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Abstract: Creativity is important for young children learning mathematics. However, much literature has claimed creativity in the learning of mathematics for young children is not adequately supported by teachers in the classroom due to such reasons as teachers' poor college preparation in mathematics content knowledge, teachers' negativity toward creative students, teachers' occupational pressure, and low quality curriculum. The purpose of this grounded theory study was to generate a model that describes and explains how a particular group of early childhood teachers make sense of creativity in the learning of mathematics and how they think they can promote or fail to promote creativity in the classroom. In-depth interviews with 30 K- to Grade-3 teachers, participating in a graduate mathematics specialist certificate program in a medium-sized Midwestern city, were conducted. In contrast to previous findings, these teachers did view mathematics in young children (age 5 to 9) as requiring creativity, in ways that aligned with Sternberg and Lubart's (1995) investment theory of creativity. Teachers felt they could support creativity in student learning and knew strategies for how to promote creativity in their practices.

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Introduction

Defining Creativity in Mathematics for Adults and Children

Creativity is critical to mathematics. Professional mathematicians, for instance, create new theories and hypotheses in doing advanced mathematics, and creative advances in mathematics underlie many breakthroughs and advances in other disciplines, including the natural and social sciences. Mathematical creativity at the adult level can be defined as “(a) the ability to produce original work that significantly extends the body of knowledge, and/or (b) the ability to open avenues of new questions for other mathematicians” (Sriraman, 2005, p.23).

However, young children are in the early stages of learning mathematics, and therefore their mathematical creativity must be defined in a different way. Applying the adult definition to young children is inappropriate, but that does not mean they cannot show creativity of other kinds (Adams & Chen, 2012). According to Sriraman (2005), creativity for school learners can be defined as “(a) the process that results in unusual (novel) and/or insightful solution(s) to a given problem or analogous problems, and/or (b) the formulation of new questions and/or possibilities that allow an old problem to be regarded from a new angle requiring imagination” (p.24). This definition applies well even to very young children, aged five to nine years (in kindergarten to grade 3 of public school)--the “emerging mathematicians” who populate the classrooms of the teachers interviewed for this study. Some might assume that there is little room for creativity in early childhood math—after all, children are learning to count, add and subtract, all things that have "right" answers. That assumption is misguided; within the early childhood era, there exists the same potential for flexible and divergent thinking and versatile use of strategies that exists at later grade levels, as opposed to seeking to solve problems in the single most efficient way. Teachers should understand that creativity flourishes when they support children’s ability to generate original ideas (Bairaktarova & Evangelou, 2012; Saracho, 2012). Gallenstein (2003) notes that effective teaching models that promote critical thinking in early childhood mathematics and science are less often used in classrooms than in those for older children, but that children aged 3 to 8 are fully capable of creative construction of math and science concepts.

Sriraman’s definition of creativity does not require children to invent new mathematical theorems or prove advanced hypotheses. Rather, it points to children transcending mechanically following procedures in order to frame their own questions, see the possibilities in mathematical situations, and/or produce unusual, novel, or insightful answers or strategies. Children can draw on their own inner resources to play with mathematical ideas. Indeed, children encounter possible problems and questions with mathematical elements every day, in and out of school. In approaching these problems, children can display mathematical thinking skills and processes emphasized in various national and state standards and principles for mathematical proficiency, including standards and principles from the National Council of Teachers of Mathematics (NCTM, 2000a), mathematical practice standards from the Common Core State Standards for Mathematics (CCSS-M, CCSS Initiative, 2010), and five strands for mathematical proficiency from the National Research Council (NRC, Kilpatrick, Swafford, & Swindell, 2001). Creativity is evidently considered essential for effective learning of mathematics, no matter the age of the student.

The NCTM (2000b) advocates that students solve problems creatively and resourcefully, but NCTM publications do not give a clear definition of mathematical creativity in school children. In fact, a simple consensus definition is lacking, both for creativity in general (Chen, 2012; Lau, Hui, & Ng, 2004; Sawyer, 2003, 2006) and creativity in the specific area of mathematics (Sriraman, 2005; Sriraman & Freiman, 2007), perhaps because the phenomenon in complex and has multiple facets. Perspectives on creativity receiving most attention today include the investment theory of creativity (Sternberg & Lubart, 1999) and the systems model of creativity (Csikszentmihalyi, 1990, 1999). We have chosen the investment theory as a theoretical framework, as it is broad, addressing different aspects of and contributors to creativity. In the investment theory of creativity, Sternberg and Lubart (1995) propose that creative persons are investors who shall "buy low and sell high" for their ideas. In buying low, they "generate and promote ideas that are novel and even strange and out of fashion" (p. 2). These ideas, at first, are not accepted by others, but creative persons persist despite of the discouragement and resistance, and finally they can sell high when these ideas are recognized and appreciated (Sternberg & Lubart, 1995).

According to the investment theory, creativity requires six different "resources" in order to develop, including *intellectual abilities, thinking styles, knowledge, personality, motivation*, and *environment* (Sternberg & Lubart, 1996). *Intellectual abilities* are concerned with the ability to apply new perspectives to view things, assess ideas, promote ideas to others, and incorporate feedback. *Intellectual abilities* involve three types of skills applicable to creative thinking: (1) experiential ability (unconventional thinking and information processing in dealing with novel problems and demands); (2) componential ability (monitoring which ideas are valuable and which are not); and (3) contextual ability (promoting a fit between one's idea and the environment through communicating, taking feedback, revising, and selling one's ideas) (Sternberg & Lubart, 1995). A person must employ all three skills in problem solving to be genuinely creative (Sternberg & Lubart, 1995). A person with only experiential ability (otherwise known as synthetic ability) can produce new and original ideas, but without an inspection process, may ignore the feasibility of the ideas. A person with only componential ability (otherwise known as analytical ability) can be a critical thinker to reason and analyze, but not in a creative way. A person with only contextual ability (otherwise known as practical-contextual ability) may be able to deliver ideas to others in an inspirational and persuasive manner, not because the ideas are of good quality but because the presentation is powerful.

Thinking styles in the investment theory of creativity refer to "how one utilizes or exploits one's intelligence" (Sternberg & Lubart, 1995, p. 7). In other words, *thinking style* is not the manifestation of intelligence, but a power to direct intelligence. For example, a person who has the *intellectual ability* to solve problems with imagination and innovation may not do so, if this person does not enjoy the process of free and unconstrained thinking, but prefers standard and safe solutions.

Knowledge in investment theory refers to the formal or informal information a person can bring to a mathematical problem. Formal knowledge refers to "facts, principles, aesthetic values, opinions on an issue, or knowledge of techniques and general paradigms" (Sternberg & Lubart, 1995, p. 150). Formal knowledge can be gained from multiple sources, including textbooks and other printed materials, speeches and lectures, or other sources of direct instruction. Informal knowledge, in contrast, is "the knowledge you pick up about a discipline or a job from time spent in that arena… [It] is rarely explicitly taught and often isn't even verbalized" (Sternberg & Lubart, 1995, p. 150). Both formal and informal types of knowledge do not equate to intelligence; instead, they are the raw materials for intellectual processes.

Personality in investment theory refers to "a preferred way of interacting with the environment" (Sternberg & Lubart, 1995, p. 205). Some creative personality traits are persistence in facing obstacles, tolerance of ambiguity, willingness to take sensible risks, readiness to grow, and firm belief in oneself. These traits are relatively stable, but can also change over time and from one environment or context to another.

Motivation in investment theory refers to "the driving force or incentive that leads someone to action… [or] the nature and strength of your desire to engage in an activity" (Sternberg & Lubart, 1995, p. 236).

Intrinsic motivation is seen when it is the work itself that motives students, and is very important for creativity because it keeps one focused on the task. However, extrinsic motivation, dependent on external rewards, can also facilitate creative work. For example, synergistic motivation is an extrinsic motivation that facilitates one's creative work through the provision of information helpful for the current task.

Finally, *environment* in investment theory refers to "a setting that stimulates creative ideas, encourages them when presented, and rewards a broad range of ideas and behaviors" (Sternberg & Lubart, 1995, p. 10).

While the investment theory of creativity has the virtue of being broad and comprehensive, it is not known which, if any, of the six resources teachers consider when they interact with young children and align teaching with national standards for mathematics. Do teachers consider any or all of them? Which seem most prominent or important to them? Do teachers think it is possible to support any or all of these resources in their own early childhood classrooms?

Statement of the Problem

The National Council of Teachers of Mathematics recommends that teachers should support creativity and flexible thinking as students learn about mathematical concepts and solve problems (NCTM, 2000a). However, teachers, especially those working with young children, may not be fully prepared to design activities that promote flexible problem-solving and creativity in the learning of mathematics (Greenberg & Walsh, 2008; Shriki, 2009). For example, they may not have adequate knowledge of mathematical content or mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008).

Teachers need to understand student thinking and the creativity within it in order to intentionally promote it. This kind of teaching requires the understanding of both the subject content matter and the students' trajectories of learning, Teachers need to employ their own mathematical knowledge and draw on their own confidence to help their students acquire a strong foundation of content knowledge and skills (Baer & Garrett, 2010; NMAP, 2008). Unfortunately, not all teachers have these abilities. As a principal from an elementary school in Vermont described, "teachers coming from college today are typically taking one or two mathematics content courses. They've memorized some formulas but they don't have a conceptual understanding of how mathematics works" (Teachers College Columbia University, 2009, p. 32). Ginsburg, Lee, & Boyd (2008) claim many teachers today are not ready to teach mathematics. The poor training they receive before teaching may make them afraid of mathematics, underestimate its importance in teaching mathematics, teach mathematics poorly, or not teach at all. Deborah Ball, noted math educator, has observed that only a small number of American mathematics educators, especially in primary schools, are well educated in this subject (Ball, Thames, & Phelps, 2008).

In addition, many teachers may hold implicit or explicit negative attitudes and beliefs about mathematical creativity. Although most teachers claim they value creativity, their implicit attitudes sometimes indicate negativity toward creative students (Runco & Johnson, 1993, 2002). Westby and Dawson (1995) found that their favorite students showed fewer characteristics identified by experts as creative than did their least favorite students. Furthermore, while the expert-identified traits for creativity included nonconformity, emotionality, impulsiveness, and trying to do what others think is impossible, the teachers in the study rated these as the least creative traits. Similarly, teachers in Scott's (1999) study also showed negative attitudes toward creative students, perceiving them as disruptive. In a study of prospective teachers, Beghetto (2007) found t they already believed that students' unique and novel responses in classroom discussions were potential distractions; this opinion was especially prevalent among preservice teachers in math education.

Such negative attitudes could be related to teachers' underlying conceptualization of the nature of mathematics. Zoltan P. Dienes, a distinguished mathematician who has powerfully influenced mathematics education, has claimed, “the real problem occurs when one doesn’t understand what mathematics is about in the first place and then tries to teach it” (in Sriraman & Lesh, 2007, p. 62). Mathematics is more than a set of rules or procedures, as many seem to think; rather, it is a way of thinking about how structures relate to one another (Sriraman & Lesh, 2007). Lockhart (2002/2008) described mathematics in “A Mathematician’s Lament” that the right way of teaching mathematics to students is:

> By choosing engaging and natural problems suitable to their tastes, personalities, and level of experience. By giving them time to make discoveries and formulate conjectures. By helping them to refine their arguments and creating an atmosphere of healthy and vibrant mathematical criticism. By being flexible and open to sudden changes in direction to which their curiosity may lead. (p. 10).

Unfortunately, many teachers do not know enough mathematics or quality math education to teach adventurously (Conference Board of the Mathematical Sciences, 2012). They do not have the expertise to provide more than just data transmission or passive reception of information without joyful creation of new ideas (Lockhart, 2002/2008, p. 11). Such limitations could be residues of their own previous schooling, since teachers tend to reproduce the kind of teaching they themselves received (Pehkonen, 1997).

Moreover, many teachers today are under occupational pressures that make them feel compelled to adopt less flexible and individualized styles of teaching (Besancon & Lubart, 2008; Depiper, 2014; Parks & Bridges-Rhoads, 2012). For one thing, acute schedule pressures make planning time a precious commodity. Elementary school teachers in the U.S. are required to teach all subjects, and they spend significantly more time teaching than do teachers in other countries, for example, in many Asian countries (Niu & Zhou, 2010). Furthermore, pressures of accountability incline teachers to follow the curriculum closely when teaching mathematics (Baer, 1999; Baer & Garrett, 2010; Beghetto & Plucker, 2006). They prioritize traditional structured curriculum and test scores (Plucker & Dow, 2010). Indeed, mathematics has long been a subject associated with textbooks and standardized curriculum (Remillard, 2005). Creativity in school may be considered relevant only to subjects like arts and literacy (Diakidoy & Phtiaka, 2002). Sosniak and Stodolsky’s (1993) studied a group of elementary school teachers who tried to enrich the textbook for literacy, yet choose to rigidly follow the mathematics curriculum. Such findings could be related to implicit beliefs about creativity or to teachers’ perceptions of mathematics and comfort levels in teaching it (Remillard, 2005). In addition, the typical mathematics curricula long used in schools are believed to be shallow and lacking in coherence (Drake, 2009). Such a curriculum is “a mile wide and an inch deep” and is filled with ill-connected knowledge and information not supportive for deep learning (Teachers College Columbia University, 2009, p. 31). These kind of curricula are neither conducive to helping children obtain a conceptual rather than procedural understanding of key concepts (Teachers College Columbia University, 2009), nor supportive of teachers encouraging creativity in the learning of mathematics.

Purpose of the Study

The purpose of this study is to explore how a sample of K-3 elementary school teachers interpret creativity in the learning of mathematics for young children and how they promote or fail to promote creativity. Three questions are explored: (1) how do teachers perceive and define different aspects of mathematical creativity; (2) what strategies teachers say they use in promoting mathematical creativity in young children; and (3) whether and if so, how, does what teachers say about mathematical creativity connect to Sternberg and Lubart’s (1996) investment theory of creativity and to the recommendations embedded in mathematical national standards and guidelines?

Methodology

This study adopted Charmaz's (2006) grounded theory methodology approach. Teachers from a public school system in the Midwest were interviewed with semi-structured questions. Coding and analyzing data were conducted simultaneously with data collection. Constant comparison and contrast of the data were conducted throughout the study. A theory was then generated to explain teachers' views and practices in regard to creativity in the learning of mathematics.

Participants

Thirty K-3 teachers from a public school system in the Midwest were interviewed. Closely matching the overall demographics of the K-3 teaching population in the area, all participants were female. All of them were interviewed in their own classroom after school except for two teachers who chose other locations. Each of the interviews lasted 30 to 45 minutes.

All teacher participants attended a professional development program called Primarily Math (PM) focused on the teaching of mathematics from kindergarten through grade three (Fleharty, & Pope-Edwards, 2013; Kutaka, Ren, Smith, Beattie, Edwards, Green, Chernyayskiy, Stroup, Heaton, & Lewis, 2016; Kutaka, Smith, Albano, Edwards, Ren, Beattie, Lewis, Heaton, & Stroup, 2016; Smith, Lewis, & Heaton, 2013). In the Discussion, we analyze how this program may have influenced the teachers' understanding of mathematical creativity.

PM is a six-course (18 credit hour), 13-month graduate program for current elementary school teachers leading to a K-3 mathematics specialist certificate. This certificate program involves three mathematics courses and three courses on math pedagogy and child development. The three mathematics courses focus on concepts of number and operations, geometry, and algebraic thinking, to develop teachers' "mathematical habits of mind" (HOMs). Mathematical HOMs refer to "curiosity, imagination, inventiveness, risk-taking, creativity, and persistence… [viewing] mathematics as sensible, useful and worthwhile and… themselves as capable of thinking mathematically… [and appreciating] the beauty and creativity that is at the heart of mathematics" (Clements et al., 2004, p. 57). The three courses on pedagogy and child development help teachers become increasingly "intentional, planful, observant, and reflective" practitioners, as stated in the course syllabi. Teachers learn about the learning trajectories of young children in mathematics, read empirical studies of effective pedagogy and teaching diverse learners, and plan, implement, and reflect on math lesson plans. Teachers situate their individual lesson planning within the mathematical ideas of the curriculum with attention to creating coherence and connections to the learning trajectories. They try out specific strategies to facilitate student learning in mathematics, for example, learning how to enhance productive math tasks, create thought-provoking questions and explanations, raise cognitive demand, increase children's variety of representations, and partner with families. However, this PM program does not specifically focus on promoting creativity in young children's learning of mathematics.

In this study, the primary data were collected from interviewing, which is "a flexible, emergent technique; ideas and issues emerge during the interview and interviewers can immediately pursue these leads" (Charmaz, 2006, p. 29). The focus of the study was on interpreting teachers' feelings and views to collect rich, in-depth data for purpose of theory-development. The interviews were audio taped and transcribed verbatim by the first author.

Data Analysis and Verification Procedures

Coding took place after each interview to inform interviews that followed. For example, the interview with the first teacher was conducted, and then coded, before the second interview was conducted. After the second interview was transcribed and coded, the two sets of codes from the first and second interviews were compared, and commonalities and differences were identified. The intention was to discover areas of potential interest in earlier interviews that might be expanded upon in later interviews. Such comparison was continued throughout the study to respond to the emergence of ideas of particular interest, such as teachers' different definitions of flexibility in mathematics. These ideas were explored in subsequent interviews. Teachers were always contacted after their interview to read their transcript and perhaps verify some of their ideas or provide more details for ideas. The process of constant comparison and frequent revisiting and reflection continued until theoretical saturation was reached. MAXQDA (11.0) was used for coding and analysis.

Following the approach of Charmaz (2006), the entire first round of coding was a process of open coding, during which the coding stayed close to the data, was kept simple and precise, took place at a relatively quick speed, and involved constant comparison of codes. After the round of open coding, a second round of focused coding took place; this was conducted by sifting through the large amount of codes across interviews and determining their frequency and significance as indications of their adequacy (Charmaz, 2006). The codes resulting from this round were more conceptual and selective. Potential categories and subcategories emerged. Then, these categories and subcategories were compared to determine their relationship through theoretical coding, with bonds between categories and subcategories being reestablished. In this theoretical coding, larger categories were created by elaborating and specifying a superordinate category and merging relevant categories or subcategories under it. Thus, the entire process leads to different pieces coming together to form a coherent summary story of the participants' responses.

The first author conducted the coding, sharing results with the second author and selected peer colleagues at several points. During coding, the first author also kept a series of memos. These were used to keep track of words, phrases, and sentences used by the participants. The memos also included visual notes, such as circles, squares, and arrows that helped to map possible the relationships among codes, categories, and themes. By jotting down quick notes and insights, the first author was able to examine and verify if her first impressions were confirmed by later data as the coding and analysis proceeded.

In the study, member checking and peer review were used for verification. To conduct the member check, the data analysis was shared with all the teacher participants. The goal was to ensure that the interpretation was as accurate, adequate, and true to their thoughts as possible. It also allowed both the teacher participants and the authors to revisit and reflect together on their reasoning and interpretations. In a further peer review, the analysis was shown for feedback to professors, graduate students, and scholars whose specialties were in early childhood education, developmental psychology, mathematics education, teacher education or related areas. Further details can be found in Shen (2014).

Findings

Central Phenomenon: A Six-Aspect Model

Based on the themes emergent from the teacher interviews, mathematical creativity in teaching and learning for young children can be described in six different ways: 1) *multiple approaches to solving or representing mathematical thinking; 2) creative processes of formulating a solution; 3) a resource rich* *environment: tools, materials, and curriculum; 4) conceptual understanding of knowledge; 5) motivation for persistence;* and *6) accepting risk and error.*

Multiple approaches to solving or representing mathematical thinking. In mathematics, a problem and its solution(s) rarely exist in a one-to-one relationship. Rather, alternative ways can be found for approaching and/or representing the problem and its solution. For example, there are many ways for even young children to think about or compute elapsed time, including using a "teaching clock" or drawing a representation of a clock face and hands, or using a number line. Teachers, therefore, should recognize when their students use alternative, divergent, individualized strategies, even when these lead to a partial or incorrect solution, as opposed to one standardized method or formula.

All of the 30 teacher participants (100%) stated their appreciation for student use of multiple strategies for thinking about and representing mathematical problems. Many of them (77%) put an emphasis on the quantity of strategies students might generate. These teachers found mathematical creativity in students' multiple ways of solving a problem, and valued students' abilities to think of a variety of strategies or to approach a problem from various perspectives. For example, Ms. Smith (note: each teacher is given a pseudonym), a kindergarten teacher, employed an anecdote to describe her recognition for the number of strategies in defining mathematical creativity, "I have a girl who always solves problems in different ways. I just mean she's not lazy, she just likes to keep going and doing things, and so, she'll keep herself going."

Ms. Taylor, a 1st grade teacher, said:

> I would say in terms of math, what I see creativity from students would be, being able to represent or show things in a variety of ways... [S]ome students were able to show it in multiple ways, so I felt like they kind of took that creativity or that freedom to express it in multiple ways.

Beyond describing sheer frequency of solutions, a number of teachers (70%) spoke about students' production of ideas that were different from others. These teachers respected individual exploration in the learning of mathematics. They allowed and encouraged differences and diversity among students. These teachers believed student learners should take ownership or show individuality and independence in learning in order to develop mathematical creativity. For example, Ms. Hult, a kindergarten and preschool teacher, said, "We want them to be independent… and we want to teach independent thinking. We want to teach self-gratification. And I think that's hard to feel good about yourself and feel confident when everybody else is doing the same thing." Another teacher, Ms. Williams, a 1st grade teacher, stated, "I would say that children need to be able to explore their own methods of problem solving." A third teacher, Ms. Clinton, a kindergarten and 1st grade teacher, described the importance of personal exploration with the mention of open-mindedness:

> And one other thing I do encourage is: you can do it in whatever way you want, you can do it with an equation, you can do it with a picture, and so this student just chose to do all of these ways on their own…They don't feel like 'okay, this is the only way that I can solve the problem,' They're really thinking about the problem and then, you know what makes sense to [them].

A few teachers (43%) even spoke explicitly about students producing unconventional strategies or methods of representations. These teachers recognized the value of fluency in the sense of being able to generate multiple and unusual ways to think about and solve mathematics problems. For example, Ms. Miller, a 2nd grade teacher, described atypical thinking through this anecdote:

> I have a little boy named Anthony. He is very creative. He often thinks out of the box, like, he's the one we can often call on to show us... He'll do it in a different way than the other kids do. He doesn't go straight to what you would see the answer should be. He would take different approaches to solve it. So

> we often ask him to share. And he always kind of does unique ways of doing things. He sees it differently, you know… And of course we give them models of doing it, but how he'll decompose a number, or how he'll... he's just a little bit different.

In sum, all thirty teachers spoke about mathematical creativity as involving multiple approaches to thinking or representing mathematics. Making a basic observation, most of these teachers spoke about the *quantity* of approaches or perspectives students produced, while perhaps in a more sophisticated way, some teachers described the *quality* of thinking shown in alternative solutions—their originality, independence, or unconventionality.

***Creative processes in formulating a solution*.** Teachers also pointed to creativity inherent in the processes involved in going from deciding upon a solution to communicating it. In these processes, students go through three different steps in problem solving: first, students formulate, try out, and select a solution; second, students check and adjust the solution; and third, students express, refine, and communicate the solution

One or more of the three steps were described with reference to creativity in the interviews of 28 of the 30 teacher participants (93%). For example, there were 19 teachers (63%) whose focus was on the initial development of the strategy. These teachers recognized the importance of retrieving previous knowledge. For example, Ms. Cook, a 1st grade teacher stated, "I guess maybe applying knowledge that they have and using it in a new way to solve a problem...so [students] take what they know and then expand on that." In addition, 18 teachers (60%) focused on comparing and adjusting strategies. These teachers described the importance of choosing and checking strategies. For example, Ms. Durbin, a 1st grade teacher, said:

> [Mathematical creativity is] not only solving in many different ways but as a way kind of checking yourself… if you solve an equation, and being able to solve it with a picture or a diagram, or a math knot, or something like that… so having that flexibility and being able to see when all these different ways add to the one and then make that choice of which one is better for you… which one works for you.

Finally, 18 teachers (60%) focused on expressing and communicating ideas. For example, Ms. Miller, a 3rd grade teacher, said, "[Mathematical creativity is] students expressing their math processes, in ways that make sense to them, with accuracy and precision." These teachers emphasized the ability to explain the thinking process.

***Resource rich environment: Tools, materials, and curriculum*.** The teachers also spoke about how the physical context influences and interacts with students in the learning of mathematics with creativity. Twenty-six teachers (87%) emphasized the role of the environment in mathematical creativity. They talked mostly about curriculum content and structure and about the manipulatives and other learning tools available in the classroom.

For example, Ms. Smith, a kindergarten teacher, commented that her school's new curriculum supported students to achieve deep understanding:

> Since we adopted the *Math Expressions* program a couple of years ago, I think that program has helped us see the importance of that deep understanding, um, and in our old curriculum, we kind of moved very quickly through concepts, and if it seemed like they [students] had it, we moved on to the next thing. And we introduced many new things very quickly.

Ms. Anderson, a primary school instructional coach, described that tools and materials facilitated students in thinking about complex problems. She said:

> If they are good at drawing, especially in drawing pictures, or even using some manipulatives or something to build what it is they're thinking about, they can figure out complex problems, just with

> those materials. Even though it's not been taught to them yet, they worked it [the problem] out. We have to allow them to have those opportunities.

These teachers valued the resources from the environment that students can utilize to nurture creativity.

Conceptual understanding. A conceptual as opposed to purely procedural understanding of mathematics (Rittle-Johnson, Siegler, & Alibali, 2001; Skemp, 1976) requires the ability to retrieve, connect, and expand on what one is learning and has already been taught. This ability or predisposition leads to deeper comprehension as well as better long term retention of mathematical vocabulary and ideas. It provides a cement that holds together the pieces of knowledge the student is acquiring. The importance of deep or conceptual understanding of knowledge was supported by 17 teacher participants (57%) when they described their definition of mathematical creativity. For example, Ms. Brook, a kindergarten teacher, said, “I think creativity in math is probably the exploration of concepts...going deeper into those concepts, [and] really getting the meaning behind the concepts.” Some teachers contrasted conceptual understanding with quickness or readiness of responding; they respected deeper understanding more than command of procedures. For example, Ms. Stone, a 1st grade teacher, stated:

> It’s more about students making sense of the math, more so than trying to find the correct answer or procedure that fits the mode…the students are trying to analyze, generalize, make connections, and show relationships among the concepts that they're learning, versus just trying to get that answer, or just trying to figure what is it that the teacher is teaching or what correct response that I need to put on my paper.

These teachers were able to recognize students’ thinking processes of making sense of and creating connections among concepts.

Motivation for persistence. Some teachers recognized creativity is manifested in the motivation or drive students show in working on a mathematical question or problem even when students are experiencing a hard time. Some students are persistent in struggling with challenging problems, showing unwillingness to give up and wanting to push through to figure problems out even after other students have moved on. There were 16 teachers (53%) who supported the importance of being motivated to persevere as part of mathematical creativity. Some of the teachers emphasized the inner states involved in this motivation, such as the positive emotion associated with doing mathematics. For example, Ms. Dior, a 1st grade teacher, stated, “If you’re creative, you enjoy… looking at it in different ways [that] makes you happy.” Ms. King, a 2nd grade teacher, in contrast, spoke of the motivation that derives from response from sharing with others. She talked about sharing a unique strategy in public was attractive to students:

> The kids always want to share, and if someone has already shared a way, they want to share a different way. So they’ll solve a problem, and show in a few different ways on that white board, and then they are hoping that they’ll get chance to show one of their ways.

The teachers recognized different motivators in students’ creative endeavors.

Acceptance of risk and error. The final theme to emerge was teachers’ recognition of accepting risk and error as a necessary part of mathematical creativity. This idea was expressed by 9 teachers (30%) who spoke about students learning to accept, even welcome mistakes and false conjectures, as a normal and expectable part of the process of seeking to understand concepts and generate correct solutions. For example, Ms. Edwards, a previous kindergarten teacher and now teaching special education, said, “I’ve seen kindergarteners be very adventurous about numbers. They are not concerned if they make a mistake. They try anything, [and] they want to share what they’re thinking.” Ms. Monroe, a 2nd grade teacher, said, “…they are not afraid of making a mistake. The kids who are really afraid of making mistakes are very dampened... they can’t be creative very well because they’re so afraid. They cannot try anything.”

These teachers argued students should not be afraid of taking risks and making mistakes, but instead be willing to grow and learn from mistakes. For example, Ms. Stuart, a 2nd grade special education teacher, said:

> And I think these kids... they are okay with making a mistake. And they are open to looking at their strategy and say what went well and what didn't go so well. They’re willing to take [risks]… and let peers teach them. They’re willing to let me teach them.

It was implied by the teachers that what really mattered for students was the process of trying, with less concern about the final answer. After all, even adult mathematicians’ conjectures are sometimes proved wrong. Making mistakes sometimes are important stepping stones to successful creative solutions. For example, Ms. Stuart stated, “They are more willing to risk ....they are all about the work and not necessarily the answer. And I think that’s where you show real growth...it’s kind of enjoying the journey, you know, until when you get there.” Such teachers were most interested to see their students engaged in the process of trying and exploring the unknown.

Strategies to Promote Mathematical Creativity

Besides being asked to define and describe what they saw as mathematical creativity in their young students, the teachers were also asked how about their strategies to promote mathematical creativity. Based on the teacher interviews, teachers’ strategies fell into five themes: 1) *providing a nurturing environment*; 2) *enriching the curriculum;* 3) *giving students responsibility*; 4) *providing cognitive scaffolding*; and 5) *providing reinforcement and encouragement*.

Providing a nurturing environment. Teachers provide a nurturing environment for creativity by establishing a safe, supportive, and stimulating context for mathematical teaching and learning. This was mentioned by all 30 teacher participants (100%). Teachers’ responses fell into several categories: 1) helping create an emotionally safe social environment for students to speak up, take risks and be okay with making mistakes; 2) actually introducing the making of mistakes as a routine part of the daily learning of mathematics; 3) encouraging free exploration and trying out of all kinds of approaches to problem solving; 4) providing physical tools, materials, time and space, and digital technology to stimulate student thinking; and 5) working with parents to extend mathematical creativity into home environments. For example,

Ms. Turner, a kindergarten and 1st grade teacher spoke about the importance of culturing a safe environment for risk-taking:

> I think the environment has a huge effect [on whether] they [young children] can feel safe to take a gamble, even if they are unsure. I think it all starts from day one, you know, setting that environment. That’s the biggest thing. If you don’t have your environment where you kids feel safe and they can trust each other and they can trust you, and [they don’t] feel stupid, they’re never going to take a risk.

Enriching the curriculum. *Enriching curriculum* refers to a set of teaching practices whereby the teacher breathes increased life into the math curriculum in order to promote creative thinking and learning. The teacher may redesign lessons provided in the curriculum, add components to the lessons and/or create new learning experiences or encounters to expand the student’s possibilities for acquiring more and deeper mathematical knowledge, skills, and understanding. Such strategies were described by 26 out of 30 teacher participants (87%).

Teachers’ responses fell into several categories: 1) finding time and stretching it for mathematics activities; 2) purposefully selecting new content to enrich a lesson; 3) seeking out and providing open-ended challenging problems; 4) demonstrating different ways of solving a problem; 5), encouraging and making time for students to talk over their various rationales and strategies with peers or explain them to the rest of the class; and 6) exposing students to variations emerged in students' works through verbal, visual, and other modalities. In this theme, teachers' focus was on finding and exploiting any flexibility available in implementing the curriculum, with the purpose of opening students' minds and helping them achieve deeper understanding and better learning outcomes. For example, Ms. Hill, a kindergarten teacher, described:

> I love my curriculum…I really see wonderful things happen with my students, but I'll extend onto what they have, and that's how I found a way to put myself in my students' needs into learning. I'll do the activities, I'll do what's asked, but I may add on some more questioning. I may let them struggle with that longer than the book tells me to. If they say 15 minutes, we might work 20 minutes on it.

Teachers were creating all kinds of occasions to customize the curriculum in order to enrich students' learning experience.

***Giving students responsibility*.** Many teachers spoke of the need to "take one step back" and adopt more of a secondary role to let students take more responsibility in learning mathematics. This was mentioned by 22 teachers (73%). Teachers' responses fell into several categories: 1) releasing control to students so that they could take more ownership in learning mathematics creatively; 2) listening carefully and fully to students talking about their exploration in mathematics, instead of dominating the entire time of instruction; and 3) allowing students to make connections to real life to make personal sense of mathematics concepts. For example, Ms. Hill, kindergarten teacher, said, "We started the year off that way, with less of me giving strategy but more of them coming up with their own strategy. And I think that really makes a huge difference with the students." By taking one step back, teachers gave students more space to exercise creativity.

***Providing cognitive scaffolding*.** Cognitive scaffolding refers to a set of teacher practices intended to support a deeper level of student learning, by providing support during the learning process tailored to the needs of the individual students. Of the 30 teachers, 21 (70%) spoke of processes related to scaffolding learning as part of promoting mathematical creativity. Teachers' responses fell into several categories: 1) modeling creative problem solving, such as using multiple strategies; 2) facilitating students to solve mathematics problems creatively, such as by asking questions, checking in student understanding, giving hints and showing examples, and instructing and re-teaching; 3) adjusting level of complexity to match student capability; and 4) analyzing mistakes with students to help them achieve success.

Ms. Thompson, a kindergarten teacher, described how questions and hints could be cognitively helpful for young children:

> [Teachers should] help push them [young children] to keep working through it. Don't let them give up. Don't do it for them. But just continue to find a way for them to keep working. So if they're stuck in something, kind of give them a little push forward that would help them move in another direction. I just think about when we solve a math problem, some kids are having a hard time with understanding what the story is asking, I'm just kind of [asking questions for them to] step back and think about it again, 'what are we asking? How would you see it? Put it in your own words.' [So I] help them make more sense.

Ms. King, a 2nd grade teacher, stated:

> I think some kids need more scaffolding and they need [the teacher] asking them questions, I guess. So if they're totally stuck, you know, you can ask them a question to help them [or] probe them along to trying something, or to think about something in a different way. So questioning is really important.

In this theme, teachers' focus is on facilitating students' cognition to bring them up to the next level.

***Providing reinforcement and encouragement*.** Finally, in speaking of strategies they used to support creativity, teachers discussed reinforcement and encouragement as ways they used to motivate students to learn mathematics creatively. This was discussed by 19 teachers (63%). Teachers' responses fell into several categories: 1) praising or showing approval and appreciation to students when they showed creativity in learning mathematics; 2) encouraging students when they struggle with mathematics and inviting them to keep thinking and trying; and 3) encouraging students to try different possibilities and use an open mind in their work. For example, Ms. Campbell, a kindergarten and 1st grade teacher, described that teachers should make students aware that exploring various strategies are recognized and encouraged:

> [Teachers should] give them [young children] the confidence to try different ways… One of the things we work on is just helping kids build confidence and knowing that they don't always have to have the right answer… As long as they're trying different ways, and that's what we're looking for.

And Ms. Monroe, 2nd grade teacher, stated:

> They need a lot of praise for taking small steps. They need to be recognized in front of the class when they do something correctly, or when they just put forth effort. So they may not reach the correct answer of the problem, but you can still find a way to praise that effort they've made in their steps they took. When you continue to do that with children, they start to grow confidence in math, whether or not...even if they're not really [getting the correct answer] you know.

These teachers were aware of the power of positive comments on students' creativity in doing mathematics.

Table 1.

Numbers of Teachers Reporting Strategies to Promote Different Aspects of Mathematical Creativity.

Strategies / Aspect of Mathematical Creativity	Providing a nurturing environment (30)	Enriching the curriculum (26)	Giving students responsibility (22)	Providing cognitive scaffolding (21)	Providing reinforcement and encouragement (19)	All Strategies
1. Multiple approaches to solving or representing mathematical thinking (30)	23	22	17	8	14	30
2. Creative processes of formulating a solution (28)	13	12	6	8	3	23
3. Resource rich environment: tools, materials, and curriculum (26)	30	8	2	4	1	30

4. Conceptual understanding of knowledge (17)	9	20	3	15	2	26
5. Motivation for persistence (16)	5	4	1	1	5	8
6. Accepting risk and error (9)	7	3	3	3	2	10

In sum, the thirty teachers felt they could use five different types of classroom strategies to support their students' mathematical creativity. Connecting back to the six aspects of math creativity they described, all five strategies were believed to be involved with the promotion of all six aspects of creative math thinking (see Table 1). The strategy of *providing a nurturing environment* was especially important for the first three aspects: *multiple approaches to solving or representing mathematical thinking*, *creative processes of formulating a solution*, and *resource rich environment: tools, materials, and curriculum*. The strategy of *enriching the curriculum* was especially important for *multiple approaches to solving or representing mathematical thinking*, *creative processes of formulating a solution*, and *conceptual understanding of knowledge.* The strategy of *cognitive scaffolding* was most important to *conceptual understanding.* The strategies of *providing reinforcement* and *giving students responsibility* were most important for *multiple approaches to solving or representing mathematical thinking*.

Looked at from the reverse perspective, supporting students in *multiple approaches to solving or representing mathematical thinking* can be supported by all five teaching strategies (with *providing cognitive scaffolding* least important). *Creative processes of formulating a solution* are most supported by *providing a nurturing environment* and *enriching the curriculum*. A *resource rich environment* is an environmental matter, supported by an overall nurturing environment. *Conceptual understanding* most depends on an enriched environment and cognitive scaffolding. *Motivation for persistence* is mostly about providing reinforcement and nurturing environment, while help students to be *accepting risk and error* also arises from the nurturing environment.

Discussion

The first point of discussion is the striking similarity of the mathematical creativity themes emergent from our analysis with Sternberg and Lubart's (1995, 1996) investment theory of creativity, though the teachers used their own words and images for description and omitted or placed different emphases on specific features. .

For example, the theme, *multiple approaches to solving or representing mathematical thinking*, resembles in a simplified way the concept of thinking style in the investment theory of creativity. In that theory, thinking style, in terms of supporting creativity, refers to the flexibility to view things from multiple perspectives. Thinking style is a choice regarding how to apply one's intellect. In mathematical creativity, then, it would be a choice of directing oneself to explore the richness in mathematics. Teachers' descriptions of *multiple approaches to solving or representing mathematical thinking* overlaps with some of the specific creative thinking styles discussed by Sternberg and Lubart, for instance, legislative thinking style, flexible thinking style, independent thinking style, convention-breaking thinking style, and thinking with existing knowledge (Sternberg & Lubart, 1995). For example, people with legislative thinking style are described as liking to do things in their own way and to enjoy exploration (Sternberg & Lubart, 1995).When solving a division problem, for instance, a student with legislative thinking style may explore using a pictorial representation to accomplish division through repeated subtraction.

Creative process of formulating a solution corresponds quite well to the idea of intellectual abilities in the investment theory of creativity. Intellectual abilities, with respect to creativity, refer to three several kinds of thinking, including viewing things in novel or original ways (experiential ability); comparing, adjusting, and evaluating the quality or worth of idea (componential ability), and communicating and promoting the ideas to others and then utilizing and responding to outside feedback (contextual ability) (Sternberg & Lubart, 1995). In this study, no teacher specifically addressed all three of these kinds of thinking process, but many described something closely resembling one of them. Sternberg and Lubart claimed that experiential ability is the most important for creativity. However, in our study, there was not a clear preference among the teachers for the first stage of problem solving, that is, to the kind of creativity that the investment theory categorizes as experiential ability, although all teachers valued fluency of ideas, that is, multiple approaches to representing and solving problems, which seems to bear some connection.

The aspect of *resource rich environment*: *tools, materials, and curriculum* corresponds almost exactly to the idea of the environment in the investment theory of creativity. Environment refers to “a setting that stimulates creative ideas, encourages them when presented, and rewards a broad range of ideas and behaviors” (Sternberg & Lubart, 1995, p. 10). In this study, teachers put stress on the textbook content and organization, scheduling and timing, home environment, and mathematics tools and manipulatives.

Conceptual understanding of knowledge can be compared to knowledge in the investment theory of creativity, in particular, to formal knowledge. In mathematics, formal knowledge should include the knowledge of standard approaches to solving a problem, as well as basic skills and strategies that enable students to calculate and organize information. *Conceptual understanding of knowledge,* however, as the teachers described it, goes beyond the memorizing of standard approaches and strategies, rather, it is the quality and depth of understanding of mathematical concepts that matters.

Motivation for persistence corresponds closely to motivation in the investment theory of creativity. Motivation is the driving force to action. Consistent with the investment theory, teachers put emphasis on both intrinsic and extrinsic motivation. They recognized, for example, intrinsic motivation accumulates after students make sense of mathematical concepts and create a personal relationship with what they are learning. Students enjoy mathematics as they explore. When it comes to extrinsic motivation, for example, a tangible motivator would be blue star tickets as rewards for creative ideas. An intangible example would be the social recognition from peers and the teacher for a novel idea.

Finally, *accepting risk and error* corresponds to two aspects of personality in the investment theory of creativity. According to this theory, a creative person usually shows a series of personality traits, including perseverance in the face of obstacles, willingness to take sensible risks, willingness to grow, tolerance of ambiguity, openness to experience, and belief in oneself and the courage of one’s own convictions (Sternberg & Lubart, 1995). Among all the personality traits associated with creativity, teachers in this study mostly focused on risk taking and willingness to grow. These teachers encouraged students to take risks to try new ways regardless of being wrong. Teachers believed the willingness to try things without too much concern about the correctness of the answer was critical for mathematical creativity. They argued students should welcome mistakes and appreciate the learning opportunities from making mistakes.

The second point of discussion concerns the difference between our findings and past studies of teachers’ recognition and support for mathematical creativity in the classroom. These K-3 teachers interviewed in this study interpreted mathematics as much more than just memorizing and practicing facts, applying rules, and following procedures. Instead, the learning of mathematics involves a great deal of creativity, which can be promoted by different types of teaching strategies.

Why were the findings of this study so different from past work that has suggested many limitations in teachers' competence to understand and foster creativity in the early childhood classroom? One possibility is the teachers in this study had been supported in understanding and promoting mathematical creativity indirectly, from their professional development in the Primarily Math (PM) program. From the three pedagogy and child development courses, for example, teachers learned specific skills and strategies that would help promote creativity in mathematics, even though these were not discussed in terms of "creativity." Teachers learned about how to sustain productive math talk (Chapin, O'Connor, & Anderson, 2009), provide cognitively challenging tasks, incorporate various ways of representation, create thought-provoking questions and explanations, support individually different needs of young children, and place young children in the center of learning mathematics. As teachers explained, they believed these practices promote mathematical creativity for young children.

From the three mathematics courses in the PM program, furthermore, teachers learned to understand more deeply the nature of mathematics and view themselves and their students as "mathematicians." Teachers were exposed to student-centered learning and challenging math problems that were intended to promote "mathematical habits of mind" (Cuoco, Goldenberg, & Mark, 1996). Even though their PM instructors did not explicitly label these experiences as being about "creativity," it seems likely the teachers may have generalized or extended their new knowledge to better understanding and valuing mathematical creativity in their students. The challenging "habits of mind" problems the teachers worked through in their mathematics courses were intended to give them personal experience with coming up with multiple approaches to representing and solving math problems, formulating solutions at all steps from selecting and checking to communicating, using a resource rich environment, seeking conceptual understanding, working persistently (even enjoying the struggle), respecting others' perspectives and attempts, and accepting risk and error.

The teachers in the professional development program also studied national standards and principles from the leading professional organizations, and these parallel the six aspects of creativity emergent from their interviews. For example, in the NCTM process standard of *problem solving*, students should be able to apply learned strategies to *new* and *unfamiliar* problems and situations either inside or outside the mathematics classroom context (NCTM, 2000a). This advocacy corresponds to *creative process of solving problems*. In the NCTM standard of *connections,* students should be able to recognize and make use of connections among mathematical ideas, to understand how different ideas *interconnect*, and to build the idea on one another to construct a *unified whole* (2000a), which is quite like the theme *conceptual understanding of knowledge*. According to the National Research Council (Kilpatrick, Swafford, & Swindell, 2001), a productive disposition is advocated, that is, students should adopt "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 5). This advocacy bears a similarity to *motivation for persistency*.

Thus, in general, the findings of this study support the interpretation that the PM program enriched teachers' interpretation and promotion of mathematical creativity by deepening teachers' understanding of the nature of mathematics and supporting teachers to be intentional practitioners who catch teachable moments and address students' individual needs.

The study has several limitations. This grounded theory study is exploratory, and the methodology does not allow for any causal inferences. It is only speculation, for example, why these particular teachers had such articulate ideas about mathematical creativity for young children, and whether any aspects of the PM program resulted in their interpretation of mathematical creativity and their suggested strategies for promoting it. We know that the PM program did not explicitly teach about mathematical creativity or how to promote it, but we do not yet know what particular readings, assignments, or class discussions informed teachers about math teaching and learning that they themselves could translate and apply when asked about creativity in their interviews. In addition, the data collection methods have limitations. What teachers say in the interviews may not accurately represent their teaching practices in the classroom. Certainly, teachers in this study represent a specific sample of teachers who have volunteered to participate in an intensive professional development program, and thus one should be cautious when applying the findings to teachers in general.

Conclusions

The K-3 teachers studied in this research differed from some teachers in past studies in their strong capacity to articulate creativity as central to young children's mathematical learning in the early grades, including kindergarten. They were not limited to interpreting their teaching task as helping students memorize and practice math facts and acquire procedural knowledge, such as rules and algorithms. Instead, teachers believed the learning of mathematics can involve many kinds of creativity. Indeed, in K-3 mathematics where teachers are teaching children to count, add, subtract, multiply and divide, there is still plenty of room for creativity in problem solving and developing strategies within that framework of mathematical "facts". Teachers also felt they could use many strategies for promoting creative thinking and a resource rich environment nurturing of creativity.

One implication of these findings for those designing mathematical professional development is not to underestimate teachers' potential. Teachers, like children, are ready for critical thinking about mathematics, including a deeper understanding of the nature of mathematics and a conceptual approach to mathematical problem solving from generating solutions to comparing, revising, and communicating them. Teachers also recognize the value of encouraging children to persist and accept challenge, responsibility, risk and error.

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