The Mathematics Enthusiast

Volume 14
Number 1 *Numbers 1, 2, & 3*

Article 28

1-2017

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Recommended Citation
Shriki, Atara; Barkai, Ruthi; and Patkin, Dorit (2017) "Developing mental rotation ability through engagement in assignments that involve solids of revolution," *The Mathematics Enthusiast*: Vol. 14 : No. 1 , Article 28.
DOI: https://doi.org/10.54870/1551-3440.1410
Available at: https://scholarworks.umt.edu/tme/vol14/iss1/28

Developing mental rotation ability through engagement in assignments that involve solids of revolution

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Abstract: Spatial ability is essential for succeeding in the STEM (Sciences, Technology, Engineering and Mathematics) disciplines, especially mental rotation. Research points out that spatial ability is malleable, and therefore calls for developing learners' ability by engaging them in appropriate assignments, starting from kindergarten. Given this, our paper presents several assignments designed for mathematics prospective teachers with the aim of fostering their mental rotation skills. Specifically, these assignments deal with solids of revolution, three-dimensional shapes formed by revolving a planar shape about a given axis that lies on the same plane.

Keywords: Spatial ability; 3-D geometry; STEM; mental rotation; learning of geometry

Our paper also describes a small-scale study that examined the manner in which mathematics prospective teachers cope with various types of assignments in the field of spatial ability, including mental rotation. We found that the participants' engagement with tasks that involve static shapes might not guarantee the development of required skills for dealing with tasks that involve dynamic shapes. Moreover, dealing with specific tasks that include dynamic shapes might not ensure success in coping with other similar tasks. In other words, it appears that transfer of skills is limited. This raises questions related to the nature of assignments that are more effective than others in developing spatial ability, and the optimal duration of training necessary for developing the capability to transfer associated skills.

1. Introduction

Spatial ability is amongst the major cognitive abilities manifested by varied aspects of daily life as well as in the various disciplines. Hence, these abilities are perceived as affecting both

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everyday and professional conduct of people (Halpern, 2005; Linn & Petersen, 1985; Sorby, Wysocki, & Baartmans, 2002).

Specifically, spatial ability distinctively predicts achievements and attainments in the STEM disciplines, as well as specializing in these disciplines in higher education (Uttal & Cohen, 2012; Uttal, Meadow, Tipton et al., 2012). On the other hand, deficiencies in spatial ability constitutes an obstacle for STEM studies (Harris, Hirsh-Pasek & Newcombe, 2013; Shea, Lubinski, & Benbow, 2001; Wai, Lubinski & Benbow, 2009). Uttal and Cohen (2012) therefore concluded that spatial ability serves as a gateway for entry into STEM fields.

As attested by the meta-analysis carried out by Uttal et al. (2012) and Uttal and Cohen (2012), spatial ability is malleable in people of all ages, and training generally has a positive and continuous impact on them. Furthermore, training was proven as having strong effects of transfer from one assignment to another. The researchers therefore concluded that in order to increase learners' chances of succeeding or specializing in one of the STEM disciplines, they should be provided with a targeted training.

Obviously, relating to all possible types of training would be impossible. Acknowledging the close connection between succeeding in mentally manipulating objects and the STEM disciplines (Uttal et al., 2012), this paper focuses on training concerning mental rotation, paying special attention to solids of revolution.

2. Spatial ability

The research literature is neither consistent nor uniform regarding the terminology, definitions and meaning of spatial ability as well as its associated skills, processes and the resulting outcomes. Moreover, attempts to define and classify spatial ability relate mainly to psychometric indices and particularly to the relations between performances of test items perceived by various researchers as characterizing spatial skills. Since tests in the field of spatial ability were not grounded in a theoretical basis or a certain accepted definition, it is not surprising that there is no consensus about definitions, abilities and interpretations of findings in the field (Uttal et al., 2012).

Despite the lack of uniformity as far as definitions are concerned, great importance is attributed to the various aspects of spatial ability in the context of creativity in the various disciplines. Shepard (1988) gathered autobiographies of prominent figures in the fields of science, arts and literature. According to these autobiographies, the involved experts believe that a considerable part of their original ideas stems from the use of ‘spatial imagination’. Einstein for example, portrayed his thinking in terms of mental images, claiming that words play only a secondary role in his thinking. He described the way he applied for developing the theory of relativity as a process directed mainly by a cognitive experiment whereby he imagined, in his mind, the properties of space and time while travelling at different speeds.

In this section we do not discuss the current controversy in this field. Rather, we briefly mention some of the existing definitions and then focus on one of the major skills of spatial ability - mental rotation.

2.1 Types of spatial ability

According to Linn and Petersen (1985), “*Spatial ability generally refers to skill in representing, transforming, generating, and recalling symbolic, nonlinguistic information*” (p. 1482). Relating to typology of spatial skills, the researchers distinguished between spatial perception, mental rotation, and spatial visualization, based on the processes required to solve problems that relate to each ability: **Spatial perception** requires individuals to determine spatial relationships with respect to the orientation of their own bodies; **mental rotation** skill is the ability to mentally rotate a two- or three-dimensional figure quickly and accurately; and **spatial visualization** is associated with the ability to recognize complicated, multistep manipulations of spatial information. Tasks that call for spatial visualization may involve the processes required for both spatial perception and mental rotations. However, they are differentiated by the possibility of multiple solution strategies (e.g. finding embedded figures, finding representations of unfolded papers, and more).

Drawing on studies in the field of linguistic, cognitive, and neuroscientific research, Uttal et al. (2012) relate to a classification system of spatial ability that makes use of two fundamental distinctions: intrinsic and extrinsic information, and static and dynamic tasks. **Intrinsic information** is what one normally has in mind while thinking of an object (e.g. the specification of its parts and their relations); while **extrinsic information** relates to the relations between objects in a group, either relative to one another or as a whole. As for the second distinction, clearly objects can move or be moved (e.g. rotate in place, change the position) and movement might modify their intrinsic specification. Taking both distinctions together yields a 2*2 classification of spatial skills as indicated in Table 1:

	Intrinsic information	Extrinsic information
Static tasks	The recognition of a certain object	Thinking about relations between locations in the environment
Dynamic tasks	Mental rotation of an object	Thinking about the way relations

		between objects change as they move through the environment

Table 1 - a classification system of spatial ability

As indicated by Uttal et al. (2012), scientists are highly likely to excel at intrinsic–dynamic skills, which are related to mental rotation, and these skills are typically associated with the STEM disciplines.

2.2 Mental rotation skills

Mental image or **mental picture** is a representation of an object that individuals see in their mind. Mental images can be either static or dynamic. Piaget and Inhelder (1967) were the first to point out that young children are capable of generating mental images of moving objects when they reach the stage of performing concrete actions (age 7-8). This was contrary to a previous premise that young children can only generate mental images of static objects. Some years later, Marmor (1975, 1977), found that even 4-year old children are capable of creating a mental rotation of objects in order to deal with various assignments.

Shepard and Metzler (1971) conducted one of the earliest studies in the field of mental rotation. This study was a precursor in enhancing insights about the topic within the framework of cognitive psychology, paving the way to additional studies of mental rotation. The researchers defined 'mental rotation' as a cognitive process whereby one imagines some rotating object. In their pilot study, participants were presented with pairs of 3-dimensional objects (in a 2-dimensional representation) positioned in different orientations or 'cut into' cubes. The experiment aimed at measuring the reaction time required for determining whether two objects are identical. Shepard and Metzler (1971) found that the reaction time increased linearly with the angular difference in the portrayed orientation of the objects. The relation between the reaction time and the angle of rotation was investigated over the years in many additional studies, resulting in similar findings (Uttal et al., 2012). Attempting to understand how subjects determine whether two differently oriented objects the identical, Shepard and Metzler (1971) illustrated that:

> *all subjects claimed (i) that to make the required comparison they first had to imagine one object as rotated into the same orientation as the other and that they could carry out this "mental rotation" at no greater than a certain limiting rate*; *and (ii) that, since they perceived the two-dimensional pictures as objects in three-dimensional space, they could imagine the rotation around whichever axis was required with equal ease* (pp. 701-702).

Later, Shepard and Cooper (1982) reported that their participants sensed they were performing an operation on an analog spatial representation, in which the intermediate stages of the internal process have one-to-one relation to intermediate stages of the corresponding external process.

However, despite the numerous studies of mental rotation, its underlying mechanism is still controversial today, in particular the question whether mental rotation depends on an analog spatial representation (Uttal et al., 2012).

In recent years there has been a growing tendency to use brain imaging for the purpose of exploring the functional role of visual images and more specifically their role in dealing with assignments that require mental rotation. According to Zacks (2008), this enables

> *…to test the hypothesis that mental rotation, in particular, operates in virtue of analog spatial representations… If spatially mapped regions of the brain consistently show increases in activation with increasing amounts of mental rotation, this would provide good evidence that mental rotation depends on analog spatial representations* (p. 1).

In his meta-analysis, Zacks (2008) found that foci of brain activity related to mental rotation have been identifies in every lobe of the cerebrum and in the cerebellum, and that results of brain scans support the hypothesis that mental rotation depends on analog spatial representations.

3. Improving spatial ability through training

Although all people have spatial ability, individuals utilize it in different ways and at different degrees of success. Moreover, people often manage to perform assignments requiring spatial ability in a certain area yet fail to perform such assignments in another (Amorim, 1999).

Can spatial ability be improved through appropriate training? If so, how can this ability be improved? Answers to these questions are essential for providing learners with a proper support which will eventually lead to the improvement of their spatial ability.

The research literature discusses varied attempts to improve spatial ability through targeted intervention programs. These endeavors include, among others, engaging students of various ages in diverse learning materials for different durations. To mention some: 3-dimensional objects are represented as 2-dimensional ones, or using cubes for building 3-dimensional objects on the basis of 2-dimensional views of constructs presented at different angles and different types of drawings; numbering cubes in various drawings, building models and detecting cube constructs displayed in different orientations by means of a 2-dimensional representation (e.g. Ben-Chaim, Lappan & Houang , 1988, 1989; Medina, Gerson & Sorby 1998; Patkin & Fadalon, 2013; Sorby & Baartmans, 2000; Turos & Ervin, 2000); making use of isometric drawing and projections and nets of solids on a dotted plane (e.g. Patkin & Dayan, 2013); and numerous others. Although each of the mentioned intervention program has its unique characteristics, research findings illustrate that their contribution to developing learners' spatial ability is prominent. Nevertheless, it should be pointed out that the literature review carried out by Uttal et al (2012) attests to a relation between the initial level of spatial skills and improvements. Participants who started the training at lower levels of performance improved relatively more in response to training than those who started at higher levels. Uttal et al. (2012) assumed that this might be attributed to the existence of some 'ceiling' for possible improvement. That is, the malleability of spatial ability is limited, although one can presume that it is related to the characteristics of the training and its duration.

This leads to further questions, such as: Is the improvement of spatial ability following training maintained over a long period of time? Does training a specific spatial ability in a certain context affect other spatial skills? Is training at young age more effective than training at older age? In their meta-analysis, Uttal et al. (2012) related to these questions, viewing them as important when considering whether or not to invest efforts in improving spatial skills for increasing STEM achievements. The researchers pointed out that most of the studies that focused on the long-term impact of the training, found an effect of retention. They also showed that transfer of the practiced skills to other skills associated with spatial ability was possible if sufficient training or experience was provided.

Obviously, retention and transfer were not observed in all intervention programs. With respect to transfer, Sims and Mayer (2002) investigated differences between college students who were highly skilled in the Tetris game and those who were not skilled in this game. Playing Tetris, one has to rotate, move, and drop shapes of different colors that 'fall' at an increasingly growing speed from the upper end of the screen. The objective is to arrange them at the bottom of the screen without leaving empty spaces between them. In fact, winning the game depends on players' ability to quickly rotate the shapes and mentally predict how they should be rotated. Sims and Mayer (2002) were interested to learn whether players with high ability of mentally rotating the Tetris shapes would transfer this ability to mental rotation of shapes which are different than those of the Tetris. Their research findings indicated only a limited transfer. The Tetris experts were indeed better than the others in rotating shapes that were similar but not identical to the Tetris ones. However, no evidences of a more considerable transfer were found. That is, the Tetris experts did not demonstrate an advantage over the others when mentally rotating shapes essentially different than those included in the Tetris game, although in some cases this concerned familiar shapes such as letters. Other studies (e.g., Bethell-Fox & Shepard, 1988; Lohman & Nichols, 1990; Tarr & Pinker, 1989) found that when participants received a focused training, their performance improved particularly in context that involved the objects included in the training. They also exhibited a transfer of the acquired abilities to simple and unfamiliar objects. However, in many cases, participants found it difficult to generate mental representations or perform mental rotation of complex objects not included in the training.

Evidence of inability to make a transfer was also identified among our students, high school mathematics prospective teachers (Barkai & Patkin, 2014). Seventeen students participated in an annual course of spatial geometry in one of the academic colleges of education. During the first semester, the students engaged in proving theorems and solving advanced problems in spatial geometry, taken from matriculation exams. Analysis of these assignments indicated that they could be characterized as static tasks (utilizing both intrinsic and extrinsic information, as specified in Table 1). Dynamic tasks were not discussed during the first semester. However, one of the questions in the final test was characterized as a dynamic task utilizing intrinsic information. Students were required to mentally rotate a 2-dimensional shape around an axis of rotation. This question was taken from a collection of assignments developed by Walker, Winner, Hetland, et al. (2011):

> *Imagine holding a small square card by the diagonal corners and spinning it around the diagonal. What shape would be carved out in the air? Describe your answer in words as best you can* (p. 26).

As illustrated in Figure 1, the rotation of the square (which is a 2-dimensional shape) about the diagonal (which constitutes the axis of rotation) results in a solid comprised of two cones with a common basis. In fact, it is a **solid of revolution**, namely a 3-dimensional solid figure formed by rotating a plane area about a given line (the axis) that lies on the same plane.

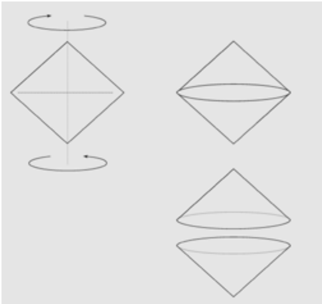


Figure 1

While most of the students responded correctly to the test questions that were associated with the learnt contents, only 7 out of the 17 students (about 41%) correctly answered this question. Six students (about 35%) gave a wrong answer, arguing that the obtained shape was a sphere, cube or triangle. Four students gave no answer. The results led us to assume that students' engagement with static tasks might not guarantee the development of skills required for dealing with dynamic tasks. In other words, only a limited transfer of the practiced abilities to other abilities was observed. We will elaborate on this in the next section.

Finally, a few words about the issue of the optimal timing for spatial skills training. The prevalent premise is that young people's thinking is more malleable than that of grownups. Hence, ostensibly, spatial ability training might have a more meaningful impact on young children than on adolescents or adults. However, the literature review made by Uttal et al. (2012) illustrated that no significant statistical differences were found between the ages following such training. This indicates that spatial ability can and should be developed at any age.

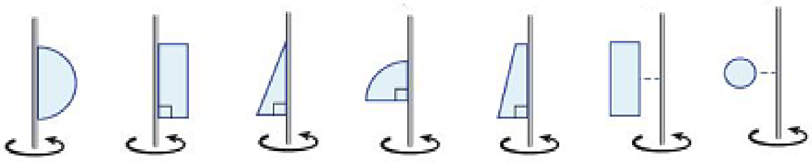
4. Solids of revolution- Assignments designed to develop mental rotation skill

As mentioned, although our students had sufficient skills for managing high level static tasks, they exhibited a limited transfer of these skills to the situation that necessitated a mental rotation skill. Realizing the need for deliberate attention to developing students' mental rotation skills, we designed a series of related assignments, four of which deal with solids of revolution. We present them below, as assignments of this type are not common. The students engaged in Assignment I, II, and III in the first, seventh, and last lesson of the second semester, respectively. In each lesson a discussion was held, following the students' experience (two examples of "didactic suggestions" are described below). Two weeks after completing the course, the students were given Assignment IV as part of their end-of-course exam with the aim of checking whether they would exhibit the required transfer of mental rotation skill in order to succeed in managing it. The assignments are based on Patkin and Barkai (2015), each is intended for a 90-minute session.

Assignment I – Matching 2-dimensional shapes to their representation as solids of revolution

Assignment I[2]

Below are seven 2-dimensional shapes.
Each of these shapes rotates about a given line, as described in the drawing.



1 2 3 4 5 6 7

Nine representations of solids are given below (A-I).
For each of the rotated figures 1-7 above, please match the representation of the solid obtained as a result of the rotation.
Note that two of the solids do not comply as a solution.

								
A	B	C	D	E	F	G	H	I

Describe in words the manner in which you approached the assignment.

Did you use the representation of the solids (A-I) for generating a 'mental image' of the rotated shapes (1-7)? In case you did use the representations, please indicate at which

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[2] Assignment I is based on an activity developed for elementary school teachers by Hadas (2006).

stage of answering the question: before, during or after generating the 'mental image' (in order to ensure that the generated image does indeed match one of the given solids).

Solution of Assignment I

First, it should be noted that in order to prevent students from guessing their answers, the number of solids displayed in the assignment is greater than the number of the 2-dimensional shapes (solids A and I do not represent solids of revolution of one of the 2-dimensional shapes). Part of the solids of revolution obtained from the rotation of the displayed 2-dimensional shapes are more common and familiar to students, e.g. a cylinder, a cone and a sphere, whereas others are less familiar solids, such as a hollow cylinder, torus and truncated cone.

Table II indicates the solution of the assignment and the name of the obtained solids of revolution.

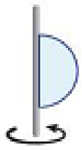
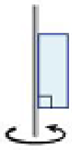
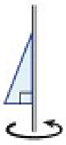
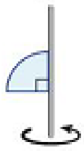
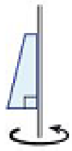
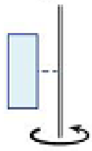
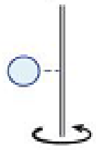
The 2-D shape							
	1	2	3	4	5	6	7
Solid of Revolution							
	C	E	G	F	H	I	B
Name of the obtained solid	Sphere	Cylinder	Cone	Hemisphere	Truncated cone	Hollow cylinder	Torus

Table II - solution of Assignment I

Didactic suggestions for implementation in class

1. After collecting students' answers to Assignment I, it is recommended enabling students to share the approaches or strategies they employed in order to cope with the assignment, the difficulties they experienced, and their misconceptions.

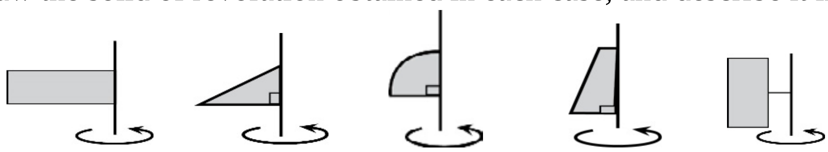
2. Students can be divided into two groups. One group will be provided with the representations of the solids of revolution (A-I), as described in Assignment I, while the other group will not be provided with these representations, and will be asked to draw them (this case appears in Assignment II). A following discussion might focus on the differences in the involved processes and level of difficulty in performing the assignment in both cases and the underlying reasons for these differences.

Assignment II – Drawing representations of solids of revolution of 2-dimensional shapes, with different position of the axes of rotation

This assignment concerns two cases of five 2-dimensional shapes and axes of rotation. In the first case the axis of rotation is vertical and in the second, it is horizontal. The position of the 2-dimensional shape is identical in both cases.

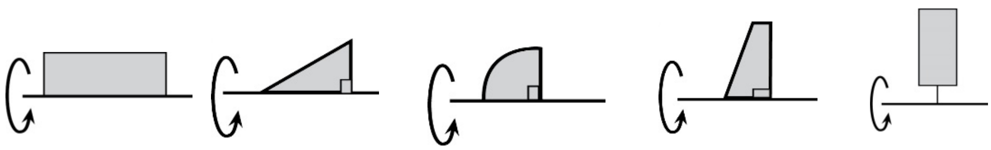
Assignment II

(i) Below are five 2-dimensional shapes.
Each of the shapes rotates about a vertical axis.
What is the solid of revolution that is obtained as a result of the rotation?
Please draw the solid of revolution obtained in each case, and describe it in words.



1 2 3 4 5

(ii) Below are five 2-dimensional shapes.
Each of the shapes rotates about a horizontal axis.
What is the solid of revolution that is obtained as a result of the rotation?
Please draw the solid of revolution obtained in each case, and describe it in words.



1 2 3 4 5

(iii) What are the main differences between Assignment I and Assignment II?

Solution of Assignment II

Changing the position of the axis of rotation does not necessarily affect the type of obtained solid but it might affect its volume. The change in volume depends on the characteristics of the axes of symmetry of the 2-dimensional shape. In both cases the solid of revolution obtained from shape No. 3 is a hemisphere with the same volume. For the 2-dimensional shapes Nos. 1, 2 and 5, the solids of revolution in both cases are of the same type but they have different volumes. Note that for the two cases of shape No. 4 the location of the axis of rotation has changed in relation to the right-angled trapezium, but it is not just its position: in the first case the axis of rotation is the short side of the trapezium while in the second case the axis of rotation is the longer base of the trapezium. A different solid of revolution will therefore be obtained in each of the cases. This case whereby the axis of rotation is the longer base of the right-angled trapezium is also the only one that was not included in Assignment I.

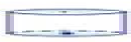
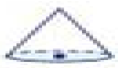
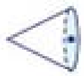
	Rotation about the vertical axis		**Rotation about the horizontal axis**	
The 2-dimensional shape	The solid of revolution	Name of obtained solid	The solid of revolution	Name of obtained solid
1		a cylinder		a cylinder
2		a cone		a cone
3		a hemisphere		a hemisphere
4		a truncated cone		a cylinder and a cone with a common base
5		a hollow cylinder		a hollow cylinder

Table III - solution of Assignment II

Didactic suggestions for implementation in class

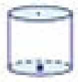
1. The request to verbally describe the obtained solids aimed to develop students' use of accurate mathematical language, while paying attention to critical properties of these solids. Subsequent to collecting students’ responses, it is recommended asking students to reflect on their difficulties to verbally describe the solids.
2. Students should explain how changing the position of the axis of rotation affects the type of obtained solid and/or its volume, and why the type of solid and/or its volume depend on the characteristics of the axes of symmetry of the 2-dimensional shape.
3. Attention should also paid to the differences between Assignment I and Assignment II, focusing on the abilities required in the case whereby the representation of the solid is or is not given.

Assignment III – Drawing 2-dimensional shapes and axes of rotation based on their representations as solids of revolution

In Assignment III the students are asked to draw for each representation of solid of revolution the 2-dimensional shape whose rotation resulted in the solid, as well as its axes of rotation. The aim of this assignment is to further enhance the mental rotation ability, since in assignments I and II the students had ‘to imagine’ solids of revolution obtained from rotating a 2-dimensional shape about an axis, but not the other way around. Moreover, the representations of the solids of revolution displayed in this assignment are already familiar to the students from Assignment I and II. Nevertheless, in those assignments students are not given the option of generating identical solids of revolution from different 2-dimensional shapes or by changing the location of the axis of rotation. Hence, one of the objectives of this assignment is to develop insights regarding the fact that some solids of revolution can be generated in more than one way.

Assignment III

Below are seven representations of solids of revolution.
For each one of them please draw the 2-dimensional shape from which the solid of revolution was obtained, as well as the location of the axis of rotation.
In addition, describe the drawing in your own words.
In case you think that a certain solid of revolution can be obtained from more than one 2-dimensional shape and/or an axis of rotation, please draw all the possible options.

						
1	2	3	4	5	6	7

Solution of Assignment III

As indicated above, in some cases the solid of revolution displayed in the assignment can be generated in more than one way. For example, the cylinder (solid 1) can be obtained from a rectangle whose axis of rotation is one of the rectangle's sides or when the axis of rotation is a straight line passing through the midpoints of two opposite sides of the rectangle. The verbal description of all the possible cases develops the ability to generalize solutions and enhance insights about the symmetry of solids of revolution. Following are the solutions:

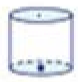
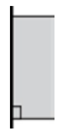
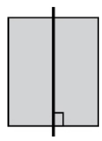
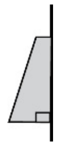
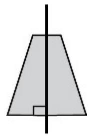
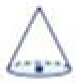
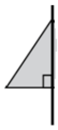
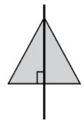
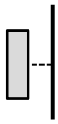
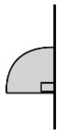
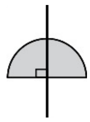
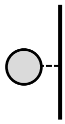
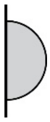
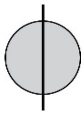
Representation of the solid of revolution		The 2-dimensional shape and axis of rotation		Representation of the solid of revolution		The 2-dimensional shape and axis of rotation	
1				5			
2				6			
3				7			
4							

Table IV - solution of Assignment III

Assignment IV – Reflecting a right-angled trapezium across an axis of rotation displayed in five different positions

In this assignment students are asked to describe the solid of revolution obtained when a right-angled trapezium is mentally rotated about a certain axis of rotation. The assignment includes five cases. For two of them the axis of rotation is the large base (trapezium 1 – vertical axis of rotation, trapezium 4 – horizontal axis of rotation), for two of them the axis of rotation is the small base (trapezium 2 – vertical axis of rotation, trapezium 3 – horizontal axis of rotation), while for one of them (trapezium 5) the axis of rotation is the small side which is perpendicular to both trapezium bases.

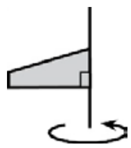
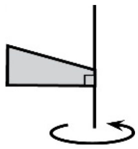
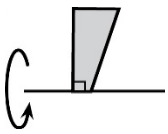
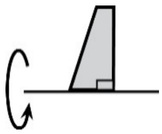
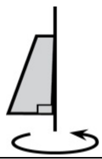
It should be pointed out that some of the cases are familiar to students from previous assignments: rotating a right-angled trapezium about the small side when the axis of rotation is perpendicular appears in shape 5 of Assignment I, in shape 4 in the first part of Assignment II, and in shape 5 of Assignment III (similarly to shape 5 in Assignment IV above). The case of rotating a right-angled trapezium about the large base when the axis of rotation is horizontal appears in shape 4 of the second part of Assignment II (similarly to shape 4 in Assignment IV). The cases described in shapes 1, 2 and 3 of Assignment IV were not discussed in previous assignments. Thus, in order to cope with them there is a need for a transfer of mental rotation skill. A transfer is also required in order to arrive at the conclusion that a simultaneous change of the trapezium position and axis of rotation does not change the type of the solid of revolution obtained and the volume thereof. On the other hand, changing only the location of the axis of rotation might affect the type of solid of revolution obtained as well as its volume.

It should be noted that asking students to describe the solid of revolution obtained in case where the axis of rotation is the longer side of the trapezium is a complex demand, which requires a very high level of mental rotation malleability. This case is not included in the assignment, and the readers are invited to try drawing the solid obtained in this case.

Assignment IV

Please indicate whether among the following five 2-dimensional shapes there are shapes whose rotation about their axis of rotation will result in identical solids of revolution, namely a solid of the same type and the same volume.

For each of the following shapes please specify the name of obtained solid of revolution, describe it in your own words, and try to draw it.

				
1	2	3	4	5

Solution of Assignment IV

Rotation of trapezium 1 and trapezium 4 results in a combination of a cylinder and a cone with a common base, while rotation of trapezium 2 and trapezium and trapezium 3 results in a cylinder from which a cone with a common base has been 'extracted'. Rotation of trapezium 5 results in a truncated cone.

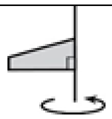
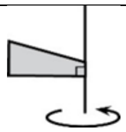
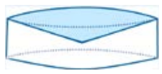
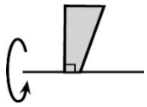
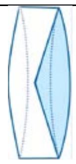
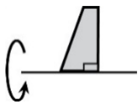
	The trapezium and the axis of rotation	The obtained solid of revolution	Description of the obtained solid of revolution
1			a cylinder and a cone with a common basis
2			a cylinder from which a cone has been 'extracted'
3			a cylinder from which a cone has been 'extracted'
4			a cylinder and a cone with a common basis
5			a truncated cone

Table V - solution of Assignment IV

As mentioned, Assignment IV was part of the students' end-of-course exam. Interestingly, for the case depicted by shape 5, which was familiar to the students from Assignments I and III, all the answers were correct (both the identification and drawing). The case depicted by shape 4 was familiar to the students from Assignment II, and 12 of them responded correctly. The new cases (shapes 1, 2, 3) were solved correctly by 11 students, while 6 students provided incorrect answers in all three cases. Students' incorrect responses related to shape 1 stated that the obtained solid of revolution was a truncated cone, or a solid comprised of two cones, or a cone whose base is subtended by a cylinder. In the case of shape 2, one of the students wrote that *'what was obtained following the rotation has no name'*. Others portrayed the obtained solid as *'a truncated cone'*, *'a cylinder to which an inverted cone was attached*' or *'an inverted cylinder subtended from a cone'*. The incorrect descriptions in the case of shape 3 were identical to those given in the case of shape 2.

Regarding the question whether the rotation of the shapes about an axis would result in the same solids, the findings showed that 13 students were able to identify that by rotating shape

1 and shape 4, the same solids would be obtained and that the difference was only in the position of the shapes and the solids. Hence, the shapes and the solids would also have the same volume. The same answer and the same argument were given with regard to shapes 2 and 3. One of the students did not settle for a verbal explanation and supported his argument by calculating the volume. The other 4 students claimed that the obtained solids of revolution were not identical and therefore the volumes could not be compared.

5. Discussion

Spatial ability is essential for both daily life and the professional conduct of people (Halpern, 2005; Linn & Petersen, 1985; Sorby, Wysocki, & Baartmans, 2002). As mentioned in the introduction section, research has found that a two-way relationship exists between spatial ability and attainments in the STEM disciplines, and that deficiencies in spatial ability constitute an obstacle for STEM studies (Harris, Hirsh-Pasek & Newcombe, 2013; Shea, Lubinski, & Benbow, 2001; Uttal & Cohen, 2012; Wai, Lubinski & Benbow, 2009). This close relationship raised awareness of the need for proper training: "*The available evidence supports the claim that spatial training could improve STEM attainment…Thus we think that an investment in spatial training may pay high dividends*." (Uttal & Cohen, 2012, p. 177). Indeed, research indicates that spatial ability is malleable in people of all ages and training generally has a positive impact on its improvement (Uttal et al., 2012; Uttal & Cohen, 2012). However, while it is reasonable to assume that training has a positive impact on the development of spatial ability, the research literature is not unequivocal when it comes to issues related to transfer and retention of skills. These seem to be associated with the characteristics and the duration of training. Nevertheless, we were unable to find any conclusive findings or recommendations concerning these issues, thus indicating the need to carry out a systematic exploration of the field. The importance of developing insights regarding transfer and retention is derived from the limited time that can be allocated for training learners' spatial ability in the context of school or the academy. Therefore, identifying the characteristics of effective training in terms of contents and duration is highly important.

Transfer of skills was one of the central themes observed in our course, as described above. In fact, we identified two 'types' of limited transfer:

1. *Transfer of skills required for managing static tasks to skills needed for performing mental rotation*: As mentioned, during the first semester the students solved high-level problems associated with what Linn and Petersen (1985) termed as "spatial perception". These problems were in the form of 'static tasks' and utilized both intrinsic and extrinsic information (see Table 1). The limited transfer was observed when the students had to cope with the task described in Figure 1. Only 7 out of the 17 students were able to solve it correctly. Since we did not carry out a controlled study, we cannot say whether those who completed the task successfully implemented a transfer of skills they had developed during the first semester, or perhaps they had the required skills from the outset. A further study is needed in order to examine this question and understand whether or not practicing static tasks can support the development of mental rotation skills.
2. *Transfer of acquired mental rotation skills to performing mental rotation of unfamiliar shapes*: Realizing the need to pay deliberate attention to developing students' mental rotation skills, we designed a series of related assignments. Four of the assignments specifically deal with solids of revolution, as this topic is often ignored while studying spatial geometry although it is part of the students' Calculus studies. Assignment IV was included in the students' end-of-course exam, and provided us with the opportunity to examine what we termed as "inner transfer", namely transference of practiced skills related to mental rotation of solids of revolution to mentally rotating unfamiliar solids of revolution. In some sense, this situation is similar to the experiment carried out by Sims and Mayer (2002), in which subjects engaged in unfamiliar Tetris-like shapes. As pointed out by the researchers, Tetris experts were better than others in mentally rotating shapes that were similar to the Tetris ones, but they were not better than the others when mentally rotating shapes different than those involved in the Tetris game. Our findings relating to Assignment IV are similar to those of Sims and Mayer (2002). As could be seen, the more the students were exposed to a specific solid of revolution in a certain task, the better they managed to cope with a task that involved this shape. Limited "inner transfer" raises questions concerning the optimal duration of training needed for developing the malleability of mental rotation skills to enable a better transfer.

To conclude, given the connection between spatial ability and STEM disciplines, and the fact that research findings relating to the various aspects of spatial ability are not consistent, there is a need for a wide-scale study aiming to address issues such as: the meaning of ‘appropriate training’; characterization of the nature of activities which optimally support the development of spatial perception, mental rotation, and spatial visualization; types of assignments that assist in maintaining the acquired skills over time and enable learners to transfer associated skills; adaptation of assignments to the different age groups; determining the duration of appropriate practice; and more. It is also recommended to consider suitable ways of exploiting recent findings in the field of brain research for this purpose.

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