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Abstract

There are many approaches to task design (Watson & Ohtani, 2015) from a large number of local and global design heuristics. The purpose of this paper is to present how mathematical modeling cycles, a popular way of describing mathematical modeling processes, were used as a task design heuristic.

Keywords: Mathematical Modeling, Task Design

Since there are many theoretical perspectives on modeling (Sriraman, Kaiser, & Blomhoj, 2006) – and no consensus as to a definition of mathematical modeling (Cai et al., 2014) -- assessments of mathematical modeling tend to be ad hoc or based on experience rather than theoretically grounded (Frejd, 2013). Therefore, the research presented here sought to take advantage of modeling task design principles in order to generate tasks that would evoke students' mathematical modeling processes. Typically in research reports, the smallest amount of space is accorded to instrument development despite the fact that all data, results, interpretations, and conclusions are derived from the instrument and therefore dependent upon its genesis. This paper attempts to make transparent the considerations and decisions that are present in a researcher's logs but that typically are cut from the dissemination of results. I then use these ideas to reflect on the domain-specific theory of modeling cycles and the methods used to design the tasks.

Theoretical Perspectives

Situating the Task Design Project

Kieran, Doorman, and Ohtani (2015) reviewed of sets of principles for task design in mathematics education studies. The review was organized around two dimensions. The first dimension was the scope of the theory informing the research objectives (grand, intermediate, or domain-specific) and the second was whether the design could be characterized as *design as implementation* or *design as intention*. Design as implementation studies focus attention on "the process by which a designed sequence is integrated into the classroom environment and subsequently is progressively refined" (Kieran et al., 2015, p. 28). This is consistent with how design research projects are understood(Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). In contrast, design as intention studies "address the initial formulation of the design" and use well developed theoretical frames in order to provide clarity and coherence to the intention (Kieran et al., 2015, p. 28). Grand theoretical frames explain learning in general (e.g., cognitive constructivist or social constructivist theories). Intermediate-level frames can be applied across many mathematical areas and domains (e.g., theory of didactical situations). Finally, domain-specific frames specify reasoning processes (e.g., conjecturing or modeling) or content (e.g., place value, geometry).

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Using Kieren et al.'s dimensions, I can situate this task design project as *design as intention* using a *domain-specific theory*. The objective of the research program was to study mathematical thinking from within a mathematical modeling paradigm. The participants were to be undergraduate engineering students in a course on differential equations. One sub goal of this project was to create an observational rubric that could be used to systematically observe students' modeling activity as it unfolded. Thus, domain-specific theories related to mathematical modeling processes were adopted. The creation of the rubric was to be guided by microanalysis of students' work on modeling tasks within a one-on-one interview setting. This required a "bootstrapping" approach that design studies are able to handle (DiSessa, Cobb, & Disessa, 2004, p. 85). The tasks and the rubric were developed through an iterative design process grounded in the students' mathematical modeling. The development of the rubric through qualitative analysis of the students' work on the tasks is presented elsewhere (Czocher, 2016).

The tasks designed needed to satisfy a particular intention, namely, evoking students' modeling processes so that those processes could be documented systematically. The goal in designing the tasks was not to create a direct measure of students' competencies nor to teach modeling. Instead, it was an attempt to study and explain in greater detail components of mathematical modeling identified by the literature as central to the mathematical modeling process or as difficult for students. The objective was to create items that could be adapted and extended within an interview setting that would parallel the mathematical and cognitive activities predicted by mathematical modeling cycles. Then a student's responses to the tasks and to the interviewer could be used to document her thinking as she mathematically modeled. Subordinate to this objective was the desire that these items would be theoretically compatible with other instruments developed for examining mathematical and cognitive activities.

Definitions of Mathematical Models and Mathematical Modeling

For this work, I define a mathematical model as a quadruplet (S, Q, M, R) where S is a system, Q is a question relating to S, and M is a set of mathematical statements M={Σ_1, Σ_2, …, Σ_n} which can be used to answer Q (Frejd & Bergsten, 2016). Then mathematical modeling is a process of rendering a real world problem, Q, as a mathematical problem that can be answered through analysis of those mathematical statements M. The process creates a relation R mapping the objects and relationships of the situation S to the mathematical entities M (Blum & Niss, 1991).

The process of forming the relation R can be described by a mathematical modeling cycle. Modeling cycles are just one view of mathematical modeling, but they are a very popular view and underlie the descriptions of modeling found in curriculum benchmarking documents, research reports, practitioner journals and international assessments (Anhalt & Cortez, 2015; Blum & Leiß, 2007; Borromeo Ferri, 2007; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010; OECD, 2012; Zbiek & Conner, 2006). Though there are many perspectives used to study students' mathematics learning during mathematical modeling (Kaiser & Sriraman, 2006), modeling cycles allow a focus on cognition and a means for understanding how to trace individuals' thinking (Ärlebäck, 2009; Borromeo Ferri, 2006, 2007; Czocher, n.d.).

There is a consensus that modeling is a nonlinear process. A mathematical modeling cycle is one way of describing it. One such modeling cycle (MMC) is shown in Figure 1. This MMC was selected because it is compatible with the definition given above: it distinguishes between the individual's idealized representation of the problem situation S and its mathematical representation M. The *real situation* [a] occurs in the real world. Working to *understand* [1] the problem produces a *situation model* [b], a conceptual model, in the mind of the modeler. *Simplifying/structuring* [2] refers to identifying, introducing, and specifying variables and conditions. This specifies the *real model* [c] (which likely has internal and external components). Through *mathematizing* [3], the modeler represents the real model mathematically. Zbiek and Connor (2006) make explicit that the modeler must first specify conditions and assumptions of the physical situation before transforming them into properties and parameters of mathematical systems. The *mathematical model* [d] itself is an expression, in formal mathematics, of relationships among key variables. *Working mathematically* [4], or performing analysis, produces *mathematical results* [e], which can then be *interpreted* [5] in terms of the real model in order to get *real results* [f]. These results are then *validated* [6] by checking them against the situation model [b]. Lastly, the individual *exposes* or shares his model with others. In other theoretical models, this last stage is known as *communicating*

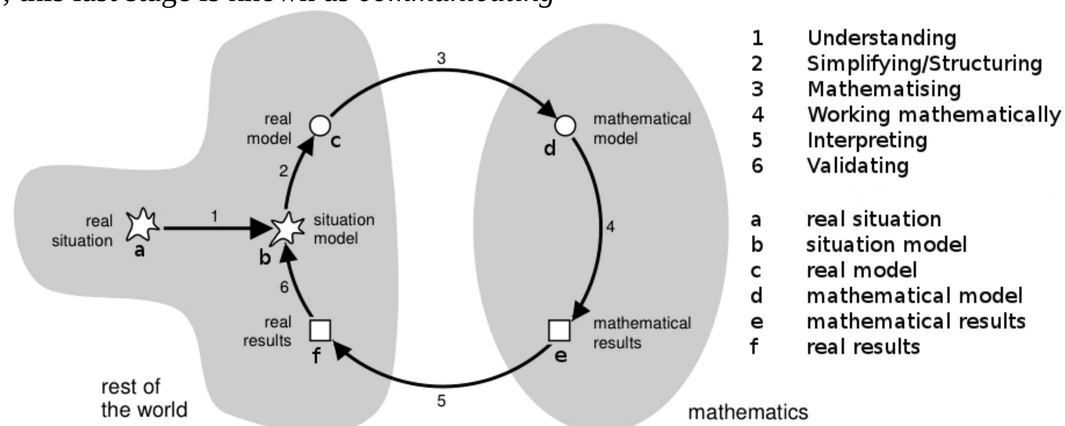

Figure 1 Mathematical modeling cycle (Blum & Leiß, 2007)

Modeling Task Design

Frejd (2013) reported on a comprehensive survey of the mathematical modeling literature to present a review of the kinds of assessments used to evaluate mathematical modeling activities. He found that written assessments tended to focus on an atomistic view of modeling whereas projects tended to be more holistic.

For example, Haines and Crouch (2001) and Haines, Crouch, and Davis (2001) wished to examine the development of students' mathematical modeling skills as a consequence of an instructional intervention. They created and administered a multiple-choice questionnaire whose items targeted individual stages and transitions of the modeling cycle in Figure 1. The atomistic approach allowed for the researchers to explore a variety of modeling competencies while avoiding “distortions from dependence upon a single model or a particular real-world problem” (p. 130). The possible answers to the questions were designed to assess the students’ competencies in discrete sub processes of the modeling cycle. The researchers developed a variety of items to assess modeling competencies of undergraduates studying differential equations (Haines & Crouch, 2005) and have found that the most challenging sub process of modeling is transitioning from the real world to the mathematical model (simplifying/structuring and mathematizing) (Crouch & Haines, 2004).

At the other end of the spectrum, Model-Eliciting Activities (MEAs) “zoom out” to study how mathematical modeling tasks lead to the development of significant mathematics learning. They are activities designed to allow the researcher (or teacher) to observe the student creating mathematical models (both formal external models and conceptual internal models) in order to learn targeted mathematics or statistics concepts. The six design principles for creating MEAs (Lesh, Hoover, Hole, Kelly, & Post, 2000) do not draw explicitly on the sub processes laid out in modeling cycles because the object is to study students' development of mathematical ideas, over many revisions, rather than their development of modeling competencies. Therefore, the tasks tend to be of wider scope and may be extended into multi-lesson classroom modules (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003).

Other large-scale modeling projects designed to teach mathematics such as the Interdisciplinary Lively Applications Projects for differential equations (ILAPs, Huber, 2010) share similar goals and require 8 – 10 hours of student work outside of the classroom. To use such problems over many interview sessions would result in bias from relying on only one or two contexts and would shift the focus from modeling processes to mathematics learning. Thus medium-grain tasks were needed for the current project.

An example of a medium grain task might be a Fermi problem. A famous Fermi problem is "How many piano tuners are there in Chicago?" They are useful, pedagogically, for clarifying assumptions and conditions that arise from making educated guesses about the circumstances of the problem (Sriraman & Knott, 2009). The modeler must make explicit how her mathematical decisions depend on real-world knowledge (Ärlebäck, 2009). Thus the modeler must make many decisions based on limited real-world knowledge and relatively simple mathematical concepts (like proportions), revealing her approach to model construction.

Design Categories

Many attempts have been made to develop design categories or classification schemes for word problems, applications problems, and modeling problems (Blum & Niss, 1991; Bock, Bracke, & Kreckler, 2015; Maaß, 2010) in order to make a distinction between word problems and modeling tasks. Elaborating on this distinction is beyond the scope of this paper. For this project, I adopted Maaß's (2010) scheme because it identified five factors that could be used to classify modeling tasks in order to aid in task design. She identified the scope of the modeling process (whole process or sub process), the amount of data provided (superfluous, inconsistent, missing, matching), the nature of the task's relationship to reality (level of authenticity or artificiality), the contextual situation (personal, occupational, public, scientific), and the type of model used (descriptive or normative). I used these categories, along with target mathematics content, to assemble a multidimensional set of tasks.

Because the focus of this research was on students' modeling processes, and specifically how they were connecting their real world and mathematical knowledge, the best approach seemed to use tasks with a medium scope. Thus, tasks were selected that targeted multiple transitions (sub arcs of the modeling cycle) as well as the whole cycle, but that could be completed within a single interview session. The task selection procedure is described in detail below.

Methods

Literature Search

I began by surveying the global research and instructional literature on mathematical modeling to compile a list of modeling tasks. Tasks were selected for further study if they seemed *a priori* capable of evoking more than just the *working mathematically* and *interpreting* transitions of the modeling cycle. The resources included mathematics education research journals, Lively problems (Arney, 1997), resources listed at CODEE (Community of Ordinary Differential Equations Educators, 2012), the biennial ICTMA publications and ZDM (2006) special issues, mathematics and engineering education research journals, differential equations textbooks (Baker, n.d.; Boyce & DiPrima, 2009), mathematical modeling textbooks designed for engineers (Dym, 2004; Edwards & Hamson, 2007) or for mathematics majors (Huber & College, 2012; Huber, 2010), and teacher resources (NCTM, 1991; Mason & Davis, 1991).

I also read the literature first to see if there were any existing instruments for microanalysis of students' modeling activity and to glean principles of task design. While there were many studies of mathematical modeling, few (e.g., Ärlebäck, 2009; Borromeo Ferri, 2007) had taken a cognitive approach to deeply understanding modeling as a dynamic process. The coordinated literature review allowed me to articulate objectives for task design: (i) the tasks should be capable of evoking (visible) modeling processes, (ii) the tasks could be used to trace students' modeling processes via microanalysis in order to develop an observational rubric, (iii) the tasks should be compatible with existing instruments (e.g., Haines & Crouch, 2004), and (iv) the tasks should be solvable by an individual.

Task Evaluation

Four rounds of evaluation were conducted to end with a total of 17 tasks from a variety of mathematical backgrounds (ranging from arithmetic to partial differential equations). I used Blum and Leiß's (2007) modeling cycle to filter the tasks from the item pool. The goal was to select a variety of tasks (mathematics and contexts) that focused on arcs of the modeling cycle. First, a panel of mathematics educators evaluated the tasks for face validity – that they would evoke particular phases of the modeling cycle – by checking them against Maaß's (2010) design categories and the modeling cycle. For example, a task was marked for the arc mathematical representation → mathematical results → real results if it was anticipated that a task would evoke the *working mathematically* and *interpreting* phases of the modeling cycle.

Next, a panel of mathematicians who were experienced in mathematical modeling and in teaching differential equations to STEM majors evaluated the tasks for content validity – that the correct mathematics was targeted. The tasks were then field tested with both mathematics educators and with engineering undergraduates. Items that did not fit the four goals were removed. Items that were unclear were reworded and field tested again. For example, #18 was a problem about continental drift that could not be solved without extensive prior knowledge of geography, geology, and scale that many of the undergraduate engineering students did not have. This task was replaced with #19 (described below).

Finally, the items were critiqued by mathematics educators in the research community at a national conference. This resulted in collapsing tasks #8 and #9 because their mathematics content overlap and modeling overlap was substantial (both tasks were about exponentials and population growth). The final 17 tasks were evaluated against the modeling cycle and the design categories.

Table 1 shows each task mapped against the anticipated stages and transitions of the modeling cycle expected to be evoked. Entry to the problem is shown in bold/underline. Problems 4, 16, 17, and 19 were intended to evoke the full modeling cycle whereas the others were intended to concentrate on only a sub arc of the modeling cycle.

Table 1 Task analysis according to stages (top) and transitions (bottom) of the modeling cycle. An underlined x indicates where the student was anticipated to enter the cycle.

Task Number	1	2	3	4	5	6	7	8	10	11	12	13	14	16	17	19
Real Situation				x̲		x̲	x̲							x̲	x̲	x̲
Situation Model	x			x		x	x	x	x̲					x	x	x
Real Model	x̲	x̲		x	x̲	x	x	x̲	x	x	x̲	x̲	x̲	x	x	x
Mathematical Model	x	x	x̲	x	x	x		x	x	x̲	x	x	x	x	x	x
Mathematical Result		x	x	x				x	x		x		x	x	x	x
Real Result				x							x		x	x	x	x
Understanding				x			x							x	x	x
Simplifying/ Structuring				x		x	x							x	x	x
Mathematising	x	x		x	x	x			x	x	x	x	x	x	x	x
Working Mathematically		x	x	x		x		x	x		x		x	x	x	x
Interpreting		x		x				x	x		x		x	x	x	x
Validating				x				x			x			x	x	X
FULL				x										x	x	x

Thus, the final tasks were capable of eliciting sub processes of modeling as defined by a mathematical modeling cycle. They were positioned between Lesh, et al.'s model-eliciting activities (which focused on macroscopic conceptual systems) and Haines and Crouch's multiple choice assessments (which focused on single stages of model construction), making them suitable for one-on-one cognitive interviews.

Sample Tasks

In this section I share some sample tasks and their analysis according to Maaß's (2010) task design categories. I also share some student work from interviews with engineering undergraduates in order to demonstrate the various modeling processes being carried out.

#7: The Wrecking Ball (Edwards & Hamson, 2007). *A wrecking ball is the most efficient way to raze a concrete frame structure. Consider the following question (but do not try to solve!): Which part of the building should be hit so the building is brought down most efficiently*?

In the Wrecking Ball problem, the real situation S consists of the heavy equipment base for the wrecking ball and the building to be razed. The question Q concerns toppling the building efficiently. In order to develop a set of mathematical statements M capable of answering the question, a number of assumptions must be made, conditions must be articulated, and variables must be identified. The problem focused on a sub arc of the modeling process, had missing data, presented a realistic situation with a scientific focus, and requested a descriptive model. In asking the students to consider the question (rather than answer it), they did not need to find M but rather begin developing the relation R connecting S, Q, and M. The task was intended to target the understanding and simplifying/structuring phases.

Below is the work of an undergraduate majoring in environmental engineering.

> **Student**: So I would assume there's, based on the structure's volume, or based on its general shape there's gonna be support structure or support beams at certain locations based on the volume. So you'd have, say you have a volume V you're gonna have main supports every X feet. You're also gonna try and strike at, not the very bottom of it but the middle part of it so that once the beam is bent, this will all collapse. I'm pretty sure there's an equation I could do from [mechanical engineering class] with just the deflection of the maximum shear strain the beam can take before it shears, but uh, we always end up doing horizontal beams with the horizontal force. And I'm pretty sure it'd end up being the same with a vertical beam.

The student began with this extended statement, drawing on knowledge from his engineering courses and particularly a course on statics to make a series of assumptions and identify important variables. As he spoke, he developed an idealized version of the problem situation, which is shown in Figure 2. The student's work shows an equation for shear, a horizontal beam with force applied (downwards arrow ¼ distance from the right edge of the beam), a vertical building wall with force applied 1/3 from the base, and a schematic of a wrecking ball with the dimensions along which the ball is constrained.

Thus the task evoked the intended transitions in the modeling cycle, understanding and simplifying/structuring and due to the student's recent study of materials and statics, he was able to recall a related formula.

Figure 2 Undergraduate engineering student's work on #7, the Wrecking Ball.

#12: Baker's Yeast. *Baker's yeast is a type of fungus that reproduces through budding. Each cell reproduces once every 30 minutes. To grow yeast for baking bread, you have to proof it first – allow it to form a colony – in a bowl of warm water. Suppose that in a particular bowl, after six hours, the surface of the water is covered with yeast cells. When was half of the surface covered?*

According to Maaß's (2010) design categories, the Baker's Yeast problem was supposed to elicit an arc of the modeling process, gave just the right amount of data, was embedded in reality (the task requires only knowledge of mathematics to solve; knowledge about yeast growth is unlikely to be helpful), dealt with a realistic or scientific situation, and requested a descriptive model. It was anticipated that students could use recursion, multiplication, or exponential growth to generate an answer to the question.

So what ensures that students would need to engage in modeling in order to answer the question? On this task there is a tendency to focus on the surface area of the bowl or on the size (number or surface area) of the initial population. The models that depend on surface area cannot be used to answer this question. Mathematically, the time at which half the surface of the water is covered in yeast depends only on the doubling time. This requires a recursive formula or an exponential growth model.

For example, one undergraduate engineering student began with a recursive model based on doubling. He obtained the series $\{1, 2, 4, 8, \ldots 2048\}$ before realizing that he needed to compare the surface area of 2048 yeast cells to the total surface area of the water in the bowl, which would vary from bowl to bowl. Another student immediately recognized the distinction between percent change and amount (magnitude) change. He determined that knowing an exact number of cells would be unnecessary.

#19: Piano Tuners. *How many piano tuners are there in the city of New York?* Task 19 was a Fermi problem, and was intended to evoke the full modeling cycle with an emphasis on *simplifying/structuring* since no information was given. The task's profile according to Maaß's (2010) framework was: a whole modeling cycle tasks with missing data, that was intentionally artificial, requiring knowledge of societal factors, and that requested a descriptive model.

Two students' written work is shown in Figure 3 and Figure 4 for comparison. They obtained two different models, one based on a ratio comparison between two large cities (New York and Chicago) and the others based on successive rates and proportions. The student who used successive rates validated their models based on whether or not a single piano tuner would be able to make a living. A third student (whose work is shown in Figure 5) approached the problem began by finding the proportion of the population able to work and then considered how many of those could be supported by piano tuning as a profession.

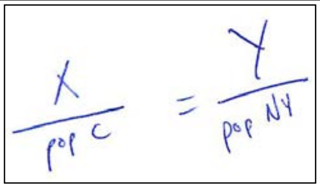

Figure 3 Proportion of piano tuners in Chicago is equal to the proportion of piano tuners in New York

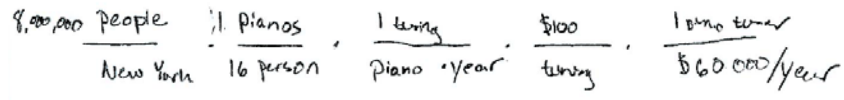

Figure 4 Successive rates: 8 million people in New York, 1 piano per 16 people, 1 tuning per piano-year, $100 per tuning, livable salary $60,000 per year

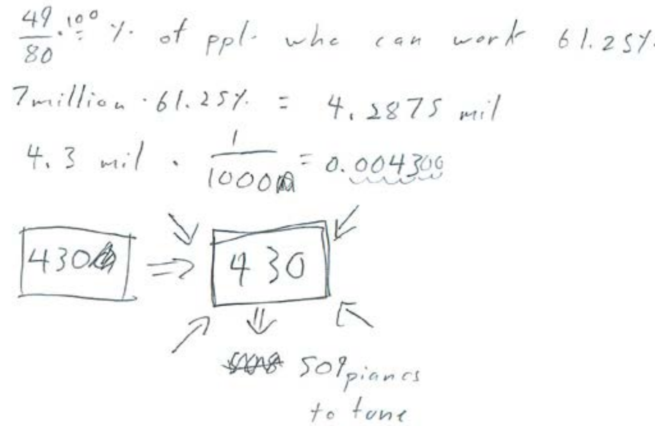

Figure 5 Successive rates: 7 million people in New York, 61.25% eligible to work, 1/1000 people getting their pianos tuned annually

Reflections

One can view the scope of a mathematical modeling task along two continuous dimensions. The first is the strength of connection between the mathematical and real worlds. The categories *word problem, application problem*, and *modeling problem* (Blum & Niss, 1991) offer a way to think about where a particular task may fall along a continuum from *could be solved without reference to the real world* to *could be solved without mathematics*. Some of the tasks designed for this set, such as the Baker's Yeast problem, take on the appearance of "real world problems" but present situations that could be modeled without reference to any real world considerations. Others, such as some of the Fermi problems, could be solved without mathematics if only the appropriate measurement could be taken. Thus, the categories along the spectrum should not be taken as discrete, mutually exclusive categories because modeling processes (as defined by Blum and Leiß (2007)) can be observed even as a student solves wholly artificial word problems that merely dress up mathematics (Czocher & Maldonado, 2015).

A second dimension is formed from considering the complexity of the task in terms of a mathematical modeling cycle. By placing single-stage tasks at one end of the spectrum and whole-cycle or multiple-cycle tasks at the other end, one can focus on individual modeling competencies or on the entire modeling processes as a skill (Maaß, 2010). One outcome of the task design project was adding the idea of eliciting sub arcs of the modeling cycle when studying modeling in order to look more closely at the processes and knowledge involved in carrying out modeling activities. The items I designed for the purpose of generating an observational rubric are positioned between the two ends in order to highlight the cognitive and mathematical activities carried out as a modeler transitions between stages in the modeling cycle.

Having a range of research tools available to study different aspects of cognitive phenomena positively impacts the field and increases the potential of making our descriptions of human processes richer. Specifically, a selection of tools and mutual compatibility among those tools allows researchers to sharpen focus on different layers of complexly interwoven aspects of mathematical thinking. By revealing the reasoning and decision-making that went into creating the specific items used in creating and validating the rubric, other researchers will be better able to judge whether it suits their needs and philosophical commitments.

Using modeling cycles along with design categories as a task design heuristic is an example of employing a domain-specific theory (Kieran et al., 2015). Task design and selection decisions should not be made in a vacuum as there is mounting evidence that a task cannot be expected to accomplish instructional goals on its own (Czocher & Maldonado, 2015; Kieran et al., 2015). How the tasks are intended to be used should be a determining factor in their design. Domain-specific theories and design heuristics need to be augmented in order to account for how the task is implemented or carried out. Reports that compare different theories or sets of design principles within mathematical domains would be useful for tracking how the principles evolve and the kinds of learning ecologies in which they are successful and not successful.

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