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Positioning Pre-service Teacher Beliefs along the Traditional-Reform Continuum: An Examination of Normative Beliefs and Discursive Claims

Mary Elizabeth Riley Lloyd

Malia Howell

Positioning Pre-service Teacher Beliefs along the Traditional-Reform Continuum: *An Examination of Normative Beliefs and Discursive Claims*

Mary Elizabeth Riley Lloyd[1]
College of Charleston
&
Malia Howell
System Wide Solutions, Inc.

Abstract: This study examined a typical sample of K-8 pre-service teachers (PTs) enrolled in typical teacher-preparation programs (TPPs) to provide insight about where on the traditional-reform continuum PTs' beliefs are positioned. To ensure accuracy in results, a sequential explanatory design was utilized in the examination of the PTs' normative beliefs assessed three times throughout their TPPs (using Likert items and open-ended questions), relationships between other PTs' and in-service teachers' (ITs') beliefs, and alignment between normative beliefs (what PTs believed they should do) and discursive claims about their teaching (what PTs claimed to do). Results of this study – particularly related to beliefs that are not positioned as far along the trajectory toward reform – are intended to assist mathematics educators and PD developers in targeting future instruction to meet PTs and ITs where they are. Beliefs related to "The power of students' ideas"; "Critical thinking, problem solving, and understanding, justifying, and communicating processes and their connections to resulting answers";

[1] lloydb@cofc.edu

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"Relevance"; and "Productive struggle" were identified as being strong-reform, positioning them the farthest along the continuum toward reform. Beliefs related to "Telling students answers"; "Process v. answer"; and "Collaboration" were classified as being mid-to-weak reform, positioning them lower on the continuum. Beliefs related to "Computation" and "Expository teaching" were categorized as being strong-traditional, positioning them closest to the traditional end of the continuum.

Keywords*:* pre-service teachers; mathematics education; normative beliefs; discursive claims; traditional-reform continuum

Introduction

For over 25 years, the National Council of Teachers of Mathematics (NCTM, 2000) has articulated and worked toward a vision in which all students have access to excellent mathematics instruction intended to develop 21st-century skills such as critical thinking, problem solving, evidence-based argumentation/ reasoning, and communication (National Research Council [NRC], 2012). Despite NCTM's efforts, actualization has been slow, as attaining said vision requires a shift in the dominant culture's deeply-rooted beliefs related to mathematics and the teaching and learning of mathematics (hereafter, "MLT beliefs"). Such cultural shifts are part of an evolutionary process that is gradual and requires time (Hefendehl-Hebeker, 1998).

While NCTM's vision is well articulated and known, as are the desired shifts in beliefs and practices toward its realization, according to Sarason (1996), attempts at desired shifts are often made without considering where the objects of change are positioned. An important question to consider then is, after nearly three decades of reform efforts, where are current beliefs situated within the evolutionary process? Answering this question in totality is beyond the scope of one paper, as the unit of concern is culture which requires examination of all constituent beliefs, such as those of parents, pre-service teachers (PTs), in-service teachers (ITs), students, administrators, and policy makers. However, to contribute to answering this complex question, the unit of analysis for this study was PTs' beliefs. Specifically, this paper aimed to answer the questions: Where along a continuum of traditional-reform beliefs do a sample of typical PTs' beliefs fall? What potential implications might this study's data imply related to a cultural shift toward more innovative beliefs?

Context and Frameworks

Why Cultural?

Culture is an elusive term that is difficult to define, making it challenging to study. Most anthropologists agree that aspects of culture – such as beliefs – are shared, dynamic and evolving, learned or transmitted socially through generations, and patterned (Spradley & McCurdy, 1972; Jacob, 1995; Mead, 1951; Spindler, 1963; Tylor, 1871). Most definitions reveal the interplay between individuals and culture (Spindler, 1980). Culture as an entity exists, in which individuals learn the rules either through enculturation, if born into a given culture, or acculturation, if introduced to a new culture. Individual beliefs, behaviors, and actions assist in the process of cultural transmission or, if different from the dominant culture, influence change in existing cultural norms over time in the process of cultural transformation.

MLT beliefs are cultural; among the dominant culture, there are shared meanings about the nature of mathematics and shared rules and expected behavioral patterns related to how mathematics should be taught and learned. These shared meanings, shared rules, and patterns of behaviors have been transmitted and learned over the course of generations. These beliefs have gradually changed somewhat over time – with early studies reporting that traditional beliefs were "resolute" and more recent studies reporting minor successes in shifting mathematical beliefs of PTs during TPP; such gradual change is indicative of the dynamic and evolving nature of these beliefs (Cross Francis, Rapacki, & Eker, 2015, p. 340; Fives, Lacatena, & Gerard, 2015, p. 261). Gradual change – part of the process of cultural transformation –

indicates that individual beliefs and ideas influence cultural beliefs and ideas just as much as cultural beliefs influence "socially constructed and enacted" individual beliefs – that is, they co-construct one another (Seeger, Voight, & Waschescio, 1998; Cross Francis et al., 2015, p. 337). Given the cultural nature of MLT beliefs, the realization of NCTM's vision requires more than convincing legislature or district administrators and having them mandate innovations that trickle down to classrooms; rather, a cultural shift must occur in which the long-standing, traditional beliefs about mathematics must evolve, as well (Seeger, Voight, & Waschescio, 1998).

Continuum

For the purpose of framing this study, Spindler's (1963) framing of cultural value shifts along a "transformation line" from "tradition-centered" to "emergent" core values was adapted (pp. 136-137, 142). Though a simplistic framework for complex contexts, knowing the trajectory of a shift along a "transformation-line" or continuum can be helpful in understanding position within an evolutionary process (p. 142).

Traditional v. reform-based cultural beliefs. Cultural beliefs about mathematics and the teaching and learning of mathematics can be situated along a continuum ranging from (1) the dominant culture's predominant traditional beliefs, also known as unproductive beliefs, which are named as one of the main impediments to the realization of the NCTM's vision, to (2) reform-based, innovative beliefs, also known as productive beliefs toward the actualization of this vision (NCTM, 2014).

A powerful traditional, unproductive cultural belief is that "some people are good at mathematics while others are not" (Lloyd, Veal, & Howell, 2016, p. 362; Frank, 1990). This belief can potentially restrict equal opportunity for *all* students to acquire 21st-century skills. Even when teachers believe that all children can learn mathematics, their beliefs about the nature of mathematics and how it should be taught may be unproductive. Traditionalists view mathematics as being comprised of unchanging truths, consisting of basic facts and procedures (Oakes and Lipton, 2003; Wood, 1995). Fixed math content is transmitted by the teacher and "received, but not interpreted, by students" (Spillane, 2002, p. 380; Smagorinsky et al., 2004).

Traditionalists believe that content is transmitted through whole-class, teacher-centered, show-and-tell lecturing in which students are passive recipients, as opposed to student-centered, small-group work in which students actively participate (Wilson & Lloyd, 2000; Cuban, 1984; Mewborn, 2001; Sirotnik, 1983; Romberg & Carpenter, 1986; Gregg, 1995; Thomson, Turner, & Nietfeld, 2012). The expository transmission of content is via established and organized routines, focused on clear instructional objectives, affords students frequent feedback and reinforcement, and is sequenced from simple to complex skills (Spillane, 2002). Those holding reform beliefs suggest that frequent reinforcement, though seemingly productive to learning, actually limits the productive struggle necessary for mathematical meaning making (NCTM, 2014).Orderly, teacher-led transmission of knowledge is perceived to be depersonalized and isolating; whereas the productive belief is that the learning of mathematics is a social process (Cobb & Yackel, 1998; Seeger, Voigt, & Waschescio, 1998).

The traditional view is that an observable appropriate and correct response by a student indicates that the content has been learned and the student is developing toward mathematical proficiency (Darling-Hammond, 1997). The belief is that mathematical learning and teaching should focus on correct answers and memorized procedures, in contrast to the reform-based focus on mathematical processes such as posing and solving problems, representing, communicating, reasoning, and making mathematical connections (Oakes & Lipton, 2003; NCTM, 2000; Ernest, 1998). The consequence of operating with such traditional beliefs is that mathematical learning is superficial (Cobb & Yackel, 1998; Donovan & Bransford, 2005). Connections are not made, or expected to be made, between mathematical concepts and procedures in a way that students make meaning of the mathematical concepts and attain deep understanding (Donovan & Bransford, 2005). Nor are connections made between mathematical concepts and the real world in a way that conveys mathematical relevance to students (Confrey, 1986, in Cobb & Yackel, 1998).

The reform view is that procedural fluency and mastery of facts are important aspects of mathematical proficiency but not in isolation (Kilpatrick, Swafford, and Findell, 2001). To attain deep understanding of the mathematical content, NCTM (2014) explains that procedural fluency should be built upon conceptual understanding. Along with procedural fluency and conceptual understanding, strategic competence, adaptive reasoning, and productive dispositions are interdependent “strands" necessary for mathematical proficiency (Kilpatrick et al., 2001). If an individual possesses strategic competence, s/he can derive and successfully carry out a plan to solve the problem. Possessing adaptive reasoning entails being able to “think logically about” and justify how mathematical facts, procedures, concepts, and solution strategies connect (p. 129). Productive disposition encompasses (1) the belief that mathematics is worth doing, that is, relevant in the real world, and (2) confidence in one’s ability to do the mathematics.

With a focus on correct answers, the step-by-step, memorized processes, and teacher-directed instruction, traditionalists use the IRE discourse pattern (initiation by the teacher; response by student; and evaluation by teacher), believing that it is best practice (Greeno, 2003). Wood (1995) explains the consequence to assessing student learning within this pattern of discourse: teachers often do not truly listen; rather they “hear” the anticipated correct answers in their students’ responses. Contrary to this belief about best practice, NCTM (2014) advocates that teachers facilitate student-student discourse in which students analyze, compare, and critique one another toward the goal of co-constructing mathematical meaning.

Normative-Discursive Disconnect

While there are shared rules and meanings associated with traditional and reform-based (hereafter, “reform” or “innovative”) MLT beliefs, positioning along the continuum of cultural beliefs is complicated by the fact that individuals are inconsistent. What they believe they ought to do (ie, their normative beliefs) may differ from what they believe and claim they do (ie, their discursive claims) (D. Hoffman, personal communication, September 8, 2004; Lloyd et al., 2016; Gibbard, 1996; Buehl & Beck, 2015). Not as widely discussed within the literature is the fact that such disconnects between normative beliefs and discursive claims can delay the cultural shift toward reform beliefs. A teacher, for example, may articulate having productive normative beliefs; however, if his/her discursive claims indicate the use of traditional practices, the traditional views are more likely to be transmitted to students over the reform views (Lloyd et al., 2016). Investigating discursive claims along with reported normative beliefs assists in attending to the complexities related to positioning beliefs (Cross Francis et al., 2015). Such an examination assists in accurately interpreting the strength of the reported normative beliefs, allowing for increased validity in their positioning.

Misalignment between beliefs and practices may occur at the conscious or subconscious level, vary in degree, and are based on internal and external factors (Cross Francis et al., 2015; Buehl & Beck, 2015). In some instances, teachers articulate that they *believe they should* be using reform practices, but they find it difficult to effectively implement certain normative beliefs well into pedagogical practice because of a lack of content and/or pedagogical knowledge (Stein, Grover, & Henningsen, 1996; Henningsen & Stein, 1997; Beswick, Watson, & Brown, 2006; NRC, 2012). In such instances, the disconnect is at the conscious level, attributed to the internal lack of knowledge.

Theoretically, in such instances, because the misalignment is remedied with the acquisition of knowledge on how to effectively utilize reform instructional practices, disconnects are likely adaptable, that it, contextual and not fixed in nature. Lloyd (2009) found that PTs believed that an ideal practice was to use all class time for instructional purposes but chose, in practice with students who resisted engaging in lessons, to use time as a negotiation tool to motivate students, allowing students to finish working before class was over if they worked productively up to that point. This was deemed an adaptable misalignment because the practice was used situationally. In other classes, PTs' normative beliefs did align with their discursive claims and observable practices. In another instance, a PT articulated a belief in using problem solving to help students attain deep, conceptual understanding, but, when faced with student resistance, relied on more traditional teacher-directed instruction. Gradually, as his students became more accepting of his reform methods, he implemented practices aligned with his beliefs more frequently, resolving the belief-practice disconnect (Lloyd, 2009).

Internal factors contributing to normative-discursive (hereafter, "N-D") relationships include knowledge (as seen earlier), emotions, and learned behavior, among others (Buehl & Beck, 2015). The transfer literature claims that horizontal transfer (ie, "recognition") is made often, such that PTs and novice teachers can implement the specific activities in their classrooms that were modeled in their TPPs or PDs and have an understanding of how to talk about appropriate mathematical teaching and learning (Ensor, 2001, p. 314); however, vertical transfer (ie, "realization") is much more challenging to obtain because, according to the transfer literature, novices do not possess the deep knowledge of the theories – and/or subsequently do not fully buy into the reform philosophies – espoused during their TPPs and PDs required to be able to apply theoretical, reform practices to more general pedagogical activities (p. 314). Fives et al. (2015) explain, "…teachers may be learning the appropriate language of educational contexts and appropriating it without actually committing to these [reform] beliefs" (p. 256). In these instances, disconnects may be at the subconscious level, which ultimately hinders realignment.

In instances in which there are belief conflicts, individuals may feel emotional suffering. To minimize suffering, they may reflect or "succumb" to an opposing belief, potentially resulting in a disconnect between beliefs and practices (Cochran-Smith et al., 2015, p. 113). Others may navigate through conflict, challenging the emotional beliefs and norms of the setting, and reach "emotional freedom," likely resulting in authentic, strong beliefs aligned with practices (Zembylas, 2005, p. 477).

PTs may reflect the beliefs of their professors' during coursework, their cooperating teachers' during field placements, and in-service colleagues' once hired into a new school to avoid emotional suffering or because they are being *good* learners. As good learners, PTs mirror the traditional idea that what the teacher disseminates is absolute truth and is the information students should *learn* and report (Wood, 1995). Consistent with the earlier claim by Fives et al. (2015), if PTs – the learners – have been traditionally conditioned to memorize and regurgitate what they *learned*, during their TPP classes, in the field, and within their in-service settings, they may be inclined to report the rhetoric of the respective teachers without actually adopting the beliefs (Beswick, 2006). Further, PTs have learned to behave with professional decorum where they are expected to respect colleagues, students, parents, administrators, professors, even when faced with conflicting viewpoints; in some instances, PTs and novices with malleable belief structures may adopt practices reflecting the points of view of those that they respect and hold in high regard (Richardson, 1996).

External influences such as "traditionally-oriented educational environments," curricular demands, lack of resources, classroom management issues, student resistance/lack of motivation (seen in earlier example), and colleagues' conflicting teaching methods and/or beliefs have been cited as more influential on instructional decisions than normative beliefs (Lloyd, 2013; Handal, 2003, p. 53; Cross Francis et al., 2015; Buehl & Beck, 2015). If cooperating teachers have different beliefs about practices, PTs may adopt practices despite conflicting normative beliefs simply because PTs are guests in the cooperating teachers' classrooms and have limited authority in instructional decision making. PTs with malleable belief structures may revert back to traditional pre-training MTL beliefs acquired through their "apprenticeships of observation" as K-12 students (Lortie, 1975, p. 21), resulting in traditional N-D alignment (Zeichner & Tabachnick, 1981; Staton & Hunt, 1992; Richardson, 1996; Beswick, 2006; Wood & Turner, 2014).

Cross Francis et al. (2015) describe a typical situation, illustrating both internal and external factors influencing a N-D disconnect. A teacher's district chose to adopt a traditional curriculum, and she was "compelled to teach with fidelity due to her own work ethic and the district guidelines" (p. 344). Here both district guidelines and the understanding of "work ethic" to mean compliance to those guidelines are more influential than her reform beliefs. Did this teacher have a deep understanding of her professional responsibilities and work ethic -- related to and measured by student learning outcomes rather than compliance to administration? If so, would such a misalignment have occurred? These questions are meant to illuminate the fact that within a single misalignment, knowledge exists and decisions are made at the conscious level (here she knows that there is a misalignment and explains it due to district guidelines and work ethic), but there are also deeper issues that occur at the unconscious level (implication that compliance is professionally more important than student-learning outcomes), which makes the rationale for exploring N-D relationships all the more important when positioning beliefs along the traditional-reform continuum (hereafter, “continuum”).

Cross Francis et al. (2015) explain, “[T]here are still open questions about the kinds of beliefs that are most influential to pedagogical decision-making” (p. 340). An examination of the relationship between normative beliefs and discursive claims can provide insight related to these questions. Where aligned, whether reform beliefs with reform discursive claims or traditional beliefs with traditional claims (Buehl & Beck, 2015), positioning is easier; these beliefs potentially have greater impact on decision making or are simply easier to actualize within the social context. Misalignment may suggest that a belief is not as strong and unable to compete with external pressures or that there is a lack of deep knowledge related to beliefs and corresponding practices, making positioning of self-reported normative beliefs more complex.

Pre-service and In-service Teacher Beliefs[2]

To make sense of where this study’s PTs’ beliefs are positioned in the evolutionary process of shifting cultural beliefs, knowing and drawing comparisons to what the literature reports on other PTs’ beliefs related to similar belief constructs is important. Are there certain traditional beliefs that develop toward reform throughout the TPP? Are there other traditional beliefs that are more enduring? Of those that develop toward reform, do some regress following clinical internship [hereafter, “CI”]?

As mentioned, NCTM (2014) explains that conceptual understanding must be at the foundation of procedural fluency for students to acquire depth in their understanding of mathematical content. In a study following PTs throughout their TPP and into the second year of in-service teaching, by the end of the TPP, this normative belief, along with corroborating observable classroom practices, was strong among the cohort. By the second in-service year, however, participants either altered the belief, claiming that drill and practice lead to deep understanding or continued to articulate the belief but acknowledging its

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[2] In this section, along with several other studies, we highlighted findings from Beswick (2006), Beswick, Watson, and Brown (2006), and Author (2016), as these studies used similar instruments, belief constructs, and/or methodologies, making comparisons more concrete.

misalignment with classroom practices (Lloyd, 2013). Reasons cited for misalignment were that lessons focused on conceptual understanding take much longer to plan and perform; high-stakes test questions are not based as much on conceptual understanding; and colleagues do not teach for conceptual understanding so it is challenging to teach in such a way in isolation (Lloyd, 2013). Also, as they progressed through their TPP, PTs moved away from the reform notions, increasing in their belief that an effective teaching tool was to tell the students answers (Beswick, 2006).

Moving toward reform notions, throughout the TPP, PTs' beliefs that teachers should be fascinated by student thinking increased (Beswick, 2006). Interestingly, though PTs' normative reform belief that mathematical processes were just as significant as answers and grades increased, when PTs tried to enact this belief into observable action by spending class time discussing different processes for completing problems that were frequently missed on assessments, the students were disengaged, suggesting that ultimately all that mattered was the grade they got on the test based on one right answer (Lloyd, 2009).

Given that culture is dynamic and evolving from generation to generation, knowing and drawing comparisons to what the literature reports on ITs' beliefs related to similar belief constructs is important and not widely examined within the literature. Are the beliefs of entering PTs similar, more traditional, or more reform-based than those ITs that might represent their former teachers[3]? Related to how and what mathematics should be taught, literature on middle and secondary ITs examining similar belief constructs and utilizing similar methodologies revealed that at the normative level, persistent reform beliefs included: teachers should be fascinated with student insights; slight frustration by students can lead to a productive struggle resulting in deep learning; student-generated ideas should not be ignored;

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[3] Lloyd et al. (2016) examined and analyzed the beliefs of a sample of experienced middle and secondary teachers from the same area as the participants within this study, using a similar instrument. The beliefs of the IT teachers may be similar to the beliefs of middle and secondary teachers this study's in-state participants recently would have had.

justifications to solutions are an important part of mathematics; and students should develop inquiry skills when doing mathematics (Lloyd et al., 2016; Beswick, Watson, & Brown, 2006). Reform discursive claims included the assigning of problems that could be solved in multiple ways, using multiple representations and the sharing of varying solution strategies among peers (Lloyd et al., 2016).

Collaboration was a major N-D disconnect; teachers believed that collaboration, theoretically, was a valuable pedagogical practice, but claimed that it was challenging to implement. Interestingly, though ITs reported both at the normative and discursive level the reform belief that mathematical processes were just as significant as answers, they also very traditionally indicated that teacher-centered, expository teaching with a procedural focus was important to effective mathematics teaching and learning (Lloyd et al., 2016; Beswick, Watson, & Brown, 2006), putting into question if the mathematical processes they valued as much as the solutions were the mathematical procedures that were taught through direct instruction.

Related to the relevance of mathematics in the real world, ITs reported that being numerate was as important as being literate and that mathematics is relevant for everyone (Lloyd et al., 2016; Beswick, Watson, & Brown, 2006).

Purpose

Though one of the most prolific categories of educational research, much of the PT belief literature examines either (1) the impact of particular program innovations and provides general evaluations based on PT belief changes or (2) the links between espoused beliefs and practices. Though changes in beliefs and belief-practice relationships were examined as part of this study, these were used as means toward the study's ultimate goal: to provide insight about where on the continuum PTs' beliefs are positioned. To contribute meaningfully and accurately to understanding where beliefs are situated within the reform evolution, this study combined the examination of a cohort of K-8 PTs' entering normative beliefs, normative beliefs at the conclusion of two mathematics-education (hereafter, "ME") courses, and exiting normative beliefs and discursive claims (using Likert items and open-ended questions), relationships between other PTs' and ITs' beliefs within the literature, and alignment between participant PTs' reported normative beliefs and discursive practice claims. Because literature on teacher beliefs consistently reports a strong link between beliefs and pedagogical change (Czerniak et al., 1999a, 1999b; Fishman et al., 2003; Knapp 2003; Lakshmanan et al., 2011), the results of such an examination are important both for the future planning of effective in-service professional development (PD), particularly PD during the induction in-service year, and for the future planning of ME courses offered during TPPs (Levin, 2015). Too, these results address Sarason's (1996) concern that too often "those who attempt to introduce change rarely, if ever, begin the process by being clear as to where the teachers *are*" (p. 232).

Methodology

A sequential explanatory design was utilized for this examination. First, statistical analysis of three iterations of a Likert-scale survey – administered upon entry into TPPs (A1), following completion of two ME courses (A2), and at the completion of TPPs (A3) – was conducted. Next, assisting in the interpretation of these data (Siwatu & Chesnut, 2015; Boone & Boone, 2012), open-ended questions, included as part of the survey iterations, were analyzed.

The use of Likert-scale surveys to study beliefs presents some methodological concerns such as questions of reliability and validity when data gathered are self-reported; the fact that such scales are not likely to provide the depth and richness necessary to fully or accurately assess and capture the nuances of teacher beliefs and practices; and the limitation of such scales to provide insight into the development of teachers' beliefs (Levin, 2015). This study focused largely on PTs' normative beliefs – that is, what PTs believed they should do. Therefore, it was necessary to collect self-reported data. To assist in the depth of understanding about the development and strength of these self-reported normative beliefs, normative beliefs were examined across time (Turner & Drake, 2016; Levin, 2015), open-ended questions were included and analyzed "to solicit further explanation in teachers' own words" to capture nuances (Levin, 2015, p. 16), and the relationship between normative beliefs and discursive claims was analyzed. Additionally, for increased accuracy in the positioning of PTs' beliefs, the use of Likert data allowed for "a comparative baseline across different studies" so that participants' beliefs could be compared to those of ITs and PTs who responded to similar survey items and allowed for "a comparative baseline" within this study so that PTs' beliefs could be examined longitudinally (Schraw & Olafson, 2015, p. 92; Hoffman & Seidel, 2015). Though the use of such methods is supported within research findings (Geer, 1988) and among researchers (Patton, 2002) and the open-ended questions provided clarity and depth, mitigating the potential for misinterpretations of the Likert data, the inclusion of these items may have contributed to the significant attrition on A3 (Dillman, 2011).

Sites and Sample

Though findings vary related to the effectiveness and influence of TPPs, TPPs and their enrolled PTs are significant to and necessary for the desired cultural shift to occur (Seeger, Voigt, and Waschescio, 1998). Based on this, PTs' beliefs were chosen as the unit of analysis in studying the current positioning of MLT beliefs. All PTs (n=85) entering the undergraduate, K-8 licensure TPPs within a nationally-recognized, mid-sized, Southeastern, public, liberal arts and sciences college as juniors in August 2011 were asked to participate. The sample was limited to K-8 PTs, as they could be examined in aggregate based on the similarities among their TPPs allowing for a larger sample size[4]. Participants in aggregate were not exceptional (Levin, 2015) in that they included predominantly white (87%), female (87%), and traditionally-aged college juniors (with the exception of one non-traditionally-aged male student returning to college for vocational rehabilitation using the GI Bill). The average SAT and ACT scores for this cohort were 25 and 1701, respectively. Of the 85 PTs within this cohort, eight switched majors or left the college all together; one opted not to apply for state licensure; two did not take the necessary Praxis

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[4] Acknowledging that there may be variations among the beliefs and practices of elementary and middle-level PTs, prior to full analysis, PTs were grouped by major to identify statistical variations between majors. Though there was variation on a few specific item (A1_BT7, A2_BT7, and A3_BT9 – See Appendix C), overall, there were no statistical differences between majors; as such, analysis proceeded on the participants in aggregate.

exams to be recommended for state licensure; one did not initially pass the necessary Praxis exam to be recommended (though now certified and teaching); and the remaining 73 passed the necessary Praxis exams and were recommended for state licensure.

Akin to many TPPs, this college's K-8 undergraduate TPPs consisted of four semesters. Coursework and practicum experiences were the focus of the first three semesters; PTs completed their CIs during the final semester. These TPPs emphasized depth in mathematical content knowledge and innovative pedagogical practices by offering two distinct ME courses, one devoted to each goal and offered during the first two semesters.

Based on a review of course syllabi, assessments, activities, texts, and lesson plans; student, peer, and chair evaluations; and in-class observations and discussions with the mathematics education faculty (two tenure-track and one adjunct faculty), mathematics educators within these programs appeared to be dedicated to reforming traditional beliefs and teaching practices, like the majority of their contemporaries. As such, like most, they followed many of the recommendations for helping PTs develop reform belief. Courses and activities were aligned with the NCTM (2000) Process Standards and NCTM (1991) Standards for Teaching Mathematics; course instructors explicitly taught and modeled a variety of reform instructional strategies, which required communication and collaboration; problem solving and critical thinking; challenging, questioning, and exploring alternatives; reasoning; and making connections. Gavin, Casa, Adelson, and Firmender's (2013) "talk moves" ("repeat and check, agree/disagree and why, add on, think time, and partner talk") were used to facilitate student-led discourse (p. 486). Course time was devoted to exploration of beliefs, and PTs negotiated cognitive disequilibrium related to traditional and reform beliefs and practices (Kagan, 1992; Lloyd et al., 2016; Levin, 2015). As modeled, PTs were expected to self-reflect on their beliefs and (Caudle & Moran, 2012; Brownlee & Chak, 2007, in Fives et al., 2015). PTs were given authentic opportunities to develop their pedagogical skills within ME courses by writing and rehearsing lesson plans, critiquing other PTs' plans, and revising plans based on peer critiques (Siwatu & Chesnut, 2015).

Instruments and Data Collection

Between August 2011 and May 2013, PTs were asked to complete three iterations of a Likert-scale survey. Participation was voluntary and PTs were informed that participation would not affect their grades, that data would be de-identified following analysis, and that their responses would not be analyzed until they concluded their coursework to minimize inauthentic responses. In August 2011, 83 PTs consented and completed the first survey (A1), which was administered in hard-copy format at the beginning of the first ME course, a required course for all K-8 PTs. Sixty-four participants completed the second survey (A2) which was administered in hard-copy format at the end of the second ME course in May 2012. The final survey (A3), completed by 43 participants, was administered through the online survey software Qualtrics at the completion of the TPP – following PTs' CIs – in May 2013.

All three iterations included Likert items written in third person in the form of normative belief statements. The final iteration included the addition of Likert items written in first person as discursive practice statements to elicit information about what PTs claimed about their teaching (Lloyd et al., 2016). Discursive claims were only measured in the final survey as this followed their prolonged CI of sustained teaching, at which time PTs were able to make claims about their teaching. Items related to beliefs about what and how mathematics should be/is taught, beliefs about the nature of mathematics in the real world, and efficacy and were derived from the Mathematics Teaching Efficacy Belief Instrument (MTEBI; Enochs, Smith, & Huinker, 2000), the Nature of Mathematics Survey for Teachers (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d.), the Attitudes and Practices to Teaching Math Survey (McDougall, 2004), Beswick, Watson, and Brown's (2006) Mathematics and Numeracy in Everyday Life and Mathematics and Numeracy in the Classroom, or Showalter's (2005) Teacher Interview Protocol[5]. Within

[5] Adapted from Enochs, Smith, & Huinker, "Establishing factorial validity of the Mathematics Teaching Efficacy Beliefs Instrument" in School Science and Mathematics. Reprinted by permission of Larry Enochs.

all three iterations, PTs could provide additional comments and were asked open-ended questions related to perceptions of content and pedagogical-content knowledge and mathematics instruction in general. In the final iteration, PTs were asked open-ended questions related specifically to their teaching. This paper focused on responses to items related to beliefs about what and how mathematics should be/is taught and the relevance of mathematics in the real world. (See Appendix A for review of the full survey.)

Data Analysis

Likert scale survey items were coded on a scale from one to five, such that strongly disagree was coded as one and strongly agree was codes as five. For most items, a value of one was equivalent to highly traditional and a value of five was equivalent to highly innovative/reform-oriented. Items which asked about reform beliefs in a negative manner or asked about traditional beliefs in a positive manner were reverse coded.

Of the 85 PTs in the study, 36 (42%) completed all three iterations, 26 (31%) completed the first and second iterations, six (7%) completed the first and third iterations, 15 (18%) completed the first iteration only, and two (2%) completed the second iteration only. Attrition between the first and second administrations is believed to be largely due to the fact that the course during which the survey was administered was not mandatory for non-mathematics, Middle-Level (ML) PTs. Thus, if the participant

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Adapted from Showalter, Teacher interview protocol in the doctoral dissertation The effect of middle school teachers' mathematics teaching self-efficacy beliefs on their students' attitudes toward mathematics. Reprinted by permission of Betsy Showalter.

was not a ML mathematics major, then the individual was not present in the class where the survey was administered. The greatest drop in response rates occurred at the third iteration, following PTs' CIs. Of the original 85 participants, eight switched majors or left the college prior to the last survey request, and despite several emails asking participants to complete the survey which was made more accessible using Qualtrics, it is believed that the loss of face-to-face interaction with participants and potentially the use of open-ended survey items resulted in severe attrition in the final survey.

Due to high attrition between survey iterations, before calculating means, standard errors, and running subsequent t-tests, methodological decisions had to be made regarding whether or not to dismiss all survey responses except those that came from participants who responded to all three iterations. Such dismissal would diminish the power of the study and have implications related to the study's representation of the cohort; however, the use of all of the data might skew the results. Therefore, an analysis of the patterns of non-response was conducted to determine the appropriate treatment method for missing data (Pigott, 2001; Peugh & Enders, 2004). Analysis of only the complete cases (n=36) could potentially lead to a less powerful analysis, and analysis of available cases (pairwise) has the potential to provide biased estimates in cases where the variables are highly correlated (Pigott, 2001; Raghunathan, 2004). The average responses on all items for PTs who did not respond to the third survey are not significantly different from the average responses for PTs who did respond to the third survey. Further, the rate of response on the third survey is not statistically associated with the program (EC, EL, MID) in which the PT is enrolled. Therefore, it appears that the non-response did not result from a measured variable and it is likely that the missing data occurred completely at random (MCAR). Little's MCAR test is not statistically significant ($X^2=389.7,\ df=468,\ p=0.997$) and the hypothesis could be rejected that a pattern exists in the missing data. It is possible that PTs who did not respond to the survey had a different reaction to the internship, in terms of how it changed their beliefs, than that experienced by PTs who completed the third survey. However, it is more likely that the attrition was related to the difference in data collection methodology for the third survey. Therefore, the assumptions for imputing missing data have been met.

Imputation of missing data was conducted using the Expectation Maximization (EM) algorithm for obtaining Maximum Likelihood parameter estimates. The EM technique estimates the value of the missing data using the values of other items on the survey. Data was imputed separately for all missing normative items then all missing discursive items for all PTs who completed at least two administrations of the survey (n=68). All means and statistical tests include actual or imputed responses for 68 PTs. The means and standard errors for the data items post-imputation were similar to the means and standard errors pre-imputation. The direction of the relationships between items were also similar, though the strength of the relationship changed in some cases. The similarity in responses and relationships pre and post imputation further justifies the use of imputation to strengthen the power of the analysis.

Paired t-tests were conducted on the Likert data to identify if there were differences between iteration responses. Item means and t-test results were used in the initial positioning of categories of beliefs. For example, if an item had a low mean on each survey iteration, it was initially categorized as being weakly positioned along the continuum; items positioned similarly were then compared and grouped accordingly. Following data collection, several PT comments on the open-ended questions prompted the researchers to examine several of the items more closely. Items which were identified as having the potential for response error were excluded from the analysis. Too, an anonymous external reviewer highlighted potentially questionable items that "may not always differentiate reform vs. traditional teaching" (anonymous correspondence, October 2016). Some of these responses were removed from the study's analysis altogether. Those that remained in the study, as they seemed to offer important information in conjunction with the relevant open-ended questions, were highlighted as being ambiguous in the findings. See Appendix B for items' imputed means and standard errors, categorized into one of four tables: *What and How Mathematics Should Be Taught* (normative items), *What and How I Teach Mathematics* (discursive items), *Mathematics in the Real World* (normative items), and *My Teaching of Relevance of Mathematics in the Real World* (discursive items).

After completing the statistical analysis of the Likert items, analysis of open-ended questions was conducted for the purposes of providing depth to the Likert findings, avoiding potential misinterpretations, and clarifying ambiguities. Responses to a single open-ended question within each survey iteration were analyzed initially in aggregate, identifying the categorical responses to the respective question (Green et al., 2007). Counting coding was employed to obtain the frequency of each categorical response (Miles & Huberman, 1994). To understand the development of the normative beliefs over time, particularly to attend to any misinterpretations that may have occurred due to attrition, individual PT's responses were examined over time (looking at responses to a given question in all three iterations), individual's belief changes were categorized, and themes among categorizations of change were identified and reported in aggregate once again. Findings from open-ended questions and related Likert items were compared, determining if findings supported or contradicted the Likert data analysis.

Results and Discussion

Positive Positioning: The Power of Students' Strategies and Ideas

Related to beliefs about what and how mathematics should be taught, the most innovative entering belief was that teachers should be fascinated with how students think and intrigued by their alternative strategies (Item III.2). The mean value for this item increased following PTs' ME coursework and became significantly more traditional following CI. Though a vacillation in this belief, perhaps because it was espoused more within coursework than clinical practice, this belief remained significantly more innovative than it was at the beginning and was the second most innovative belief reported on A3. (See Table 1.)

Table 1: Normative Beliefs Revealing Positive Positioning Related to Student Generated Ideas and Strategies

Likert item	Survey Iteration/Time	Mean	SE	Paired Sample t-test t	(*p*)
III.2. Mathematics teachers	Beginning of TPP (A1)	4.26	0.08		

"should be fascinated with how students think and intrigued by their alternative strategies" (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 3).	After Methods Course (A2)	4.73	0.06	5.3	<0.001
	End of TPP (A3)	4.48	0.08	2.98	0.004
	Change from A1 to A3			2.11	0.038
Similar Likert item in the literature [hereafter "Similar"]: Primary and EC (n=155); assessed at beginning and end of TPP (Beswick, 2006, Table 2)		3.95 4.08			
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		4.24			
III.7. "Ignoring the mathematical ideas that students generate themselves can seriously limit their learning" (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 8).	Beginning of TPP (A1)	3.82	0.11		
	After Methods Course (A2)	4.54	0.09	5.3	<0.001
	End of TPP (A3)	4.39	0.19	2.98	0.004
	Change from A1 to A3			2.11	0.038
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		4.20			

According to another study of PT beliefs that utilized a similar survey item on a five-point Likert scale both at the beginning and end of the participants' TPP, PTs' beliefs became more reform-based on this item (Beswick, 2006, Table 2). Similarly, among ITs, this was one of the strongest normative beliefs (Lloyd et al., 2016, Table 1).

Another survey item related to the power of students' ideas was that student-generated ideas should not be ignored as this might limit their learning (III.7). Though the PTs' entering beliefs on this item were not as strongly reform-based, they grew significantly following ME coursework. By the end of the program, this belief had not changed significantly but was measured as the third most reform-based. (See Table 1 above.) This, too, was one of the strongest reform beliefs among the sample of local ITs (Lloyd et al., 2016, Table, 1).

Though normative beliefs were strongly reform-based related to the power of student-generated strategies and ideas, on A3, PTs expressed that in practice, they felt unprepared in handling their students' alternative responses, revealing a disconnect between normative beliefs and discursive claims. This, too, may have been a reason for a small regression in their reform belief between A2 and A3. Concerns related to implementing these beliefs were apparent both within the discursive Likert items and the open-ended questions. When asked to rate the discursive Likert item (II.11 reverse coded*), “I do not like to assign open-ended tasks because I worry that I may not be prepared for unpredictable results” (McDougall, 2004, Item 15),* beliefs tended to be more traditional than on the normative counterparts (mean=3.67).

Though the lower discursive value may indicate a reduction of momentum along the trajectory toward reform, when asked openly on A3 what they wanted to learn more about in their continued professional development, four of 33 PTs indicated explicitly, to be better prepared, they wanted to learn more about students’ potential solutions, “alternative strategies,” and potential questions. Four may not seem like many, however, this response was among many varying types of answers such as lesson planning, finding problem-based activities, and improving specific mathematical content such as probability or statistics. Therefore, having four PTs acknowledge that they wanted to know more about students’ alternative strategies suggested that with increased knowledge about these strategies the gap between beliefs and practice may be reduced, suggesting that this disconnect may be adaptable more than something fixed.

Based on the strength of the normative beliefs of PTs in this study and among those within other studies, PT beliefs about valuing student-generated ideas are situated toward the reform end of the continuum. If PTs are able to gradually align their practices with these strong reform normative beliefs, their future students will be exposed to reform practices, which will facilitate continued progress toward reform in this category of beliefs.

What Are the Most Important Goals in Teaching Mathematics?

To be mathematically proficient, students need to have procedural fluency, conceptual understanding, adaptive reasoning, strategic competence, and productive dispositions (Kilpatrick et al., 2001). Developing deep conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition along with procedural fluency can occur via problem solving and inquiry with a focus on sense making: critically think about the problem and solve, question solutions, determine the reasonableness of solutions by considering the process taken to solve and the end results, and communicate explanations and justifications for processes chosen to achieve the answer. As explained earlier, the literature reports, however, that the traditional focus by many teachers and desire of many parents and students is that students master basic facts, develop procedural fluency without necessarily making connections to conceptual understanding, and provide correct answers on quizzes, tests, and standardized examinations.

Positive Positioning: Critical thinking; problem solving; and understanding, justifying, and communicating processes and their connections to resulting answers. Related to beliefs about what and how mathematics should be taught, the second most reform-based entering belief was that the justification of mathematical ideas and statements is an important part of mathematics (III.8). The literature reports this was one of the strongest normative reform beliefs among ITs (Lloyd et al., 2016, Table 1). The mean value for this item increased significantly following PTs' ME coursework. The belief stayed relatively the same following CI, perhaps due to consistency between courses and field, which would imply that in the field the cultural shift is occurring in this direction as well (Beswick, 2006). It was the second most reform-based belief reported on A3. (See Table 2.)

Table 2: Normative Beliefs Revealing Positive Positioning Related to Critical Thinking; Problem Solving; and Understanding, Justifying, and Communicating

Likert item	Survey Iteration/Time	Mean	SE	Paired Sample t-test	
				t	(*p*)
III.8. Justification "of mathematical ideas and statements is an important part of mathematics" (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 9).	Beginning of TPP (A1)	4.12	0.08		
	After Methods Course (A2)	4.58	0.07	4.26	<0.001
	End of TPP (A3)	4.51	0.09	-0.76	0.453
	Change from A1 to A3			3.47	0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		4.28			
III.10. An attitude of inquiry should be developed through the teaching of mathematics (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 12).	Beginning of TPP (A1)	3.84	0.07		
	After Methods Course (A2)	4.31	0.09	4.04	<0.001
	End of TPP (A3)	4.37	0.09	0.60	0.552
	Change from A1 to A3			5.25	<0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		4.19			

Another survey item related to the goals in teaching mathematics is that students should develop an attitude of inquiry. Though the PTs' entering beliefs on this item were not as strongly reform-based (III. 10), they became significantly more so following ME coursework and continued to develop in a reform direction, though not significantly. (See Table 2 above.) This belief, too, was one of the strongest reform beliefs among ITs reported in the literature (Lloyd et al., 2016, Table 1).

Analysis of the open-ended responses revealed that PTs articulated that deep conceptual understanding, strategic competence, and adaptive reasoning were important goals in the teaching of mathematics, all citing on A2 (n=62) and A3 (n=35) the need for greater focus on problem solving, critical thinking, sense making, and conceptual understanding.

PTs articulated that they could always use more assistance in helping their students develop these mathematical competencies. After ME coursework and concurrent practicum experiences, PTs commented specifically that they wanted to improve on ways to facilitate increased critical thinking and problem-based learning and improve on making effective problems and choosing "appropriate activities to achieve deep understanding" (n=11). One PT expressed wanting to improve on "how to help students see beyond the algorithm." Others expressed the need to improve on teaching with multiple representations and reaching the needs of all learners (n=7). Following internship, open-ended responses mirrored those from A2, indicating a focus on the goals of thinking and sense making. For example, one PT wanted to know where and how to find good "challenging critical thinking questions that can't be found in the textbooks." With increased knowledge about strategies, they likely will strengthen connections between their strong normative beliefs about these goals and their teaching practices, suggesting that potential disconnects are adaptable rather than fixed.

PTs expressed, both on discursive Likert items and open-ended responses on A3, that they were developing conceptual understanding, adaptive reasoning, and strategic competence by having students critically think, problem solve, and understand, justify, and communicate processes and their connections, implying a connection between their normative beliefs and discursive practices. Every discursive Likert item related to problem solving, communicating mathematical ideas, and reasoning had mean values above four, indicating on average that PTs agreed to using all of these (see Table 3).

Table 3: Discursive Claims Revealing Positive Positioning Related to Critical Thinking; Problem Solving; and Understanding, Justifying, and Communicating

Discursive Item	Mean A3
II.1. I like assigning problems that can be solved in multiple ways (McDougall, 2004, Item 1).	4.52 (0.06)
II.3. I provide time and encourage students to share their differing strategies for completing the same problems (McDougall, 2004, Item 3).	4.52 (0.07)
II.6. I encourage students to use multiple representations or alternative resources (i.e., manipulatives, technology, etc.) to communicate their mathematical ideas to me and their peers (McDougall, 2004, Item 10).	4.39 (0.08)
II.9. Instead of answering students' math questions, I ask them additional questions to help them reason through their initial question (McDougall, 2004, Item 14).	4.02 (0.08)
II.13. "I teach students how to communicate their mathematical ideas" (McDougall, 2004, Item 17).	4.10 (0.07)

When asked if they were able to utilize strategies to help students gain depth in content, 27 out of 30 PTs reported that they were. This suggested that while all indicated that they needed additional assistance in implementing practices related to helping their students develop conceptual understanding, adaptive reasoning, and strategic competence, as implied above, some PTs were gradually able to minimize the gap between their normative beliefs and instructional practices by the end of their CIs, at least according to their perceptions of their practices. Two PTs' responses provided evidence of this adaptable process of aligning beliefs and practices, citing student resistance to communicating explanations and justifications as being the obstacle in this adaptable process:

> By requiring students to explain their work, they have to have more than a basic formula to compute. This is a huge change for my class this year, as previously it's been a plug and chug environment. They don't always like me because of it, and I sometimes get a lot of blank looks, but slowly they have started to really grasp the concepts. They also remember information more if they can teach it to someone else. (fifth-grade clinical intern)

> When I started teaching my students this semester, I don't think that many of them were used to explaining how they solved a problem, or why their answer was justified. I have used a lot of discussion in my class, and students are now used to not only giving me an answer but explaining how or why they did something. (third-grade clinical intern)

Positive Positioning: Productive disposition related to the relevance of mathematics in the real world. As stated, to be mathematically proficient, students need to develop a productive disposition about mathematics, which includes confidence in their mathematical ability along with a belief that mathematics is relevant and worth doing (Kilpatrick et al., 2001). Therefore, a major goal in teaching mathematics is to help students see the relevance of mathematics. To that end, the reform ideology is that teachers should believe that mathematics is relevant and utilize practices that convey this belief.

Related to beliefs about the relevance of mathematics in the real world, all entering normative belief means were between 3.5 and 3.9, with the exception of the strong reform belief that people use mathematics in the daily decision making (IV.6). Each of the beliefs in this category grew significantly following ME coursework. This growth continued or was sustained throughout CI, suggesting consistency between coursework and fieldwork. Exiting mean values were between 4.18 and 4.60, indicating that PTs believed strongly in the relevance of mathematics. (See Table 4.) These beliefs, too, were reportedly reform based among ITs (Lloyd et al., 2016, Table 2).

Table 4: Normative Beliefs Revealing Positive Positioning Related to Mathematics in the Real World

Likert item	Survey Iteration/Time	Mean	SE	Paired Sample t-test t	(*p*)
IV.1. To be an intelligent consumer, one must be numerate. (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 1).	Beginning of TPP (A1)	3.68	0.09		
	After Methods Course (A2)	4.18	0.11	4.61	<0.001
	End of TPP (A3)	4.19	0.09	0.10	0.920
	Change from A1 to A3			4.80	<0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 2)		4.03			
IV.2. Understanding mathematics is increasingly important in today's society. (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 4).	Beginning of TPP (A1)	3.85	0.09		
	After Methods Course (A2)	4.38	0.07	5.50	<0.001
	End of TPP (A3)	4.51	0.10	1.08	0.283
	Change from A1 to A3			4.75	<0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 2)		4.19			
IV.3. To function in today's society being numerate (having quantitative literacy) is equally as necessary as being literate (Beswick,	Beginning of TPP (A1)	3.63	0.10		
	After Methods Course (A2)	4.16	0.12	4.32	<0.001
	End of TPP (A3)	4.29	0.13	0.79	0.432

Watson, & Brown, 2006, p. 71, Table 3, Item 5)	Change from A1 to A3			4.30	<0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 2)		4.25			
IV.4. Mathematics is necessary to understand media claims (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 7).	Beginning of TPP (A1)	3.60	0.10		
	After Methods Course (A2)	3.88	0.12	2.05	0.044
	End of TPP (A3)	4.18	0.11	2.14	0.036
	Change from A1 to A3			3.96	<0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 2)		3.92			
IV.6. Often people use mathematics in their daily decisions (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 10).	Beginning of TPP (A1)	4.25	0.08		
	After Methods Course (A2)	4.56	0.07	4.13	<0.001
	End of TPP (A3)	4.60	0.09	0.36	0.722
	Change from A1 to A3			3.33	0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 2)		4.47			

Analysis of discursive claims and open-ended responses revealed consistency with the end-of-program normative beliefs related to the teaching of mathematical relevance. PTs indicated that they had their students complete relevant problems (II.2 $mean_{A3}$=4.22) and that they disagreed with the statement about telling their students that "a lot of what we learn in mathematics is not much fun, of interest, or relevant" (McDougall, 2004, Item 20)* (II.14 reverse coded* $mean_{A3}$=4.51, indicating strong disagreement based on reverse coding).

Responses to the open-ended questions supported the Likert data. In particular, when asked for reasons why students should take mathematics courses (open-ended item #14), the same interwoven themes reappeared in all three iterations (A1, A2, & A3 total responses n=79, 64, & 34). These included that mathematics must be learned: "to get by on a daily basis" (n_{A1}=42, n_{A2}=35, n_{A3}=24); "to function in today's society – plain and simple" and in preparation to be "effective" and "competent" members of society (n_{A1}=17, n_{A2}=14, n_{A3}=6); to set the groundwork for present and immediate future success in school (as mathematics "overflows into all other subjects") and future success in the job force (n_{A1}=9, n_{A2}=3,

n_{A3}=1); and to develop critical thinking skills, problem solving skills, and reasoning (n_{A1}=10, n_{A2}=10, n_{A3}=3). Others responded, "To relinquish any fear of the subject and to motivate children to learn" (n_{A1}=1); "Because it is fun" (n_{A2}=1); and "Why do students go to school?" (n_{A2}=1). Clearly, by the end of the TPPs, findings indicated that PTs did believe that there are important reasons for why students should learn mathematics beyond simply because it is what has always been taught. Contrary to the ideas of many parents who argue not to change what and how mathematics is taught, most PTs acknowledged that traditional instruction needs to change, ensuring the accessibility of content for all students given its necessity for success in everyday life.

Weaker positioning: Content coverage sans sense making; answer v. process; computation. While findings about the goals of mathematics teaching thus far indicate positioning toward reform, some concerns, ambiguities, and questions presented in the analysis.

Content coverage sans sense making. PTs were asked to indicate their level of agreement with two statements related to the "most important part of instruction," curriculum content and sense-making (III.12* and III.13). Acknowledging the external reviewer's comment that these items may be questionable to use to assess beliefs on the traditional-reform continuum in isolation, with the corresponding open-ended responses, inclusion of these data seemed important. While PTs' reform belief that the most important goal of mathematics teaching is sense making grew to 4.07 by A3, indicating agreement on average, they remained somewhat neutral or only in slight disagreement with the traditional belief that the most important part of instruction is the content of the curriculum (reverse coded*$mean_{A3}$=3.44). (See Table 5.)

Table 5: Normative Beliefs Revealing Weaker Positioning Related to Content Coverage and Computation sans Sense Making and Computation

Likert item	Survey Iteration/Time	Mean	SE	Paired Sample t-test t	(*p*)
III.1. "Mathematics is computation." (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 1)*	Beginning of TPP (A1)	2.51	0.12		
	After Methods Course (A2)	2.79	0.10	1.83	0.072
	End of TPP (A3)	2.75	0.13	-0.23	0.816

	Change from A1 to A3			1.42	0.159
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		3.12			
III.12. "The most important part of instruction is the content of the curriculum" (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d., Item 11).*	Beginning of TPP (A1)	3.19	0.10		
	After Methods Course (A2)	3.16	0.13	-0.21	0.837
	End of TPP (A3)	3.44	0.16	1.77	0.081
	Change from A1 to A3			1.61	0.112
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		3.29			
III.13. "The most important part of instruction is that it encourages sense-making or thinking. Content is secondary" (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d., Item 11).	Beginning of TPP (A1)	3.90	0.10		
	After Methods Course (A2)	3.99	0.10	0.78	0.437
	End of TPP (A3)	4.07	0.14	0.52	0.608
	Change from A1 to A3			0.99	0.327
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		3.78			

Discursive comments revealed that PTs felt the pressure of content coverage for the passing of tests, so while they wanted to believe that sense making is the most important part of instruction, external pressures created some shakiness in their belief system. On the final survey, 14 out of 31 PTs indicated that while they were able to integrate some reform strategies to develop deep understanding of mathematical concepts, they were constrained in doing so as frequently as they would have liked, citing the curriculum or too many concepts within the mandated standards to cover as persistent, fixed obstacles unlike the waning student-resistance obstacle reported above. One PT explained, "Teachers have to … get through so much in the year that there's no time to spend a week on a whole concept. …Students are taught the basic procedures and not why they are doing what they're doing so that it makes sense." Several PTs conveyed the consequences of this practice: Students get "left behind" because teachers' concerns with coverage results in "miss[ing] that their students don't understand."

Though concerning based on the perceived fixed nature of these obstacles that may potentially create a disconnect between normative beliefs and instructional practice, 17 of the 31 PTs felt as though they were able to teach content in depth consistently, 11 of whom cited that the district scope and sequence plans, which drew on both formal textbooks and curriculum and supplemental materials established for teachers to utilize in the coverage of content, were a major support. These findings indicate that supportive and appropriate teaching tools do exist that can facilitate deep learning, so the curriculum is not necessarily a fixed obstacle for actualizing this belief. Furthermore, four of the 14 PTs who reported not being able to utilize reform practices as frequently indicated that the shift from the state standards (which they claimed had too much content and did not allow for enough depth) to the Common Core State Standards was helping to address the issue of greater depth of content. One PT explained,

> What we are required to teach with STATE standards rarely allow for knowledge depth. For example: USE the formulas for area of …; USE the formula for volume of …. This standard is written so that students are given the formulas on the STANDARDIZED test as opposed to truly understanding the formulas so that they don't truly have to memorize them--rather, they should be able to understand and reason through them without being given the formulas. The movement towards Common Core is a relief because state standardized tests will soon reflect them and this idea of knowledge depth as opposed to simple knowledge accumulation. (Fifth-grade clinical intern)

These findings indicate that the issue with "too much to cover" in the standards may not be a fixed obstacle for actualizing this belief either. Therefore, as with student resistance, if obstacles are not fixed and dissipate, PTs may be able to minimize the gap between their normative beliefs and instructional practices, modeling to their future students that the goals of mathematics instruction include conceptual understanding, adaptive reasoning, and strategic competence.

The normative belief that deep conceptual understanding and sense making should be the foundation of mathematical learning (NCTM, 2014) was strong at the end of PTs' TPPs; however, it is unclear as to whether this strength can be maintained. In a study of ITs, by the second year of in-service teaching, this belief either regressed or was consciously not translated into instructional practice because of the time it took to plan conceptual-based lesson, the fact that high-stakes test questions were not typically conceptual but procedural, and colleagues do not teach in accordance with this belief rather they teach for coverage of procedures (Lloyd, 2013). This begs the question, will the strength of these reform beliefs about sense making as the most important part of mathematics, toward the attainment of mathematical content knowledge, persist or will they regress toward more traditional beliefs as indicated in the literature, given mention of some of the same external obstacles?

Answer v. process. PTs were asked to indicate their level of agreement with two statements related to the emphasis they placed on correct answers versus the processes used to acquire the answers (II.7 and II.8). Responses indicated that on average they remained somewhat neutral or only slightly in agreement with the claim that on graded tasks, they emphasized process ($mean_{A3}$=3.33) and slightly agreed with the emphasis on process on non-graded tasks ($mean_{A3}$=3.76). Findings from the sample of ITs revealed a somewhat stronger reform belief related to graded tasks (mean 3.6) and similar findings related to non-graded tasks (mean=3.81) (Lloyd et al., 2016, p. 377).

As previously indicated, analysis of the open-ended responses revealed that during their CIs, PTs felt the pressures and constraints of testing and content coverage, therefore, similar to beliefs reported about content versus sense making, external pressures may have created some shakiness in PTs' belief systems. If other PTs' beliefs and practices from the literature shed any light on this weakness toward reform beliefs, student behavior also may be an influence to explore in greater depth through observations. As previously cited in the literature, while the belief that mathematical processes were just as significant as answers became increasingly more reform-based among a cohort of PTs, when PTs tried to enact this belief into observable action, their students were disengaged, implying that they were only concerned with the answers and their grades (Lloyd, 2009). Regardless of the reasons, these lack-luster results among PTs and ITs suggest that there is room to develop toward reform.

Computation. Again, while most of the normative and discursive findings suggested strength in reform beliefs and practices related to the goals of mathematics teaching, to the contrary, the second most traditional entering belief on all items was that mathematics is computation* (III.1). Analyses of normative beliefs of the sample of ITs revealed that this was the second most traditional belief as well; however, the belief was not as highly traditional among these ITs (Lloyd et al., 2016, Table 1). For this study's PTs, the mean value for this item increased slightly following ME coursework, and the traditional belief was maintained following CI. (See Table 5.) Though there may be some growth toward reform, on average, most PTs still traditionally believed mathematics to be computation.

How Should Mathematics Be Taught?

For students to become mathematically proficient in all areas, the reform literature advocates "an upside down approach" (Van de Walle, Lovin, Karp, Bay-Williams, 2014, p. 12) in which students begin instruction of a particular content with a problem. During the process of working through problems, students grapple with productive struggle, collaborate, and engage in meaningful student-student discourse. They develop mathematical language and conceptual understanding. Procedures are learned through doing mathematics and in connection with conceptual understanding. As explained earlier, however, the literature conveys that mathematics instruction typically moves from direct instruction, guided and independent practice, and potentially the application of the learned procedures in the solving of word problems (Van de Walle et al., 2014). In this expository method, there is little collaboration or meaningful student-student discourse that results in in-depth understanding, rather collaboration and student-student discourse revolve around practicing procedures to arrive at correct answers. Little productive struggle or reasoning occurs, as teachers find it more efficient to tell students answers or confirm if answers are correct or incorrect. This traditional process makes assumptions that all students are at the same level with the same prior knowledge; since only providing one process to acquire the answer – the teacher's process, there is very little differentiation necessary to reach all learners.

Positive positioning: Productive struggle. Though on average PTs initially were neutral or only slightly in agreement with the benefits of having students experience productive struggle (III.4 $mean_{A1}$=3.44), following ME coursework, and sustained throughout CI, on average PTs were in agreement with this belief ($mean_{A2}$=4.20; $mean_{A3}$=4.21). Following ME coursework, this belief revealed a large and highly significant change toward innovative (mean difference=0.76, t=5.44, p<0.001). Analyses of normative beliefs among a sample of ITs revealed that this was also among the strongest reform beliefs (Lloyd et al., 2016, Table 1).

Weaker positioning: Expository teaching; telling answers; and collaboration.

Expository teaching. Interestingly, despite strong normative beliefs and discursive claims related to the goals of developing critical thinking; problem solving; and understanding, justifying, and communicating processes and their connections to resulting answers, analysis of normative and discursive Likert items revealed a very strong alignment between normative beliefs and discursive claims about instructional practices related to the expository style of teaching. The most traditional entering belief out of all items surveyed was that the best method for teaching mathematical concepts is an expository style (III. 5 *). Analysis of ITs' beliefs, using a similar belief instrument, revealed that this was the most traditional belief among these ITs (Lloyd et al., 2016, Table 1).

Though the mean value for this item showed a large, significant change toward reform following ME coursework, it remained the most traditional of all items on A2. It then became slightly more traditional, though significantly similar, following CI, remaining the most traditional of all items with most PTs on average agreeing that expository teaching is the best method for teaching mathematics. (See Table 6.)

Table 6: Normative Beliefs Revealing Weaker Positioning Related to Expository Teaching; Telling Students Answers; and Collaboration

Likert item	Survey Iteration/Time	Mean	SE	Paired Sample t-test t	(*p*)
III.3. It is an efficient way to facilitate student mathematical learning by telling students answers (Beswick, Watson, & Brown, 2006, p. 72, Table 4, #4).*	Beginning of TPP (A1)	3.91	0.11		
	After Methods Course (A2)	4.39	0.10	3.69	<0.001
	End of TPP (A3)	3.92	0.15	-2.79	0.007
	Change from A1 to A3			0.07	0.945
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		4.19			
III.5. The best method for teaching mathematical concepts is an expository style (i.e., demonstrating, explaining, describing, providing examples) (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 6).*	Beginning of TPP (A1)	1.53	0.07		
	After Methods Course (A2)	2.36	0.12	6.07	<0.001
	End of TPP (A3)	2.18	0.18	-1.05	0.299
	Change from A1 to A3			3.42	0.001
Similar: In-service local middle (n=12) & secondary (n=47) teachers (Lloyd et al., 2016, Table 1)		2.44			
III.14. Students must have opportunities to work together to get an in-depth understanding of the content (Adamson, Burtch, Cox, Banks,	Beginning of TPP (A1)	3.82	0.11		
	After Methods Course (A2)	4.26	0.07	3.71	<0.001
	End of TPP (A3)	4.02	0.09	-2.49	0.015

Judson, & Lawson, n.d., Item 13).	Change from A1 to A3			1.56	0.124
III.15. Working together is problematic because a teacher cannot assess what each individual understands and oft one student does a majority of the work (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d., Item 13).*	Beginning of TPP (A1)	3.03	0.13		
	After Methods Course (A2)	3.43	0.12	2.48	0.016
	End of TPP (A3)	3.67	0.13	1.42	0.159
	Change from A1 to A3			4.56	<0.001

Review of PTs' discursive claims about what and how they teach revealed that on average they preferred to have students master the basics before problem solving (II.12 reverse coded* $mean_{A3}$=2.12), antithetical to the upside-down approach to teaching which begins with complex problems as a vehicle for fact mastery and procedural fluency.

Following the discursive open-ended questions asking about the utilization of strategies to teach content in depth, PTs were asked to describe their typical day and, subsequently, answer if this was how they wanted to be teaching and if they believed this was effective teaching. Thirty-four responded, but due to the way the item was written ("Describe a typical day of teaching in your classroom."), 13 of the responses were not specific to mathematics instruction. Only seven of the 21 mathematics specific PT responses described some leaning toward reform teaching practices. One PT wrote,

> 1. Check HW and allow students to ask questions if needed (daily) …
> 2. Introduce the next topic and ask the students general questions about what they think the topic means in terms of mathematics and why it might be meaningful for them
> 3. Pose a question/problem concerning the topic to see if the students can figure it out and generate strategies to reach the answer.
> 4. Build on the strategies they came up with to reach the standard algorithm
> 5. Provide more questions/problems for them to practice the standard algorithm
> 6. Provide challenge questions that cause them to still use the standard algorithm, but to think about it in a different and more complex way (fifth-grade clinical intern)

Each of these PTs cited administrators, curriculum coaches, their cooperating teachers, other teachers within the school, and/or resources such as textbooks and technology as major supports in being able to teach utilizing reform practices.

The remaining 14 descriptions were extremely traditional, almost verbatim describing the expository style of demonstrating, explaining, describing, and providing examples and then having students practice.

Six of these 13 PTs' responses indicated that they held reform beliefs but, admittedly were unable to teach in accordance with these beliefs, citing constraints such as not enough time, the curriculum, classroom management obstacles, lack of teaching freedom, and/or a test-centric district culture. They were conscious of the misalignment in their beliefs about best practice and their actual practices. One EC PT explained, "The school I student taught in was extremely concerned with test scores. Therefore, there was little room for creative lessons and the teachers were forced to follow scripted lessons." She elaborated in her next response, "I do not want to be teaching like this. I purposefully found a job in a school that would allow me to create lessons that I felt were best for my classroom" (first-grade clinical intern).

The remaining eight of these 14, making up more than a third of the relevant responses, believed that this was how they wanted to be teaching, as they believed it to be effective.

> I use the Smartboard to introduce the material. Students take some notes if there are any terms to define. I *demonstrate* [emphasis added] how to do a problem …. Students ask questions. I do a problem … and ask students to help me solve it. When I feel they have a good idea on how to do the problem, I let them work with their learning partners to solve problems …. I walk around the room to see how students are solving the problems and to *see if their answers are correct* [emphasis added]. I have students work with partners for the majority of the class time. At the end, I will have students answer a question independently so I can determine their individual understanding. (fifth-grade intern)

She elaborated on the following question, "Yes, this is how I want to be teaching. I feel that it is effective because my students learned the material and enjoyed the learning process." The reality is that those who liked what they were doing and found it effective – despite the traditional nature of the expository, teacher-led instruction – are less likely to question their practices and more likely to continue to transmit beliefs that this is how mathematics should be taught.

Strong traditional normative beliefs favoring the expository style of teaching, discursive claims that students master basic facts prior to solving complex problems, and a third of the relevant open-ended questions revealing the use and approval of the expository method indicate that this is an area of weakness toward reform that needs to be pinpointed and explicitly addressed within TPPs and PD. However, all is not bleak. The fact that one-third of the relevant open-ended questions regarding the typical day revealed reform teaching shows potential progress. Too, so does the fact that the remaining third acknowledged that the traditional expository style was not how they wanted to be teaching.

Additionally, as reported in the prior section, PTs and ITs reported strong normative beliefs and discursive claims related to the goals of mathematics teaching, while also indicating traditional beliefs and practices related to expository teaching. Most of the of the thirteen PTs who described the traditional expository, teacher-centered instructional practice and a focus on "correct processes" within the description of their typical day also indicated that they integrated group work, questioning strategies, the expectation that students provide explanations or justifications for answers, student-student discourse, or class discussions. One PT wrote,

> A typical day begins with students doing a warm up that consists of 5 questions …. I then collect the warm up and we go over it as a class. I call on a student to provide an answer and that student has *to explain their reasoning* [emphasis added]. After this I engage in activating prior knowledge where I ask students a variety of questions from the previous lesson. This helps to refresh our memory and allows for us to have some discussion time. In this time I may also give students some practice problems to work on. While they work I am walking around checking students' *processes* [emphasis added] and answers. I even have minute conferences with students where I ask well *how did you get this and what did you do* [emphasis added]. I then move into my lesson, during this time I am doing a lot of modeling and explaining. I continue this into my guided practice. When students are doing independent practice, they are working and I am observing what students are doing and making sure they are using the correct process. When then come back together as a full group and I call on a student to give the answer and explain their reasoning. As a class we vote on whether or not we *agree on the answer* [emphasis added]. Closure consists of what I call "Mediation", where students place their heads on their desk and I have them rank their understanding of the lesson on a scale of 1-5. This makes students responsible for their learning and allows me to see who is still struggling.
> (6^{th}-8^{th} clinical intern)

The literature corroborates that teachers may reveal inconsistencies and contradictions in their beliefs and practices (Handal, 2003; Lim & Chai, 2008. in Buehl & Beck, 2015). So, it may be that PTs and ITs are truly utilizing reform practices amidst the reported expository style. However, these contradictions evoke questions about the authenticity of the reported reform practices integrated within the expository teaching style. For example, in the previous quotation the *reasoning* that is required may simply be to tell what steps were taken while working through a standard formula or algorithm; this is vastly different than justifying why these steps were taken or why an algorithm was chosen. The process in this scenario is simply procedural. Further exploration utilizing observations of PTs' classroom practices is necessary to determine if these reported practices were truly reform practices integrated within a traditional structure or simply traditional teaching articulated using reform rhetoric (Fives et al., 2015; Beswick, 2006).

Telling students answers. The belief that telling students answers is an efficient way to facilitate student mathematical learning (III.3*) significantly grew toward reform following ME coursework; this growth was reflected in the open-ended questions as well when asked what they wanted to learn more about (on A2: "I want to learn more about what kind of things to say if a student is stuck without giving the answer"). Subsequently, the belief significantly regressed between A2 and A3, ending at a point statistically similar to where the PTs' beliefs were at the beginning. (See Table 6 above.) Ending mean values indicated that on average most PTs disagreed with this statement. This regression suggests that this belief may have been explicitly deemphasized within coursework but supported during clinical practice, making it more challenging for mathematics educators to enact change toward reform related to this belief. Interestingly, in another study of PTs who were asked to respond to a similar Likert item, PTs traditional beliefs increased following CI as well, indicated by a decrease in means given the reverse coding of this item (Beswick, 2006, Table 2, #11).

Collaboration. Gradual increase occurred in the disagreement with the normative belief that group work can be problematic for assessment purposes (III.15). Though this indicated reform growth, on every iteration, this belief was among the lowest quarter of beliefs reported on the entire survey. The normative belief that students must have opportunities to work together increased significantly following ME coursework (III.14). It became significantly more traditional after clinical practice compared to at the completion of ME coursework, ending slightly higher but significantly similar to entering beliefs. (See Table 6 above.) This regression suggests that this belief may have been explicitly emphasized within coursework but less supported during clinical practice, making it more challenging for mathematics educators to enact change toward reform related to this belief. Too, PTs disagreement with the discursive item indicating that group work was unproductive in their classrooms was not very strong (II.4 reverse coded* mean_{A3}=3.84).

Related to collaboration, one PT conveys a clear understanding of how she *should* teach aligned with what was modeled during coursework, but, as seen in the literature (Stein, Grover, & Henningsen, 1996; Henningsen & Stein, 1997; Beswick, Watson, & Brown, 2006; NRC 2012; Handal, 2003; Lloyd, 2012), to meet the reality of "doing what works" for students to pass the test and/or class, she acquiesces to more traditional beliefs, particularly related to collaboration. She explained,

> While working in groups is great and I practiced that in my classroom, it is also extremely important to practice independently. One of the critiques I had this semester was that I didn't do enough independent work and I did too much group work. When kids take the [end-of-year assessment], they aren't going to be working in groups and MUST know how to do the problems on their own, ... Also, while it would be nice to stray from the curriculum in order to focus on deeper understanding, it isn't always realistic - kids need to know the curriculum to succeed on state tests.

This example provides insight into why there may have been a regression from A2 to A3 related to the belief that students must have opportunities to work together. To avoid further critique and emotional suffering, as indicated in the literature on emotions, this PT may have begun to reflect more traditional beliefs within her clinical practice setting. Such disconnects make positioning more challenging and further research on collaboration through observations, surveys, and interviews is needed to better understand these disconnects..

Positive Positioning?: Student Ideas, Inquiry, Relevance, and Productive Struggle

Not ignoring student ideas, inquiry, relevance, and productive struggle were all categorized as "positive positioning." Each grew significantly following ME coursework. Following CI, this growth was sustained or continued toward reform, with the exception of a minor regression in the "not ignoring student ideas" belief. This implies that TPPs can influence these beliefs, that there exists some consistency between coursework and fieldwork allowing for growth to be sustained or improved, and that the cultural shift related to these beliefs is occurring in the same direction within the field (Beswick, 2006).

Another connection is that PTs' entering beliefs about the value of not ignoring student ideas (III.7, mean=3.82), inquiry (III.10, mean=3.84), mathematical relevance – with the exception of one normative belief (mean values between 3.5 and 3.9), and productive struggle (mean=3.44) were considerably lower than those reported in the literature among ITs (Lloyd et al., 2016). These relationships are interesting as they imply that while ITs may learn and possess these beliefs and even translate them into practices, these beliefs and the value of aligned practices may not be recognized by students during their apprenticeships of observation. Without knowing the nuances and rationale, students may find the practices frustrating or irrelevant rather than recognizing the gains such practices may have had in their learning.

While PTs and ITs may buy into these beliefs and implement aligned practices *based on what they have learned in TPPs and PD*, students and parents conceivably are not undergoing transformations in their beliefs about the value of student ideas, productive struggle, inquiry, and solving relevant problems. This means, related to these beliefs, the cultural shift may be more challenging and take longer to attain, than what the positive exiting beliefs may imply because the transformations appear to be attributable to some form of teacher preparation or development rather than general enculturation. Further research is necessary to test these interpretations.

Summary

Many studies on PTs' beliefs are aimed at evaluating the implementation of a specific innovation or program, measured in part by a change in PTs' beliefs or examining the relationship between beliefs and practices. Instead, this study examined a typical sample of PTs enrolled in typical TPPs with the aim of providing insight about where on the continuum PTs' beliefs are positioned. Beyond the examination of one set of data to position beliefs, to ensure accuracy in results, this study uniquely combined the examination of the same normative beliefs at significant times throughout PTs' TPPs (both using Likert-scale items and open-ended questions), relationships between other PTs' and ITs' beliefs reported in the literature, and the alignment between PTs' normative beliefs and discursive claims about teaching. Attending to Sarason's (1996) critique, results of this study – particularly related to beliefs that are not positioned as far along the trajectory toward reform – are intended to assist mathematics educators and PD developers in targeting future instruction to meet PTs and ITs where they are.

In the following summary, beliefs have been categorized as strong-reform, positioning them the farthest along the continuum toward reform; mid-to-weak-reform, positioning them lower on the continuum; and strong-traditional, positioning them closest to the traditional end of the continuum. There were several beliefs that were not positioned based on ambiguities in the interpretations of the collective findings and, therefore, require further research and examination.

Strong-Reform Positioning: The Power of Students' Ideas; Critical Thinking, Problem Solving, and Understanding, Justifying, and Communicating Processes and Their Connections to Resulting Answers; Relevance; and Productive Struggle

"The power of students' ideas" and "Critical thinking; problem solving; and understanding, justifying, and communicating processes and their connections to resulting answers" (hereafter, "C-P-U") entering (means $\geq$3.82), developing (means$\geq$4.31), and exiting (means$\geq$4.37) means were among the highest among the normative beliefs related to what and how mathematics should be taught, indicating strong-reform positioning. These beliefs were also deemed to be strong-reform because N-D misalignments were perceived to be adaptable toward reform alignment with increased knowledge about reform instructional strategies. Among "C-P-U," the growth in beliefs following ME coursework was sustained following internship; indicating consistency between coursework and fieldwork which facilitates productive belief transformations toward reform.

Within "The power of student ideas," beliefs vacillated, increasing after ME coursework and decreasing after internship; given that ending beliefs were stronger than entering beliefs, the regression from coursework was not too concerning. Nonetheless, the perceived uneven emphasis placed on these beliefs within coursework and fieldwork may evoke the need for mathematics educators and PDs developers to focus on ways to increase TPP and PD power in reforming these particular beliefs.

Though "Relevance" and "Productive Struggle" entering normative beliefs were lower than others (means$\geq$3.60 and mean=3.44, respectively), exiting beliefs were high (means$\geq$4.17 and mean=4.21). Related to both, sustained growth throughout the program indicated that PTs likely were receiving consistent messages in coursework and fieldwork, which presumably assists in the reforming of beliefs. Adding support to the strong-reform categorization of the "Relevance" beliefs was the alignment between normative beliefs and discursive claims.

Of note, all four of the categories assessed as being strong-reform had at least one item with a lower entering mean than the sample of local ITs. As previously explained in detail, this relationship may suggest that despite seemingly strong positioning, evolution toward continued reform may be delayed. Further research is necessary to explore this relationship and its implications on reforming beliefs.

Mid-to-Weak-Reform Positioning: Telling Answers; Answer v. Process; and Collaboration

Beliefs categorized as mid-to-weak reform may not be terribly low, but relatively speaking, they are not as strong as those in the aforementioned section and will likely require more attention by mathematics educators and PD developers to facilitate the desired shift toward reform. The entering normative belief for "Telling students answers," when reverse coded, was not terribly low (mean=3.91); in fact, it was among the highest on all survey items. What was troubling was the fact that it was among the lowest third of all survey items at the completion of the program ($mean_{A3}$=3.92). Despite considerable growth following ME coursework, it regressed essentially to the same level following internship. Significant regression following internship was also found within the literature (Beswick, 2006, Table 2, #11). Such findings suggest inconsistent messages, explicitly or implicitly, being conveyed in courses and the field that make evolution toward reform more challenging.

Related to "Answer v. process," responses to discursive claims revealed that PTs were neutral or only slightly agreed that they emphasized process used to acquire answers versus simply correct answers (means=3.36 and 3.79). These means were among the lowest half calculated. Open-ended responses implicated perceived *fixed* external obstacles such as test-centric district mentalities which reinforce an answer-oriented culture. The fixed nature of these perceived obstacles informed the positioning of "answer v. process."

Related to "Collaboration," associated exiting normative and discursive means were not terribly low (means=3.67, 4.02, and 3.84). However, one normative item, on all three iterations, had mean values among the lowest quarter of all normative means calculated ($mean_{A1}$=3.03, $mean_{A2}$=3.43, & $mean_{A3}$=3.67). The exiting mean for this item, along with the discursive mean (mean=3.84) suggest neutrality or slight agreement related to the value and use of collaboration. The other normative item revealed considerable growth following ME coursework; however, it regressed considerably following internship, suggesting inconsistency about collaboration within course and field settings which, consequently, can slow reform.

Strong-Traditional Positioning: Computation and Expository Teaching

Both “Computation” and “Expository teaching” beliefs were persistently traditional among PTs and ITs. They ranked among the lowest normative beliefs on all survey items on all three iterations. With mean values less than 2.79, reverse coded, on average PTs believed that mathematics is about computation and is best taught using an expository style. In the descriptions of typical teaching, 14 out of 21 responses described expository teaching. However, despite the persistence of expository teaching in both normative beliefs and claimed practices, there was a glimpse of hope toward reform. Out of the 21 responses describing typical days, six PTs described expository teaching but expressed that they did not care for this type of teaching, citing external pressures for the disconnect between their normative beliefs and discursive practices; seven described reform teaching, rather than expository teaching.

Further Research Needed: Content Coverage sans Sense Making and In General

The beliefs related to “Content coverage sans sense making” are challenging to position, particularly based on ambiguous results obtained, in part, by questionable survey items. On average most PTs agreed that sense making was most important ($mean_{A1}$=3.90, $mean_{A2}$=3.98, & $mean_{A3}$=4.07). However, they were not in total disagreement with the statement that content was the most important part of instruction (reverse coded* $mean_{A1}$=3.20, $mean_{A2}$=3.16, & $mean_{A3}$= 3.44). As sense making and content are both important parts of mathematical instruction, and certainly should not be mutually exclusive, it is no surprise that PTs, on average, did not fully disagree with the latter (Cross Francis et al., 2015).

The results may be indicative of a reform ideology that supports making sense of the content of the curriculum; nonetheless, the open-ended responses revealed an N-D disconnect. Uncertain was whether or not these disconnects were fixed or adaptable based on the nature of the external obstacles cited – fixed or flexible and waning – and whether or not a strong normative belief about sense making was strong enough to overcome external pressures. Existing literature suggests that normative beliefs are not strong enough to overcome these pressures, resulting in the regression of reform beliefs about sense making once PTs become ITs (Lloyd, 2013). To better position beliefs about "Content coverage sans sense making," it is clear that further research and exploration are needed.

This study reviewed PTs' beliefs over time and N-D relationships to inform how beliefs are positioned along the continuum toward reform. Additional research conducted using observations and interviews, along with surveys and open-ended questions, both during TPPs and into in-service teaching would be useful. The addition of observations would allow for the relationships between normative beliefs, discursive claims about practice, and actual observable practice to be examined. The same fundamental idea would help position beliefs; for example, where all three are aligned, there is strength in the belief and likely consistency among TPP and in-service school settings. Extending exploration into in-service settings would provide information related to the endurance of particular beliefs following TPP.

As the beliefs of this study are cultural in nature, they are dynamic and shift over time, albeit slowly. Therefore, research needs to continue to examine how beliefs are positioned to assess progress toward reform. As positioning shifts, appropriate changes in TPPs and PDs should be implemented to meet PTs and ITs "where they are" (Sarason, 1996).

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Appendix A

Preservice Teacher Beliefs about Mathematics Survey

I. Background Information

Name: ______________________________ Date: __________________

Race/Ethnicity/Gender: ____

Certification area(s) desired:______________________

Content area(s) for which you feel most comfortable teaching:_____________________________

Content area(s) for which you feel least comfortable teaching: ____________________________

Content area(s) for which you feel most comfortable learning: ____________________________

Content area(s) for which you feel least comfortable learning:_____________________________

Grade(s) taught during clinical internship (along with any other experiences teaching mathematics – give grade and math subject area if you taught a specific math content area such as Pre-Algebra, Algebra I, Geometry):

II. Practices (P) – ONLY ASKED ON THIRD ADMINISTRATION (A3)

Based on your clinical-internship experience and other field experiences, please indicate the degree to which you agree or disagree with each statement by circling the appropriate letters.

SD	**D**	**N**	**A**	**SA**
Strongly Disagree	Disagree	Uncertain or Neutral	Agree	Strongly Agree

1. I like assigning problems that can be solved in multiple ways (McDougall, 2004, #1).		SD	D	N	A	SA
2. Often I have students complete relevant problems of interest (#2).		SD	D	N	A	SA
3. I provide time and encourage students to share their differing strategies for completing the same problems (#3).		SD	D	N	A	SA
4. Usually, it is not very productive when my students work together (#6).		SD	D	N	A	SA
5. “Every student should feel that mathematics is something he or she can do” (#7).		SD	D	N	A	SA
6. I encourage students to use multiple representations or alternative resources (i.e., manipulatives, technology, etc.) to communicate their mathematical ideas to me and their peers (#10).		SD	D	N	A	SA
7. On graded tasks, I put more emphasis on correct answers than on the process to get to an answer (#11).		SD	D	N	A	SA
8. On non-graded tasks, I put more emphasis on correct answers than on process (#11).		SD	D	N	A	SA
9. Instead of answering students’ math questions, I ask them additional questions to help them reason through their initial question (#14).		SD	D	N	A	SA
10. I do not like to assign open-ended tasks because I am		SD	D	N	A	SA

concerned that I will not cover the material for a unit in the designated time.						
11. I do not like to assign open-ended tasks because I worry that I may not be prepared for unpredictable results (#15).		SD	D	N	A	SA
12. I prefer that my students master basic procedures before tackling complex problems (#16).		SD	D	N	A	SA
13. "I teach students how to communicate their mathematical ideas" (#17).		SD	D	N	A	SA
14. I frequently have to remind my students that a lot of what we learn in mathematics is no much fun, of interest, or relevant to their lives, but that it is important to learn anyway (#20).		SD	D	N	A	SA
15. "When preparing lessons, I generally follow the textbook and/or the proscribed curriculum" (Adamson, Burtch, Cox, Banks, Judson, & Lawswon, n.d., #14) .		SD	D	N	A	SA
16. "When preparing lessons, I generally modify the textbook approach and supplement it with additional problems and/or activities" (#14).		SD	D	N	A	SA
17. "I mainly see my role as a facilitator. I try to provide opportunities and resources for my students to discover or construct concepts for themselves" (#10).		SD	D	N	A	SA
18. "I mainly see my role as a transmitter of knowledge. I try to assist students in arriving at a point of independence and mastery from which they can proceed on their own" (#10).		SD	D	N	A	SA

Please feel free to make additional comments about your practices in the space provided:

III. Beliefs about How Mathematics Should Be Taught (BT)						
1. "Mathematics is computation" (Beswick, Watson, & Brown, 2006, Table 4, #1).		SD	D	N	A	SA
2. Mathematics teachers should be fascinated with how students think and intrigued by their alternative strategies (#3).		SD	D	N	A	SA
3. It is an efficient way to facilitate student mathematical learning by telling students answers (#4).		SD	D	N	A	SA
4. Having students experience slight frustration and tension when solving a problem can be beneficial – even necessary – for learning to occur (#5).		SD	D	N	A	SA
5. The best method for teaching mathematical concepts is an expository style (i.e., demonstrating, explaining, describing, providing examples) (#6).		SD	D	N	A	SA
6. Mathematical concepts need to be presented in the correct sequence (#7).		SD	D	N	A	SA
7. "Ignoring the mathematical ideas that students generate themselves can seriously limit their learning" (#8).		SD	D	N	A	SA
8. Justification of mathematical ideas and statements is an important part of mathematics (#9).		SD	D	N	A	SA
9. To be an effective teacher of mathematics, one must enjoy learning and doing mathematics (#10).		SD	D	N	A	SA
10. An attitude of inquiry should be developed through the teaching of mathematics (#12).		SD	D	N	A	SA

11. Grade-nine mathematics is best taught to groups which are heterogeneous based on ability (#13).		SD	D	N	A	SA
12. "The most important part of instruction is the content of the curriculum" (Adamson, Burtch, Cox, Banks, Judson, & Lawswon, n.d., #11).		SD	D	N	A	SA
13. "The most important part of instruction is that it encourages sense-making or thinking. Content is secondary" (#11).		SD	D	N	A	SA
14. Students must have opportunities to work together to get an in-depth understanding of the content (#13).		SD	D	N	A	SA
15. Working together is problematic because a teacher cannot assess what each individual understands and oft one student does a majority of the work (#13).		SD	D	N	A	SA

Please feel free to make additional comments about your beliefs about how mathematics should be taught in the space provided:

IV. Beliefs about the Nature of Mathematics in the Real World (BRW)						
1. To be an intelligent consumer, one must be numerate (Beswick, Watson, & Brown, 2006, Table 3, #1).		SD	D	N	A	SA
2. Understanding mathematics is increasingly important in totals society (#4).		SD	D	N	A	SA
3. To function in today's society, being numerate (having "quantitative literacy") is equally as necessary as being literate (#5).		SD	D	N	A	SA
4. Mathematics is necessary to understand media claims (#7).		SD	D	N	A	SA
5. "Mathematics is not always communicated well in the media" (#9).		SD	D	N	A	SA
6. Often people use mathematics in their daily decisions (#10).		SD	D	N	A	SA

Please feel free to make additional comments about your beliefs about mathematics in the real world in the space provided:

V. Efficacy – Based on your clinical internship and other teaching experiences (E) -- #2, 3, & 6 ONLY ASKED ON THIRD ADMINISTRATION (A3)						
1. "When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort" (MTEBI, #1).		SD	D	N	A	SA
2. "I am continuously finding better ways to teach mathematics" (#2).		SD	D	N	A	SA
3. "I know the steps to teach mathematics concepts effectively" (#5).		SD	D	N	A	SA
4. "If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching" (#7).		SD	D	N	A	SA
5. "The inadequacy of a student's mathematics background can be overcome by good teaching" (#9).		SD	D	N	A	SA
6. "When a student has difficulty understanding a mathematics concept, I am usually at a loss as to how to help the students understand it better" (#19).		SD	D	N	A	SA
7. "I do not know what to do to turn students on to		SD	D	N	A	SA

mathematics" (#21).						

Please feel free to make additional comments about your effectiveness as a mathematics teacher in the space provided:

VI. Open-ended Questions -- #3, 5, 8-12 ONLY ASKED ON 3rd ADMINISTRATION (A3)

Please answer the following questions.

1. Do you feel as though you have a deep understanding of the content you are required to teach?

2. What about your content do you need to learn more about in order to help your students achieve a deep understanding?

3. Do you feel as though what and/or how you are required to teach (including the scope and sequence) can result in a depth of knowledge for your students in this content area? Can you provide any examples?

4. Do you feel as though you know the teaching strategies to teach your students in depth in this content area? Provide examples.

5. If yes to the previous question, do you feel as though you are able to utilize these strategies in order to help your students gain depth in this content area?

6. "How well do you think you can explain the concepts [in this content area] as opposed to just the rules or procedures?" (Stowalter, 2005, #2)

7. Mathematics curricula and teaching in this country is said to be "a mile wide and an inch deep." Do you agree with this? Why or why not?

8. Describe a typical day of teaching in your classroom.

9. Is this how you want to be teaching? Do you feel effective in your teaching? Why or why not?

10. If you answered yes to the previous question, what supports are in place for you to teach how you want to teach and achieve this effectiveness?

11. If you answered no, what constraints can you identify to why you can't teach how you want to be teaching?

12. "To what extent do you feel responsible for your students' learning?" (Showalter, 2005, #1a)

13. "Do you think students are excited about mathematics?" (Stowalter, 2005, #6a)

14. "Why do you think students should take mathematics?" (Stowalter, 2005, #7a)

Appendix B

Survey Items' Imputed Means and Standard Errors: Tables 7-10

Table 7: *What and How Mathematics Should Be Taught*

Normative Item	Mean A1 *SE*	Mean A2 *SE*	Mean A3 *SE*
III.1. "Mathematics is computation." (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 1)*	2.51 (0.12)	2.79 (0.10)	2.75 (0.13)
III.2. Mathematics teachers "should be fascinated with how students think and intrigued by their alternative strategies" (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 3).	4.26 (0.08)	4.73 (0.06)	4.48 (0.08)
III.3. It is an efficient way to facilitate student mathematical learning by telling students answers (#4).*	3.91 (0.11)	4.39 (0.10)	3.92 (0.15)
III.4. Having students experience slight frustration and tension when solving a problem can be beneficial – even necessary – for learning to occur (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 5).	3.44 (0.12)	4.20 (0.10)	4.21 (0.11)
III.5. The best method for teaching mathematical concepts is an expository style (i.e., demonstrating, explaining, describing, providing examples) (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 6).*	1.53 (0.07)	2.36 (0.12)	2.18 (0.18)
III.7. "Ignoring the mathematical ideas that students generate themselves can seriously limit their learning" (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 8).	3.82 (0.11)	4.54 (0.09)	4.39 (0.19)
III.8. Justification "of mathematical ideas and statements is an important part of mathematics" (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 9).	4.12 (0.08)	4.58 (0.07)	4.51 (0.09)
III.9. To be an effective teacher of mathematics, one must enjoy learning and doing mathematics. (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 10).	3.69 (0.13)	3.64 (0.13)	3.53 (0.19)
III.10. An attitude of inquiry should be developed through the teaching of mathematics (Beswick, Watson, & Brown, 2006, p. 72, Table 4, Item 12).	3.84 (0.07)	4.31 (0.09)	4.37 (0.09)
III.12. "The most important part of instruction is the content of the curriculum" (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d., Item 11).*	3.19 (0.10)	3.16 (0.13)	3.44 (0.16)
III.13. "The most important part of instruction is that it encourages sense-making or thinking. Content is secondary" (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d., Item 11).	3.90 (0.10)	3.99 (0.10)	4.07 (0.14)
III.14. Students must have opportunities to work together to get an in-depth understanding of the content (#13).	3.82 (0.11)	4.26 (0.07)	4.02 (0.09)
III.15. Working together is problematic because a teacher cannot assess what each individual understands and oft one student does a majority of the work (#13).*	3.03 (0.13)	3.43 (0.12)	3.67 (0.13)

*These questions have been reverse coded such that a response of "Strongly Agree" was coded with a 1; as a 1 is indicative of highly traditional beliefs.

Table 8: *What and How I Teach Mathematics* (McDougall, 2004; Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d.)

Discursive Item	Mean A3
II.1. I like assigning problems that can be solved in multiple ways (McDougall, 2004, Item 1).	4.52 (0.06)
II.3. I provide time and encourage students to share their differing strategies for completing the same problems (McDougall, 2004, Item 3).	4.52 (0.07)
II.4. Usually, it is not very productive when my students work together (McDougall, 2004, Item 6).*	3.84 (0.09)
II.6. I encourage students to use multiple representations or alternative resources (i.e., manipulatives, technology, etc.) to communicate their mathematical ideas to me and their peers (McDougall, 2004, Item 10).	4.39 (0.08)
II.7. On graded tasks, I put more emphasis on correct answers than on the process to get to an answer (#11).*	3.33 (0.10)
II.8. On non-graded tasks, I put more emphasis on correct answers than on process (#11).*	3.76 (0.10)
II.9. Instead of answering students' math questions, I ask them additional questions to help them reason through their initial question (McDougall, 2004, Item 14).	4.02 (0.08)
II.10. I do not like to assign open-ended tasks because I am concerned that I will not cover the material for a unit in the designated time.*	3.27 (0.09)
II.11. I do not like to assign open-ended tasks because I worry that I may not be prepared for unpredictable results (McDougall, 2004, Item 15).*	3.67 (0.10)
II.12. I prefer that my students master basic procedures before tackling complex problems. (McDougall, 2004, Item 16).*	2.08 (0.08)
II.13. "I teach students how to communicate their mathematical ideas" (McDougall, 2004, Item 17).	4.10 (0.07)
II.15. "When preparing lessons, I generally follow the textbook and/or the proscribed curriculum" (Adamson, Burtch, Cox, Banks, Judson, & Lawswon, YEAR, #14)	2.86 (0.09)
II.16. "When preparing lessons, I generally modify the textbook approach and supplement it with additional problems and/or activities" (#14).	4.14 (0.07)
II.17. "I mainly see my role as a facilitator. I try to provide opportunities and resources for my students to discover or construct concepts for themselves" (Adamson, Burtch, Cox, Banks, Judson, & Lawson, n.d., Item 10).	3.56 (0.09)

*These questions have been reverse coded such that a response of "Strongly Agree" was coded with a 1; as a 1 is indicative of highly traditional beliefs.

Table 9: *Mathematics in the Real World*

Normative Item	Mean A1 *SE*	Mean A2 *SE*	Mean A3 *SE*
IV.1. To be an intelligent consumer, one must be numerate. (Beswick,	3.68	4.18	4.19

Watson, & Brown, 2006, p. 71, Table 3, Item 1).	(0.09)	(0.11)	(0.09)
IV.2. Understanding mathematics is increasingly important in today's society. (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 4).	3.85 (0.09)	4.38 (0.07)	4.51 (0.10)
IV.3. To function in today's society being numerate (having quantitative literacy) is equally as necessary as being literate (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 5)	3.63 (0.10)	4.16 (0.12)	4.29 (0.13)
IV.4. Mathematics is necessary to understand media claims (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 7).	3.60 (0.10)	3.88 (0.12)	4.18 (0.11)
IV.6. Often people use mathematics in their daily decisions (Beswick, Watson, & Brown, 2006, p. 71, Table 3, Item 10).	4.25 (0.08)	4.56 (0.07)	4.60 (0.09)

Table 10: *My Teaching of Relevance of Mathematics in the Real World* (McDougall, 2004)

Discursive Item	Mean A3
II.2. Often I have students complete relevant problems of interest (McDougall, 2004, Item 2).	4.22 (0.06)
II.14. I frequently have to remind my students that a lot of what we learn in mathematics is not much fun, of interest, or relevant to their lives, but that it is important to learn anyway (McDougall, 2004, Item 20).*	4.51 (0.06)

*These questions have been reverse coded such that a response of "Strongly Agree" was coded with a 1; as a 1 is indicative of highly traditional beliefs.

Appendix C

Survey Items Revealing Statistical Differences for Middle-Level Majors

The three ML Math majors that took all three surveys scored significantly lower than [all others] on the first and second administrations of BT7 and significantly higher on the third administration of BT9. There were no other significant differences among middle-level participants.

Item	ML Math Majors			All others*				Independent t-test		
	Mean	SD	SE	Mean	SD	SE	n	t	df	P
A1_BT.III.7	2.6667	1.52753	.88192	3.9740	.82676	.09422	77	-2.61	78	0.011
A2_BT.III.7	3.3333	2.08167	1.20185	4.5862	.62223	.08170	58	-2.93	59	0.005
A3_BT.III.9	5.0000	.00000	.00000	3.1944	1.03701	.17284	36	2.98	37	0.005

*all others excludes MS math majors