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Direct instruction and problem-solving

Critical examination of Cognitive Load Theory
from the perspective of mathematics education.

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Introduction

On the first page of his well-known "*How to solve it*" Pólya (1945) writes about problem-solving:

> One of the most important tasks of the teacher is to help his students. This task is not quite easy; it demands time, practice, devotion, and sound principles.
> The student should acquire as much experience of independent work as possible. But if he is left alone with his problem without any help, he may make no progress at all. If the teacher helps too much, nothing is left to the student.
> The teacher should help, but not too much and not too little, so that the student shall have a reasonable share of the work. (p. 1)

Striking in this quote is his balanced view on explication and exploration in the course of problem solving. The classic problem of designing a 3x3 magic square may be used as a didactic model to illustrate what Pólya exactly means with "not too much and not too little". This model can also show how the teacher, without extensive explication, is able to put even young children on the right track of problem solving.

Draw a 3x3 square and place therein the digits 1 to 9, as exemplified below.

1	3	7
8	4	5
6	9	2

> How can we construct a number square in which the sum of each row, column and diagonal is the same?

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If you are going to work on this task in the classroom, use number cards to easily permute the different possibilities. Let the children first try to find a solution themselves. The chance of success is not large: by blindly selecting, only 1 in 45,000.

Then we cast the problem in the context of a story about King Square. He and his arithmetic masters also failed to either find a magic square or to prove that such a square does not exist. He therefore promises a high reward to anybody who can discover that magic square.

It doesn't take long before a finder arrives: "I have discovered the magic square, but will not directly show it. I only give you a hint. You are the highest in the country, that's why I put the 9 in the centre." The king feels flattered and very happy with the tip. He thinks the magic square is coming into view.

But before handing over the reward to the finder, he decides to present the tip to his masters in order to check that he is not dealing with a cheater. The king's arithmetic masters … are the children, who examine whether the finder deserves the reward. They try in groups of four to complete the magic square with 9 in the centre, but no group finds the solution. The children are able to explain why they didn't succeed: the difficulty is that you can't place the big numbers: if 9 is in the centre then must be a threesome which also includes, for example, 8, and this obviously is impossible.The same occurs with 8, 7, and 6 in the centre. And with 1 in the centre ("Your majesty, you are the number 1 in the country") you likewise get stuck with the numbers; the same with 2, 3, and 4 (for example, no threesome with 4 and 1 is possible). But for 5 it is different! 1)

All children start working again solving the puzzle. They need little time: a magic square has been discovered.

6	1	8
7	5	3
2	9	4

To conclude the lesson several solutions are shown.2)

In addition to "place 9 in the centre", there are, of course, more possibilities to present and guide solution of the problem. For older children and (young) adults, for example, the general suggestion to first examine what number might be in the centre could be suitable.

One can opt for: [a] solve the problem independently, [b] explain through direct instruction how one can come up with a solution or [a, b] an alternating mixture of both. These are the different didactic possibilities on the sliding scale of [a] via [a,b] to [b]. And it depends on the type of problem which didactic work form one chooses, as Pólya indicates in the opening of his book.

His view on problem solving, as exemplified wih the magic square, serves as a contrasting background to the concept of problem-solving within Cognitive Load Theory (CLT) that is the main focus of this article. This theory explictly chooses the explanation method of [b], thereby opposing the independent problem solving of [a], and without including intermediary forms [a,b] of flexible didactic aid in both the theoretical analysis and the comparative research.
First I explain what this theory entails. Then I describe the model-experiment from which the theory developed. Subsequently the CLT's original concept will be analysed didactically.

Cognitive Load Theory

Cognitive psychologist Sweller is the founder of Cognitive Load Theory. Sweller doesn't like the usual approach to problem solving in mathematics instruction, because it loads working memory so heavily with new cognitive operations that the pupil is locked up in the solution process. As a result, new information cannot be transferred to long-term memory, so there is no question of learning. For this reason the teacher can show better through direct instruction how to solve problems than to let students solve problems themselves. Other possibilities are exemplifying problems with worked examples and with open tasks – old didactical tools …

> The worked example effect, according to which learners who study worked examples perform better on test problems than learners who solve the same problems themselves, also derived from the reasoning that conventional problem solving interfered with learning because it concentrated on reaching a problem goal rather than transferring knowledge to long-term memory. (Sweller, 2016, p. 4)

In his "Story of a research program", Sweller (2016) describes the experiment whose outcome led to the design of his theory.

The CLT model experiment

The students were confronted with problems in which a given number had to be converted into a given target number via two operations.

> The problems required students to transform a given number into a goal number where the only two moves allowed were multiplying by 3 or subtracting 29. Each problem had only one solution and that solution required an alternation of multiplying by 3 and subtracting 29 a specific number of times. (Sweller, 2016, p. 2)

However, the students did not know that the solution could only be found by applying the couple [x3, -29] two or more times in succession. The assignment to convert 15 to 16 goes as follows: 15 **x 3** = 45, 45 **- 29** = 16. With two operation couples 15 is transformed into 19, and with three into 28.

In these examples, the first move 'x 3' is forced and also the alternating follow-up takes place automatically, because with a second move with a repeated 'x 3' you wander far away from the target number, so follows '–29' and so on. In other examples, the solution also proceeds that way.

It is not surprising that the outcome of the experiment is that "my undergraduates found those problems relatively easy to solve with few failures" (Sweller, 2016, p. 2). The researcher, however, observes something remarkable:

> While all problems had to be solved by this alternation sequence because the numbers were chosen to ensure that no other solution was possible, very few students discovered the rule, that is, the sequence of alternating the two possible moves. (Sweller, 2016, p. 2)

From the fact that the students did not discover the alternating solution with [x 3, -29] Sweller drew the remarkable conclusion that it is better to show students how to solve problems than to allow them to solve problems themselves:

> Cognitive load theory probably can be traced back to that experiment. My objections to the many variations of discovery and problem-based learning also have a similar source. (…) We give students problems to solve in subjects such as mathematics with the expectation that they would learn to solve such problems. If my experimental results were generalizable to educational problems, that expectation may not be realized. Perhaps we should be showing students how to solve problems rather than having them to solve the problems themselves? (Sweller, 2016, p. 3)

Problem solving loads the working memory too much and thus impedes the storage of the intended knowledge and skills – in this case the rule of the alternating operation couple. New in this view is the connection between, on the one hand, working memory and long-term memory and, on the other hand, problem solving – or, rather, the decoupling of them.

The fact that Sweller wants to keep discovery learning away from education is shown once again by the following quote:

> It was obvious to me that if I had simply informed students to solve each problem by alternating the two moves until they reached solution, they would have immediately learned the rule and would have been able to solve any problem to them no matter how many moves were required for solution. Of course, since these were problem-solving experiments, I had not informed participants of the alternation rule and most failed to discover the rule for themselves. (Sweller, 2016, p. 3).

In his "Story of a Research Program", Sweller (2016) writes about the difficulties he experienced in the 1980's to convince researchers that problem solving is a problem:

> Most of the field leapt enthusiastically on the problem solving bandwagon. The research on worked examples was treated either with hostility or more commonly, ignored, a state of affairs that lasted for about two decades. (Sweller, 2016, p. 5)

The outcome of Sweller's experiment has nothing to do with mathematics, because that does not impose random, subjective, rules and procedures: the hidden rule – which was not even asked for in advance – cannot be proved with mathematical induction.
And then again: is problem solving the detection of a rule found by induction?

Calculate and prove

Sweller states that all problems had to be solved by an alternating sequence of operations because the numbers were chosen to ensure that no other solution was possible. This claim leads to the task.

1. Try to find one or more examples of target numbers that can only be reached with the operations 'x 3' and '-29' from a given number.

Many students will – just like Sweller – first use the chosen starting numbers to achieve the target number by applying the operations alternately and then non-alternately. At first glance the latter does not seem to work, but on closer inspection it does.
This then raises the question of showing that for every alternating sequence that works a non-alternating sequence can always be found:

2. Prove that each target number produced by alternating the couple [x 3, 29] can also be reached via a non-alternating sequence of those operations on the given starting number.

Label the starting number *'s'*.
The number reached after two alternating couples is: $3(3s - 29) - 29 = 3^2 s - (4 \times 29)$. From this formula you can immediately see that you can reach this number by a non-alternating sequence by two multiplications, followed by four subtractions, so in six moves.
The formula for three couples is $3^3 s - (13 \times 29)$, which means that the goal number can also be reached here in 16 steps (or in 14, 12, 10, or 8 steps) as well as the six steps of the alternating sequence. Sweller thus happened to show the same behaviour as his test subjects: he overlooked a general rule when composing the task. Was this due to overloading of his working memory? Was insufficient knowledge of the algebraic method stored in his long-term memory? The answer to both questions is in the negative.
If the solution of the second question immediately is given in this way – so by labeling the starting number 's' - via direct instruction or with a worked example, than it is difficult to understand that Sweller has overlooked it: there hardly seems to be a problem. Note that

'seems', because in fact the application of the general method is indeed a find: one had to foresee that an alternative series can be derived from that formula. But that discovery is completely nullified by "name the starting number , s" !

With this we touch the Achilles heel of the explanation of the teacher and of a worked example. If they are not preceded by exploration of the pupils, and where necessary with hints from the teacher, the problem and the discovery can not be valued as such.

In short, this sum of couples can serve as an example of a flexible, interactive lesson about problem solving – an approach that is at odds with Sweller's teaching theory.

Critical reflection

The experiment from which the concept of the CLT originates is mathematically as well as educationally below an appropriate level, the assignment contains a mathematical error, and the fact that no prior request was made for the (alleged) solution rule is also problematic.

The conditions for solving the problem correctly were fulfilled. But the right solution was not found. The reason for this has just been indicated: the solution was not dealt with at a general level. An important heuristic guideline was not followed. Accordingly, the described experiment can not function well as a basis of an educational theory. What is most striking, however, is that Sweller completely ignores the flexibly guided teaching approach to problem-solving.

Pólya and CLT

In the preface to the first edition of *How to solve it* Pólya (1945) notes the following about problem solving:

> A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.
>
> Such experiences at a susceptible age may create a taste for mental work and leave their imprint and character for a lifetime. Thus, a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity.

For him, problem solving is not only a means but also a goal-in-itself. The general image of the tightly guided pupil does not fit in there. Pólya differentiates four phases in solving a problem:

> First, we have to *understand* the problem; we have to see clearly what is required. Second, we have to see how the various items are connected, how the unknown is linked to the data, in order to obtain the idea of the solution, to make a *plan.* Third, we *carry out* our plan. Fourth, we *look back* at the completed solution, we *review* and discuss it. (Pólya, 1945, p.5)

In fifty examples, he pays a lot of attention to the heuristic strategies that can lead to the solution. In the foregoing, these general guidelines, such as those of reasoned trying, working systematically, schematizing, and so on, have already been discussed. He also exemplifies the flexible role of the teacher in solving a problem, such as the suggestion, after the solution, to ask the pupils to come up with related problems.

How do Sweller and his colleagues respond to the ideas of Pólya about problem solving? This question is answered in an article with the telling title “Teaching general problem-solving skills is not a substitute for, or a viable addition to, teaching mathematics”. The general strategies pursued by Pólya add nothing to mathematics education, write the three cognitive psychologists in “Notices of the American Mathematical Society”:

> For example, many educators assume that general problem-solving strategies are not only learnable and teachable but are critical adjuncts to mathematical knowledge. The best known exposition of this view was provided by Pólya. (Sweller, Clark, & Kirschner, 2010, p. 1303)

Pólya, however, claims and demonstrates exactly the opposite in a way that can hardly be misunderstood:

> It may be good to be reminded rudely that certain aspirations are hopeless. Infallible rules of discovery leading to the solution of all possible mathematical problems would be more desirable than the philosophers’ stone, vainly sought by alchemists. Such rules would work magic; but there is no such thing as magic. (…)
>
> A reasonable sort of heuristic cannot aim at unfailing rules; but it may endeavor to study procedures (mental operations, moves, steps) which are typically useful in solving problems. Such procedures are practiced by every sane person sufficiently interested in his problem. They are hinted at by certain stereotyped questions and suggestions which intelligent people put to themselves and intelligent teachers to their students. (Polya, 1945, p. 172)

Sweller, Clark and Kirschner criticize reform curricula that Pólya follows: not only is the role of problem-solving misunderstood in them, but reformists also think that by teaching general solution strategies the content can be reduced, while one should know better:

> There is no body of research based on randomized, controlled experiments that such teaching leads to better problem solving. (Sweller et al, 2010, p. 1303)

According to them, research shows that domain-specific problem solving can be taught:

> How? One simple answer is by emphasizing worked examples of problem solution strategies. (p. 1304)

In a much-discussed article by the aforementioned authors, they give a theoretical explanation for this positive worked-example effect that we already encountered earlier:

> Solving a problem requires problem-solving search and search must occur using our limited working memory. Problem-solving search is an inefficient way of altering long-term memory because its function is to find a problem solution, not altering long-term memory. (Kirschner, Sweller, & Clark, 2006, p. 80)

Against this, it may be asserted that by offering examples – without those examples being preceded by independent problem solving – the essential mathematical activities are, in fact, reduced to solving routine problems. On this point, Pólya notes:

> Routine problem may be called the problem to solve the equation $x^2 - 3x + 2 = 0$ if the solution of the general quadratic was explained and illustated before so that the student has nothing to do but to substitute the numbers – 3 and 2 for certain letters which appear in the general solution. Even if the quadratic equation was not solved generally in 'letters' but half a dozen similar quadratic equations with numerical coefficients were solved just before, the problem should be called a 'routine problem'. In general, a problem is a 'routine' problem if it can be solved either by substituting special data into a formerly solved general problem, or by following step by step, without trace of originality, some well-worn conspicuous example. (…) Routine problems, even many routine problems, may be necessary in teaching mathematics but to make the students do no other kind is inexcusable.(Pólya, 1945, p. 171)

Together with the solution of linear equations, this example is the very subject on which research related to worked examples first focused and on which the CLT authors set so much store.

The final words of that article on direct instruction and mathematical problem-solving may be read in this context:

> For novice mathematics learners, the evidence is overwhelming that studying worked examples, rather than solving equivalent examples, facilitates learning. Studying worked examples is a form of direct, explicit instruction that is vital in all curriculum areas that many students find difficult and that are critical to modern societies. Mathematics is such a discipline. (Sweller, Clark, & Kirschner, 2010, p. 1304)

After the foregoing critical analysis of CLT with regard to problem solving, the question arises, of course, as to how it is possible that an important application of that theory could yield such positive results – at least according to the authors mentioned.

Research

In the CLT approach, worked examples function as the most important instructional tool. It is true that examples were often used to illustrate new rules and procedures, but that use is too limited, according to Sweller (1999, p. 73):

> It would be an unusual (not to mention incompetent) teacher who did not use worked examples. Similarly, textbooks universally use worked examples to illustrate new concepts. The suggestion being made here goes beyond this limited use of worked examples. Rather than using them merely to demonstrate how to use a mathematical or scientific rule, the proposal is that they should be used in large numbers as a form of practice. In other words, instead of practicing by solving many problems (an activity engaged in by most conscientious students), it is proposed that many of these problems could profitably be replaced by worked examples.

It is the intention that worked examples will be interwoven much more with (series of) tasks that the students have to solve independently. In this form, the research on worked examples has been set up. The general paradigm is that the performances of [a] a group of pupils who solve a series of problems independently after a short explanation, are compared with those of [b] a group of pupils who tackle part of the same collection of problems, but through worked examples studied by the students.

The results of these comparative studies, which began in the 1980s with solving linear and quadratic equations, appear to show an advantage for the [b] groups using worked examples. The key question, however, is how the comparison between the guided [b] group, and a flexibly guided group combining approaches [a] and [b] with different weightings, would work out. Or, more broadly, how the performances of students under direct instruction relate to those given flexibly guided problem solving. For Sweller, Kirschner, and Clark the answer to this question is settled: the [b] group would also be the best in this comparison, because as new material is involved, new skills and concepts will have to be provided and processed. Unguided problem solving for that reason has no added value, and is therefore not very efficient and effective.

This theoretical position is controversial within its own CLT circle: there are CLT theoreticians who do not endorse these extreme views.

> While we concur with the authors about the failure of minimally guided instruction for novice learning in structured domains, in this commentary we will argue that problem-based learning (PBL) is an instructional approach that cannot be equated with minimally guided instruction. On the contrary, we contend that the elements of problem-based learning allow for flexible adaption of guidance, making this instructional approach potentially more compati-

> ble with the manner in which our cognitive structures are organized than the direct guided instructional approach advocated by Kirschner, Sweller and Clark. (Schmidt,et al, 2007, p. 91)

Their teaching approach to problem-solving is in line with the views of Pólya.

A meta-analysis of hundreds of studies (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011) showed that independent problem solving is less effective than direct instruction (d = -0.38). However, a different meta-analysis (Lazonder & Harmsen, 2016) concluded that if the problem solving investigation is carried out with appropriate support, it turns out to be more effective than direct instruction (d = 0.30).

Kalygua and Singh mention the relationship between instructional forms and educational learning goals in different phases of instruction:

> One of the consequences of this reconceptualization is abandoning the rigid explicit instruction versus minimal guidance dichotomy and replacing it with a more flexible approach based on differentiating specific goals of various learner activities in complex learning. In particular, it may allow reconciling contradictory results from studies of the effectiveness of worked examples in cognitive load theory (supporting the initial fully guided explicit instruction for novice learners) and studies within the frameworks of productive failure and invention learning that have reportedly demonstrated that minimally guided tasks provided prior to explicit instruction might benefit novice learners. (Kalyuga & Singh, 2016, p. 831)

In complex learning, the authors distinguish three phases, namely those of exploration, explication, and inventive application. As an objection to traditional CLT-version they indicated that this theory has restricted itself too much to the second instruction phase, whose outcomes can be described in terms of domain-specific objectives.

Take the aforementioned example of solving quadratic equations of the type $x^2 + ax + b = 0$ by factorization. In the first phase, the pupils are instructed to solve some of these problems even before the decomposition of factors has been dealt with. Only the method of substitution is at their disposal, substituting for x values 1, 2, … and so on until (one) solution is found – a cumbersome and incomplete method. Thereafter follows the second phase in which the technique of factorization is explained and practiced with some examples, and reduced to a routine. In the third phase this routine can be used to solve more complex quadratic equations, such as $x^4 - 13x^2 + 36 = 0$. This phase often has a high aha-level with a lot of scope for application of heuristic guidelines. In their article, the authors refer to various researches from which it emerges that the independent problem solving of phase 1 positively influences the results of 2.

What is remarkable about their phasing is that there is no room therein for independent problem solving in phase [b] of the (direct) instruction. From the above example, however, it appears that decomposition in factors can largely be detected by the students themselves. This is done by reversing the earlier learned multiplications

$(x + a)(x + b) = x^2 + (a + b)x + ab$ and the relevant product- and sum- rules from statements such as $x^2 - x - 6 = (x + 2)(x - 3)$. This rule can then be practiced with self-produced tasks – all to be realized, of course, under appropriate, in this case minimal, guidance from the teacher. However, this approach is not in line with the concept of explicit, direct instruction of CLT 1.0 in which learning of knowledge and skills precedes problem solving.

In the following it will be argued that interactive didactics with the *alternating* [a] and [b] elements best fits conceptual mathematics education – an educational approach that is not in line with the CLT position of Sweller and his colleagues.

A paradigm of interactive problem-solving

The following Sudoku example of determining number of permutations is related to the aforementioned coupling problems of the CLT experiment, in that both involve deriving a general rule from a number of examples.

2	4	9
5	7	3
6	8	1

How many different squares of 3 by 3 are possible with the digits
1 to 9?

However, the essential difference is that we now ask for this general rule in advance. In the exploratory phase, if necessary, the problem is converted into an analogous task and simplified at the suggestion of the teacher.

Transformation: read the given number square as 249573681 and interpret the sequence as a number. The question then becomes how many numbers can be made with the nine digits.

Simplification: instead of nine, choose three digits first, then four and five.

We have the numbers 1, 2, and 3. Use them all.

a) Make as many numbers as possible using these three digits once each.

b) Do the same with 1, 2, 3 and 4. And then with 1, 2, 3, 4 and 5.

c) How many possibilities are there with the numbers 1 to 9?

With three figures given, six numbers are possible – working systematically, pupils in the middle and upper classes of primary school find this answer relatively easy. Organizing the numbers according to size is most obvious. With four digits, 24 numbers can be made. There are pupils who draw this conclusion directly from the answer of a) on the basis of symmetry considerations. Others, however, are going to rearrange all the figures and come to the conclusion that a shortcut is possible.

How a similar 5-digit problem is solved is highly interesting. Do more students now make use of the answer that they have found at 4 digits? If not, a hint can be given. Finally, in phase c) it is possible to express the rule in a short, general permutation formula, where the letter g can be given as an abbreviation of a general number: g different numbers can be put in order in 1 x 2 x 3 x 4 ... x g different ways. This can be visualized with a tree diagram. With this, one can get a view of the wider field of combinatorics: For the first number there are 9 possibilities, we can combine each with the 8 for the second number and so on. The number of possibilities is thus 9 x 8 x 7 x ... .1 = 362 880 – a multiplication that can be executed easily (9x8x7 = 504, ...). All squares on the number square are filled systematically.

The theory-loaded, complex permutation problem that can be at the beginning of a new course for combinatorics is not "goal-free" but goal-oriented, and it is not aligned with worked examples, but is set within a interactive educational approach. In the solution, a number of general heuristic guidelines can also be applied and identified.

In conclusion

In all the examples mentioned in this discussion on problem solving – magic square, sequences of arithmetical operations with source and target numbers, solving quadratic equations, and detecting the permutation rule – the teaching method of the so-called alternating [a,b]-couple of independent (group)work within (horizontal and vertical) interactive instruction was clearly visible. This interactive method is distinguished from both [a] the dominant constructivist and [b] the instructivist teaching approach. However, this approach is not in line with the concept of the explicit, direct instruction of the CLT-version in which learning knowledge and skills precede problem solving. There are numerous studies that show that the interactive approach leads to better results than the aforementioned extremes.

Notes

1). In virtually all mathematics books a completely different solution path is indicated to show that 5 must come to the center, the even numbers on the corners and the remaining odd numbers in the middle of the sides. The children are instructed to write the number triplets.

15 = 9 + 1 + 5 = 9 + 2 + 3. The 7, 3 and 1 are also in two triples.

15 = 8 + 1 + 6 = 8 + 2 + 3 = 8 + 3 + 4. Plus 6, 4 and 2 in three triples.

15 = 5 + 1 + 9 = 5 + 2 + 8 = 5 + 3 + 7 = 5 + 4 + 9. And 5 in four triples.

From this it follows that only with 5 in the centre the requested sum of 15 can be made horizontally, vertically and twice diagonally.

From a problem solving point of view, however, this is not a productive approach: the core of the problem is given away in advance – a didactic issue that is also discussed by Pólya.

2) After the solutions of the wizard square have been inventoried, different versions appear to exist.

Suppose the students have found six.

Then the question arises whether all possibilities have been exhausted - a question that can be raised in the last years of primary school and be answered through interactive problem-solving.

Determine the number of possible 3 by 3 magic squares

Two solutions are available when answering this question.

The first is based on the insight that with 5 in the center and one placed 9 and 8 the magic square is fixed. For each placed 9, two places remain for 8. And since 9 can be in four places, the number of possible squares is 4 x 2 = 8

(8)		**(8)**
	5	
	9	

This solution can be represented with a tree diagram.

The second solution consists of turning and mirroring. Do the students come up with this strategy? If not, a hint can be given in that direction.

From four possible spins and four reflections it can not be directly deduced that there are eight different magic squares in total. After all, we still have to investigate whether a combination of turning and mirroring might produce a new variant - which after research does not appear to be the case.

What if we had nine other integers? For example the odd nunbers from 1 through 17. can we find a magic square for them?

And if we add or multiply all the cells in a magic square by a constant, can we get a new magic square?

The problem of the magic square is therefore richer than it seemed at first glance, not only arithmetically but also geometrically. Mathematics isn't just about mastering facts and procedures. (Schoenfeld, 2013, p. 27)

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