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Exploring Secondary Students' Modelling Competencies

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Abstract: *Mathematical modelling is a very important component in the teaching and learning of mathematics. In Malaysia, educators require more exposure to mathematical modelling it is still a new pedagogical method in classrooms. Modelling tasks that require students to construct a mathematical model would be a good start in developing modelling competencies. In this study, the authors investigate three mathematical modelling competencies of secondary students in the state of Selangor through a qualitative analysis of 20 students (divided into four groups) responses based on one modelling task developed by the researchers. The three modelling competencies we focus in this study are (1) making assumption; (2) computing and interpreting solution; and (3) mathematical reasoning. Our findings indicate that most students were basic users while a small percentage reached the proficient users level. With more engagement with the modelling tasks, students would able to acquire the modelling competencies and also develop their modelling skills. This would contribute to the meaningful learning of mathematics.*

Keywords: mathematical modelling, modelling competencies, secondary students

Introduction

Good problem solvers but also manage to reason or interpret real life problems. Mathematics is integral on our daily lives. By learning mathematics, students are expected to apply their knowledge in their daily lives. Mathematics modelling offers a platform to use mathematics in solving real world problems. Learners also develop mathematical competencies through mathematical modelling such as making assumptions, computing and interpreting solution and mathematical reasoning. Mathematical modelling has contributed in providing students the opportunity to develop their skills and competencies in the twenty-first century learning by solving real-world problems (English and Sriraman, 2010). The appropriate use of mathematics and model can help students develop modelling competencies in their understanding of the

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problem situation and the realization of different stages in the modelling process. In the end, students not only become

It is important to assess secondary students modelling competencies as these skills are related to the 21st century learning skills. By knowing the level of students modelling competencies, teachers would be able to plan their mathematical lessons by including modelling tasks that help students develop the modelling competencies. In addition, mathematical modelling requires attention from teachers in classroom interventions to assist students' modelling process (Leiss, 2005). Mathematical modelling has contributed in providing students the opportunity to develop their skills and competencies in the twenty-first century learning by solving real-world problems (English and Sriraman, 2010). By engaging in modelling tasks, students have the opportunity to develop mathematical reasoning processes such as constructing, explaining, predicting and conjecturing (English and Watters, 2005).

This paper describes the mathematical modelling competencies among 4 groups of Form 4 (Year 10) students in Selangor, Malaysia. Students' modelling competencies were assessed from the aspect of identifying variables, making assumptions, mathematics reasoning and interpreting solutions. An analytical scale of rubric was used to assess students' modelling competencies. In addition, this study also reflected on the challenges students faced when doing the modelling task and the advantages of teaching through the modelling task.

Theoretical Framework

The Models and Modelling Perspective (MMP) is the underpinning theory guiding this study. In emphasizing the need of mathematical thinking and usage of mathematics in order to solve real world problems, MMP (Lesh & Doer, 2003) moves beyond the skills in solving only traditional word problems in textbooks. MMP suggests that learners should have the ability to transform the real world problem into mathematical terms and construct models as a way to solve them. While students are involved in modelling, their way of thinking is assumed not be in a single route but appears to work in cycles, in which the mental models are created and tested. (Lesh & Doer, 2003)

Literature Review

Mathematical modelling is a cyclic process of translating a real-world problem into a mathematical problem by formulating and solving a mathematical model, interpreting and evaluating the solution in the real-world context, and refining or improving the model if the solution is unacceptable (English, 2007).

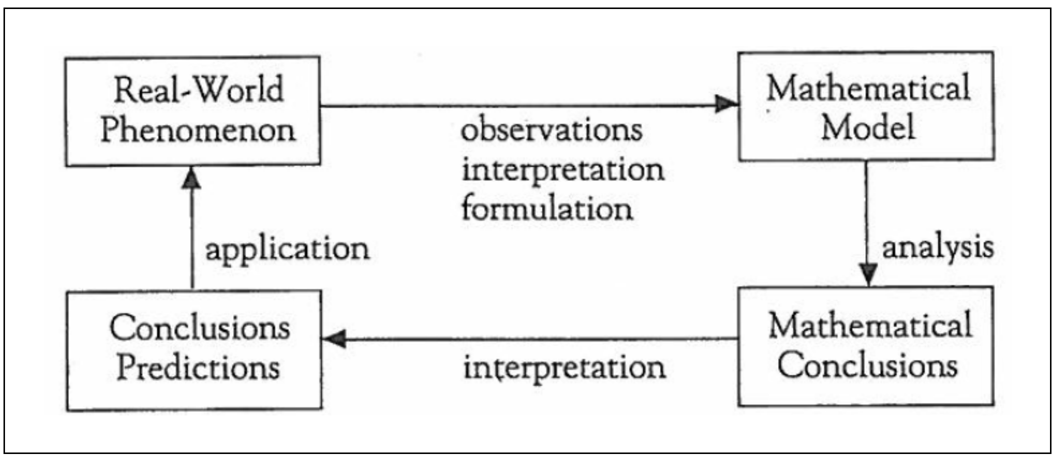


Figure 1. Mathematical modelling process (Swetz and Hartzler, 1991, p. 3).

Figure 1 shows a simple mathematical modelling process. Swetz and Hartzler (1991) presented a set of stages for modelling that could be used to assist pupils:

1. observing a phenomenon and delineating the problem;
2. conjecturing the relationships among factors and interpreting them mathematically (mathematizing);
3. applying appropriate mathematical analysis to the model; and
4. obtaining results and reinterpreting them in the context of the phenomenon (p. 3).

It can be simplified as a process that include four stages, namely: Observation, Analysis, Interpretation and Application (Ang, 2001; Swetz & Hartzler, 1991). All modelling process begins with the real world problem that can be formulated into a mathematical problem. The mathematical solutions obtained are usually interpreted in the real-world context before they can be accepted. Similarly, mathematical modelling also refers to a process of representing real-life problems using mathematical terms while understanding and finding solutions to the problem (Ang, 2010). Mathematical modelling is a bridge to show how mathematics is connected to everyday life in the classroom. Thus, mathematical modelling provides a learning environment where students are given the opportunity to solve real world problems through mathematics (Barbosa, 2006). Learning environment refers to the development of certain activities by students through suitable classroom conditions (Skovsmose, 2001). The problems the students are expected to solve or formulate about daily life is called the mathematical modelling task.

Maass (2006) defined modelling competencies as "skills and abilities to perform modelling processes appropriately and are goal oriented as well as the willingness to put these into action." (p. 117). Specifically, modelling competencies are based on the actions involved within movements between the real-world and the mathematical world. She identified sub-competencies in the modelling process such as to understand the real problem and set up a model based on reality, to set up a mathematical model from the real model, to solve mathematical questions within the mathematical model, to interpret results in a real situation and to validate the solution.

Niss, Blum, and Galbraith (2007) highlighted the mutual impact of mathematical and modelling competencies. In other words, growth in modelling competence is dependent on mathematical competence and at the same time helps develop mathematical competence. This puts forth the argument that modelling is a potential vehicle for student learning of mathematics and that learning mathematics develops competency in applying mathematics and developing models. When students are engaged in modelling activities, it is not just a case of students expressing their mathematical competencies but is one that concurrently develops students' competence further (Lingerfjard, 2006; Swan, Turner, Yoon, & Muller, 2007).

Mathematical modelling has been one of the six general mathematical competencies that were introduced in Germany beginning in 2003 (Greefrath, 2016). It is an educational standards required in the mathematics at the primary level, intermediate level and upper secondary level. Students are required to develop the ability to translate between reality and real world in both directions. Studies by Blum (Blum & Leiß, 2007) have described a more detailed way on how to translate between reality and mathematics as well as comparing, analysing and assessing the mathematical models developed. Using the modelling cycle, the indicators and detailed descriptions of the sub-competencies involved in modelling were developed such as constructing, simplifying, mathematising, interpreting, validating and exposing (Greefrath, 2015). To reduce complexities for students and teachers, modelling can be further reduced to partial processes. This also helps in creating suitable exercises for classroom use. By using this paradigm of training partial competencies for leaner enables the mastery of modelling competencies in the long term (Kaiser & Brand, 2015).

According to the study by Chan, Ng, Widjaja and Cynthia (2012), mathematical competencies of primary five pupils were found to be at the beginner (Level 1) and intermediate level (Level 2). The participants of the study were from a mixed ability class who worked in two groups of five pupils each. Pupils in both groups had difficulties in making assumptions, interpreting solutions and mathematical reasoning.

Methodology

Sample and Location

This study was carried out in a private secondary school in Selangor, Malaysia. Twenty students from a mixed-ability class in Form Four 4 (Grade 10) were involved in the research.. At first, students tried to solve the modelling task individually. Next, the students worked in a group to discuss their findings of the modelling tasks. The students were randomly assigned into four groups. Each group consists of five students. Students were allowed to access the internet to search for important information that is required to solve this real life modelling task. The students had limited experience with modelling tasks as they had only been exposed to one modelling task previously. The students were randomly assigned into groups so that they will be able to work together with others which might not be their close friends. The outcome showed that students were able to work together in a group to construct a model even though it took some time for them to warm up to one another.

Modelling Task

The modelling task used for this study was the basketball trajectory. This modelling task was designed based on the 7-step modelling process by Galbraith (1989, 1995) (Figure 3). These steps were arranged into 5 stages which are real world problem, make assumptions, formulate mathematical problem, solve and interpret and verify the model and predict.

MODELLING TASK

In basketball game, a player will be awarded a free throw when a foul is committed. Estimate the angle at which a player should throw /shoot the ball in order to get a perfect free throw (shooting angle).

Figure 2. Mathematical modelling task.

In this modelling task (Figure 2) students were required to estimate the angle for a player to shoot the ball in order to get a prefect free throw. This task required some knowledge of physics and the class selected for this study was science stream students. The teacher went through the modelling task with several mathematics and physics teachers to gather feedback before conducting the class.

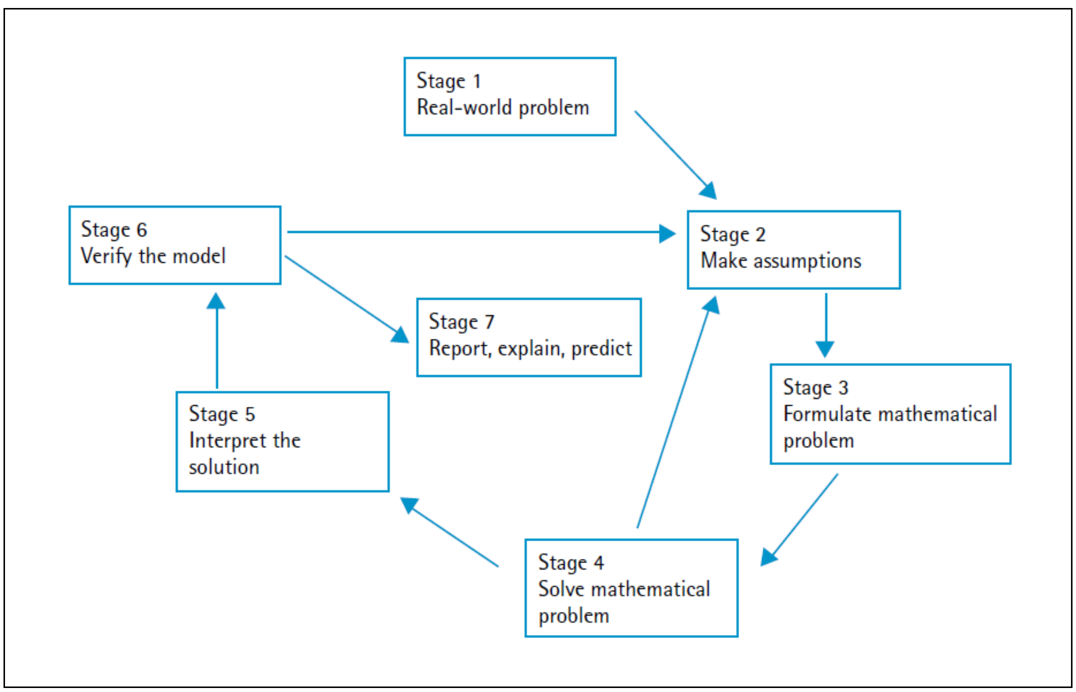


Figure 3. Mathematical modelling cycle process (Galbraith 1989, 1995).

Data Collection and Analysis

This is an exploratory qualitative analysis of students' response based on the modelling task developed by the researchers. Each student does the research individually and then worked in groups to solve the task. In addition, students' responses were also analyzed to assess their modelling competencies.

Understanding of Students' Mathematical Competencies

Several elements of modelling competencies from process by Galbraith (1995) were taken into consideration during developing the mathematical modelling competencies criteria. In this study, the researcher focused on the elements of modelling competencies such as understanding, simplifying, formulating and solving. Three main modelling competencies were highlighted which are making assumptions, computing and interpreting solution and mathematical reasoning. These three modelling competencies were assessed using the rubric developed by the researchers that adapted several elements from Chan et. al (2012)'s study. The elements adapted were the making assumptions and mathematical reasoning. The rubrics were designed in analytical scale using Level 1 to Level 4 for each competency. Several key questions were provided to help the teacher analyze students' responses.

Table 1

Analytical Scale of Rubric for Assessing Mathematical Modelling Competencies

1. Making assumption

- *Showing an understanding to the task.*
- *List out suitable variables.*

Key Question:

❖ Does the student understand the problem?

❖ Does the student think of the variables which relevant to the model?

Level	**Criteria for Scoring**
4	▪ Comprehensive list of variables or assumptions made based on real-world interpretations of task. ▪ Assumptions stated are relevant to model.
3	▪ 2 variables or less and assumptions made based on real-world interpretations of task. ▪ Assumptions stated are relevant to model.

	Level	Criteria for Scoring
	2	▪ Less than 2 variables listed or assumptions made based on real-world interpretations of task. ▪ Assumptions stated are irrelevant to model.
	1	▪ No keywords listed. ▪ No variables listed. ▪ No assumptions made. ▪ Incorrect notions of assumptions.

2. Computing and interpreting solution

- *Applying mathematical concept and solve for the problem.*

Key Question:

❖ Does the student develop a mathematical model?

❖ Does the student able to find the solution for the task/problem?

Level	Criteria for Scoring
4	▪ Develop the-most appropriate mathematical model. (able to connect the variables to form the suitable mathematical model) ▪ Computation is clear and accurate. ▪ Show evidences of more than 2 real-world consideration in examining variables that will impact interpretation and solution of modelling task.
3	▪ Attempt to develop a proper mathematical model. (able to connect the variables to form an initial mathematical model that might be inaccurate) ▪ No errors shown in computation. ▪ Show evidences of 2 real-world consideration in examining variables that will impact interpretation and solution of modelling task.
2	▪ Show little attempt to develop mathematical model. (fail to connect the variables to form the mathematical model) ▪ Minor errors shown in computation. ▪ Show evidence of 1 real-world consideration in examining variables that will impact interpretation and solution of modelling task.

	Level	Criteria for Scoring
	1	▪ Never show attempt to develop mathematical model. ▪ Errors shown in computation. ▪ There is no evidence of real-world constraint in the presentation of work.

3. Mathematical reasoning

Explaining the reasoning (the "why") using diagrams, symbols or vocabulary.

Key Question:

- ❖ Does the student apply a correct mathematical concept?
- ❖ Was I able to understand the student's thinking easily?
- ❖ Did I have to make a guess about what students were trying to do?

Level	Criteria for Scoring
4	▪ Mathematical reasoning is logical. ▪ Correct use of mathematics (no computational errors) ▪ Each stage is connected.
3	▪ Mathematical reasoning is logical. ▪ Appropriate use of mathematics (minor computational errors) ▪ Some stages are connected.
2	▪ Mathematical reasoning is somewhat logical. ▪ Inappropriate use of mathematics. ▪ Some stages are connected.
1	▪ Mathematical reasoning is not logical. ▪ Inappropriate use of mathematics. ▪ No connection for each stage.

Findings

This section presents the assessment of the four groups of students concerning their mathematical modelling competencies using the analytical scale of assessment rubrics in Table

1. The assessment was carried out by investigating students' written work. This analytical scale of assessment rubrics used the 4-point scale from Level 1 to Level 4.

Table 2

Analytical Assessment for 4 Groups of Students

Group	Making assumption	Computing and interpreting solution	Mathematical reasoning
1	Level 2	Level 2	Level 3
2	Level 3	Level 3	Level 3
3	Level 2	Level 2	Level 2
4	Level 2	Level 2	Level 2

The results show Group 1 (G1), Group 3 (G3) and Group (G4) students mostly reach level 2 for these three dimensions of modelling competencies whereas Group 2 was the only one can achieve level 3 for all the competencies. The author divided these groups of students into 2 categories which are Category A (G1, G3, G4) as basic user and Category B (G2) as proficient user for the discussion later.

Exemplification of Basic User for Mathematical Modelling (Category A)

Group 1, 3 and 4 students were assessed to be basic user of mathematical modelling as they only took one of the factors for making assumptions and students did not validate their results for the real world situation. Several aspects show this category of students had difficulties in relating and validating their mathematical model to real world situation.

Competence in making assumptions. Generally, these students did not have much difficulties in understanding the problem as they were given two days to access the internet for collecting information. From this category students' response, most of the students only consider a few variables or factors for the problem and do not make much elaboration on it. For example, group 1 and group 3 students did mention the height of the player, the strength used to shoot the ball, eye level of player as their factors to calculate the best angle for the basketball shooting but they could not relate all these variables to make an assumption. This shows that students cannot connect all the real world factors and the model in stage 3 may be inappropriate.

Competence in computing and interpreting of solution. This category of students were accessed to be at Level 2 for this dimension as they only can develop a model based one real world consideration of variables.. The mathematical model developed by this group of students mostly had some minor mistakes and caused the solution to be inaccurate. Students did not sketch or explain their model using real world situation and showed the error in their sketched model.

In this modelling task, group 3 and group 4 students only sketched a mathematical model in stage 3 and they were having difficulties in solving and interpreting since they did not relate their model much to a mathematical concept, whereas group 1 students attempted to draw a model based on the eye level of the player with the ball hoop using trigonometry concept. In the diagram, group 1 students draw the eye position on the ground which at same level with the end of the ball hoop (Figure 3). This model affected their calculation and the angle found was not accurate.

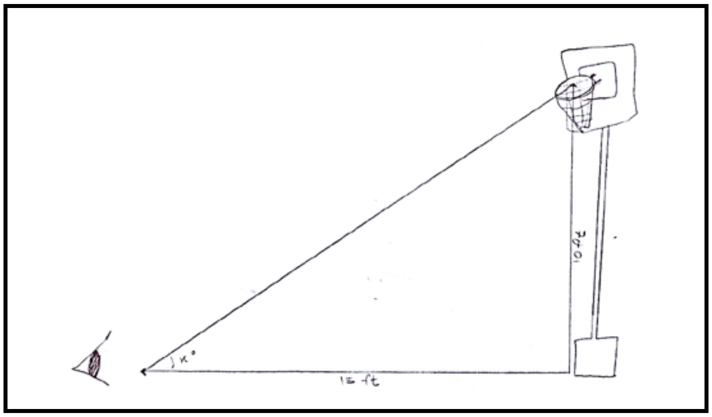


Figure 3. Mathematical model from Group 1.

In the end, the students in Category A were unsuccessful in finding the solution to the modelling task.

Competence in mathematical reasoning. The groups in Category A made inappropriate assumptions and formulated the wrong models. As a result, they could not explain or reason out their solution. At the end, group 3 and group 4 students in Category A were accessed to be in Level 2 for competence in mathematical reasoning. These two groups of students applied inappropriate mathematical concept and this caused a wrong interpretation. Whereas for group 1 students, they were assessed to be in Level 2 for competence in mathematical reasoning as they applied a logical mathematical concept although they showed errors in the model.

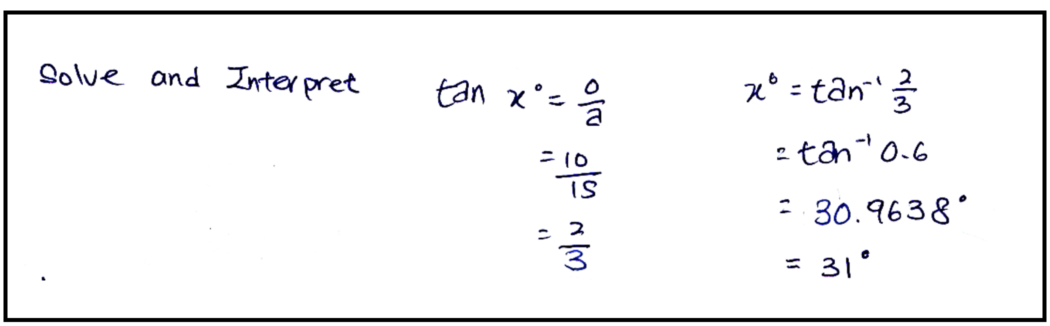


Figure 4. Solution from Group 1.

Exemplification of Proficient User for Mathematical Modelling (Category B)

Group 2 students were assessed to be proficient user of mathematical modelling as they showed more mathematizing effort and attempted to relate their solution with all the real world variables listed in stage 2. The students understand the problem very well as they were allowed to access the internet. Group 2 students listed out about 5 variables that will affect the angle of successful free throw and were able to make appropriate assumptions. This group of students formulated mathematical relationships for the variables and their mathematical reasoning was more logical.

Competence in making assumptions. Group 2 students had listed out more than 2 variables and their assumptions are relevant to the model. This caused their competence in making assumptions were accessed to be in Level 3. Students managed to list out the variables like angle of release, air resistance, gravitational forces, height of player and summation of forces. They also mentioned the information of the variables that collected from internet in a table (Figure 5). From all these variables, group 2 students attempted to relate the angle of release and the height of player with summation of forces for making assumptions. They also were the only one group wrote air resistance as the factor that affect the successful free throw.

2.	Angles of release	Around 32 degrees horizontal according for a 6 foot 6 player.	✓
	Air resistance	Affect the motion of the ball by lowering the speed of the ball.	
	Gravity	exerts a force downward on the ball, causing it to travel in an arc as it approaches the rim.	
	Height of player	big men have long arms and bigger hands make them difficult to shoot.	✓
	Summation of forces	the power of the shot	✓

Figure 5. Variables listed by Group 2 students with explanation.

Competence in computing and interpreting of solution. The students in group 2 sketched a proper mathematical model in stage 3 which is formulate mathematical problem. This model is more appropriate than the previous one because it includes the height of the player. However, although they were able to list the assumptions, they obviously had not used the assumption of gravity, hence the straight line path of the ball. The values used in the model were the information that they had collected from internet. The height of the player (up to chest position) that they used in the diagram is the average height of the members in group 2 which is 4.5ft.

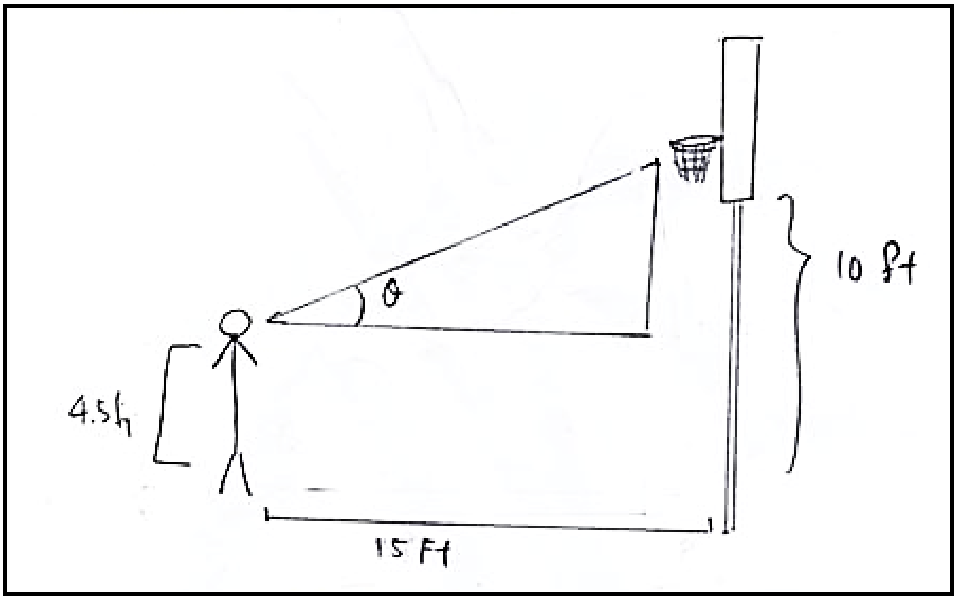
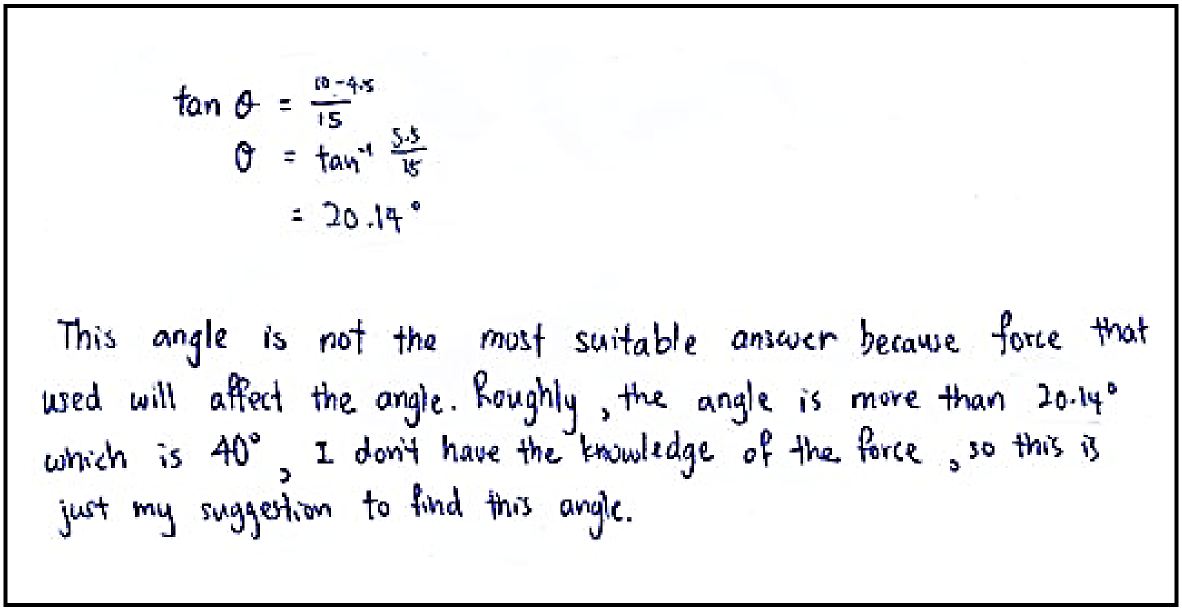


Figure 6. Mathematical model from Group 2.

There were some errors made in the model especially on the path of the ball which should not be in a straight line causing the model to be incorrect. However, the concept used was logical and made sense. Although students applied trigonometry concept to find the solution, they wrote a sentence to explain the suitable angle should be equal to 40◦ due to the forces exerted by the player. This showed that students tried to relate their solution from calculation to the assumption that they had made before. The final solution that they have found was inaccurate but they attempted to connect all the variables together. Based on the explanation given in Figure 7, the introduction of force which was not mentioned in the assumption earlier. This might be due to the students applying their knowledge on the gravitational pull and that they could have seen it is not a straight line path. This interpretation contradicts the model they proposed earlier. The students must have realised that the model they proposed earlier might be incorrect after reflecting on their knowledge of the basketball sport. However, the used this justification to support their solution of the angle so that it made sense.



$$\tan\theta = \frac{10-4.5}{15}$$
$$\theta = \tan^{-1}\frac{5.5}{15}$$
$$= 20.14°$$

This angle is not the most suitable answer because force that used will affect the angle. Roughly, the angle is more than 20.14° which is 40°, I don't have the knowledge of the force, so this is just my suggestion to find this angle.

Figure 7. Solutions from Group 2.

Competence in mathematical reasoning. The students calculated the best angle of the shooting the basketball successfully using the concept of trigonometry. Students minus the height of player the hoop of basketball as the opposite length for the angle which is 10ft -4.5ft = 5.5ft. This show students knew how to apply trigonometry concept in real life problem and draw a right angle triangle in their model. Although the solution that Group 2 students found was inaccurate, they managed to apply correct mathematics concept in solving their model.

In addition, this group of students did write a paragraph to explain their solution. The angle calculated from the equation is about 20.14◦ and students explained that their solution is not accurate as the forces will affect the angle. During the interview with this group of students, they also mentioned that the gravitational forces will pull down the angle so they decided to use 40◦ to against the gravitational pull. This showed group 2 students do not have the knowledge of motion and they tried to explain their solution by guessing. Nonetheless, they completed the stage 4 and 5 in the task and showed appropriate use of mathematics although the solution found was incorrect.

Discussion

Students from both groups initially struggled with the sequence of process they had go through before solving the real world problems. Most of them were used to solving the mathematical exercises or word problems that provides sufficient information. The struggle in making assumptions showed that students in both groups found the task challenging.

The mathematical modelling competencies for the groups in Category A showed many weaknesses in making assumption, computing and interpreting solutions and mathematical reasoning, whereas group 2 in Category B showed fewer mistakes or errors compared to the groups in Category A. Hence, students in Category A were assessed to be basic users while students in Category B were assessed to be proficient users. From the study, Category A students showed difficulties in formulating a model by relating all the variables stated. In addition, students in Category A also have problems in reasoning out the solution. However, Category B students performed slightly better in making assumptions, formulating a model and computing solution compared to Category A. Although they did not get the accurate solution for the problem, they managed to connect all the stages and reason it out. Students in the proficient user group performed better than students in the basic group similar to the findings by Chan, Ng, Widjaja and Cynthia (2012). This might be due to the ability to apply their mathematical knowledge in the right situation. Moreover formulating the model and testing it requires strong analysis and content knowledge. The results of this study also shares similar findings with the research conducted by Leong and Tan(2015).

From this study, students showed better understanding of the problem or task as they were given opportunities to look for information. In this basketball trajectory task, students need to have some knowledge of physics and the concept of motion. But this group of students have not much knowledge about this; hence they can only guess the angle by considering gravitational forces, forces used by the player and others. Students showed the abilities to think different variables for this modelling task but they were unable to connect all the variables by applying one mathematical concept. This indicated that students were only able to apply the mathematical concepts which they have learnt before.

Conclusion

Mathematical modelling competencies are not easy to develop among secondary students in Malaysia as students are used with just solving exercises questions in their mathematics classes. The development of students' modelling competencies is a continuous process and students needs regular exposure to the tasks. The modelling task used for the lesson also must be related to their syllabus so that they can apply their prior knowledge to solve the modelling task. In addition, teachers who conduct the modelling lesson must be familiar with the task so that they can guide the students in making assumption and computing the solution. Teachers have to also work through the modelling tasks so that they will understand the difficulties. Similar to science experiments, modelling tasks help students apply and use mathematics in the real world instead of just learning mathematical concepts.

In short, this study has shown that students have different levels of competence in making assumptions, computing, interpreting solutions and mathematical reasoning. Students are required to engage themselves with more modelling tasks to develop their modelling competencies. The process of formulating a model was not one of the key competencies assessed in this study. This is because it has been assessed under the computing and interpreting solutions competencies, with the question asking on the mathematical model used.   Future studies could use this formulation of model as one of the competencies to be assessed.

With more mathematical modelling lesson, teachers can also improve their modelling teaching approach whereas students can improve their thinking skills. Studies on exploring the mathematical modelling competencies should be in-depth so that we can help students improve their competencies. This is a preliminary study to understand the competencies of secondary students. Using only one task to judge the modelling competencies is the limitation of the study. In future research, modelling tasks should be used similar to doing a science experiment where students are able to investigate and make conjecture. However, in mathematical modelling the ultimate aim is to formulate the model and validating it based on the assumptions used.

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