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Understanding Teacher Noticing of Students' Prior Knowledge: Challenges and Possibilities

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Abstract: *I propose a new construct, teacher noticing of students' prior knowledge, and provide examples of a professional development intervention with the goal of supporting teacher learning. The examples demonstrate that combining discussions of animations of classroom instruction, video clubs, and lesson study can help teachers to attend to students' prior knowledge and anticipate actions that use that knowledge for promoting mathematical understanding. I discuss challenges and new questions for mathematics education researchers brought about by the new construct. I suggest that the construct is valuable for developing teachers' professional knowledge.*

Keywords: teacher noticing, students' prior knowledge, video clubs, professional development, professional knowledge, geometry

In recent years, mathematics education researchers have adopted the construct of "teacher noticing" to denote the special ways in which teachers pay attention to specific aspects of instruction (Sherin, Jacobs, & Philipp, 2011). There are some variations in the uses of this construct. Some researchers have coined related terms, such as "noticing classroom interactions" (Sherin & van Es, 2005), "professional vision," (Sherin & van Es, 2009), "professional noticing of children's mathematical thinking" (Jacobs, Lamb, & Philipp, 2010) and "noticing of student thinking" (van Es, 2011). Overall, studying teacher noticing allows for an examination of how teachers understand students' ideas during classroom instruction and through the examination of artifacts from classrooms, such as student worksheets or classroom videos in activities outside of the classroom. Most recently, researchers' examination of teacher

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noticing specifies the object of teachers' attention, such as "teacher noticing of students' algebraic thinking" (Walkoe, 2015), "noticing of students' mathematical strengths" (Jilk, 2016), and "noticing of equitable practices" (McDuffie, Foote, Drake, Turner, Aguirre, Bartell, & Bolson, 2015). Researchers' expectation about adopting a more limited focus on teacher noticing is twofold. On the one hand, having a better understanding of specific aspects of teacher noticing can help to unpack teacher thinking in relation to instructional demands. On the other hand, research on teacher noticing can support the development of teacher education initiatives for increasing teacher noticing of specific aspects of instruction and thus promote teacher learning. In general, understanding teacher thinking can guide teacher education initiatives for supporting teachers' development of capabilities for attending to student thinking in the classroom.

My work centers on understanding *teacher noticing of students' prior knowledge*. Research on learning shows robust evidence concerning the importance of prior knowledge (National Research Council [NRC], 2000). Prior knowledge affects learning (Dochy, Segers, & Buehl, 1999). People make sense of something new in light of what they already know. In addition, new knowledge becomes more meaningful and memorable when it is connected to what is known and past experiences (NRC, 2000). Recommendations for mathematics teachers state that making explicit connections with prior knowledge during instruction solidifies students' mathematical understanding (NRC, 2001). Teacher observation instruments identify attention to students' prior knowledge as a teaching action that can promote mathematical understanding (e.g., Boston & Smith, 2009). Maintaining a high-level cognitive engagement requires tasks that provide opportunities for students to connect with their prior knowledge (Stein & Smith, 1998). Typically, curriculum materials refer to students' prior knowledge in the teachers' edition of textbooks as mathematical content that students have already studied in school in the same course of study or in prior courses. Teachers recognize students' prior knowledge of school mathematics (Hohensee, 2016). However, my conceptualization of students' prior knowledge goes beyond school mathematics and considers students' experiences outside of school.

Various perspectives in mathematics education research have considered students' experiences in relation to mathematics. Work regarding students' *funds of knowledge* (González, Andrade, Civil, & Moll, 2001) and *children's mathematical knowledge bases* (Turner et al., 2012) exemplify how students' experiences, including students' cultural and linguistic backgrounds, shape mathematical problem solving. Studies on ethnomathematics (e.g., Ascher, 1994) have documented how individuals engage in mathematical activities in various non-traditional spaces. Similarly, studies on out-of-school mathematics demonstrate that mathematical problem-solving is tightly connected to contextual experiences (e.g., Nunes, Schiliemann, & Carraher, 1993). In the case of problem-based instruction, problems that are situated in a particular context can call for students to use problem-solving strategies that are aligned with their prior experiences with that context (Lubienski, 2000; van Den Heuvel-Panhuizen, 2005). For these reasons, my conceptualization of students' prior knowledge considers three main sources: *school mathematics, out of school mathematics*, and *the contexts framing the problems* (Figure 1).

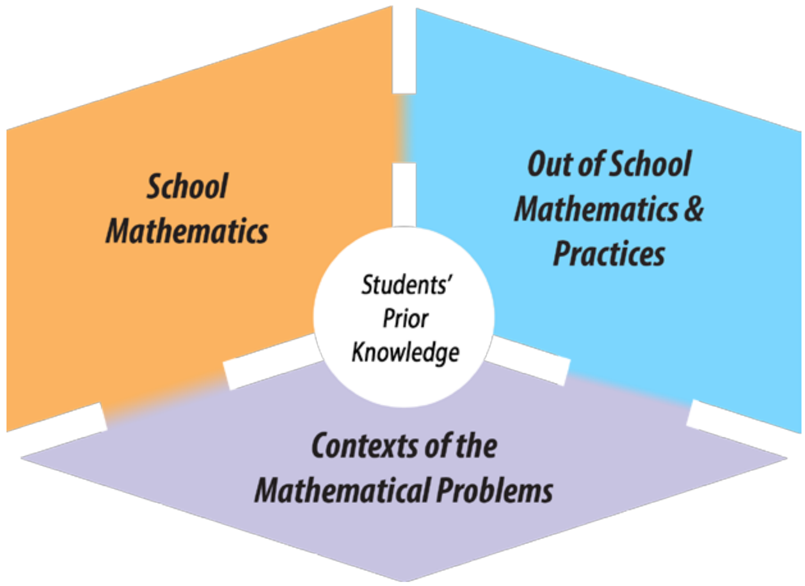

Figure 1. Three sources of students' prior knowledge (González, DeJarnette, & Deal, 2014).

I expect that by developing a better understanding of how teachers notice students' prior knowledge and how to support teacher learning to notice students' prior knowledge, teacher educators can promote instructional improvement. When teachers notice students' prior knowledge and use it in meaningful ways in their classrooms, they increase student learning opportunities. My main claim in this article is that mathematics education benefits from having the construct of teacher noticing of students' prior knowledge, as it allows to develop research to further the understanding of how teachers pay attention to specific aspects of student thinking, use student thinking in the classroom, and talk about student thinking with colleagues outside of the classroom. Research on teacher noticing of students' prior knowledge requires methodologies for examining how teachers notice and learn to notice inside and outside the classroom. I provide examples from a professional development intervention that aimed at increasing teacher noticing of students' prior knowledge, discuss critical components of the research, and propose further questions for research.

Unpacking Teacher Noticing of Students' Prior Knowledge

By teacher noticing, I refer to prior work establishing connections between what teachers see and how they act (or propose to act) in relation to classroom events (van Es & Sherin, 2002). This perspective is aligned with other studies that also consider teacher noticing as a selective process of sensory data (Sherin & Star, 2011). Jacobs, Lamb, and Philipp (2010) defined professional noticing of children's mathematical thinking with three actions: "attending to children's strategies, interpreting children's strategies, and deciding how to respond on the basis of children's understanding" (p. 172). My definition of teacher noticing of students' prior knowledge considers the actions of attending and interpreting students' prior knowledge. In addition, I consider that in settings where teachers discuss students' ideas, such as professional discussions with colleagues, teachers may not be able to act upon their interpretation of student thinking. Instead, teachers anticipate actions that they may perform as a result of analyzing students' ideas. Teachers' identification of actions that they anticipate they would perform is an example of a "re-vision" of practice (Horn, 2010). Teachers' collective interpretation of students' ideas using artifacts of instruction such as classroom videos or student worksheets leads teachers to establish connections with future work (Horn, Garner, Kane, & Brasel, 2016). That is, by noticing student thinking in discussions with colleagues, teachers can start connecting their analysis of students' ideas to specific teaching actions that consider those ideas.

Studying teacher noticing of students' prior knowledge

Examining how teachers notice students' prior knowledge requires considering whether the noticing occurs in classroom settings or in conversations among teachers. Making analogies and recalling prior knowledge when scaffolding exemplify how teachers draw upon students' prior knowledge in teaching. When making analogies, teachers establish connections between the target knowledge and a source for which students have some sort of prior experience (González, 2015). While scaffolding students who are solving problems in groups, teachers recall relevant prior knowledge needed for solving a problem (González & DeJarnette, 2015). A problem for researchers observing classroom instruction is that when teachers establish connections with students' prior knowledge they may not be labeling this action. Teachers often use linguistic markers such, "remember" or "as we did yesterday," to make the *instructional register* (i.e., the school content in the curriculum) explicit (Christie, 2002). Alternatively, teachers may rely on *metadiscourse* (Lemke, 1990) to make statements about content that students are expected to know (e.g., "you learned this last year"). However, at times, teachers do not make connections with prior knowledge explicit to their students. In addition, because the mathematics curriculum builds upon concepts and procedures previously studied, teachers may assume that students know that content when teaching new ideas. For these reasons, I consider when teachers identify the connections with students' prior knowledge during instruction, as well as when teachers make significant connections with students' prior knowledge, even if these connections are unmarked. In cases where the connections are unmarked, identifying what prior knowledge the teachers trigger—school mathematics, out-of-school mathematics, or a problem's context—can point to the actions that teachers perform to use students' prior knowledge in instruction. For example, if a teacher starts a lesson by discussing students' experiences with dance marathons before assigning a problem that uses a dance marathon as a context for teaching linear functions, then the teacher is making connections with students' prior knowledge explicit (Jackson, Shahan, Gibbons, & Cobb, 2012). Overall, identifying whether the talk is about mathematics or a problem's context can be helpful for tracking what possible connections with students' prior knowledge teachers are establishing and how.

Promoting teacher noticing of students' prior knowledge

My hypothesis is that professional development that promotes teacher noticing of students' prior knowledge in conversations with colleagues around *representations of practice* (Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson, 2009; e.g., videos or animations of classroom instruction) can support teachers' anticipation of actions for using that prior knowledge. Through discussions, teachers can develop a repertoire of actions for using students' prior knowledge during instruction. This hypothesis guided the design of a professional development intervention to promote teacher noticing of students' prior knowledge (Figure 2).

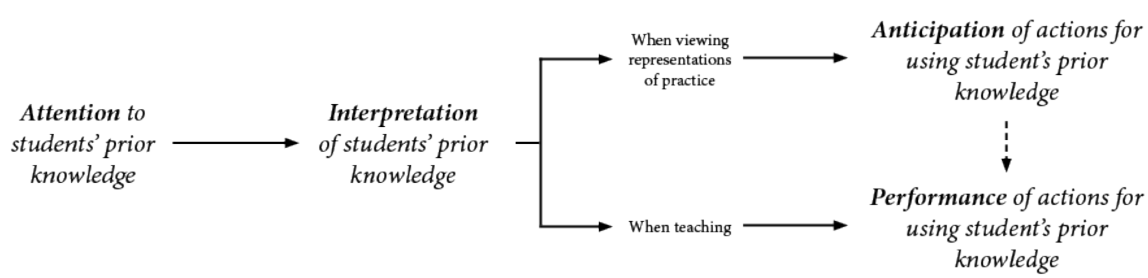

Figure 2. Teacher noticing and using of students' prior knowledge.

Two considerations shaped the design of the professional development intervention situated in the context of geometry instruction at the high school level. First, we did not have examples of geometry teachers using prior knowledge in problem-based instruction that we could use as resources for professional development in the form of videos. For this reason, my research group created tasks for eliciting students' prior knowledge and animations of classroom instruction showing possible ways in which a teacher can implement those tasks in a classroom. The animations are stories of instruction portrayed by cartoons with a voiceover (see Chazan & Herbst, 2012). In separate studies, we tested the tasks with high school geometry students and created the animations based upon our observations of how these students drew upon their prior knowledge to solve the problems (DeJarnette & González, 2016, 2017; DeJarnette, González, Deal, & Rosado Lausell, 2016; DeJarnette, Rosado Lausell, & González, 2015). The discussion of the animations in the professional development intervention supported the teachers' creation of tasks for their classrooms (González, 2017; González & Deal, in press).

Another consideration was that of promoting a common experience for the teachers to discuss students' prior knowledge. For this reason, the teachers engaged in a lesson study cycle (Fernandez, 2002) and created tasks for eliciting students' prior knowledge. Subsequent discussions of video clips from the lessons with examples of students solving the same problem intended to increase teacher noticing of students' prior knowledge. Video club is the activity of teachers watching and discussing video clips from their classrooms to study student thinking, and other researchers have used this activity to promote teacher noticing (Sherin & van Es, 2005). Replacing the live observations in lesson study by video clubs enabled us to make lesson study viable because of the difficulties of coordinating school visits. Moreover, we selected video clips from various classrooms showcasing how students' prior knowledge shaped their problem-solving strategies. The facilitators of the video club discussions promoted an inquiry stance by asking teachers to use evidence about student actions to support their analysis of students' mathematical thinking (González, Deal, & Skultety, 2016).

Figure 3 shows the cycle of activities in the two-year professional development intervention. While it is possible that the teachers started to notice and use students' prior knowledge in their first enactment of the lesson, the expectation was that, over time, the teachers would increase their attention to students' prior knowledge, in particular during the second implementation of the lesson. In the first year, the main activities were discussing animations, planning two lessons, and engaging in video clubs. During the video clubs, the teachers discussed students' prior knowledge and mentioned teaching actions for using students' prior knowledge. In the lesson revision process, the teachers modified the lesson according to their observations of student thinking (Deal & González, 2017; Skultety, González, & Vargas, 2017). The lesson revision process, informed by the video club discussion, provided opportunities for teachers to discuss teaching actions that they could perform to use students' prior knowledge. Teaching the revised lesson increased teachers' opportunities to elicit and use students' prior knowledge because the design of the task targeted students' prior knowledge based upon their observations in the first enactment of the lesson. In addition, the video club discussions helped the teachers to anticipate the interplay between students' prior knowledge and their problem-solving strategies.

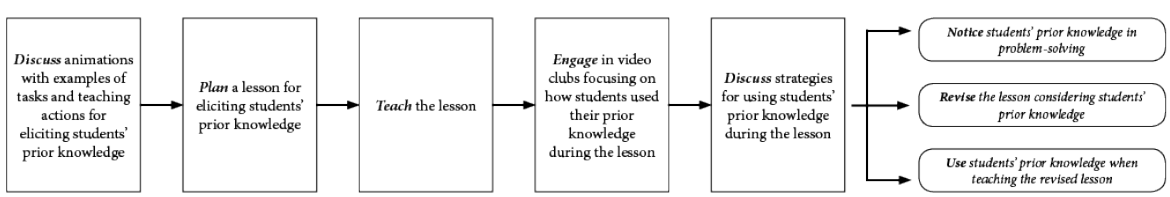

Figure 3. Cycle of activities for promoting teacher noticing of students' prior knowledge.

Overall, the challenge of not having a set of resources for illustrating how teachers can use students' prior knowledge in problem-based instruction in geometry opened opportunities for creating a professional development intervention centered on the teachers' own experiences and contexts. The combination of professional development strategies—discussing animations, lesson study, and video clubs—was intended to enable teachers to study how students use their prior knowledge in their own classroom. The teachers used representations of teaching in the form of animations to investigate examples of how teachers can use students' prior knowledge when teaching specific concepts. In addition, the videos from the lessons supported teachers' development of capabilities for analyzing student thinking as, in contrast to real time observations, they were able to pause and slow down the videos to unpack students' complex ideas (González, Deal, & Skultety, 2017). All the activities were linked by the common experience of teaching problem-based lessons that the teachers collaboratively planned. Other professional development programs, such as the Problem-Solving Cycle, combine video clubs and the experience of teaching the same tasks (Borko, Jacobs, Koellner, & Swackhamer, 2015). In our case, the teachers' engagement in the lesson study cycle supported teachers in becoming instructional designers and identifying strategies for maximizing the opportunities for using students' prior knowledge in the lesson. In addition, the opportunity to teach a revised lesson supported teachers in noticing and using students' prior knowledge, as they were better able to manage the uncertainty that characterizes problem-based instruction by anticipating students' solution strategies (Cohen, 2011).

An Example of Noticing Students' Prior Knowledge in Professional Development

As part of a project funded by the National Science Foundation, five high school geometry teachers participated in the professional development intervention for increasing their noticing of students' prior knowledge. The project is situated in the Midwestern portion of the U.S. All the teachers, Alexa, Clara, Erin, Gian, and Madeline, taught in high-needs schools, which are diverse schools in terms of social class and ethnicity, and had between 4 and more than 20 years of teaching experience.[2] The teachers met in 20 study group sessions. The sessions occurred after school hours and had a duration of 3 hours. Two facilitators with experience teaching geometry led the sessions, a lead facilitator (Facilitator 1) and the author, who is also the project director (Facilitator 2). The teachers engaged in the cycle of activities for planning and teaching a lesson twice. As a result, they created two lessons centered on geometry concepts in the Common Core Standards for Mathematics (Common Core State Standards Initiative, 2010). One lesson was on the concept of a perpendicular bisector and the other lesson was about the concept of dilation. For the purpose of illustrating how the professional development intervention promoted teacher noticing of students' prior knowledge, I use examples from the discussion of the dilation lesson. My goal here is to identify changes in teacher noticing of students' prior knowledge and in their anticipation of teaching actions for using students' prior knowledge in pre- and post-lesson discussions. I examine two questions: (1) *What prior knowledge did the teachers notice?* (2) *What teaching actions did the teachers identify to trigger or use students' prior knowledge?* The teachers' discussion of the dilation lesson provides an opportunity to examine issues pertaining to the goal of building *a professional knowledge base for teaching* (Hiebert, Gallimore, & Stigler, 2002).

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[2] In compliance with research procedures approved by the Institutional Review Board, all the names of persons, institutions, and locations in the study are replaced by pseudonyms.

Teacher noticing of students' prior knowledge in a dilation lesson

Teaching dilation is new in the geometry curriculum as a result of the implementation of the CCSSM. The CCSSM emphasize geometric transformations in the geometry curriculum. Dilation is a geometric transformation, together with reflection, rotation, and translation. A dilation can change the size of an object but not its shape. In order to perform a dilation, one needs to determine a center of dilation and a scale factor. The standard states, "A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged" (CCSS.MATH.CONTENT. HSG.SRT.A.1.A). Figure 4 shows an example of a dilation where O is the center. The segments that do not pass through O are parallel, namely $AB \parallel A'B'$, $AC \parallel A'C'$, and $BC \parallel B'C'$. The lines that pass through the center of dilation AA', BB', and CC' do not change. The triangle ABC was dilated using O and, as a result, we obtained triangle $A'B'C'$. One part of the definition considers the lines passing through the center of dilation and the other part considers the lines that do not pass through that center.

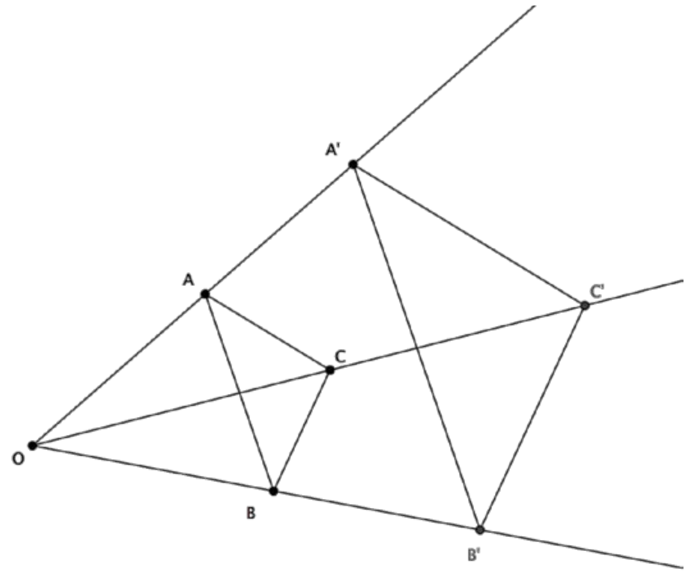

Figure 4. An example of a dilation.

In sessions 5 and 6, the teachers discussed animations with examples of two problem-based lessons to teach dilations. The first lesson in the animations concerned making a shadow puppets show (DeJarnette, Rosado Lausell, & González, 2015). The second lesson used the context of one-point perspective drawings (DeJarnette & González, 2017). The teachers decided to adapt the one-point perspective lesson for the lesson study cycle.

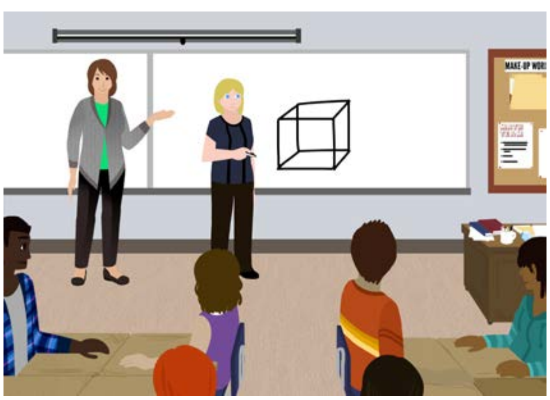

Figure 5. Scenes from the one-point perspective animations *Launch 4* and *Explore 3*.

Figure 6 shows the main diagram in the problem shown in the animation. The problem asked students to determine whether objects in the diagram were the same or different in real life. The key idea in the lesson was that whether corresponding points of pairs of objects in the diagram were collinear with the vanishing point, the objects had the same size in real life. In this case, the houses were the same height, but the trees were not.

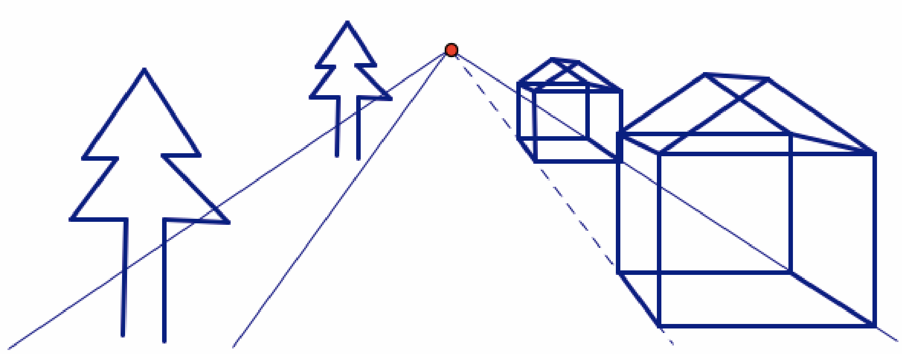

Figure 6. Diagram in the one-point perspective lesson (DeJarnette & González, 2017).

In four other sessions (7, 8, 18, and 19), the teachers had video club discussions about examples from students' work during the one-point perspective lesson. Here, I use examples from the first-year study group (sessions 6 and 7) to contrast teacher noticing of students' prior knowledge during the discussion of the animations, which occurred before the teachers taught the lesson and during the first video club after some teachers taught the lesson.

Teacher noticing during animation discussions. The discussion of the one-point perspective lesson started in session 6 with an animation showing an example of the teacher launching the problem (*Launch 4*). The teacher-character tells the student-characters that they will need to approach a problem like artists and invites a student-character to the board to draw a box. The teacher-character asks the class about experiences seeing paintings of landscapes. One student-character recalls a drawing of a sailing boat by her sister. The teacher-character elicits characteristics of perspective drawings and identifies important keywords for the lesson including “vanishing point” and “horizon.” The teacher-character refers to the example of a sailing boat because it has a horizon, which is also used in one-point perspective drawings.

In the session, the teachers’ initial reaction to the problem was positive. Alexa said, “I have no idea what the problem is, but I’m loving where this is going.” Erin stated that the context of art for teaching dilations was surprising: “It’s interesting, ‘cause when I think of dilation I don’t think of perspective and three-dimensional.” The teachers expected students to be intrigued by the problem, especially the students who like to draw. Alexa said, “we’re not saying anything ‘mathy’,” suggesting that by eluding references to mathematical ideas, the problem would be interesting for students.

The first mention of students’ prior knowledge in the session was by the facilitator[3] with the following question: “What different strategies did you see the teacher using to try to pull out kids and use their **prior knowledge** of drawing, like you said, not necessarily of mathematics, because we didn’t go there yet?”[4] The facilitator asked teachers to identify teaching actions in the launch for eliciting students’ prior knowledge. In response, the teachers identified the prior knowledge that the teacher-character elicited, namely, knowing how to draw a box, the drawing of a sailing by the student-character’s sister, and knowledge of architecture. The teachers also valued that the teacher-character made references to art history. When the facilitator asked for disadvantages of the launch, the teachers mentioned that they were unsure about the lesson’s connections with dilations, which made it difficult for them to assess possible disadvantages. Erin said, “I guess I don’t feel like I can say anything as far as warning, ‘cause I don’t know where it’s going.” The facilitator asked if students would be familiar with the term “perspective.”

[3] Unless noted, references to the facilitator are to Facilitator 1.
[4] In the transcripts, I note in bold explicit references to students’ prior knowledge. The transcription conventions are: overlapping speech is in parenthesis, explanatory notes within brackets and “—“for interruptions.

The teachers stated that the students have prior knowledge of perspective from their art classes and from playing the game Minecraft™ when they were younger.

The discussion of an animation showing an example of how the students solved the problem during the exploration part of the lesson (*Explore 3*) included limited comments about students' prior knowledge. Most comments pertained to understanding students' problem-solving strategies and the underlying mathematical ideas in a one-point perspective diagram. Nevertheless, the facilitator drew the teachers' attention to the possibility that students' prior knowledge of perspective drawing affected their problem-solving strategy.

Facilitator 1:	So, you think they kind of figured it out by, like, using the structure of the questions. So, the first question helped them, I don't know, hypothesize this is how it is. Do you think it's possible maybe she had **experience with drawing a perspective drawing in the past** too, maybe?
Erin:	Maybe because taking that approach of drawing that.
Gian:	But pattern matching too, if you establish a pattern where all lines hitting that point and one doesn't, you would kind of say, "this doesn't match the other pattern, so there's something different."

Even though Erin found plausible that students could rely on their experience with perspective drawing, Gian proposed an alternative that considered students' recognition of patterns in the diagram. Gian's idea that students could expect that in a one-point perspective diagram the perspective lines will be concurrent suggests that students may rely upon their prior experience working with diagrams. However, Gian did not explicitly refer to students' prior knowledge.

The teachers' concern about eliciting the concept of dilation from students' work on the problem led them to consider pedagogical moves for enabling students without prior experience with perspective drawing to identify important features of a one-point perspective diagram. Specifically, Alexa suggested showing a photo of houses lining up in one-point perspective during the launch, after the student draws the box on the board. She provided the example of a location in the peripheries of her school where one could see such view. The purpose of showing the photo would be to expose students to a one-point perspective diagram during the launch without making explicit references to dilation. Alexa mentioned an important idea regarding one-point perspective diagrams: "if somebody stood at the end of the street and took a picture, 'cause you would see, you know, as they look further away they look smaller." This idea is important because it underscores the experience of perceiving objects that are far away from an observer's point of view as smaller. In future sessions, the teachers would elaborate on this idea. She said, "Cause for the kids that haven't done the drawing, they are not going to get this." With this comment, Alexa called attention to some students' lack of prior experiences with perspective drawing. Showing the photo was a pedagogical move to support these students.

The facilitator made another reference to students' prior knowledge during the discussion of Explore 3. The teachers had been discussing whether the student-characters' discovery of two possible vanishing points constituted a solution to the problem. The two vanishing points in the diagram were the result of connecting corresponding points of the trees, which resulted in a vanishing point different than the vanishing point in the diagram (the red point in Figure 6). Both facilitators clarified that in a one-point perspective diagram there must be only one vanishing point, which is the center of dilation. Facilitator 1 stated that a class discussion about the possibility of having two vanishing points in the diagram could highlight the differences between dilation and similarity. This topic is relevant, as geometry students usually study similarity in previous years and possess prior knowledge of similarity. If the corresponding points of two similar figures and the vanishing point are not collinear, they are not in dilation.

That is, two similar objects do not require a center of dilation. However, the teachers did not discuss further the ideas of using students' prior knowledge of similarity.

The teachers' consideration of students' prior knowledge was more substantial during the discussion of an animation showing a possible summary for the one-point perspective lesson, Summary 1. In the summary, the teacher-character uses the context of the problem to introduce the definition of dilation. The teacher-character adds the perspective lines (i.e., the lines connecting the corresponding points of two similar objects and the vanishing point) to show that the trees are not in a dilation with respect to the same vanishing point established by the perspective lines connecting the corresponding points of the houses (Figure 7). The teachers anticipated students' confusion with the notion that the houses are not the same height in the diagram but represent houses that are the same height in real life. Alexa suggested a pedagogical move, "Well, I think you need to distinguish between are you talking about what it's representing or the actual drawing." Even though the teachers did not show connections with students' prior knowledge explicitly, they referred to the contrast between reading a one-point perspective diagram and everyday experiences. In the diagram, objects that have the same size in real life are represented with different sizes. According to the teachers, the way in which real life objects are represented in a one-point perspective diagram can cause student confusion.

Figure 7. Scene from Summary 1 where the line passing through the treetops does include the vanishing point.

In the session, the teachers discussed two other animations with examples of student exploration of a follow up problem and another version of the summary. However, these discussions did not elicit comments about students' prior knowledge. The main discussion themes were an analysis of the task in relation to curricular standards, unpacking a student's solution that relied upon measurements (based on DeJarnette & González, 2017), and assessing the viability of using the problem to introduce scale factors and proportions.

Teacher noticing during video club discussions. In session 7, following the session where the teachers discussed the one-point perspective animation, the teachers engaged in the first video club to discuss video clips from the dilation lesson taught by Alexa. In all the video clubs, the video-clips showed students working on the problem in groups with very limited or no teacher intervention at all. The criteria for selecting video-clips followed researchers' recommendation (Sherin, Linsenmeier, & van Es, 2009). The facilitators' goal was to promote an understanding of students' thinking using evidence from the video, the transcript, and copies of students' worksheets. Considering that the professional development intervention integrated various professional development strategies, the activities for session 7 varied, including a video club, discussion of animations showing other alternatives for the one-point perspective lesson, and discussion of revisions to the dilation lesson. Each activity had a duration of approximately 1 hour. Because of differences in the schools' curricular sequence, not all the teachers had taught the one-point perspective lesson by session 7, and the last portion of the session was meant to support others in preparing for teaching the lesson in the following weeks.[5] The video club discussion in session 7 was the first opportunity for the teachers to demonstrate noticing of students' prior knowledge in the one-point perspective lesson that they taught.

The facilitator started the discussion by showing the research hypothesis that the teachers had formulated in the previous session when planning the lesson: *Students will use their prior knowledge of one-point perspective to identify if there is a dilation*. In lesson study, the research hypothesis guides the design of the task so that the lesson becomes an instrument for testing the hypothesis (Lewis & Hurd, 2011). In addition, the research hypothesis guides the observations and the lesson debrief. The teachers discussed three video clips. The last two video clips were from the same group of students at different moments during their exploration of the problem. After showing each video clip, the facilitator started the discussion with two questions: (1) How

[5] Elsewhere, we report how teachers' modifications to the lesson were based upon video clubs, animation discussions, and lesson observations (Skultety, González, & Vargas, 2017).

was each student thinking mathematically about the problem? (2) How was each student using their prior knowledge to work on the problem? The teachers had engaged in video clubs previously in sessions 4 and 5 for the perpendicular bisector lesson. Therefore, the facilitator had already established the norms for video clubs, such as the importance of making claims using evidence and focusing on one student at a time.

In reaction to the first video, Alexa stated that the students seemed to be relying upon their prior knowledge from art classes.

Alexa:	It seems like, I think, they, they're getting this not based on their **mathematical background knowledge or prior knowledge** but the fact that a lot of **them have done it** in art.
Facilitator 1:	Okay, so you think um—
Alexa:	But I think they used **that knowledge,** so it's not specifically math, but um—
Erin:	What make you think of art?
Alexa:	Because a lot of them had said they had done one-point perspective.
Erin:	Oh, they said it, okay.
Alexa:	Yeah, yeah.

While it is unclear whether Alexa was using information learned when teaching the lesson instead of evidence from the video, the facilitator used this opportunity to probe more into how students' knowledge from art was evident in their solution strategies. The facilitator asked them to focus on one student, Brenda.

Facilitator 1:	So, I, maybe like if we look at Brenda specifically, what did she kind of explain at the beginning of the problem? Is there anything in that sort of description that you think could come from that **art prior knowledge** that she had, or?
Alexa:	Well, I mean she, she really seems to understand the concept of what vanishing point represents, and which one's further away and that kind of stuff.

Alexa supported her statement that students were relying on their prior knowledge of art by identifying Brenda's understanding of the vanishing point and the viewers' distance from objects in the diagram. Even though Alexa could have been relying upon her knowledge of students' prior experiences in art classes, she identified specific ways in which Brenda's prior knowledge of art was evident in her solution. Similarly, in the discussion of the solution strategy of another student in the same group, Yasera, Alexa proposed that her prior knowledge of art influenced her solution. She said, "But I think that goes back to her **prior knowledge of, in art**, when they did one-point perspective, there was always a horizon." The horizon is the horizontal line that passes through the vanishing point in a one-point perspective diagram. According to Alexa, Yasera's initiative to add the horizon to the diagram was because of her prior knowledge of art. When asked to explain further whether in art class the students used the term "horizon," Alexa said that she did not know.

Facilitator 2: So, you think that the idea of vanishing point is also something, **a term that they use in art class**? 'Cause you said that she added the horizon maybe because of art class.

Alexa: Um, I think, I think in art class they refer to it as one-point perspective. I don't know if they call it anything different. I don't know. I haven't been to art class.

In the two instances, Alexa noticed students' prior knowledge of art shaping their solution strategy with specific examples of students' interactions with the diagram. According to Alexa, the students' interpretation of the vanishing point and proximity, as well as the students' initiative to add the horizon, were based upon their prior knowledge of art.

When discussing the second video, the first mention of students' prior knowledge was by Clara who asked Alexa about the mathematical ideas she had introduced in previous lessons. Clara's question surfaced in relation to their analysis of a student's solution strategy. Alexa stated that the students did not understand that a one-point perspective diagram represents real objects. She used as evidence Chris' references to scale factor.

Clara: I was gonna ask, **what did you talk about, about scale factor before**? Did you, have you talked about it? I'm assuming in class?

Alexa:	We had just started.
Facilitator 2:	I wasn't there.
Alexa:	Right. I think the day before, we had just started that target, and we just got into kinda, the terminology, you know, ratio, scale factor, and just getting into it.

Clara's question suggests that students' use of scale factor could be connected to their mathematical prior knowledge of content discussed in class prior to the lesson. Later in the discussion, the facilitator asked about the prior knowledge of another student in the group, Shawn.[6] According to the teachers, Shawn tried to establish connections with mathematical content previously discussed in class.

Facilitator 2:	So, what's Shawn's **prior knowledge**, here? How's he using it?
Gian:	Shawn's **prior knowledge** is scale factors, stuff they learned yesterday.
Alexa:	Yeah, he's trying, he's trying to—
Facilitator 2:	Shawn is—
Gian:	Trying to apply it.
Facilitator 2:	But that's Chris, the one who's saying yesterday.
Gian:	Well, he's the one that says that.
Alexa:	Yeah, but Shawn's the one that's pulling in numbers. So, I think he's, he's trying to pull in that scale factor because we talked about ratios and numbers. So, he's trying to figure out where that comes in.
Erin:	So, it's kind of interesting, if you didn't talk about ratios and scale factors, where would he have gone?
Alexa:	Right.

Erin pondered how the student's solution would be different without the teacher's mention of ratios and scale factors the day before. This comment shows that the teachers considered the interplay between students' solution strategies and their prior knowledge. In particular, given that Shawn was apparently drawing from mathematical content that the teacher had introduced the day before, the students could be more amenable to use that content.

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[6] In this point in the session and in the following excerpts, Facilitator 2 was leading the session.

While the discussion of the first video centered on students' prior knowledge of art, the discussion of the second video centered on students' prior knowledge of mathematical content taught prior to the lesson. The facilitator asked whether the students in the second video seemed to be drawing upon their prior knowledge of art, but the teachers did not see any evidence of that in their solution strategies. This point exemplifies the issue that noticing students' prior knowledge is contingent to the strategies that the students display. For this reason, designing a task that elicits different sources of prior knowledge and showing contrasting cases where students draw upon different sources of prior knowledge when solving a problem can support noticing.

Even though the teachers did not find evidence of students' prior knowledge of art in the second video, they identified students' confusion about interpreting the diagram as a representation of real life objects in relation to their prior knowledge. Erin stated that students did not possess knowledge about interpreting a one-point perspective diagram. Alexa added that students' prior knowledge of similarity also affected their interpretation of the diagram. Two similar figures that have a different size are meant to have a different size. However, in a one-point perspective diagram, two figures with different sizes can represent objects of the same size in real life.

Erin:	I, I think that they are really confused, and so talking about, like, **prior knowledge** or whatever, I think that, like, they're just not, like, that they're confused and like, "how do I know if it's the same height or a different height," because on paper it looks something and in real life—and I, I think that whole scenario, is, is, is what's like mind-boggling for them, and—
Alexa	Well, and I think—
Erin:	Then, they're like, "I don't know how to justify this!"
Alexa:	the other two units have all been about sim—here's two similar figures, set up their proportion to solve—
Erin:	Yeah.
Alexa:	for these two sides. So, we've been dealing with just two random, similar figures.

Erin:	And now you're asking them, "Are these heights the same?" And they're like—
Alexa:	Exactly.
Erin:	Ahh, like, "what?"
Alexa:	So, they're seeing, they're seeing two similar figures, they're not, like you said,
Erin:	Yeah, mhm.
Alexa:	They're not seeing it, that this is representing real life.
Erin:	Yeah, and so, I think, that's the **prior knowledge** that's probably really lacking and whatever and it's like—
Alexa:	I wonder if it would—
Erin:	like that's causing a lot of problems here.
Erin:	(And—)
Facilitator 2:	(The confusion between real life) and the representation (of real life.)
Erin:	(Yeah.)
Erin:	And especially more so if you're, if they're coming off of all this intense, like, similar, are they similar, are they not the same.
Alexa:	Right. Exactly.
Erin:	But now, we're asking, 'cause they're clearly not the same, but are they the same? Like, what are, what does that even mean?
Alexa:	Right.
Erin:	Like, whoo.
Alexa:	But I, I wonder if we had an, an actual, real picture of buildings, and asked the question, "Are these buildings the same height?"

In this excerpt, instead of noticing the prior knowledge that the students possessed, the teachers identified the prior knowledge that the students were lacking for interpreting one-point perspective diagrams. In addition, according to the teachers, students' mathematical prior knowledge affected their interpretation of the diagram because one-point perspective diagrams are different from geometric diagrams showing similar figures. At the end of the excerpt, Alexa provides a pedagogical move to address students' confusion by showing a photo of actual buildings. It seems that using the photo can trigger students' knowledge of a familiar place with buildings that have the same height but appear as having different heights in the photo. Implicit in this pedagogical move is the idea that relying upon students' prior knowledge of the location can support their understanding of one-point perspective diagrams. This example shows how teacher noticing of students' prior knowledge led to anticipating pedagogical moves for students to develop mathematical understandings.

In the discussion of the third video, students' prior knowledge surfaced when Gian established a contrast between the two groups of students the videos. Gian said, "the strategy between this group and this group, which these kids have some **prior knowledge** and these kids **didn't**, is different. 'Cause these kids are talking more about the lines and these kids are talking more how long are the lines." Here, Gian referred to students' prior knowledge of interpreting a one-point perspective diagram being different between the two groups and affecting their solution strategies. Later in the discussion, Gian proposed adopting a new task that avoids the distinction between reality and a representation of reality.

Gian: I, I keep thinking that, if we wanted to leverage this, and this is kind off topic, but [inaudible], um, if we wanted to leverage their **prior knowledge**, I think their best dilation that they could perceive is when you have a picture, and you blow it up or you put a picture on the screen, you have a picture up there, and you say "Well, in my monitor the picture is this big, up there it's a different size," you know? 'Cause then they can start to relate to the fact that they are different sizes. You don't have to worry about the reality thing, I think, is what I'm trying to go to. And they can, in their mind, they can picture that, so we talked about that, and then in relation to that we talk about, "Well would the angles change?" So, we use that to focus on the fact that you don't get a distorted picture, you get what you're talking about. Where all of the angles are gonna be congruent, but the sizes are all, changed, and so that's kinda, I don't know how to leverage that. Do you know what I'm saying? 'Cause **the prior knowledge** was missing from one group and not in the other group. But however, if you talked about taking a picture and blowing it up, it's something that most people have **prior knowledge** to [inaudible].

Alexa: Well yeah, 'cause **they've all done it** in, like, Word, when they've inserted images and that kind of stuff.

Gian: Yeah.

Alexa: Made it bigger, smaller, or whatever, yeah.

Gian: So, I don't know how to link that to a lesson, but I, I think that, you know, it'll be at least something that they have a more common theme that more people have more **common knowledge**.

Gian proposed a new task that would eliminate the confusion that surfaced with a one-point perspective diagram. The task was situated in the context of enlarging a diagram.

According to the teachers, most students possess experience with enlarging a figure, such as working with a word processor, which they could apply to this problem. This discussion shows another example where the teachers established a connection between students' prior knowledge and pedagogical moves that would support them in the classroom to develop mathematical understandings. While the notion that there is prior knowledge that some students do not possess is one that was not established in the professional development intervention and may suggest a deficit perspective on learning, the teachers used this language to contrast differences in students' experiences that may affect their problem-solving strategies. Keeping these differences in mind, the teachers strived for a context that would benefit most students. In other sessions, the teachers decided to continue to teach the one-point perspective lesson with modifications for supporting students' understanding of the context when their prior knowledge of that context was limited (Skultety, González, & Vargas, 2017).

Summary. The examples from the two consecutive sessions, pre- and post-teaching the one-point perspective lesson, show a development of teacher noticing of students' prior knowledge from less to more specific. The animations enabled the teachers to consider students' prior knowledge of one-point perspective drawing from art classes and from playing video games such as Minecraft™. In contrast, the video club discussions showed more complexity in the teachers' identification of students' prior knowledge. Specifically, the teachers identified how students' understanding of components of a one-point perspective diagram (i.e., the vanishing point and the horizon) supported students' mathematical understanding. In the animation discussions, the teachers anticipated students' difficulties interpreting a one-point perspective diagram as a representation of reality. In the video club, the teachers discussed specific examples of how this difficulty plays out in the classroom. The teachers also identified the experience of seeing objects far away as small in real life as something that could support students' understanding of one-point perspective. In contrast with the animation discussions, the video club included consideration of students' prior knowledge of mathematical content. Specifically, the teachers identified how students' prior knowledge of mathematical content such as scale factors and ratios shaped their solution strategies. In addition, the teachers contrasted students' knowledge of how to read diagrams of similar figures and how to read one-point perspective diagrams. Therefore, post-lesson discussions broadened opportunities for the teachers to identify examples of various sources of prior knowledge and to investigate how students' prior knowledge shapes their understanding of mathematical ideas.

Teacher noticing of students' prior knowledge provoked the identification of pedagogical moves to support students' mathematical understanding. These moves included ways of launching the problem, modifications to the wording of the task, using a photo of a known location, and changing the task to one involving investigating objects that had been enlarged. The discussion of students' prior knowledge in the lesson using two types of representations of teaching—animations and videos—enabled the teachers to identify specific teaching strategies for addressing specific aspects of students' prior knowledge.

Figures 8 and 9 illustrate the interplay between the prior knowledge that the teachers identified and the pedagogical moves that they would perform to use students' prior knowledge. In some cases, the teachers identified students' prior knowledge but did not connect it to pedagogical moves in the sessions discussed.[7] The arrows demonstrate that, at times, the identification of students' prior knowledge led to considering pedagogical moves and at other

[7] In other sessions, the teachers elaborated on these ideas and identified other pedagogical moves. For example, the teachers decided to use a video of MarioKart™ in the launch to support students' understanding of far and close objects in one-point perspective diagrams based upon an animation shown in another session (see Skultety, González, & Vargas, 2017).

times, the identification of pedagogical moves led to identify students' prior knowledge. In the animations and the video club discussions, the teachers identified students' lack of knowledge about understanding one-point perspective diagrams and connected to the pedagogical move of showing a photo of real objects. While talking about pedagogical moves does not warrant that teachers will perform these moves in instruction, collective interpretation of students' ideas that is connected to teachers' future work increases teacher learning opportunities (Horn, Garner, Kane, & Brasel, 2016). Our analysis of teachers' modifications to the lesson shows that the teachers drew upon their observations of student thinking to optimize student learning opportunities (Skultety, González, & Vargas, 2017). A question is whether teachers' knowledge of possible difficulties with a problem would result in lowering the cognitive demand of the tasks by eliminating challenging aspects of the problem.

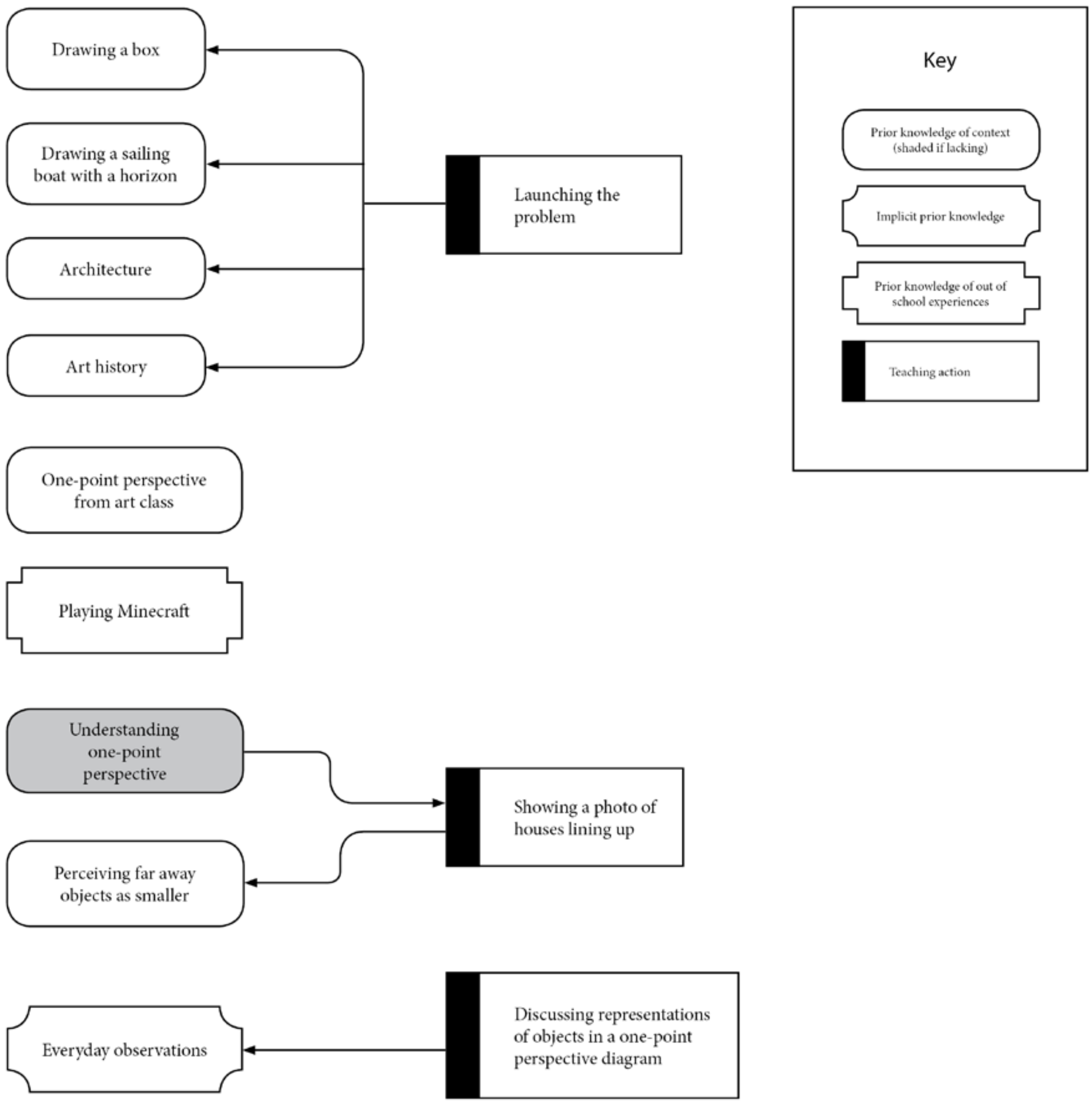

Figure 8. Interplay between noticing of students' prior knowledge and pedagogical moves during animation discussions.

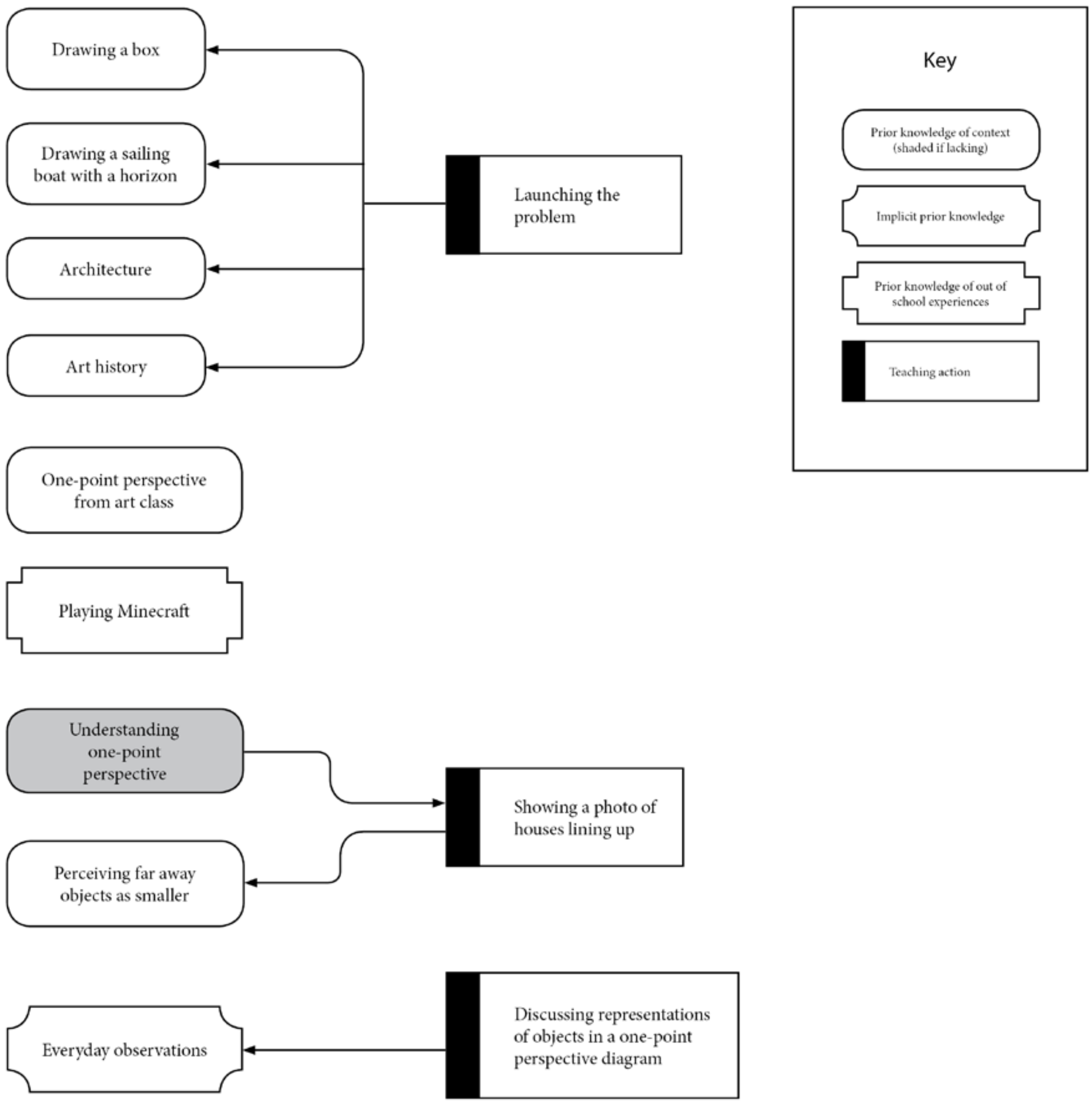

Figure 9. Interplay between noticing of students' prior knowledge and pedagogical moves during the video club.

Implications for Mathematics Education

In a piece about mathematics education research, Kilpatrick stated that new constructs could be valuable contributions to the field. He said, "A researcher makes a contribution to our field by providing us with alternative constructs to work with that illuminate our world in a new way, and not simply by piling up a mass of data and results" (Kilpatrick, 1981, p. 27). A new construct can provide opportunities for researchers to notice a phenomenon. Having a new construct also provides the language to label and classify a phenomenon, to communicate with others who are also studying that phenomenon, and to make that phenomenon memorable (Bowker & Star, 1999). In addition, a new construct supports the process of developing methodologies to further the understanding of that phenomenon. The construct of teacher noticing students' prior knowledge extends the original notion of teacher noticing, which focused on a fleeting moment by requiring teachers' extensive interpretation of student thinking (B. Sherin & M. G. Sherin, personal communication, April 28, 2017). Considering the recent increase of constructs focusing on specific aspects of teacher noticing, I suggest that the construct of noticing students' prior knowledge is valuable in light of the development of a knowledge base for the teaching profession.

Developing a knowledge base for teaching

According to Hiebert, Gallimore, and Stigler (2002), instructional improvement entails a transition from teacher knowledge to professional knowledge. They posit that teacher knowledge is detailed, concrete, and specific. In contrast, professional knowledge is public, shareable, can be stored, and subject to verification. Having a construct such as noticing of students' prior knowledge can enable teacher educators to design interventions for eliciting teachers' detailed, concrete, and specific knowledge about a particular aspect of student thinking, students' prior knowledge. When teachers attend to and identify students' prior knowledge, they can make what they noticed public. More importantly, teachers can establish connections between students' prior knowledge and pedagogical moves that elicit and use that knowledge to support students' mathematical understanding. Teachers' practitioner knowledge stems from the particular context where the teachers teach, the particular students, and the particular content of a lesson. By making teachers' practitioner knowledge public in conversations with other teachers, there is a transition to professional knowledge.

The professional development intervention that I described promoted teacher noticing of students' prior knowledge through the analysis of a lesson about dilation. The context framing the problem was one-point perspective drawing and considered students' prior knowledge of art, practices of interacting with diagrams, and mathematical concepts such as similarity and ratio. The discussion of animations and videos enabled the teachers to identify the interplay between students' prior knowledge and their solution strategies. By noticing students' prior knowledge, the teachers also identified possible pedagogical moves for supporting students' mathematical understanding. The lesson study cycle provided an opportunity for the teachers to perform some of these moves. In addition, the teachers archived their knowledge about students' thinking by modifying the lesson considering students' ideas (Skultety, González, & Vargas, 2017). The professional development model integrating animation discussions, lesson study, and video clubs supported the transition from practitioner to professional knowledge by making teachers' knowledge public through their opportunities to share insights about students' prior knowledge, verify hypothesis about student thinking, and keep a record of the lesson in their modified version of the worksheet.

Challenges

I suggest that there are three main challenges with applying the construct of teacher noticing of students' prior knowledge: the insufficient tasks for eliciting students' prior knowledge, difficulties in identifying what constitutes students' prior knowledge, and dealing with teachers' identification of prior knowledge that students are lacking. Promoting teacher noticing of students' prior knowledge requires tasks that enable teachers to elicit students' prior knowledge, beyond their prior knowledge of school mathematics. In our case, we did not have good examples of videos for engaging teachers in video clubs about students' prior knowledge. As a result, we created tasks and developed the animations with examples of implementations of the task for the teachers to design a lesson. Supporting teachers' involvement in instructional design can increase opportunities for students to establish connections between their prior knowledge and topics in the mathematics curriculum. Proponents of the Realistic Mathematics Education theory have taken into account the importance of contexts in instructional design (Gravemeijer & Doorman, 1999; van Den Heuvel-Panhuizen, 2005). Current work that supports teachers' adaptation of tasks for the purpose of eliciting students' multiple mathematical knowledge bases also supports teachers' involvement in instructional design (Drake, Land, Bartell, Aguirre, Foote, McDuffie, & Turner, 2015). Examining teacher noticing of students' prior knowledge through tasks that target various concepts, content areas, and grade levels may broaden our understanding of how to support teacher learning.

The second challenge is that there could be no limits to what is considered to be prior knowledge. Since learning something new is always based upon what is previously known, it can be hard to pinpoint the relevant prior knowledge that students use or that teachers notice. In my analysis of the study group discussions, I mostly relied upon the teachers' and the facilitator's identification of prior knowledge in their talk. However, at times I used other linguistic tokens, such as references to what students have studied before to identify the prior knowledge that the teachers noticed. Similarly, teachers' implicit references to students' prior experiences, such as their observation of local buildings that have the same height, were evidence for me of teacher noticing of students' prior knowledge. Developing methods for identifying what constitutes evidence that teachers noticed students' prior knowledge, especially in discussions of classroom episodes, can support research.[8]

A final challenge pertains to teachers' discussion of the prior knowledge that students are lacking. These discussions can support a deficit perspective where teachers focus on students' limited knowledge or experiences. While in the professional development intervention described the facilitators did not foster discussion of students' lack of prior knowledge, the teachers used that language to make references to some students' limited experiences with art. As a result, the teachers identified ways to support all students in understanding the underlying assumptions of a one-point perspective diagram. For example, in the second year of the intervention, two teachers taught a unit on one-point perspective drawing prior to the lesson. The unit provided equal opportunities for all students to draw upon knowledge of perspective drawing when working on the one-point perspective lesson. However, it is possible that in other scenarios, the teachers' focus on the prior knowledge that students are lacking reinforces stereotypes or limits the teachers' capabilities to identify their students' strengths. Considering prior research that establishes how teachers develop conceptions of students in professional conversations (Horn, 2007), professional development must attend to ways to promote conceptions that support equitable mathematics instruction.

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[8] Elsewhere, I show evidence of teacher noticing of students' prior knowledge in study group discussions (González & Skultety, 2017).

Possibilities

Despite the challenges, the possibilities of promoting the development of professional knowledge for teaching by having the construct of noticing students' prior knowledge are promising. The construct enables designing teacher education initiatives that are focused on one specific aspect of teaching and that consider the teachers' context. Professional development resources for promoting teacher noticing of students' prior knowledge, such as the animations and the lesson plans, can be available to other teachers. The development of session facilitators can consider the goal of increasing teacher noticing of students' prior knowledge to promote their capabilities for supporting teacher learning. Finally, considering that students' prior knowledge varies according to the content taught and the students' context, professional development would need to be tailored to the teachers' particular needs and, as a result, it would be more relevant to them. Professional development that draws upon teachers' expertise and promotes agency are key for supporting an inquiry stance (Cochran-Smith & Lytle, 2009).

The intervention described is an example of what Borko (2004) calls phase 1 of professional development, as it involves testing an intervention in a particular location by studying the relationship between the professional development program and the participating teachers. The intervention showed that integrating animation discussion, lesson study, and video clubs is viable. Phase 2 would involve implementing the professional development in various locations with various facilitators. Scaling up the program would enable examining how to support teacher noticing of students' prior knowledge while still attending to contextual and content issues. It will also enable to investigate how facilitation varies across sites and learning opportunities that the facilitators foster (Amador & Carter, 2015; van Es, Tunney, Goldsmith, & Seago, 2014). Since students' prior knowledge depends upon their context, examining the implementation of the same tasks in different geographical locations can allow for examining how to develop teachers' professional knowledge in relation to those contexts and experiences. Finally, the creation of professional development resources can broaden teachers' access to learning opportunities for noticing students' prior knowledge. Overall, professional development that is centered on teachers' attention to student thinking and that values teachers' practitioner knowledge is key for instructional improvement.

Further questions

My investigation of ways to promote teacher noticing of students' prior knowledge leads me to propose three new questions. The first question is whether teacher noticing of students' prior knowledge is transferable. For example, in the intervention described, the teachers' discussions of students' prior knowledge were particularly situated in the context of teaching a lesson about dilation. The prior knowledge that the teachers identified, as well as the strategies that they described for eliciting and using students' prior knowledge, were connected to the concept of dilation and the one-point perspective context. It is possible that the teachers apply to other lessons the same heuristics for designing a task that elicits students' prior knowledge and for noticing students' prior knowledge. During interviews, the teachers who participated in the intervention stated that they became more aware of how decisions about launching a problem affect the lesson and of students' complex ideas (Martin, 2015, 2016). Lesson study centers in one lesson with the intention of supporting instructional improvement (Lewis, Perry, & Hurd, 2009). Further work should examine whether teachers' experiences with one lesson result in increasing their awareness about students' prior knowledge when teaching other lessons.

A second question is how teacher noticing of students' prior knowledge affects teachers' instructional decisions. According to research on the *practical rationality of mathematics* *teaching*, teachers' decisions respond to their understanding of what they consider to be viable options in instruction (Herbst & Chazan, 2011). In the intervention described, when teachers noticed that some students had difficulties reading a one-point perspective diagram, some suggestions for helping students could have resulted in lowering the cognitive demand of the task by eliminating potential challenges that could support students' development of mathematical understandings. In addition, strategies for supporting students' difficulties can spur new instructional problems. For example, in order to address students' difficulties understanding that in a one-point perspective drawing objects that are far away are smaller than objects that are closer to the observer, some teachers showed examples of a driving video game during the launch of the problem (Skultety, González, & Vargas, 2017). In the lesson, some students assumed that the tree and the house closer to the vanishing point were supposed to be small in the diagram. These students did not problematize the issue of determining the height of the objects, which resulted in trivial answers. There was not enough motivation for the students to investigate the heights of the objects. In this case, noticing students' prior knowledge provoked a pedagogical move that affected the implementation of the lesson in ways that the teachers had not anticipated. Even though it can be challenging for teachers to anticipate all the consequences of instructional decisions, the awareness of students' difficulties can provoke changes in the implementation of a task that limit students' opportunities to engage in mathematical activity.

A third question is about the facilitation of professional development. Recent work has called attention to facilitation moves that open opportunities for teacher learning in professional development (van Es, Tunney, Goldsmith, & Seago, 2014). In relation to noticing students' prior knowledge, an open question for facilitation is how to manage when teachers either do not notice or do not make meaningful connections with instructional issues. In the case that I described, there was an instance where the facilitator suggested drawing upon students' prior knowledge of similarity to provoke the analysis of the possibility of having two vanishing points in a one-point perspective diagram. However, the teachers did not discuss this suggestion further. The example helps to illustrate cases where the facilitator of professional development sees possible connections with students' prior knowledge and should decide how to prioritize the connections in relation to the teachers' development of noticing or teachers' mathematical knowledge for teaching.

Overall, the three questions consider issues of teacher knowledge, teacher decision-making, and professional development. It could be that these questions are specific not only to noticing of students' prior knowledge but also to other research on teacher noticing as well. Advances in the study of teaching provide opportunities to create professional development initiatives for supporting teachers' professional growth. Teacher noticing as a construct enables the examination of the interplay between student thinking and teaching. The construct of teacher noticing of students' prior knowledge can be useful in the field for sustaining professional development that is centered on students.

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