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ABSTRACT: We explore the use of LaTeX to typeset solutions to calculus problems. We hypothesize that solving one problem completely, and presenting that solution properly, helps to strengthen students' overall problem solving and mathematical abilities. Several specific examples are given to demonstrate how these changes occurred within in an individual Calculus III course.

Keywords: LaTeX, problem solving

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Introduction

Last summer while recovering from a dislocated shoulder and unable to type or write, I did a lot of thinking about mathematics and student success. While talking with a colleague of mine one day about these issues, he asked me the following question: "What worked for you when you were a student? What made you a better mathematician?"

I thought about this at length, and the one thing that really stuck out was the change in my mathematics when I started typesetting in LaTeX. As any math teacher will tell you, when you collect homework from students it can sometimes be an absolute mess. Pages scribbled out, no flow to the solution, and oftentimes no final answer; just a drifting away of pencil scratches, like a song that fades out. Very often I give students full marks for their homework, as I grade on completion, but I always worry that they actually can't solve a problem from beginning to end with a clear and concise argument. The reason for this is probably because we've never actually taught them how. My hypothesis for what turned out to be an interesting, ongoing experiment was as follows:

Finishing one problem to the absolute end is better for overall understanding of key mathematical concepts than half finishing multiple problems.

A major issue at our university is the seeming inability of students to solve complex problems from beginning to end. This ability is also exactly what employers hold in high regard when searching for job candidates. To test my hypothesis, I decided to require my Calculus III students to learn the basics of LaTeX, and turn in one *perfect* solution several times a week, along with the regular, written homework on the other days.

The first round of homework assignments were quite basic. The following solution is a good representative of what was handed in.

Example 0.1. Find the equations for the plane through (1, -1, 3) parallel to the plane:

$$3x + y + z = 7$$

$$3(x-1) + 1(y+1) + 1(z-3) = 0$$
$$3x - 3 + y + 1 + z - 3 = 0$$
$$3x + y + z = 5$$

The first two lines of the text are the actual question from the book, and the last three lines are the solution.

The student didn't even copy the question down correctly from the book. The word "equations" should have been "equation". Keep in mind that I also provided the students with a template that I wanted any homework assignment to follow. The format of Example 1.1 should have been as follows:

Example 0.2. Find the equation for the plane through $(1, -1, 3)$ parallel to the plane $3x + y + z = 7$.

Solution:

Solve problem here.

I spent a considerable amount of time grading this first assignment, hoping the extensive amount of feedback would help the students to understand better what I was looking for. More importantly, I did my best to create a well crafted solution out of their work. The big question the students had when their assignments were returned was: " How do we write a solution?" To not overwhelm them, I focused on two very important ideas that exist within a good solution. Number one was to find the right answer to a given problem, and number two was how to format it in a way that flowed and was easy to follow. With these two ideas in mind we set about learning how to write solutions.

1 How Do We Write a Solution?

To help answer this question, I brought to class the Student's Solution Manual that came with the course textbook. Many students copied their homework solutions straight from this manual. Quite often these same students complained that the manual was difficult to follow because the author skipped steps. I agreed with them and attempted to remedy the situation by defining what a "skipped step" is. We also talked at length about using space and alignment in the course of a solution and about the effective use of math connectors such as "hence" and "therefore". This notion was made more difficult by the fact that the students were only just beginning to learn the basics of

LaTeX, which for a beginner can be a bit daunting. In order to help the students familiarize themselves with a "good solution" and work on their LaTeX skills, I brought in solved problems from different sources and had the students reproduce them *exactly* as they appeared in the text. This was an effective way to approach the two issues I stated above, and we did this several times throughout the semester as an in-class workshop. Naturally questions led to more questions, and the beginning of a holistic approach appeared.

Over time I saw that my approach was having a positive impact on the students' solutions. By working through one problem till the bitter end and working through the issues of LaTeX formatting, students were beginning to understand the mechanics of an acceptable solution. There came an "a-ha" moment for many when the final answer was completely sorted out and the presentation was well formatted.

2 What Is a "Skipped Step"?

When I get my course evaluations back from students I often get comments such as, "I wish Dr. Pohlmann wouldn't skip so many steps while solving problems and explain everything in greater detail." Very similar to the complaints voiced about the Solutions Manual. From the standpoint of the student, I can totally understand the confusion that arises when something is skipped. But is it? How much detail is enough?

For example, if during a Calculus III solution I factor the polynomial

$$x^2 - 4$$

into

$$(x-2)(x+2),$$

there is no work to show. You also wouldn't expect a student to write "this is a difference of squares." So we tried to come up with a baseline for a what a skipped step is. Students volunteered to allow me to project their solution to a problem on the board in class and then analyze it line by line. This was highly effective, as other students in the class began to see when more explanation was needed as they recognized these so-called "skipped steps." The end rule we came up with was, "If you think a step *might* need more explanation, then it probably does."

2.1 Editing

At midsemeter, I gave an evaluation that posed the question (amongst others): "Yes or No. Do you feel the LaTeX assignments are making you better at mathematics?" The results were pleasing, with 67 % of the class answering Yes and 33 % of the class answering No. There were various comments to go along with these answers, my favorite being, "Forces me to really justify <u>why</u> I am doing something."At this point in the semester it was not uncommon for students to use 2 <u>plus</u> pages for a problem that only needed half a page. The pendulum had swung! Now instead of too little explanation, they were writing too much.

So we began editing solutions, and this is where big changes began to take place. Confidence increased as students began to understand "skipped steps" and when more or less explanation was needed. They also began getting better with 'hence's" and "therefore's" and all the other words that are very common in mathematics, but less so in colloquial speech. One of my main goals with the experiment was not to overwhelm. To this end I made a brief list of ideas as a guide.

1. Use lots of space during the math phase. Being in MATH mode in LaTeX is designed to help with this.

2. If you think a step has been skipped, it probably *has*. Insert it and explain briefly why.
3. Ask someone else (like me) to read your solution and look for possible edits.
4. Be sure you actually answered the question being asked.

3 Conclusion and Remark

The Final Exam in the course consisted of two problems per student. The students had 48 hours to solve the problems in LaTeX and turn in their code and solutions. No two students had the same problem, which for me meant many, many hours of grading. I was looking for perfection, and for the most part, that's exactly what I got. Everything came together from our work during the semester, and the final exam average was in the 80's. I really felt the experiment worked. Here is an abbreviated final exam solution, keeping in mind the student in question is a sophomore engineering major with no previous course in foundations.

1. A triangular plate occupies the region D in the first quadrant of the xy-plane with the vertices $(0,0), (1,0)$, and $(0,2)$. The temperature at each point on the plate is given by:

$$T(x,y) = x^2 + xy + 2y^2 - 3x + 2y.$$

Find the hottest and coldest points on the plate.

Solution: Since T is differentiable, the only places where T can assume hottest and coldest temperatures (critical points) are inside the triangle where $f_x = f_y = 0$ and on the boundary.

(a) First we find the interior points.

$$\begin{aligned} T_x &= 2x + y - 3 \qquad & T_y &= x + 4y + 2 \\ 0 &= 2x + y - 3 \qquad & 0 &= x + 4y + 2 \end{aligned}$$

This yields the single point $(2, -1)$ which is not inside the given trianglular region D and therefore there are no critical points inside the triangle.

(b) Then we find the critical points on the first side of the triangle. The first side is the segment OA where $x = 0$ and $0 \leq y \leq 2$

$$\begin{aligned} T(0,y) &= 2y^2 + 2y \\ T'(0,y) &= 4y + 2 \\ 0 &= 4y + 2 \\ y &= -\frac{1}{2} \end{aligned}$$

Note: The student finished the analysis of the other two sides of the triangles and the corners. Her last step was:

f) Finally we plug all of our critical points into the original function T and conclude that the coolest point occurs at $(1,0)$ and the hottest point occurs at $(2,0)$.

In line 8 the student uses the term "therefore" effectively. Her use of space throughout the math portions is excellent. She outlines her attack and then takes each piece as a separate item, with conclusions at each step. Lastly she solves the problem and answers the exact questions asked.

From this class we built a webpage containing documents explaining the basics of LaTeX and several templates for getting started. Several of students involved in the experiment are still using LaTeX in their engineering classes, and it's quite pleasing to hear from other professors about the usage of the program for projects and homework.

We would like to remark that many of the students from this particular class are still using LaTeX to typeset projects and homework, and have informed me that certain engineering firms list LaTeX skills as a desirable qualification.

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