high school) mathematics classrooms composed of large numbers of ELLs with a view to 1) situating the development of survey items within authentic and mathematically specific teaching instances while 2) populating the testing framework with both research-based and classroom-observed aspects of the three knowledge domains. Then, based upon the testing framework and observations, a large number of survey items were developed, from which an approximately equivalent number of items across the three domains of OBST, RESRC, and STRAT were retained for the final 32-item instrument after a pilot version had been sent for review and comment to a number of mathematics educators and experts in ELL and teacher knowledge issues at different institutions in the United States. To give a sense of the final instrument which was the basis of the statistical analysis that follows, three exemplary survey items are given here.

The PCK-MELL survey item presented below was derived from a classroom observation in which a math teacher taught a geometry lesson to 9th and 10th grade students who were recent immigrants to the United States, more than 95% of which were Latinos. Both the mathematical situation and the answer options are authentic in the sense that the teaching situation and words used actually happened in a mathematics classroom. Furthermore, the mathematical topics and teaching tasks represented in this items are typical of the work of mathematics teaching. The “correct” answer to the item, as with all items in the survey, was based upon theory found in the research literature concerning effective (or ineffective) practice for teaching math to ELLs. The item below was designed to measure teachers’ knowledge of the linguistic obstacles (OBST) that ELLs may encounter in mathematics classes.

Mrs. Tash's students, many of whom are Latino English Language Learners, have been working in groups to discover how surface area changes when the dimensions of an object are changed by a scale factor. On the previous class day, students used rulers and a rectangular prism to complete the table below.

Picture of Solid	Name of Solid	Dimensions	Scale Factor	New Dimensions	New Surface Area, SA	Conclusions
			k = 2			

Which word in the table may cause the most trouble for the English Language Learners in the classroom?

- A. Solid
- B. Dimensions
- C. Conclusion
- D. Surface

Figure 2. Sample PCK-MELL survey item, OBST domain.

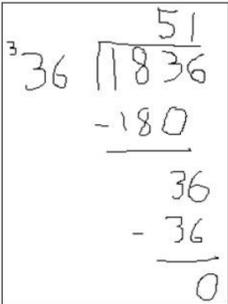
The item in Figure 2 presents several English mathematical words that could be unknown to English Language Learners. Among these four words, three of them (solid, dimensions, and

conclusion) have direct Spanish-English cognates (*sólido*, *dimensiones*, and *conclusión*) and may be more readily known to Latino ELLs as a result. Furthermore, in classrooms of Latino ELLs teachers' knowledge of the Spanish language may be helpful in answering the particular item above. To control for this effect knowledge of the Spanish language was assessed as a covariate on the survey as well. Yet it is also not difficult to imagine that even the observant, but non-Spanish-speaking teacher of ELLs may perceive *through teaching experience alone* both the English words that cause difficulty for ELLs and the words that ELLs can use to leverage their understanding. That is, non-Spanish-speaking teachers may also gain through experience an awareness of the English words that their Spanish-speaking ELLs stumble on or readily comprehend.

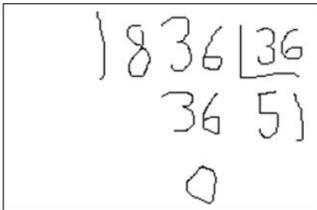
Furthermore, this item and the theory behind its hypothetically "correct" answer can be used to show why knowledge of *obstacles* encountered by ELLs in math was hypothesized to be related to knowledge of their *resources*: while language of instruction and linguistic complexity can cause difficulty for ELLs on one hand (Cuevas, 1984; Llabre & Cuevas, 1983; Martiniello, 2009), the ELLs' first language and their bilingualism can also give them access to the same mathematical ideas on the other (Gutiérrez, 2002; Khisty & Morales, 2004; Moschkovich, 2002; Sorto, Mejía Colindres, & Wilson, 2014). Teachers' knowledge of both of these functions of language in mathematics classrooms may be related. As a result, teachers' knowledge of the obstacles to and resources for ELLs' learning of mathematics may also be related.

The following figure represents a survey item in the RESRC domain.

To solve this problem, students had to divide 1836 by 36. Sara and Josué used these two different methods.



Sara's Method



Josué's Method

Who's division method is correct?

- A. Both methods are correct
- B. Josué's
- C. Sara's

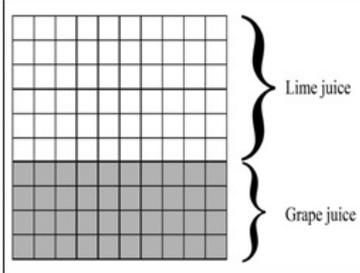
Figure 3. Sample PCK-MELL survey item, RESRC domain.

The survey item above presents to two slightly different notations of the long division algorithm and then requires the respondent to select the correct method for performing the long division. Sara's method is equivalent to the standard algorithm traditionally used and taught in schools in the United States while Josue's method is an equivalent algorithm, but that uses notation commonly found in Central America. This item was inspired by conversations with

Latino immigrants to the United States for whom this notation had been the standard for long division. The item was included in the PCK-MELL survey as a potential indicator of teachers' practical knowledge of resources that some ELLs (especially Latino immigrant students) may bring to the math classroom, namely, their prior mathematical knowledge, including valid, yet perhaps alternative, mathematical algorithms and notation.

The final item below was intended to serve as a STRAT item, measuring teachers' knowledge of effective instructional strategies for teaching ELLs.

Mr. Garza asks his class a question about the contents of the grape-limeade drink below: "How do you find out what proportion the grape juice is of the drink?"



Ciarra, an English Language Learner, responds: "Cuenta los grises y lo divides entre cien."
Which is the BEST way for Mr. Garza to respond?

- He should ask Ciarra to rephrase her answer using English
- He should acknowledge that Ciarra tried to answer his question, but then ask another student to respond in English
- He should acknowledge that Ciarra tried to answer his question, but then explain the correct answer in English so that all of the students understand
- He should ask Ciarra to give her response again, but to use the pictured image and gestures to show what she means

Figure 4. Sample PCK-MELL survey item, STRAT domain.

The survey item above, taken from a situation that occurred during the observation of a 7th grade mathematics classroom having many ELLs, involves a strategic decision on the part of the teacher: how should the teacher respond to the student who has answered the English mathematics question using the Spanish language? (And does the fact that the student actually gave a concise and mathematically valid answer affect teachers' responses to the item? Again, responses to this item may also be informed by the teachers' knowledge or ignorance of the Spanish language and this effect was controlled for in asking respondents concerning their level of knowledge of that language.) Originally, this item was intended to serve as an indicator of teachers' knowledge of mathematics instructional strategies for teaching ELLs, the STRAT domain. Yet, as the theoretically "correct" answer to this question was actually option "D", then selecting the correct also implied knowledge that ELLs' first language was a resource in mathematics classes rather than a deficit. Not surprisingly then the item was later found to correlate more strongly with other items in the RESCRC domain. Furthermore, the intersection (overlapping) of knowledge domains required by items like these hints at a particular weakness of this initial exploratory effort at defining the construct: in hindsight, the domains were not

altogether well-defined and items such as this, which may serve as indicators of knowledge in more than one domain, reflect the need for even more careful theory development and rigorous item-writing.

Evaluating the Instrument

Following the theoretical development and writing of survey items, the intention in this study was to obtain enough responses in order to be able to evaluate the extent to which the instrument developed above measured the knowledge that it indeed was designed to measure. The survey instrument described above first underwent pilot study using pre-service mathematics teachers ($N = 142$) at a major university in a southwestern U.S. state having large numbers of ELLs. Following this, responses to the final instrument were obtained via internet survey from forty-two ($N = 42$) in-service middle school mathematics teachers around the state. Thirty-one of the teachers were female, eleven male, and they ranged widely in experience from novice teachers to teachers having over twenty years of classroom experience; 40.4% of the teachers had taught for ten or more years. Teachers that took the survey were from more than twenty-three different schools drawn from more than sixteen different school districts distributed across the state. Although an initial attempt at collecting a large random sample was made, and indeed the final sample bears similar demographic characteristics to that of the teacher population of the state, the final sample of 42 teachers was altogether one of convenience as all contacts were made with teachers through the ultimate mercy of cooperative school administrators. Brief results regarding the reliability and validity of the measures obtained follows.

Reliability

As explained above, the PCK-MELL survey was built from a framework hypothesizing three underlying factors: OBST, RESRC and STRAT. Hence, the computation of Cronbach's (1951) α , a standard measure of internal consistency for surveys and which assumes the unidimensionality of the instrument, using all of the items as a single factor would in fact yield an inappropriate measure of reliability. Nevertheless, as this survey was exploratory in the sense that both the construct, the framework and the items were all novel, this alpha was computed at the outset as a starting point for further investigation of internal consistency. Cronbach's alpha, equivalent to Kuder-Richardson's formula 20 for dichotomously scored items (correct or incorrect), was .431 for the whole set of 32 items of the instrument. This alpha was taken as the first potential confirmation of multidimensionality.

To better understand the factor structure of the test, a combination of evidences from the theoretical orientation of the items, along with from Item Response Theory (IRT), factor analysis, and Cronbach's alpha were used to separate the items into different scales. First, the following two-parameter IRT model was used to compute difficulty and discrimination

coefficients for the full set of 32 items: $p(x_j = 1 | \theta, a_j, \delta_j) = \frac{e^{a_j(\theta - \delta_j)}}{1 + e^{a_j(\theta - \delta_j)}}$. In comparison with

classical test theoretic (CTT) measures, like Cronbach's alpha, which takes the observed score on the entire instrument as the unit of analysis, IRT models take the item as the unit of analysis (De Ayala, 2009). IRT is an important tool in psychometric test development. However, like Cronbach's alpha, IRT models assume unidimensionality. Hence, the initial IRT model was unstable, with about an equal number of items having positive discrimination coefficients and negative discrimination coefficients. The sign of these coefficients, positive or negative, was

then used to relegate items to one of two scales, the first concerned more with the knowledge of both obstacles and resources (OBST/RESRC) and the second concerned more with knowledge of strategies (STRAT). The numerical assignment of items (using the positive or negative sign of the discrimination coefficient) held a high degree of agreement with the theoretical orientation of the items. That is, items with positive coefficients were more closely aligned theoretically with each other than with items having negative coefficients. Furthermore, the dual classification of items achieved through the IRT coefficients was also verified by confirmatory factor analysis (Principal Component Analysis with Promax rotation) in the case of all but one of the items. Finally, eleven (11) items were excluded from this analysis entirely because of weak correlation (Pearson r) with items in either of the two scales. The resulting two scales, OBST/RESRC and STRAT, had Cronbach's alpha reliabilities of .621 and .606, respectively. These two scales were used in the following assessment of the validity of the measures.

Validity

Measurement validity of the scales was investigated using the concept of a nomological net (Cronbach & Meehl, 1955) and its implied measures of convergent (and discriminant) validity (Campbell & Fiske, 1959). The survey included several questions related to teachers' education and licensure, to teachers' experience, and to teachers' linguistic knowledge, variables hypothesized in this study to be related with knowledge for teaching ELLs. The teachers' scores on the two scales (OBST/RESRC and STRAT), computed as a simple percentage of "correct" responses, were then regressed linearly on these variables. After entering all of the teacher variables, the percentages of variance in scores on the OBST/RESRC and STRAT scales explained (R-squared) by the models given in the table below were, respectively, 38.6% and 29.5%.

Several observations can be made from Table 2. At a glance it appeared that the OBST/RESRC scale was the more 'difficult' of the two, based upon the linear regression intercept; that is, controlling for all other factors, the average score on the OBST/RESRC scale was 28.7% correct compared to 64% on the STRAT scales. IRT item characteristic and test information curves confirmed this difference in difficulty. Furthermore, Table 2 shows that, for this group of teachers, some of the education and experience variables were significant predictors of differences in scores in the two knowledge scales of the PCK-MELL survey. Teachers that had experience in teaching math classes in which more than 40% of students were ELLs scored 19.4 percentage points higher ($p < .05$) on average on the OBST/RESRC scale than did teachers without this experience. Conversely, teachers that possessed a certification to teach English as a Second Language (ESL) scored 17.1% lower on average ($p < .10$) on this scale than did teachers not having this certification. Teachers that possessed additional certifications to work in other educational settings (e.g., administration) or content areas besides mathematics scored 12.7% higher on average ($p < .10$) on the STRAT scale than did teachers who held mathematics teaching certificates alone. Likewise, teachers that had been teaching mathematics for more than 5 years scored an average of 13.9 percentage points higher ($p < .10$) on the STRAT scale than did less experienced teachers. Finally, teachers that had completed any sort of professional development concerned specifically with ELLs scored 13.7% lower on average ($p < .10$) on the STRAT scale than did teachers not having this experience.

Table 2. *Linear Regression Model of OBST/RESRC and STRAT Scale Scores on Teacher Variables.*

Measure	OBST/RESRC		STRAT	
	Coefficient	SE	Coefficient	SE
Intercept	.287*	.106	.640**	.106
Teacher Education/Licensure				
Have a Degree in Math	-.116	.080	-.054	.080
Certified to Teach Math	.053	.088	.015	.089
Certified to Teach ESL	-.171 [†]	.098	-.008	.099
Had any ELL Professional Development	.013	.067	-.137 [†]	.067
Possess Other Educational Certifications	.074	.076	.127 [†]	.076
Teacher Experience				
Taught Math for More than 5 Years	.066	.072	.139 [†]	.072
Taught More than 3 Different Types of Courses	.122	.095	.071	.096
Taught a Class with More than 40% ELLs	.194*	.072	-.004	.072
Teacher Linguistic Knowledge				
Know Spanish “Well”	-.002	.063	-.004	.063
Speak a Language Other than English or Spanish	.084	.079	-.084	.079

[†] $p < .10$; * $p < .05$; ** $p < .01$

The PCK-MELL Survey as a Measurement Instrument

PCK-MELL, the underlying hypothetical construct at the heart this survey, was proposed to be a subset of MKT (Ball, Thames, & Phelps, 2008). Furthermore, the construct was initially theorized as composite of three subdomains: knowledge of obstacles that ELLs encounter in math classes (OBST), knowledge of resources that ELLs draw upon to help them learn mathematics (RESRC), and knowledge of instructional strategies for teaching ELLs mathematics (STRAT). Based upon the limited survey data obtained in this study and upon the theoretical relationship between items in the OBST and RESRC domains, the factor structure of the survey seemed to be binary, having not three scales, but two: OBST/RESRC and STRAT.

As was seen above, the reliabilities of these two scales were low by measurement standards. Although some have argued that, for dichotomously scored items, KR-20 greater than .5 is even acceptable (McGahee, T. W., & Ball, J., 2009), alpha of .70 is often considered a lower bound on acceptable internal consistency (Nunnally & Bernstein, 1994) and considerably higher consistency would be needed for commercial usage of such a measure. This means the measurements, and interpretations thereto, obtained in this study must be taken with a degree of skepticism. Nevertheless, many of the items designed to measure knowledge of content and students (KCS), one of the larger domains to which the OBST/RESRC and STRAT domains pertain, also showed very low reliability during testing, $\alpha < .70$ (Hill, Ball, & Schilling, 2008).

Hence, this study also echoes the complexity experienced by those researchers. Precise measurement of teachers' MKT is indeed a delicate, and at times elusive, accomplishment.

A goal of this paper has been to argue that it may be valid to define knowledge for teaching mathematics to ELLs as a construct subsumed with MKT, and as a part of knowledge for equitable mathematics teaching as a subset of MKT. Although the instrument in its current form was exploratory and is not ready for commercial usage, it may offer a valuable starting point for further theory and instrument development along these lines. Furthermore, some specific directions for improvement are evident. For example, as a relatively inexpensive improvement on reliability, the Spearman-Brown prophecy formula indicates that the mere addition of fewer than ten similar items to each of the scales would elevate reliability to greater than .70 (Brown, 1910; Spearman, 1910). Writing more items is an obvious improvement. But this study also points to the need for more focused research into the specific types of, dimensions of, and relationships between both the *obstacles* and *resources* encountered by ELLs in mathematics. That is, the theory behind this strand of knowledge needs development. Qualitative studies, including careful interviews and observations of teachers in classrooms of ELLs, would seem to offer promising methods to begin to better understand the relationship between ELLs' deficits and affordances, and teachers' understanding of these.

Additionally, the linear regression models presented in Table 2 may offer limited, initial insight into how knowledge in these domains operates. For the OBST/RESRC scale, of the variables potentially related to knowledge in this domain only experience teaching larger numbers of ELLs (greater than 40%) was a significant ($p < .05$) predictor of gain in the scale score for these teachers. Yet, this variable is probably the most important one if this type of PCK is related to actual teaching experience. Similarly for the STRAT scale, experience teaching math for more than five years was a moderate ($p < .10$) predictor of gain in scale score as was possession of multiple teaching certifications. These may, respectively, be proxies for length of teaching experience and breadth of teaching experience. In this case again knowledge in the STRAT domain may be related to actual teaching experience. Finally, it is interesting to note that two of the variables, one in each of the scales, were moderate ($p < .10$) predictors of a *decrease* in scales scores. For the OBST/RESRC and STRAT scales these were, respectively, certification to teach English as a second language (ESL) and whether or not the teacher had experienced any professional development for teaching ELLs. This particular finding is alarming; one would think that both of these variables would relate to increases in knowledge in the domains being tested, not decreases. Yet, if these two variables are seen as proxies for formal *preparation* to teach ELLs, then again the knowledge called upon by this exploratory instrument may be more a matter of actual experience than of formal education. At the least, it would seem that the topics presented in the instrument of this study were different than topics that the teachers in this study had seen in their ELL professional development experiences or ESL classes.

Concluding Discussion

The PCK-MELL survey developed in this study gives an example of a way in which an aspect of knowledge for equitable mathematics teaching has been operationalized in the form of survey items yielding quantitative data and psychometric results that can lead to further theory development. Perhaps most importantly, this study has offered a way of thinking about knowledge needed by mathematics teachers of English Language Learners—and by extension, all second language learners in the mathematics classroom, including immigrant students in other countries—that builds upon and extends current research. Under the framework proposed herein,

in addition to mathematics content knowledge, teachers of ELLs need: knowledge of *obstacles* to learning that are frequently encountered by ELLs in mathematics classes, knowledge of the *resources* that ELLs bring with them to the learning of mathematics, and knowledge of instructional *strategies* for teaching ELLs. This theoretical framework takes the bold, and admittedly controversial stance, of allowing both deficit and affordance perspectives of ELLs as mathematics learners, not advancing one perspective over the other or placing them in opposition, but rather positioning these two perspectives as two *sides of the same coin*: that is, both perspectives may together offer valuable insight for mathematics teachers of ELLs who should be aware both of obstacles to learning that their students are likely to face and also of the resources within and around those very students that can effectively capacitate them to do mathematics and to communicate mathematically. While this way of thinking about learners no doubt applies to all students—regardless of their language background—the viewpoint may be of special value to the particular linguistic experiences that ELLs and second language learners in all language contexts bring to learning mathematics.

This has been an initial attempt at advancing both the theory and the tools that can lead to better understanding teacher knowledge factors that impact achievement in mathematics among ELLs. The article offers an example of an exploratory process that was used to lay the bearings of a theoretical framework of knowledge needed for teaching mathematics to ELLs and also of some steps that were made toward developing a research tool to be used to study this knowledge. This study indicates the inadequacy of current theory and tools for studying aspects of mathematics teachers' knowledge related to equitable access for diverse student populations. It is hoped that this paper may make a small contribution toward bringing more clearly into focus a research agenda from which a viable theory of knowledge for equitable mathematics teaching in the context of linguistically and culturally rich and diverse populations of mathematics students will come forth.

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