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Factors that Influence Mathematical Creativity

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Abstract: Creativity is a psychological construct that has gained research popularity (Akgul & Kaveci, 2016), however it remains a challenging one to define. The variety of definitions promulgated to understand creativity hints at the complexity of the mental process. Furthermore, as a subset of creativity, domain-specific mathematical creativity has also garnered a variety of definitions. The transdisciplinary research on creativity (Sriraman & Haavold, 2017) is seminal in this world of fast-paced innovation, invention, solution, and synthesis. Considering every human being with at least average cognitive abilities possesses the ability to think creatively (Baran, 2011), developing students' creative talents and abilities must be high on a list of educational priorities. Much of the literature surrounding mathematical creative thinking is centered on trying to quantify an individual's creative thinking abilities. There have also been studies conducted that enabled researchers to describe various traits and demonstrate multiple levels of creativity. The basis of this work will be to synthesize the characteristics of mathematical creativity, analyze the impact of specific teaching approaches on mathematical creativity, and examine the relationship between student affect and mathematical creativity.

Keywords: creativity; affect; iconoclasm; intuition; investment; impartiality; modeling; model-eliciting activities

Introduction

When one reflects on academic skills such as literacy, mathematics, science, or social studies, one may presume such skills develop based on factors such as teaching approaches, the environments in which the subjects are taught, students' affect, and a variety of other factors. But

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can a thinking process be developed or inhibited in the same manner as an academic skill? Creative thinking in mathematics is a topic that has overreach, as its implications can be connected to a great deal of progress in society. In recent years, interest and activity in general creativity research has become increasingly popular (Akgul & Kaveci, 2016), however, when one analyzes mathematical creativity, extant literature remains relatively sparse (Leikin et al., 2010; Sriraman, Haavold, & Lee, 2013).

Creativity has long been challenging to define because it is a fuzzy psychological construct. According to Vanderbos (2006), creative thinking is a mental process leading to new invention, solution, or synthesis in any area. The renowned 20th century mathematician Henri Poincaré (1948) defined mathematical creative thinking simply as discernment, or choice. Sriraman defines mathematical creativity as “the process that results in a novel solution or idea to a mathematical problem or the formulation of new questions” (Sriraman, 2005; 2013, p.217). Furthermore, Siswono (2011), using the research of Krutetskii (1976) and Torrance (1965, 1968), describes creative thinking as the mental process which someone uses to generate “new” ideas with fluency and flexibility. The variety of definitions that have been adopted to understand creativity, hints at the complexity of the mental process. To operationalize the term ‘mathematical creativity’ for this work, it is useful to understand the psychological construct as an amalgamation of mathematical fluency, flexibility, originality, and elaboration. The transdisciplinary research on creativity (Sriraman & Haavold, 2017) is seminal in this world of fast-paced innovation, invention, solution, and synthesis. Considering every human being with at least average cognitive abilities possesses the ability to think creatively (Baran, 2011), developing students’ creative talents and abilities must be high on a list of educational priorities. Much of the literature surrounding mathematical creative thinking is centered on trying to quantify an individual's creative thinking abilities. There have also been studies conducted that enabled researchers to describe various traits and demonstrate multiple levels of creativity. The basis of this work will be to synthesize the characteristics of mathematical creativity, analyze the impact of specific teaching approaches on mathematical creativity, and examine the relationship between student affect and mathematical creativity

Literature Review

Creativity and its Relationship to Mathematics

For the purpose of this review of extant literature, it is useful to evaluate several pieces of meaningful research in the world of creativity in a general-domain sense. This will illustrate how investigation into creativity has allowed for mathematical researchers to make connections from domain-general creativity to domain-specific [mathematical] creativity (Hong & Milgram, 2010).

To understand where mathematical creativity is today, one must be familiar with foundational work by an English professor and a French mathematician, Graham Wallas (1858-1932) and Henri Poincaré (1854-1912). During an insightful presentation to the French Psychological Society, Poincaré laid the framework for mathematical creativity and "not so much described the characteristics of mathematical creativity, as defined them" (Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016, p.7). Later, Wallas (1926) developed a four-stage model to describe the process of having creative thoughts that was heavily derived from the work of Poincaré. This model is known as the Gestalt Model of Creativity. In Smith's (2015) analysis of Wallas' book, *Art of Thought*, he phrases that the four-stages of creative thinking consist of Preparation, Incubation, Illumination, and Verification. The Gestalt Model is accepted and regularly used by many creativity researchers (Smith, 2015) to this day. A brief description of each of the stages presented in the Gestalt model can be viewed in Figure 1 (Wallas, 1926).

Preparation (conscious)	A problem is thoroughly investigated.
Incubation (unconscious)	The individual is not consciously thinking about the problem but the mind may be unconsciously interacting with the idea.
Illumination (unconscious)	An idea is revealed to the individual, usually abruptly and unexpectedly.
Verification (conscious)	Conscious and concrete proof to validate the illumination.

Figure 1: Four Stages in the Gestalt Model of Creativity

When considering the Gestalt Model of Creativity, preparation and verification are considered conscious stages, whereas incubation and illumination are deemed unconscious stages. Poincaré describes how the unconscious stages never bear fruit unless followed by, or preceded by, conscious work (Smith, 2015). Without the proper [conscious] preparation—education and investigation of a problem—the stage of [unconscious] incubation may never take place. Likewise, without follow-up [conscious] verification, an [unconscious] illumination will produce nothing. This interplay between the conscious and unconscious mental state is further supported by Hadamard (1945) with his descriptions of conscious work, unconscious incubation of ideas, semiconscious illumination, and finally a conscious verification process.

The Gestalt Model of Creativity is well received in the world of mathematics. Sriraman (2004) conducted qualitative research in which he analyzed mathematicians' thought process and affirmed use of the Gestalt Model.

> In trying to better understand the process of creativity, I find that the Gestalt model proposed by Hadamard (1945) is still applicable today. This study has attempted to add some detail to the *preparation-incubation-illumination-verification* model of Gestalt by taking into account the role of imagery, the role of intuition, the role of social interaction, the use of heuristics, and the necessity of proof in the creative process. (Sriraman, 2004, p.31).

This study identified the moment of conversion from incubation to illumination. Multiple mathematicians in the study described how this enlightenment occurred at unexpected times. "Many reported the breakthrough occurring as they were going to bed, or walking, or sometimes as a result of speaking to someone else about the problem" (Sriraman, 2004, p.32).

The link between creativity and mathematical ability is a relationship that has been investigated and shown to have a positive relationship (Mann, 2005; Tabach & Friedlander, 2013; Walia, 2012). One study that explores this connection thoroughly is Mann's (2005) dissertation in which he examined the relationship between mathematical creativity and mathematical ability amongst a convenience sample of 89 seventh graders. By measuring student scores on the Connecticut Mastery Test (3rd Generation) and comparing them to the scores generated from the Creative Ability in Mathematics Test, Mann demonstrated a statistically significant relationship between mathematics ability and mathematical creativity. Additional research was conducted in which the relationship between six-year olds' level of creativity and their mathematical ability was analyzed. Baran, Erdogan, and Cakmak (2011) utilized the Torrance Test of Creative Thinking (TTCT) to gather quantifiable evidence of the students' general creative thinking ability and the Test of Early Mathematics Ability-3 (TEMA-3) to measure the students' mathematical ability. The results of this study and the evidence it presents hold unique insight. The study concluded that there was not a significant relationship between the sub-categories of creativity and scores on a traditional math test (Baran, Erdogan, & Cakmak, 2011). Initially, this study appears to suggest an opposing point of view regarding the relationship between creative thinking and mathematical ability. However, when the two tests from this study are examined, it is deduced that the TEMA-3 "…focuses on measuring the formal mathematical skills of children" (p.108). The link between creative thinking in mathematics and mathematical abilities does not lie within formal algorithms and standard procedural knowledge. The relationship lies in the fluid, flexible, and novel thinking of the individual (Krutetskii, 1976). An assessment that evaluates formal algorithms and definitive procedures does not gather accurate evidence of the overall mathematical ability of a student with high levels of mathematical creative thinking.

This brief overview of creativity research has provided depth of understanding in the field and will allow for the remainder of the discussion to focus on domain –specific (mathematical) creativity (Hong & Milgrim, 2010). To begin this discussion, the four indicators of creativity in mathematics, fluency, flexibility, originality (Krutetskii, 1976; Leikin, 2007; Siswono, 2011; Sriraman, 2004; Sriraman & Haavold, 2017), and elaboration (Chamberlin & Mann, 2014; Imai, 2010; Kim, Cho, & Ahn, 2003), must be defined. These indicators, when observed, suggest that mathematical creativity is taking place. Many of the scales and assessments created to measure mathematical creativity evaluate the interplay between these indicators.

Indicators of mathematical creativity.

One of the indicators of mathematical creativity is originality (Beghetto, 2017; Silver, 1997). Mathematical originality—also referred to as novelty—according to Siswono (2011), is an individual's ability to find a solution path that is especially unique and uncommon for that individual's knowledge level. An individual demonstrates originality when one creates a solution that is out of the ordinary, rare, and novel, in a mathematical situation. Initially (Chassell, 1916), originality represented the whole of creativity. The research done by Krutetskii (1976) began to expand mathematical creativity to include other indicators.

Another indicator of mathematical creativity is fluency (Leikin, 2007; Sriraman & Haavold, 2017). Fluency, in regards to its relationship to mathematical creativity, is notably different than the term procedural fluency often used in the general mathematics classroom. Procedural fluency, as understood in the general mathematics classroom, describes using procedures "accurately, efficiently, and flexibly" (NCTM, 2014, p.1). This is in contrast to fluency, as defined in the field of mathematical creativity as an individual's ability to come up with many different responses and solution paths to a problem (Krutetskii, 1976). Often, creative fluency is quantified by the number of responses that an individual is able to construct. If the individual were able to conceive multiple solutions, regardless if they were original, then one would be demonstrating strong mathematical fluency.

Additionally, flexibility indicates mathematical creativity (Leikin, 2007; Mann, 2005). In relation to mathematics, flexibility refers to the ability of an individual to change thinking paths when encountering an impasse, or thinking obstruction (Krutetskii, 1976; Leikin, 2007). The inflexible individual will typically continue to pursue a solution path to no avail (Imai, 2010). The individual that demonstrates high levels of flexibility is likely to switch thinking paths efficiently in order to approach the problem from a new direction. A common example of flexibility in thinking may be thinking backwards from the solution to a process or changing content areas in mathematics to gain additional insight to a solution path.

The final and most recently incorporated indicator of mathematical creativity is elaboration (Imai, 2010). Elaboration describes an individual's ability to give in-depth reasoning behind a solution path. In contrast to some creative individuals who intuitively arrive at original solutions, someone who demonstrates high elaboration indicators will be able to justify mathematical reasoning and provide sound explanations for why it is an appropriate solution (Kim, Cho, & Ahn, 2003).

This work will continue to refer to these indicators of mathematical creativity. They will serve as insight, or indicators, of when mathematical creativity is occurring within an individual and thus, mathematical creativity as a whole can be interpreted as some amalgamation of the aforementioned four indicators.

Levels and classifications of mathematical creativity.

Researchers have categorized and classified mathematical creativity in various manners. Siswono (2011) describes five Levels of Creative Thinking (LCT). She bases her levels on a combination of mathematical fluency, flexibility, and originality (Krutetskii, 1976). Students that attain the highest LCT scores in mathematics demonstrate all three of these mathematical characteristics and use them to solve, represent, and pose problems, though perhaps they will not utilize each component of creativity in every single problem solving situation.

Ervynck (1991) also attempts to describe three levels, or stages, of mathematical creative thinking. The three stages are: preliminary technical, algorithmic activity, and creative (conceptual, constructive) activity. In addition to these three stages, Ervynk adds “that the context for creativity is set by a preparatory stage in which mathematical procedures become interiorized through action before they can be the objects of mathematical thought” (1991, p.42). Therefore, Ervynck constructs a fourth stage, the ‘context stage.’

Beginning with the Preliminary Stage of Ervynck’s model, an individual uses mathematics in a repeated and practical manner. The problem solver may not realize the depth of mathematical content behind the process at the time, but instead use a ‘toolkit’ to successfully complete mathematical problems. There is no proof or justification for the calculations in this stage. The individual is only verified of success in whether or not the procedure continues to work.

Ervynck coins the second stage Algorithmic Activity. This stage is defined by an individual being able to generate and calculate algorithms and describes how "all intermediate steps have to be considered, at least implicitly; if not, a serious error may occur and totally invalidate the result" (1991, p.43). The ability to solve an out-of-context algorithm is still considered part of the preliminary stage, as it does not demonstrate the ability to produce meaningful intermediate steps necessary to generate the algorithm. A conceptual understanding of all foundational skills is necessary for an individual to truly demonstrate algorithmic activity. This evidence suggests that before students move into algorithmic-based problems, they must first have internalized necessary conceptual understandings in order to fully understand the more abstract algorithms.

The final stage presented by Ervynck is the Creative (conceptual, constructive) Activity. Ervynck considers this stage the essence of true mathematical creativity. Characteristics of this stage include divergent thinking, non-algorithmic activity, hypotheses, and "deductions from the hypotheses to establish the proof of the theorem" (1991, p.43). Two distinct steps are outlined in the process of hypotheses and proofs. The first step is the creation of a valued hypothesis. The second is drawing conclusions based on the hypothesis, which leads to development of the concept or theorem. In lieu of this proposed idea of mathematical creativity, others such as Chamberlin and Mann (in preparation), Nadjafikah et. al. (2011), and Silver (1997), Torrance (2001), argue that an individual need not be an advanced mathematician to demonstrate creative thinking in mathematics. Rather, the argument is made that an eight-year-old individual who solves an addition problem in an original manner—that was never taught to him—may be demonstrating mathematical creativity to the same degree as a well-versed mathematician who is proving a new theorem. In both situations, problem solvers are creating a solution to a novel problem.

Silver (1997) contrasts this classical (genius) view of mathematical creativity with a more contemporary one. He describes the classical view as the enlightenment of a gifted individual to have an exceptional thought, whereas the contemporary view relates to “deep, flexible knowledge in content. . . an inclination to think and behave creatively” (p.75-76). The contemporary view of creativity suggests that teacher instruction is crucial in developing mathematical creativity in students and that creativity is not only for a few gifted students, but appropriately in reach for the general school population (Boaler, 2016; Silver, 1997).

Assessing mathematical creativity.

The need for valid, mathematical creativity assessments is high. As discussed previously, mathematical creativity has been categorized and classified, however scales and tests to quantify mathematical creativity appear to be scant (Akful & Kaveci, 2016). As research and interest in mathematical creativity is proliferating, valid and reliable assessments to accurately capture insight on domain-specific mathematical creativity are lacking. In light of their paucity, there have been a few tests implemented in the past 50 years that have taken aim at quantifying mathematical creativity; each has its strengths and weaknesses.

Akgul and Kahveci (2016) have recently developed a scale for quantifying mathematical creativity. The Mathematical Creativity Scale (MCS) was designed to measure responses to items that were linked to fluency, flexibility, and originality. Compiled data was then correlated to overall mathematical creativity scores. Students were given a five-item test that implemented open-ended questions to promote divergent thinking. Fluency, flexibility, and originality were graded for each item using different scoring methods. Fluency was scored by awarding one point for each strategy used to solve the problem. Flexibility was scored by categorizing the different responses into groups and awarding points for different groups that answers were produced for. Finally, originality was scored using a scale where more points were awarded for the rare responses and the point value decreased with more common responses. From the validation process of this test, there were a few noteworthy conclusions. One conclusion drawn from Akgul and Kahveci (2016) is that "originality and flexibility had a correlation value of almost 1 with creativity, which means that they can be used to assess creativity alone" (p.70). Furthermore, they added "…fluency has a minimum correlation with total [creativity] score, fluency should not be used alone." This could imply that selection of fluency alone may not be sufficient when assessing mathematical creativity. Also, considering the stance that originality and flexibility have an extremely high correlation to overall mathematical creativity, future studies may decide to focus more closely on either originality or flexibility, not necessarily both.

There were five items selected and implemented on the MCS. These five items are open-ended questions that lend themselves to multiple solutions, but fail to form a connection to the individual's life and the real-world. Lesh, Hoover, Hole, Kelly, & Post, (2000) outline six principles of design for Model-Eliciting Activities (MES's) which prove beneficial for observing students' mathematical thinking. One of these six principles is the reality principle which states that "in order to produce the impressive kinds of results . . . it is important for students to try to make sense of the situation based on extensions of their own personal knowledge and experiences" (p.610). When interpreting the findings of Akgul and Kahveci (2016), it is important to weigh the individual test items used in the scale and consider that without personal connection to the items, the students may not be able to demonstrate their true mathematical creativity.

The Creativity Abilities in Mathematics Test (CAMT) is regarded as a seminal instrument for assessing mathematical creativity. Balka's (1974) method into the development of this test incorporated the input of many content experts (Mann, 2005). The CAMT was developed with a sample of 500 middle grade students, and after a validation process, was deemed to have a Cronbach's alpha reliability of .72 (Balka, 1974). This measure of internal consistency lies in the acceptable range for an assessment of this type. The CAMT also boasts a standard error of measurement of 7.24. Mann (2005) used the CAMT as an independent measure in which to compare other factors such as gender, mathematical ability, student perception of creativity, and a student's attitude toward mathematics. This allowed him to generate conclusions about the relationship between these factors and the students' mathematical creativity. Of the many factors that were analyzed and compared in Mann's study, the strongest indicator of high achievement on the CAMT is mathematical ability. Results of this study also conclude that students' attitude toward math have a significant correlation to the CAMT score, whereas gender and self-perception of creativity do not have significant correlations to the CAMT score.

Although the Torrance Test for Creative Thinking (TTCT) is a tool for measuring domain-general creativity, its place in this discussion is elemental as it is widely recognized and utilized. This test was developed by Ellis Paul Torrance (1915-2003) as the Minnesota Test of Creative Thinking. After an initial trial, it was modified into the present day TTCT. This test incorporates two components—verbal and figural—that gives a composite score of the individual's creativity. An in-depth look into the TTCT by Hébert, Cramond, and Neumeister (2002), depicts the scoring process as it relates to five norm referenced, and thirteen criterion referenced scores. The scoring of the figural portion evaluates fluency, flexibility, and originality (Torrance, 1966, 1974). This test was designed with the understanding that it is not to be interpreted as a static measure, but a means to help nurture creativity (Hébert, et. al., 2002). The TTCT continues to be used around the world as a means to quantify an individual's general creativity.

As a general understanding of mathematical creativity has been developed, the discussion will now focus on the possibility of influencing mathematical creativity. If the mental process of creativity were an immutable one, future progress on its development in individuals may be stagnant. However, if the mental process is shown to have the capability of growth, then a focus on supporting its development would be critical.

Teacher Approaches that Promote Mathematical Creativity

Mathematical creative thinking is a mental process that can be improved upon and developed. Hong and Milgram (2010) described how the educational environment in which students are situated can affect the realization that they are capable of, and able to, think creatively. The general acceptance is that teachers are a critical component that helps foster students' creative thinking in mathematics (Nadjafikhah et al., 2012). This synchronizes with the notion that teacher practice will impact the level of mathematical creativity in students (Silver, 1970). Hong and Milgram (2010) investigated the relationship between domain-specific and domain-general creative thinking to evaluate whether educational experiences could influence creativity. They did this by investigating the long-standing assumption that domain-general creativity would have a high correlation to the individual's domain-specific creativity. On the contrary, the researchers investigated if an individual's creative thinking is primarily domain-specific, meaning that the person may be able to embody creativity in specific skills, but not necessarily in all domains. One finding from the study was a difference between the creative thinking of high schoolers and college students. This difference in creative thinking was attributed to two things: (1) years of education and (2) exposure to highly complex problem situations. Due to college students' exposure to more years of schooling than high schoolers, they demonstrated increased creative thinking in specific areas. Additionally, the college students had encountered more complex problems (requiring divergent and diverse problem-solving skills) than high schoolers. From the data, the conclusion is drawn that ". . . the inconsistent finding on age differences across studies support the *specific* nature of creative thinking and life experience that are required for problem solving in various domains." (p.285). This research supports the notion that creative thinking may be a domain-specific process that can be nurtured and developed with appropriate teaching approaches. With the supposition that mathematical creativity is a domain-specific process that can be developed, there are certain teaching approaches that require divergent thinking processes which will aid development of mathematical creativity.

Problem-solving and problem-posing tasks.

There are specific teaching approaches that support authentic and creative thinking in mathematics. The teaching approaches that will be discussed can all be viewed as Problem Solving and Problem Posing Tasks (PSPPT) as seen in Figure 2.

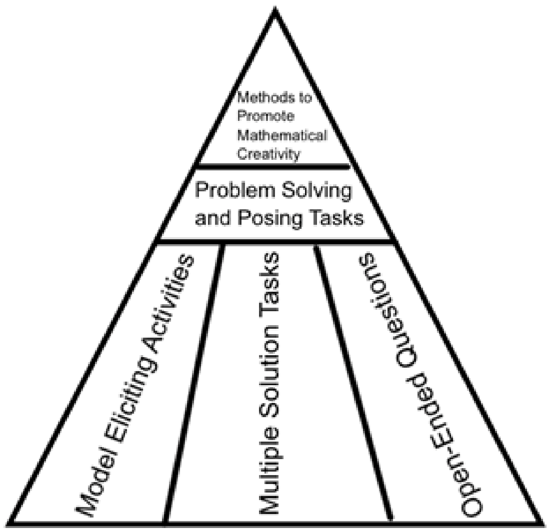

Figure 2: Teaching approaches that help support mathematical creativity

PSPPTs share characteristics in that they require to some sense, a formulation of strategy, an attempt to implement the strategy, a reformulation of strategy, and eventually a solution to the problem. This sequence of thought modification during the problem solving process plays an important role in developing an overall highly creative disposition towards mathematics (Silver, 1997). The characteristics of PSPPTs directly support fluency, flexibility, and originality, which are three traits of creativity in mathematics (Silver, 1997). Through the process of solving problems, students demonstrate the ability to solve problems by finding multiple solution paths (thereby facilitating fluency), changing of course when they encounter a stumbling block (thereby facilitating flexibility), and coming up with efficient and original ways to solve problems (thereby facilitating novelty). Through the process of posing their own problems, students practice writing a variety of problems (again, promoting fluency), create problems that lead to divergent thinking and mental impasses' (which promotes flexibility), and analyze a problem and create a problem that is different (which precipitates novelty). PSPPTs are crucial in the cultivation of creativity in mathematics (Kandemir & Gur 2007, Silver 1997).

Open-ended questions.

One specific teaching approach that is determined to have a positive relationship to mathematical creativity is open-ended questioning. After undergoing an eleven-week training on methods to promote creative thinking and problem solving, 43 secondary mathematics teachers were involved in a qualitative study by Kandemir and Gur (2007). Each educator was interviewed, the discussion was transcribed, and their responses were classified into themes. A prominent theme was that the use of open-ended questions that related to daily life supported the development of creative thinking. Another theme stated that teachers who judge skills critically—rather than generalize—can inhibit creative thinking in students. If a teacher has this characteristic, students will have a challenging time overcoming mental barriers in mathematics. To support creative thinking, teachers must be willing to look at all responses and appoint at least some sense of value to each and refrain from declaring whether a specific answer is right or wrong. Teacher practice such as this inherently reaffirms one student's answer and consequently may imply that the other students do not have worthwhile mathematical input (Kandemir & Gur, 2007). Open-ended questioning is a teaching approach that allows students to apply flexibility, fluency, and originality to their thinking (Silver, 1997). A pivotal element of the open-ended questioning is allowing students to hear their peer's mathematical justifications and reasoning (Hiebert et al., 2000). Furthermore, it gives the opportunity for students to hear multiple solution paths that were attacked in a creative or unique fashion. In this environment, creativity is not self-enclosed and manacled, but public and fostered. Students in this classroom may internalize the creative methods suggested by peers and implement similar strategies in the future.

Multiple solution tasks.

Another powerful teaching approach to support mathematical creativity is the use of Multiple Solution Tasks (MST), which are tasks that lend themselves to being solved in a variety of ways. Approaching and struggling with math problems fluently is one characteristic of creative thinking in mathematics (Leikin & Lev, 2007; Silver, 1997). Leikin and Lev (2007) conducted a study in which they posed MST's to groups of gifted (G), proficient (P), and regular (R) math students in 10^{th} and 11^{th} grade. They quantified the occurrence of three markers regularly associated with characteristics of mathematical creativity: fluency, flexibility, and originality (Krutetskii, 1974, Torrance, 1974). The purpose of this research was to perform a "systematic study that demonstrated that multiple solution tasks indeed may be used to show differences in mathematical creativity in groups of students with different ability levels" (p.161). The findings of their study elucidated how MSTs can affect creativity in mathematics. Leikin and Lev (2007) concluded that there was a consistent gap in the amount of flexibility, fluency, and novelty demonstrated in the MSTs between the G students and the P and R students. With respect to originality, it was noted that the main disparity in the data was the G-students' ability to manifest unconventional strategies on their own, whereas the proficient students relied on hints to develop these same types of strategies. This trend was also seen upon analyzing flexibility of the three groups "They, [groups P and R] were less flexible than G-students in their ability to change the direction of their mathematical thought without help" (p.167). Finally, when analyzing the data as it refers to the students' fluency, their evidence supports the claim that G-students spend less time coming up with successful solutions than their counterparts. This leads to the conclusion that it is easier for the G-students to come up with a variety of possible solution paths than it is for peers. This study illustrates the relationship between students' interaction with MSTs and their ability to create original, flexible, and fluent solutions.

Model-eliciting activities.

Encouraging young problem solvers to make sense of mathematical phenomenon through creating mathematical models is another approach to facilitate mathematical creativity. In addition to their qualities as highly open-ended problems that have multiple solutions, mathematical modeling is incredibly realistic in the respect that creating mathematical models mimics what real-life mathematicians do in a much better manner than simply solving routinized problems from a textbook (Lesh, Hoover, Hole, Kelly, & Post, 2000). A well-developed curricular approach to infusing mathematical modeling in and out of schools is by using Model-Eliciting Activities (Chamberlin, 2013; 2016). Model-Eliciting Activities (MEAs) are a curricular approach that has been presented to help students develop mathematical creativity, though that was not their initial intent. MEAs have also been shown to give valuable insight on a student's conceptual mathematical understanding. Chamberlin and Moon (2005) described MEAs as mathematical tasks that require students to create models and solve complex problems. They provide instructors invaluable insight in the form of assessment data, which enables instructors the opportunity to carefully analyze cognitive processes during mathematical problem solving episodes. A further goal of the MEAs is to "develop and identify students who are creatively gifted in mathematics" (Chamberlin & Moon, 2005, p. 37). Lesh (2000) considers an MEA to be productive if it meets but a single requirement: when students are engaged and struggling through the MEA, they explicitly reveal models that not only help the solution move forward in a mathematical sense, but also in a practical sense. MEAs allow an observer a window into the mathematical mind of an individual and gain insight on their conceptual understanding. Additionally, Lesh (2000) suggests that a productive MEA, one that meets the aforementioned goal, can help teachers plan, help researchers better understand mathematical thinking, and enables specialists to assess a wide range of mathematical capabilities. Six principles of MEA construction and design are employed (Chamberlin, 2005; Gilat & Amit, 2013; Lesh, et al., 2000) which allow the activity to possesses the appropriate amount of challenge and context to intrigue students as well as nurture all the traits of creativity.

An MEA was used in a 2013 case study by Gilat and Amit to analyze the cognitive and affective characteristics of two girls, aged ten and thirteen. The first way they analyzed these characteristics was through the students' mathematical models. The second was through an interview done with the girls after the completion of the MEA. The MEA supported three cognitive characteristics: flexibility, combinations, and analogy, along with three themes of student affect: motivation and interest, self-efficacy and persistence, and metacognition and self-reflection. The following piece of transcribed data from an interview highlights the type of mathematical thinking that may occur while one solves an MEA. "I didn't know how to apply to the task; I had to think in a different way, to think more real thinking, there was no single right solution and it made me think about other solutions, which is the best one, and not to think in a rigid way" (Gilat & Amit, 2013, p.57). This student's comment identifies how the MEA task requires students to partake in metacognition and self-reflection which may have some connection to the deeper matter at hand, mathematical creativity. Through this piece of conversational data, the student describes how she encountered challenges in flexibility (going about it in a different way), fluency (no single right way), and originality (thinking about other solutions, and making a judgment as to which is the best one). Gilat and Amit stated "The findings clearly show some cognitive and affective characteristics that could establish the foundations for creative process development methodology using MEAs" (Gilat & Amit, 2013, p.57).

In total, the use of open-ended questions, multiple solution tasks, and MEAs may have the propensity to positively influence mathematical creativity.

Student Affect and Mathematical Creativity

One of the primary motivators for the recent interest in mathematical creativity is its relationship to affect (Akgul & Kahveci, 2016). The tie between student affect and mathematics may play a critical role in the student's ability to produce creative thoughts. Mathematical affect decreases with each consecutive school year and most dramatically between the 3^{rd} and 8^{th} grade year (Tuohilampi, 2016). This trend is seen worldwide, taking into account factors in common between cultures and genders. Considering this, it is essential to evaluate the role of affect and its relationship to mathematical creative thinking. Tuohilampi created an intervention built around problem solving, open ended questions, collaboration, and student-centered mathematics (2016). She concluded that these interventions made a significant positive difference in the affect level of the students. A factor of interest in the study was "activating responses'. Activating responses were ones in which a positive emotion towards mathematics was demonstrated whereas a de-activating response was marked by a negative, or flat, emotion towards the mathematical exercise. The data analysis surfaced 54.3% of students who underwent the intervention had activating responses, whereas 16.9 % of students who were in the control group had activating responses. On the contrary, 27.8 % of students who underwent the intervention had de-activating responses, and 70.1 % of the control group had de-activating responses. This data pattern illustrates how students who were given the chance to problem solve, collaborate, and struggle with open-ended questions, maintain higher affect in mathematics than those exposed to more traditional styles of mathematical teaching.

McLeod (1992) structured the area of mathematical affect into three concepts: emotions, attitudes, and beliefs. Later on, Debellis and Goldin (1997) added a fourth concept of mathematical affect which they describe as values. When these four structures of mathematical affect are examined, the combined positive or negative existence of them makes up an individual's complete mathematical affect. DeBellis and Goldin (2006) further explain these four concepts of affect ranging from most rapidly changing to most stable. *Emotions* are very rapidly changing and vary depending on the specific situation that is presented. *Attitudes* are developed pre-dispositions based on previous experience. *Beliefs* are developed and influenced by external truths or factors that weigh on perception. Finally, *values* have deep rooted moral and ethical elements and are incredibly stable.

Pre-conditional affective states for mathematical creativity.

It is critical to analyze student affect in relation to mathematical creativity. Insightful commentary on this relationship is provided in Chamberlin and Mann's manuscript currently in preparation called *The Four Legs of Creativity* (in preparation). These four legs (pre-conditions) of creativity are affective emotional states that are suggested to, if present, increase creative output from an individual. The four pre-conditions outlined by Chamberlin and Mann are iconoclasm, impartiality, intuition, and investment (in preparation).

Iconoclasm as an affective emotional state may possibly have an influence on mathematical creativity (Chamberlin & Mann, in preparation). Iconoclasm is a mental state of an individual to break away from iconic ideas, or solution paths, and to challenge convention (Chamberlin & Mann, 2014). In current mathematics education, instructors often demonstrate a specific solution path or procedure for solving problems, which is often predicated on the use of one or more presented algorithms. For most students this is enough, but this solution path may not be deemed sufficient to the student who demonstrates high iconoclasm; this student may actively seek out a new solution path. Luchins describes the Einstellung Effect, in which "…problem solvers have repression of intuition, thus reducing the statistical likelihood that they seek a novel response" (Chamberlin & Mann, 2014, p. 2). In essence, an iconoclast is going to challenge the norm (the typical way of doing something) and may begin a venture into the unknown where the creative and modernizing ideas lie.

Impartiality is the affective emotional state in which an individual has no preference, or bias, towards one solution or another. This is posited to have a direct link to the relationship between affect and mathematical creativity. Impartial mathematicians "…may not have a proclivity to one solution because they are open to exploring alternate solution paths" (Chamberlin & Mann, in preparation, p.5). When students have found a solution path to be efficient, they can do one of two things: build a bias toward that path and repeatedly reproduce that procedure, or build no bias toward that path, continuing to look for and utilize new approaches—the latter response demonstrates impartiality.

A third pre-conditional affective emotional state for mathematical creativity to burgeon is investment. In terms of mathematical creativity, investment refers to the internal drive toward achieving a goal or purpose (Chamberlin & Mann, in preparation). This construct can be thought of as devotion to the mathematical situation for some intrinsic reason, be it dedication to one's own ideas or the motivation to persist at something challenging. When individuals show investment in mathematics, they are less likely to adopt a typical algorithm for the sake of ease, less likely to 'give-up' on a creative train of thought, and more likely to search for something unique and original. Conation is a cognitive state closely linked to this theory of investment and is regarded as the instinctual drive to complete or accomplish a goal (Gerdes & Stromwall, 2008) and the “natural tendency, impulse, or striving” (Riggs, 2007, p. 2) that individuals demonstrate in multiple facets of their lives. Certain students who demonstrate this internal drive toward achieving a goal may have a higher likelihood to produce creatively in mathematics.

Intuition is the fourth and final pre-conditional affective emotional state that is outlined by Chamberlin and Mann (in preparation). Intuition is an invisible drive that leads one toward something specific. In terms of everyday interaction, we tend to say “My intuition says not to trust that individual!” or “I have good intuition, therefore I believe we should get out of our current situation.” This indescribable impetus is often prevalent in creative mathematicians. The idea of intuition links closely to the work of Poincré (1908-1952) and the sub-conscious incubation period. He describes the preparation process for mathematical creative discoveries. Once an individual is prepared for discoveries, one’s mind may then lead the problem solver in a direction—seemingly by mere intuition—and the intuition will ‘incubate’ in their minds. Suddenly the thought can illuminate itself and be made consciously understood (Hadamard, 1945; Wallace 1926). One must have a high level of intuition to trust both the sub-conscious incubation and illumination phases of the Gestalt Model of Creativity. Hannula (2004) also describes affective competencies as “the capabilities of an individual to make effective use of affect during mathematical activity—for example, to act on curiosity, or to take frustration as a signal to alter strategy” (p.112). The idea of affective competencies is closely linked to that of intuition, as they both describe the moving toward something although the reason may be hard to describe. Why a particular student demonstrates flexibility, fluency, originality, and elaboration, is a question that still merits much research in the world of mathematical creativity. The theorized relationship of these four pre-conditional affective emotional states being building blocks of mathematical creativity may provide answers as to why some students demonstrate higher levels of mathematical creativity than others.

Discussion

Implications of the Literature

The research in mathematical creativity has implications to cognition and ultimately education. Not only does the research imply that changes may be needed in our education system, it also implies that changes may be needed in the general view of mathematics, a view that nourishes mathematical creativity in the classroom. The presented literature suggests that students' mathematics learning be fostered through support of their creativity due to their positive relationship (Mann, 2005; Tabach & Friedlander, 2013; Walia, 2012). The body of research acknowledges that teacher approaches, paired with an environment in which certain affective traits are nurtured, may support the emergence mathematical creativity.

The need for promulgating mathematical creativity in education.

The corpus of research supports the need for mathematical creativity to be nurtured and cultivated in students. As Sriraman (2004) stated "It is in the best interest of the field of mathematics education that we identify and nurture creative talent in the mathematics classroom" (p.32). Recent research implies that promoting creativity in mathematics will benefit student math learning and math knowledge (Mann, 2005). Nadjafikhah, Yaftian, and Bakhshalizadeh (2012) support this claim in stating "… it is necessary to improve teachers' ability to plan and implement educational environments that provide a secure atmosphere that students are encouraged to take risks, make mistakes and interact with others and share their point of view" (p.290). It is feared that without the nurture of mathematical creativity, students may suppress their creative thoughts and develop a distaste for mathematics overall (Mann, 2006). Similarly, the environment in which a student learns mathematics can support, direct, and guide creativity (Baran, 2011). Teachers play a vital role in student development by supporting creativity through teaching methods and by nurturing students mathematical affect in the classroom. Mathematical creativity is important to students, teachers, and crucial for the future progress and development of the field of mathematics.

Teaching approaches foster mathematical creativity.

Specific teaching methods should be in place to help develop students' creativity in mathematics. Just like best practice in reading, writing, or classroom management, there are specific teaching strategies that best support creative thinking in mathematics. Four specific teaching methods are discussed and endorsed to promote creative thinking (a) problem-solving and problem-posing tasks (PSPPT); (b) mathematical modeling (through MEAs); (c) multiple solution tasks (MST); (d) open-ended questions. Tasks that encourage creativity have certain characteristics that include constructions, connection to reality, self-assessing, documentation, multiple solutions, and justifications. Moreover, in the context of mathematics, they support divergent thinking at times and convergent thinking at other times. These characteristics in mathematical teaching transcend some current and traditional problems. Lesh, et. al (2000) describe how traditional problems most often seen in the current education system and textbooks tend to assess and endorse the acquisition of low level facts. The methods in which educators administer and evaluate students' mathematics only have the potential for measuring specific skills rather than deeper level understanding. On the contrary, teaching approaches presented in aforementioned work may give insight into students' mathematical thinking, promote deep level mathematics understanding, and support mathematical creativity.

Shared characteristics occur between these four teaching approaches, making them somewhat facile to implement. One common thread amongst these instructional approaches is that they do not encourage one single correct response or solution. Educators play an important role by refraining from providing verified and directed responses. Employing these teaching approaches allow students to create solution paths that may lead to technically accurate or less accurate solutions. The mathematical art evident in the process of developing the strategies is where the learning takes place, more so than the summative answer derived from the process. Furthermore, these teaching approaches espouse students' original thinking and celebrate struggles within the problem. By participating in any of these tasks, students have failures and struggles embedded in their mathematical thinking. Boaler (2016) suggests that individual struggle is when the mind is most active and engaged. These teaching methods provide students opportunity to encounter struggle, thus creating an environment where their mind is developing. In addition, these teaching approaches relate to a student's experience and their schema of knowledge. Mathematical instruction should be meaningful to students so they develop a relationship between the content and an experience in their lives.

Potential affective states that support mathematical creativity.

Based on affective characteristics, certain individuals are more prone to exhibit creative thoughts in mathematics than others are. This implication lends itself to classroom adaptations which support certain affective states (which possibly support mathematical creativity) and curtail certain negative affective states (which possibly hinder mathematical creativity).

When looking at the entire picture of mathematical understanding in an individual, affect may play a larger role than previously assumed. It is critical that educators can use this knowledge to develop a culture of mathematics in the classroom that develops certain affective traits within students, thereby supporting mathematical creativity.

Future Research

This analysis of mathematics research leaves questions that require further investigation. Boaler (2016) and Hiebert, et al. (2000) discuss the importance of meaningful mathematics instruction in schools. Boaler, for instance, argues that student engagement in mathematics comes from seeing the creativity and different views of mathematical solutions, that precise thinking "…combined with creativity, flexibility, and multiplicity of ideas, the mathematics comes alive for people" (p.59). Continued research in mathematical creativity could allow for this love and life to be brought into mathematics for individuals. Research in creativity in mathematics still has a plethora of avenues in which it needs to be explored.

Assessment tools for affective emotional states.

Empirical research is needed to pursue concrete answers in the discussion revolved around the influence of affective emotional states. Given the fact that many of the tenets of this paper are based on the synthesis of literature, empirical data must be collected in an experimental or quasi-experimental setting. Whether a relationship exists between mathematical problem solvers' affective states and creative output in problem solving tasks appears to be an answered question. However, precise states may be in question and exact levels of said states may also be in question. For instance, if iconoclasm (Chamberlin & Mann, 2014) is a psychological construct, what levels must be attained for creative output to be revealed?

Empirical documentation of mathematical creativity development.

A necessary direction for future research could be focused on empirical studies in which data are gathered, and relationships are identified between domain-specific mathematical creativity and specific teacher approaches. Certain teaching approaches have been examined and mathematics psychology researchers claim positive effects on students' mathematics creativity such as open-ended questions, Model-Eliciting Activities, and Multiple Solution Tasks (Lesh 2000, Chamberin & Moon 2005, Leikin 2009). What is yet to be documented, however, is the quantifiable score of creativity that was assessed as a pretest/posttest model when these instructional approaches are implemented in a holistic manner. For example, a classroom that is solely implementing these strategies in their mathematics instructions needs be compared to a control classroom, one that is not implementing any of these creativity-nurturing approaches. This would give insight into the correlation between creative output in a classroom that implements these strategies and one that implements an algorithmic/procedural teaching approach. Data from this research would encourage the mathematics education community to reconsider the way that mathematics is being taught in some schools. Sheets of problems that stimulate no creative thinking processes would therefore come under reconsideration for value and perhaps be mitigated in use. Said research may illuminate the understanding that students' mathematical creative thinking ability, an essential for future growth, will be nurtured most effectively by implementing thought-provoking and problem solving and problem posing tasks.

Re-tooling mathematical creativity assessments.

The validity and reliability of tests used to quantify domain-specific mathematical creativity has been brought into question. Although there are tests in this area of research that do attempt to assign a score to mathematical creativity, they lack the necessary psychometric properties and structure to truly examine mathematical creativity (Akgul & Kahveci, 2016). As outlined previously, each test has its own strengths and weaknesses, but none appear to directly assess the fundamentals of mathematical creativity based on updated research. It is noted that the most recent attempt at a mathematical creativity assessment is the MCS. Although it gives a score for creative thinking in mathematics, this scale does not meet the specific needs of measuring mathematical creativity, as the items throughout the test are not linked to real-world problems and do not lend themselves to the MEA principles of design (Chamberlin & Moon, 2005; Lesh et al., 2000). This area begs further investigation. Currently, the TTCT and the CAMT are being used in many settings. Although these tests have proven worthwhile and contributed to the body of research, they are nearly half a century old and lack updated scoring criteria that are based on new components of mathematical creativity such as the elaboration process. Huck (2012) suggests that instruments that have not been normed in the past seven years should undergo another round of validation.

Furthermore, there is an incongruity between extant literature in mathematical creativity and the tests used to assess mathematical creativity; the factor of incongruity is the indicator of elaboration. Elaboration is the fourth indicator of mathematical creativity (Leikin, 2007) and currently instruments designed to assess mathematical creativity do not incorporate elaboration in their data collection. Taking this into consideration, it is time to develop an up-to-date assessment that meets current needs in mathematical creativity. There has been a growing amount of interest in the field and many researchers are continuing to contribute to the current corpus of literature. This recent surge in scholarly work will provide valuable information that must be imbedded as foundation for any future mathematical creativity test.

Concomitant factors to cultivate mathematical creativity.

A conclusion drawn from current literature is conceivably powerful to the field. Research explicated contains two factors that influence mathematical creativity—student affect and teacher approaches. The obscurity lies in the power of considering these two factors jointly.

Missing research conceals a possible relationship between these two factors (when harmoniously supported in the classroom) and mathematical creativity. The question remains unanswered: Will a change in student affect or a change in teaching methods be enough to influence mathematical creativity? E. Paul Torrance would argue the essence of creativity cannot be quantified and a mere measurement of current indicators will not guarantee that an individual acts in a creative manner (Hébert, et. al., 2002). However, to depict a student's creativity more comprehensively, one possible next step is to look at an individual's affect combined with pedagogical experiences in a combined manner to evaluate its relationship to the student's mathematical creativity.

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