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The Ideational Meaning of Diagrams in the Malaysian and Singaporean Mathematics Textbooks

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Abstract: *A mathematical text is multimodal with different modes of communication, namely verbal language, algebraic notation, visual forms and gestures. This paper aims to compare and discuss the ideational meaning of visual forms in worked examples from Malaysian and Singaporean Grade Seven Mathematics textbooks on Lines and Angles. There are two structures in ideational meaning, namely narrative (with action) and conceptual (without action). Action diagrams represent ongoing mathematical activity whereas, without action diagrams represent mathematical objects. Document analysis and coding were carried on 57 geometrical diagrams found in the textbooks used in a 20-year period. The properties to identify a narratively and a conceptually structured diagram were based on grammar to 'read' geometrical diagrams. The Malaysian textbook used from the year 1997 to 2002 consisted of some narrative diagrams and the Singaporean textbooks consistently gave importance to conceptual diagrams. Further, there are differences in the classification, identifying and spatial relations between geometric elements among the series of textbooks and country. The geometrical diagrams in the Singaporean textbooks had given much importance to attributive letters compared with the Malaysian textbooks that had given much importance to letters to identify objects. Besides, the Singaporean textbooks had represented relations with 'shapes' whereas, the Malaysian textbooks had represented relations with 'points'. The findings provide valuable information for educators in general to 'read' the ideational meaning of geometric diagrams and to construct better visual representations, especially in school textbooks.*

Keywords: ideational meaning, visual forms, worked examples, mathematics textbooks

Introduction

Geometry that involves points, lines, angles, shapes, planes, surface and space is one of the five main areas of the Malaysian school mathematics. The five major areas are number and operations; measurement and geometry; relationship and algebra; statistics and probabilities; and discrete mathematics (Ministry of Education, 2015). Hence, the importance of geometry had

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influenced school curriculum and is tested in both national and international examinations. International examinations like Trends in International Mathematics and Science Study (TIMSS) under the auspices of the International Association for the Evaluation of Educational Achievement (IEA) conducts studies on educational achievements for eighth graders. Among the participant countries, Singapore was consistently ranked in the top three and their achievement had interested a number of researchers (e.g. Menon, 2000; Fan & Zhu, 2007; and Erkan, 2013). In contrast, the Malaysian students with a similar cultural background as the Singaporeans performed poorly especially in the content area of geometry in TIMSS evaluations (Noraini Idris and Tay, 2004). Apart from the Malaysian researchers, the students' poor performance in geometry in TIMSS has caused concern among educators (Fujita and Jones, 2003; Chen, 2006; Chen, Reys, & Reys, 2009) and various efforts have been taken to find out the possible factors that deal with this issue. Among the various factors, the significance of textbooks in students' achievement in TIMSS evaluations has become a topic of interest (Valverde and Schmidt, 1997; Haggarty and Pepin, 2002; Valverde, Bianchi & Wolfe, 2002 and Tornroos, 2005). This is especially important when one considers that teachers from all over the world use textbooks as their main reference in teaching and learning process (Kulm, Roseman & Treistman, 1999). Besides, the TIMSS 2011 Encyclopaedia highlights that most teachers often use textbooks as the basis for instruction (Mullis, Martin, Minnich, Stanco, Arora & Centurino, 2011). Hence, school textbooks that influence students' performance in assessments especially in geometry, play a crucial role in transmitting curriculum content into practice or as mentioned by Johansson (2005) as the potentially implemented curriculum.

In mathematics school textbooks, topics are arranged in a sequential order by introducing a chapter, geometrical concepts, formulas, worked examples, exercises and enrichment activities.

The worked examples are intended to guide students to understand a geometric concept by displaying a problem with step by step solutions. The significance of worked examples especially in textbooks has interested several researchers. For example; Atkinson, Derry, Renkl and Wortham (2000) mentioned that worked examples encourage learners by providing direct practice. In general, low-performing students experience less anxiety in understanding a mathematical concept when there are worked examples and prefer worked examples in textbooks compared with high performing students who prefer problem-solving questions. Thus, worked examples are aimed to familiarize students especially the novices with skills and techniques to build confidence in answering exercises and assessments.

Most of the worked examples in the topic of geometric highly depend on diagrams to visualise the geometric elements and relationships involved. The geometrical diagrams reflect the visual mode of communication between the textbook writers and readers. However, the misuse of visual mode could distress the communication of the concepts intended to be presented (Pinto and Ametller, 2002). Besides, according to Kress and Van Leeuwen (1996), '…educationalist everywhere have become aware of the increasing role of visual Communication in learning materials of various kinds, and they are asking themselves what kind of maps, charts, diagrams, pictures and forms of layout will be most useful for learning. To answer this question they need a language for speaking about the kinds of meanings of these visual learning materials' (p.12).

Thus, the meanings of geometrical elements or relationships presented in geometrical diagrams would help textbook writers to construct readable geometrical diagrams. Hence, the following research question was examined:

What are the ideational meaning of geometrical diagrams presented in the worked examples on Lines and Angles found in the Malaysian and Singaporean mathematics textbooks in a period of more than 19 years?

Literature Review

The Malaysian Mathematics Curriculum

The Malaysian formal education system started since the year 1957, after gaining independence from the British and the medium of instruction was in the English language. However, in the 1970s' the medium of instruction at the secondary level was changed to the national language, Malay with emphasising on technology and science. Besides, 68 technical and vocational schools were established in the late 1970s' (Abdolreza Lessani, Aida Suraya Yunus, Rohani Ahmad Tarmizi & Rosnaini Mahmud, 2014). Further, there was a revision in the Malaysian mathematics curriculum at the end of the 1980s'. Since then, the Malaysian Integrated Curriculum for primary schools (KBSR) and the Malaysian Integrated Curriculum for secondary schools (KBSM) was implemented in the year 1983 and 1989 respectively. In the context of mathematics; the curriculum had highlighted the importance of problem-solving (Noor Azlan Ahmad Zanzali, 2011) and the medium of instruction was still in the Malay language. In the year 2001, there was a curriculum review with the medium of instruction changed to the English language, and national school examinations were in the bilingual text.

However, the Malaysian government was convinced that the English medium is not working and the teaching and learning of mathematics were replaced again with the Malay language since the year 2012 (Saadiyah Darus, 2010). However, an option was given to schools to use either the new textbooks with Malay medium or the one with English medium (used from the year 2003) for the mathematics and science subjects. In addition, a new curriculum was introduced since the year 2016 in the Malaysian education system in aligning to the 21st-century learning. The KSSR (Primary School Standard Curriculum) and KSSM (Secondary School Standard Curriculum) was implemented to develop an individual who can think logically, critically, analytically, creatively and innovatively (Ministry of Education, 2015). The word “standard” was emphasised in the new curriculum to help all the children to achieve a required standard.

The Singaporean Mathematics Curriculum

Singapore, a small country compared to Malaysia, adapted mathematics curriculum from several nations at the interest of different schools in the 1950s’. For example, the Chinese schools tailored to the curriculum from China. The first syllabus in mathematics known as syllabus B was implemented in the year 1959 with little consideration given to differences in the mathematical skills of pupils (Kaur, 2014). The syllabus went through a revision in the 1960s’ and named as the syllabus C at the end of 1970s’ in response to Math Reform. There was another review in the Singaporean mathematics curriculum at the end of 1970s’ known as syllabus D (Kaur, 2014). The syllabus D was used since the 1980s’ for the secondary pupils. The curriculum covers arithmetic, mensuration, algebra, graphs, geometry, statistics and trigonometry (Kaur, 2014). The syllabus was used in school textbooks from the year 1981 with weaker students were streamed into the normal curriculum and the good students into the express curriculum. However, the syllabus was revised again in the year 1989, and the new curriculum focused on mathematical problem-solving by using a pentagon model (Cheow, 2008). The five sides of the pentagon model are interrelated with concepts, skills, processes, attitudes and metacognition (Ministry of Education Singapore, 2006). Since then, in a period of six years or so, the Singaporean mathematics syllabus prepares pupils for the future in line with the national objectives (Kaur, 2013). Meanwhile, in the year 2003, 'Teach Less, Learn More' was launched to improve the quality of interaction between the teachers and pupils focussing on quality. For example, students were given more opportunity to explore and learn mathematics through classroom interaction (Pak, 2008; Fogarty and Pete, 2010; & Kaur, 2013)

Worked Examples in School Textbooks and Geometric Diagrams

Review of cross national studies suggested that textbooks could be a good point of comparison in students' mathematics performances (Erbas, Alacaci & Bulut, 2012; Choi and Park, 2013; Hong and Choi, 2014). For example, Erbas, Alacaci & Bulut (2012) compared Grade Six mathematics textbooks of Turkey, Singapore and America in terms of visual design, text density, internal organization, weights of curriculum strands, topics covered and content. They found that the Singaporean textbooks mirrored simple features of text density and enriched use of visual elements, a fewer number of topics and easier inner organization. Whereas, American textbooks were mainly designed as reference books while the Turkish textbooks reflected a design that valued active student learning. Hence, the Singaporean textbooks have better visual design compared with the textbooks from Turkey and America.

Besides, the topics in school textbooks, especially in geometry, are arranged in a sequential order by introducing a chapter, geometrical concepts, formulas, worked examples, exercises and enrichment activities. The worked examples are intended to guide students to understand a geometric concept by displaying a problem with step by step solutions. The significance of worked examples especially in textbooks has interested several researchers. For example; Atkinson, Derry, Renkl & Wortham (2000) mentioned that worked examples encourage learners by providing direct practice. Low performing students prefer worked examples and experience less anxiety to understand a mathematical concept compared with high performing students who prefer problem solving. Thus, worked examples aimed to familiarize students especially the novices with skills and techniques to build confidence in answering exercises and assessments.

In the content area of geometry, most of the worked examples can be communicated through geometric diagrams. As stated by Gal and Linchevski (2010), visual representations play an important role in understanding geometry and Jones (2013) mentions that complex geometric process and structures can be presented holistically in a geometric diagram. In a geometric diagram, the complex geometric process and structures would be represented through spatial relations that give the ideational meaning. Hence, in worked examples, geometry diagrams are intended to make the geometrical problems simpler by embedding the problem and concepts in the diagrams. However, geometrical worked examples without diagrams would be difficult for readers to realise the geometrical relationships as it would be presented in the verbal mode of communication. Meanwhile, in a comparison between learning with text and diagrams, it was found that learning with diagrams shows a good self-explanation effect among students (Ainsworth and Loizou, 2003). Hence, the use of diagrams is essential in the learning process, especially in mathematics. In geometrical diagrams, geometric elements or objects were used to represent the geometrical relationship to help students to understand and solve the problem. Thus, it is important to observe the geometrical elements on how it helps reader to construct meanings and make the diagrams readable.

Reading the meaning of objects in diagrams had interested several researchers (e.g., Winn, 1991; Ametler and Pinto, 2002; Pinto and Ametler, 2002, Alshwaikh, 2011; and Dimmel and Herbst, 2015). For example, Winn (1991) presented a theoretical framework for learning from maps and diagrams. The theoretical framework on varying spatial relationships among objects and concepts lead to the predictions that maps and diagrams were:

"(a) particularly effective for showing physical layout, how things are put together, and how they work; (b) can serve as schemata that help to organise information; (c) can make abstract ideas more concrete; and (d) allow people to use their spatial skills" (Winn, 1991, p.213).

However, according to Winn, the focus of the components in maps and diagrams will be affected if the number of details was increased. Thus, in worked examples, spatial relationships in diagrams should be in align with the verbally stated questions. Pinto and Ametler (2002) mentioned that the design of the compositional structure is important for students to read images. Besides, modality of images is necessary as it could not only help students to understand the image yet, it helps them to interpret other similar images. However, found that teachers' have a low degree of awareness of students' difficulties in reading images.

In another study, Dimmel and Herbst (2015) led a semiotic inquiry to conceptualise geometric diagrams as mathematical texts that include choices from different semiotic systems and used it to analyse diagrams from 22 textbooks used before and after 1950. Each textbook that was listed under chapters, units, or sections that covered triangles, triangle congruence, and proofs involving triangles. Variations in weight, style and colour in diagrams were observed to understand the interpersonal meaning. Found that the newer textbooks have more visually varied diagrams with colours, markings, and specific labels than the earlier geometry textbooks. Hence, from the studies, it is essential to understand and explore the meaning of objects in diagrams as it would help to construct the mathematical relationships and help readers to appreciate diagrams since they are the visual mode of communication that is often used in mathematics especially in the content area of geometry.

Meanwhile, according to Alshwaikh (2008), the inclusion of geometric diagrams in verbally stated questions represent the ideational, interpersonal and compositional meanings. However, this study focused only on the ideational meaning that conveys either narrative (with action) or conceptual (without action) diagrams. In other words, ideational meaning refers to the representation of mathematical activities and objects in geometric diagrams (Alshwaikh, 2011). The narrative diagrams could be identified through the directional, dotted, shaded and construction structures; and by looking at the sequence of diagrams. Meanwhile, the conceptual diagrams represent the classification, identifying and spatial process. Hence, the ideational meaning of geometric diagrams is essential to be explored in order to identify the mathematical activities that were presented in geometrical worked examples. Consequently, the main aim of the present study is to analyse and compare the ideational meaning (mathematical activities represented) of diagrams between the Malaysian and Singaporean Grade Seven textbooks on the topic of 'Lines and Angles'.

Theoretical Framework

The Systemic Functional Linguistics (SFL) approach suggested by Halliday (1985) argued that any text fulfills ideational, interpersonal and textual functions. While the ideational function represents the idea as a whole, the interpersonal function represents the relationship between the writer and the readers and the textual function is the compositional meaning on the whole as between the verbal and visual mode of communication.

Initially, Halliday's (1985), approach on Systemic Functional Linguistics (SFL) framework was applied on the verbal mode of representation and later interested Kress and Van

Leeuwen (2006) on the visual mode. Consequently, the ideational, interpersonal and textual functions were further developed by Alshwaikh into a grammar to read geometric images based on earlier frameworks on verbal (Morgan, 2006) and visual modes (Kress and van Leeuwen, 2006). Alshwaikh (2011) suggested an analytic framework to read geometrical diagrams by considering diagrams as a semiotic mode of representation and communication. An iterative methodology was tested with data from classrooms in the UK and the Occupied Palestinian territories and from textbooks. The analytic framework for reading geometrical diagrams illustrates the ideational (representational) meaning that represents the mathematical activity and objects, the interpersonal meaning explaining the position of the viewer and the textual (compositional) meaning reflecting the unity or coherence of the textual and visual meaning. In the ideational meaning, geometrical diagrams were classified into either narrative or conceptual structured diagrams. Narrative diagrams involve temporality whereas, conceptual diagrams do not present time factor. Hence, narrative diagrams represent ongoing human activity, for example, measuring the length of a side in a polygon. Besides, narrative diagrams expose the mathematical activity and the conceptual diagrams present the mathematical objects (Alshwaikh, 2011). Hence, narratively and conceptually structured diagrams could be differentiated. According to Alshwaikh (2011), there are six properties to identify a narratively structured diagram; directional structure (arrows), dotted structure, shaded structure, a sequence of diagrams and construction structures. Meanwhile, in a conceptually structured diagram, three are three types of processes involved, namely classification, identification and spatial relations. Classification refers to categorising the same kind of relation. For example, readers need to classify congruent figures from polygons given. Besides, identifying refers to recognising geometrical objects such as indexical letters, arrows and symbolic words. Spatial relations are the positional relations involving geometrical objects in a diagram such as lines, points and angles; comparison and measurement based size relations; and labels and colours. In this study, Alshwaikh's analytic framework to read geometrical diagrams will be used to identify the ideational meaning (narrative or conceptual) in the non-verbal mode of communication constructed in the Malaysian and the Singaporean textbooks for a period of 19 years.

Method

Adopting the content analysis method, 57 worked examples with diagrams from Malaysian and Singaporean Grade Seven mathematics textbooks for the past 19 years were analysed using Alshwaikh's framework. Table 1 shows the textbook series with the number of worked examples with diagrams from the topic of Lines and Angles. Each diagram was categorised into the narrative or conceptual and the conceptual diagrams were further analysed by looking into the classifying, identifying and spatial relations. The coded diagrams according to the properties of the narrative and conceptual structure were given for checking to experts. In this study, there were four experts involved for validation purposes; a senior lecturer on engineering mathematics from Nilai, Malaysia; a mathematics lecturer from Penang, Malaysia; a Professor from Kristiansand, Norway and Assistant Professor from Birzeit, Palestine. The experts check according to Alshwaikh's framework and gives feedback on the coding done. Direct discussion with experts and coming up to a mutual conclusion.

Table 1

The Malaysian and Singaporean Mathematics Textbook Series with Number of Worked Examples

Textbook		Malaysian (M)	Singaporean (S)
Series One (S1)	Year of usage	1997-2001	1997-2001
	No. of worked examples with diagrams	18	8
Series Two (S2)	Year of usage	2002-2011	2002-2007
	No. of worked examples with diagrams	10	7

Series Three (S3)	Year of usage	2012-2017	2008-2012 and 2013-2017
	No. of worked examples with diagrams	9	6

Analysis

Conceptual and Narrative Diagrams

The analysis shows that there are seven out of the 58 analysed diagrams from all the three series of the Malaysian and Singaporean textbooks that are classified as narrative diagrams. All the seven narrative diagrams are from the Malaysian Series One textbook. For example, diagram M161a, (Figure 1) involves a clock with arrowed lines (hands of a clock) showing the time as 8 o'clock. The arrowed lines represent the measurement of angles from 12 o'clock to 8 o'clock that gives a temporal factor of before and after. As well, the other six narrative diagrams involve either with humans or physical objects. The other textbooks from the Malaysian series and all the three series of the Singaporean textbooks are not in favour of using narrative diagrams. Both countries had emphasised to use conceptual diagrams as in Figure 2 and Figure 3 that show geometric objects without a temporal factor of before and after. For example, diagram S141a (Figure 2) is a conceptual diagram with arrowed lines AB and CD that express geometric relations of parallel lines. Hence, the pair of lines do not signify temporal factor of before and after.

Figure 1. Diagram M161a (Form One Mathematics, KBSM syllabus).

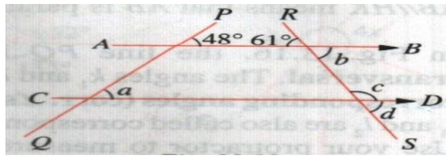


Figure 2. Diagram S141a (New Syllabus, Mathematics 1).

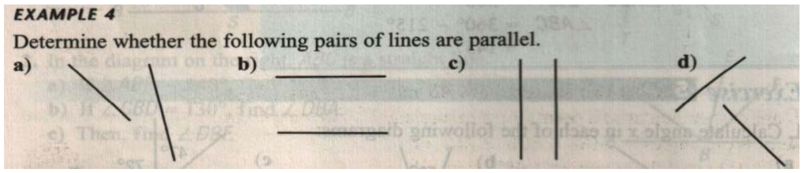


Figure 3. Diagram M142a (Form One Mathematics, KBSM syllabus).

Classification

Found that there are only one out of the 36 conceptual diagrams involve in the process of classification. The diagrams involving classification of parallel and non-parallel lines are from the Malaysian Series One textbook (Figure 3). Hence, in this series of textbook, there is an opportunity given for readers to classify given diagrams according to geometrical relations involving parallel and non-parallel lines. However, none of the 21 diagrams from the Singaporean textbooks shows the process of classification. Perhaps, in normal, teachers are intended to explain and introduce certain new geometric elements with respective properties to students by asking questions orally, discussing and giving samples. These are also considered to be the process of classification. Thus, worked examples involving the classification processes would be helpful for students who did not attend school. Worked examples involving the process of classification is important to build understanding on the properties and relations of the geometrical concepts.

Identifying Processes

The identifying processes involve all the 51 conceptual diagrams (sum of conceptual diagrams from the Malaysian and Singaporean textbooks). For example, diagram S141a as in

Figure 2, is expressing identifying objects and attributes. The capital letters AB and CD are representing a pair of parallel lines, and PQ and RS are intersecting lines on the pair of parallel lines. Here, students could read out that AB is parallel to CD. However, small letters a, b, c and d are used to identify attributes, illustrating specific angles that students need to find. Meanwhile, diagram M163a, as in Figure 4 has identifying words and identifying attributes. A note box on the right of the diagram states that 'Hasil tambah pada garis lurus ialah 180° [The sum of angles in a straight line is 180°']. The first statement of words mentions that 'the sum of angles on a straight line is 180°', this word applies to any straight lines and shows identifying words. Then given that PQT + SQT + RQS = 180°, showing specific angles in the geometrical diagram representing attributive words. Symbolic words are very useful for readers especially for students to make connections within geometrical objects.

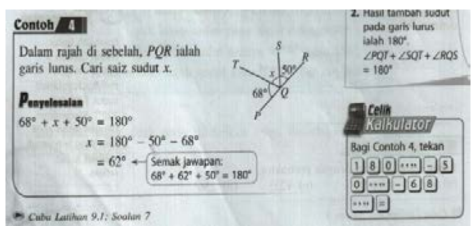


Figure 4. Diagram M163a (Form One Mathematics, KBSM syllabus).

Figures 5, 6, 7 and 8 represent the percentage of the identifying objects, identifying attributes, identifying words and attributive words respectively from the diagrams analysed. Attributive and identifying arrows were not discussed as they were not found in any of the textbook series. The findings on identifying objects indicate that the Malaysian Series One and Series Three textbooks gave more importance to capital letters to identify objects compared with the Singaporean textbooks. Capital letters are used to present points, vertex and lines in the diagrams. It helps readers to read geometrical diagrams and communicate during discussions. However, the Singaporean Series Two textbook used more indexical letters to identify objects compared to the Malaysian Series Two textbooks. The use of indexical letters is significant as it would help readers to make connections between geometrical objects. Meanwhile, the Singaporean textbook in Series One did not give much importance to identifying objects compared to other textbooks. The Malaysian Series One, Three and the Singaporean Series Two and Three textbooks provide a better opportunity for students to read the geometrical diagrams.

Findings from analysing small letters to present the identifying attributes show that the Singaporean textbooks had given more importance compared to the Malaysian textbooks. Surprisingly Figure 6 reveals that all the three series of Singaporean textbooks had identifying attributes in their geometrical diagrams. The Malaysian textbooks had shown a lot of improvement in the use of small letters from textbook Series One (0%) to textbook Series Three (88.9%) even though the percentage is less compared to the Singaporean textbooks.

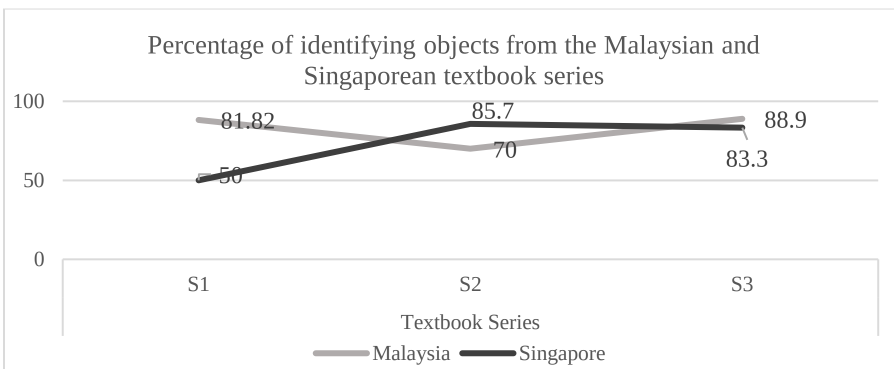


Figure 5. Comparison of identifying objects between Malaysian and Singaporean textbooks.

Besides, the Singaporean textbooks in the three series had sufficiently emphasised the use of unknowns in representing the problems that need to be solved. Hence, readers using the Singaporean Series One to Series Three textbooks would probably learn to use unknowns to represent geometrical problems for example for unknown angles in their diagrams on problem-solving questions. Besides, the Singaporean readers would have more opportunity to guess the angle that needs to be solved by identifying attributes compared with the Malaysian textbooks.

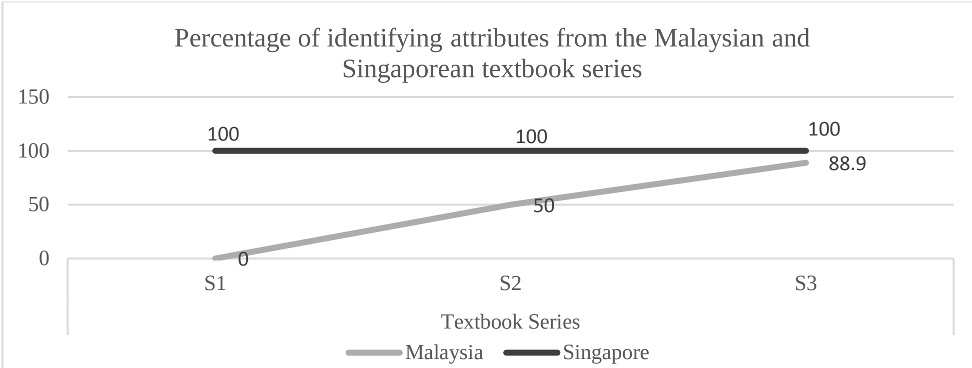


Figure 6. Comparing identifying attributes between Malaysian and Singaporean textbooks.

Furthermore, in Figure 7 and 8, it was identified that both countries do not have symbolic words in Series One textbooks to show identifying and attributives of geometrical objects in the worked examples. However, there is a small percentage of identifying words in Series Two textbooks from both countries. Besides, the use of identifying words had increased from Series Two to Series Three in the Malaysian and Singaporean textbooks. There are no attributive words in the Singaporean Series Three textbook compared with the Malaysian textbook with a percentage of 22.2%.

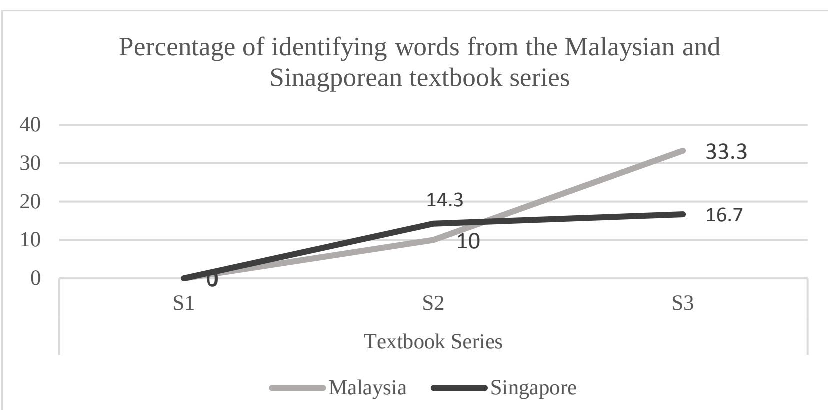


Figure 7. Comparing identifying words between Malaysian and Singaporean textbooks.

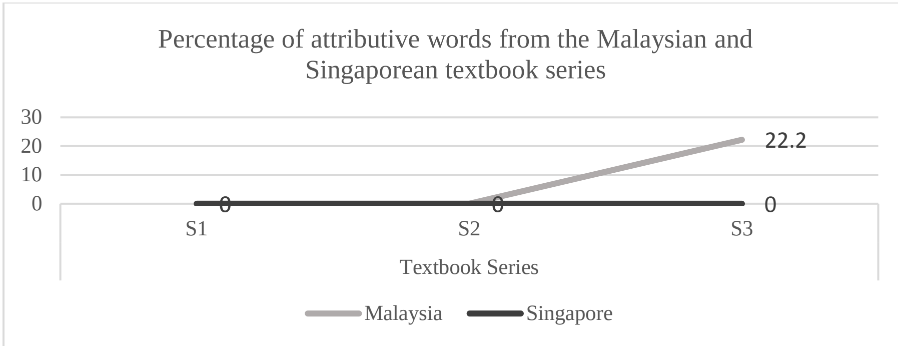


Figure 8. Comparing attributive words between Malaysian and Singaporean textbooks.

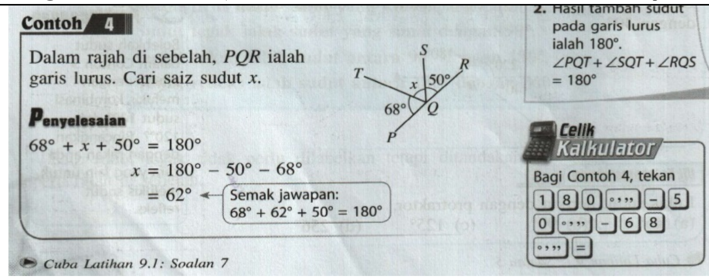
The result indicates that worked examples were guided by symbolic words since the Series Two textbooks from both countries. These words would be helpful for readers to understand the geometrical concepts used in step by step solution of worked examples. Besides, objects in geometrical diagrams could be described with symbolic words reflecting the attributive words as used in the Series Three of the Malaysian textbook. Perhaps, the use of attributive words would help readers of the textbook to understand and construct geometrical relationship between specific geometrical objects in the geometrical diagrams. The added words would enhance their understanding and would probably motivate them to work on similar exercises.

Spatial relations: Positional relations

Spatial processes in a visual representation can be identified through positions and size of objects in a diagram (Alshwaikh, 2011). The position of objects in a diagram can be identified if there is a relation between Point and Point, Point and Line, Point and Angle, Point and shape, Line and Line, Line and Angle, Line and shape, Angle and Angle, Angle and shape; and Shape and Shape. As a sample of analysis, Table 2 and 3 illustrates the positional relations involved in the textbooks from both countries.

Table 2

Positional Relations of Diagram M163a (Form One Mathematics, KBSM syllabus)

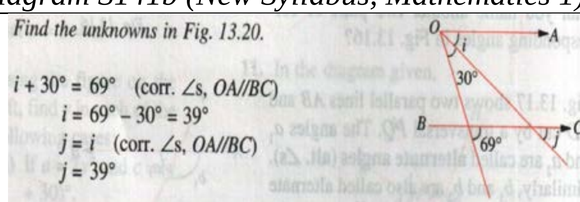


Question: In the diagram on the right, PQR is a straight line. Find the value of angle x.

Point & Point	P, Q, R, T and S are distinct
Point & Line	P, Q and R, T and Q and S and Q lie on the same line respectively
Point & Angle	Q is the vertex of ∠ PQT, ∠ TQS and ∠ SQR
Line & Line	Line PQ and TQ are concurrent, Line TQ and SQ are concurrent, Line SQ and QR are concurrent, Line TQ and QR are concurrent and Line PQ and SQ are concurrent
Angle & Angle	∠ PQT, ∠ TQS, ∠ SQR, ∠ PQS and ∠ TQR share the same vertex
Line & Angle	Line PQ and TQ are sides of ∠ TQP; Line PQ and SQ are sides of ∠ PQS; Line TQ and QS are sides of ∠ TQS; Line TQ and QR are sides of ∠ TQR and Line SQ and QR are sides of ∠ SQR

Table 3

Positional Relations of Diagram S141b (New Syllabus, Mathematics 1)



Point & Point	O, A, C and B are distinct
Point & Line	B,C and O,A lie on the same line respectively
Point & Angle	Angle 30° and *i* share the same vertex at O
Point & Shape	O is one of the vertex in the triangle Points A, B and C lie outside of the triangle
Line & Line	Line OA and lines from vertex O and intersects line BC are concurrent at O
Line & Angle	The two lines from O forms angle 30° + *i*
Line & Shape	The two lines that intersect at 30°and the line on BC forms a triangle
Angle & Angle	Angle 30° and *i* share the same vertex
Angle & Shape	The interior angles in a triangle

Table 4 shows the percentage of positional relations between the analysed diagrams from Malaysian and Singaporean mathematics textbooks on Lines and Angles. The number of worked examples that are involved in textbook Series One is 11 because seven out of the 18 diagrams were narratively structured. However, worked examples from other series of the Malaysian and Singaporean textbooks were conceptually structured. The findings show that the percentage of positional relations of Point and Point; Point and Line; and Point and Angle increased from Series One to Series Three in the Malaysian and Singaporean textbooks. There is only 37.5% of the positional relations (Point and Point; Point and Line; and Point and Angle) in the Singaporean and 81.8% in the Malaysian Series One textbooks. However, the positional relations with points had increased to 100% in the Series Two and Series Three of the Singaporean textbooks.

Table 4

The Comparison of Positional Relations between the Malaysian and Singaporean Textbooks

Positional relations	Malaysian Textbook Series (%)			Singaporean Textbook Series (%)		
	One	Two	Three	One	Two	Three
Point & Point	81.8	70.0	100.0	37.5	100	100
Point & Line	81.8	70.0	100.0	37.5	100	100
Point & Angle	81.8	70.0	100.0	37.5	100	100
Point & Shape	18.2	0.0	0.0	37.5	0	0
Line & Line	100	100.0	100.0	100	100	100
Line & Angle	100	90.0	100.0	100	100	100
Line & Shape	18.2	0.0	0.0	37.5	0	0
Angle & Angle	100	90.0	88.9	100	100	100
Angle & Shape	18.2	0.0	0.0	37.5	0	0
Shape & Shape	9.1	0.0	0.0	0	0	0

In contrast, the percentage of positional relations had dropped by 11.8% in the Series Two of the Malaysian textbook and increased to 100% in the Series Three. The analysis indicates that both countries had improved their diagrams in textbook Series Three with capital letters to show the positional relations involving points. The positional relations with points would help readers to construct more geometric relationships in the geometrical diagrams. Hence, students using the textbooks, specifically the Series Three from both countries would help them to give more geometrical details in the diagrams and perhaps students would be able to learn to construct geometrical diagrams with positional relations emphasising on points. Moreover, all the three Series of textbooks from both countries shows a 100 percent for Line and Line; and Line and Angle relations except for Malaysian Series Two textbook has only 90% of the diagrams with Line and Angle relation. This is due to the existence of a diagram on classifying parallel lines from the Malaysian Series Two textbook. The diagram did not present any angle in the diagrams for readers to construct meaning. All the other textbook Series show a 100% for Line and Line; and Line and Angle relation probably because this topic is mainly about Lines and Angles. For the Angle and Angle relationships, all the analysed diagrams (100%) from the Singaporean textbooks are involved, but there is a small decrease in percentage in the Malaysian textbooks, Series Two and Three. The Angle and Angle positional relations would help students to differentiate and compare the value of angles.

Apart from positional relations involving points, lines and angles, another geometric element is 'shape'. Positional relations involving shapes are Point and Shape; Line and Shape; Angle and Shape; and Shape and Shape. However, these relations are very less in all the textbook series. For example, the Point and Shape; and Line and Shape relations found in the Malaysian Series One (18.2%) and Singaporean Series One (37.5%) textbooks, but Series Two and Three books do not show the relations. The Angle and Shape relationships found in the Malaysian and Singaporean textbook Series One but the Shape and Shape relations could be determined only in the Singaporean textbooks. The positional relations involving shapes would probably help textbook writers to construct questions with higher order thinking skills.

Comparison and Measurement based size relations

Besides, comparing Line and Line; and Angle and Angle, the Shape and Shape that represent the size relations found in all the diagrams from the Singaporean textbooks but the percentage had dropped by 10% in the Malaysian textbooks, from Series One to Series Two and another 0.1% from Series Two to Series Three (Table 5).

For the measurement based size relations, all the textbook series were not involved with the Line and Angle; Line and Shape and Point and Point relations. However, only the Angle and Shape relations exist in a small percentage in the Malaysian and Singaporean Series One textbooks. Textbooks in Series One from both countries have given the opportunity for their readers to enhance their thinking skills to find the sum of angles inscribed in a polygon. The other textbooks did not present this relation perhaps the Grade Seven students might find it difficult to understand the relations.

Beside labels, colours too offer geometric relationships in diagrams. However, as the offer, colours are limited to equality such as equal angles, sides or areas (Alshwaikh, 2011). The analysis presents that none of the geometrical diagrams on Lines and Angles from series of textbooks from Malaysia and Singapore has colours on equality of angles.

Table 5

The Comparison and Measurement based Size Relations between the Textbooks from Malaysia and Singapore

Size		Malaysian Textbook Series (%)			Singaporean Textbook Series (%)		
		One	Two	Three	One	Two	Three
Comparison	Line & Line	0.0	0.0	0.0	0	0	0
	Angle & Angle	100.0	90.0	88.9	100	100	100
	Shape & Shape	0.0	0.0	0.0	0	0	0
Measurement	Line & Angle	0.0	0.0	0.0	0	0	0
	Line & Shape	0.0	0.0	0.0	0	0	0
	Angle & Shape	18.2	0.0	0.0	37.5	0	0
	Point & Point	0.0	0.0	0.0	0	0	0
Labels		100.0	80.0	88.9	100	100	100
Colours		0.0	0.0	0.0	0	0	0

Discussion

Narrative Diagrams

The findings on the ideational meaning show that seven geometrical diagrams from the Malaysian textbook Series One are narrative diagrams. Besides, two out of the seven narrative geometrical diagrams included human figures while four of them included physical objects. According to Pinto and Ametller (2002), students are more interested in real world objects than mathematically symbolic objects in the textbooks. Hence, readers, especially students would be more attracted to use the Malaysian Series One textbook than the other textbook series as it displayed narrative diagrams as real world objects.

The inclusion of human figures and physical objects express an action that is happening which depicts the characteristics of a narrative diagram. However, according to O'Halloran (2008), during the seventeenth century, the views of two mathematicians, Descartes and Newton had influenced the 'Modern Western' mathematics by eliminating human figures while emphasizing mathematical symbolism in geometrical diagrams. This was explained that "based on the notion of Plato that senses are not a reliable source of knowledge, since they may deceive or mislead the perceiver" (Alshwaikh, 2011, p. 43). Besides, the significance of a symbol as a semiotic tool had "became the centre of mathematics, and diagrams became a companion and aid to symbols" (Alshwaikh, 2011, p.134).

The existence of humans within the geometrical diagrams could be identified by observing temporal factors (such as before and after) in geometrical diagrams as based on Alshwaiksh's framework. For example, the narrative diagram in the Malaysian Series One textbook depicts a clock diagram with arrows showing the hands of a clock. Although this diagram did not show the existence of humans, the role of human as doing the measurement of angle could still be identified by the arrow shown. Thus, the arrow in the clock represents a measurement of an angle with a temporal factor of before and after.

However, the remaining 51 geometrical diagrams from both the Malaysian and Singaporean textbooks emphasized only mathematical symbols and geometric elements. The temporal factor of before and after were eliminated in these diagrams. Thus, these diagrams were considered as conceptual diagrams. Perhaps, the significance of mathematical symbols and geometric elements in representing geometric relationships compared to narrative diagrams with temporal factors might be the reason for the high number of conceptual diagrams in the analysed textbooks. This finding is not surprising because teachers and learners prefer the conceptual approach that hides the personal aspects of mathematician work (Alshwaikh, 2011). Likewise, this might be related to the role of mathematics as impersonal and formal (Morgan, 2006).

Conceptual Diagrams

In conceptual diagrams, the mathematical symbols and objects were used to guide students to understand the geometric diagrams. As well, such diagrams would be able to guide students to make reasoning on geometric concepts and gave them suggestions to draw such diagrams in the future. Besides, conceptual diagrams could be easily drawn by students as compared with narrative diagrams which contain human or physical objects. Nevertheless, as suggested by Alshwaikh (2011) that even though conceptual diagrams are more mathematical and symbolic, a richer image of diagrams in worked examples with a combination of narrative and conceptual diagrams would engage students and teachers with different kinds of mathematical meanings.

In a conceptual diagram, the geometric relations could be identified through the relational process of classifying, identifying and spatial relations. However, the classifying of geometric objects was found in only one of the 51 conceptual diagrams. The geometrical diagram was related to the geometrical concept of parallelism in the Malaysian textbook Series Two. Through the process of classifying, students would be able to identify the properties of a particular geometric concept. Meanwhile, Fujita and Jones (2007) mentioned that classifying examples would help students to recall the definitions and grasp the differences between geometric concepts. However, the "absence of the classificational process is not a surprise, since this kind of relation has few examples in geometry and is most commonly used to 'show' students a wider view of relations between diagrams" (Alshwaikh, 2011, p.167). Hence, in this study, the lack of geometrical diagrams of worked examples representing classifications in the Malaysian and Singaporean textbooks is expected.

In addition to classifying, identifying objects could also be determined in conceptual diagrams. The result of this study shows that there were differences in the identifying process for the geometric concepts in the topic of Lines and Angles between the different series of textbooks from both countries. The differences are in the use of letters (either capital or small letters) to identify objects (e.g., A or B) and attributes (e.g., a or x). Capital letters used to identify objects were given priority in the Malaysian textbooks whereas, the small letters were used to identify attributes in the Singaporean textbooks.

Capital letters that present identifying objects in a geometrical diagram were less dominant in the Singaporean Series One and Malaysian Series Two textbooks. However, the other series of textbooks has a percentage of more than 80% presenting the use of capital letters to identify objects. Identifying objects in geometrical diagrams are considered to be important as it is useful for the readers especially novices to interpret the diagrams. In aligning with Netz (2003), lettering diagrams are used to specify objects (lines, angles and planes) in diagrams and subsequently "lettered diagrams make an infinite diagram finite" (Netz, 2003, p.47). Moreover, specifying letters in geometric diagrams would help readers to locate geometric elements (Dimmel and Herbst, 2015). For example, if a question mentions that AB is the radius of a circle, readers will have to imagine a circle with A and B as the possible centre of a circle. However, if the same statement is communicated through a diagram, the writer could specify the centre of the circle with capital letters (such as A or B). Thus, the centre of the circle becomes transparent.

Meanwhile, letters used to identify attributes were more dominant in the Singaporean textbooks than in the Malaysian textbooks. The Singaporean textbooks series have consistently shown a 100% use of letters to identify attributes. Identifying attributes helps students to recognize the problem without referring to the verbally stated questions. Hence, all the geometric diagrams in the Singaporean textbooks were more significant compared to the respective verbally stated questions. Perhaps students reading the Singaporean textbooks might save time in understanding the problems that need to be solved. Furthermore, letters used to identify attributes provides the size relations (Alshwaikh, 2011) in a spatial process, such as two angles with a value of x and 2x will symbolise the difference in size.

Apart from representing with indexical letters, all the diagrams analysed in textbooks of both countries do not have the indexical arrow. Arrows in geometric diagrams may present various meanings such as vectors; showing a temporal factor of 'before' and 'after'; or as an indicator referring to a particular geometric element in a diagram. The various use of arrows could confuse the understanding of students especially the novices as noted by Ametller and Pinto (2002) that some students faced difficulty in reading geometric objects that have various meanings. Hence, this finding shows that the textbook authors of both countries were well aware of the possible confusion by students. Thus, there is no inclusion of indexical arrows.

Furthermore, the symbolic words were given less priority than the indexical letters in both countries' textbooks series. Nevertheless, there were some symbolic attributive words found in the Malaysian Series Three textbook. The symbolic attributive words are helpful as they would be able to guide readers to understand better the geometric problems posed. However, none of the worked examples from the Singaporean textbooks have symbolic attributive words. Nevertheless, the Singaporean and the Malaysian Series Two and Three textbooks had included symbolic identifying words to help readers to understand the underlying geometric concept. The inclusion of symbolic words in geometrical diagrams would help students to identify and recognise geometric elements and geometric concepts quickly. This was as pointed out by Kress and Van Leeuwen (2006) that symbolic process in an image makes a statement about what the object means or is. Thus, the textbooks from both countries used since the year 2003 till present especially in the Malaysian series had considered providing a better opportunity for students to understand geometric concepts or relationships.

Apart from that, with analysing the spatial relations, it was found that the Malaysian Series Three and the Singaporean Series Two and Three textbooks represent the highest percentage of relationships with points compared with the other textbook series. Thus, these textbooks had the highest percentage in using capital letters to denote points and lines that represent geometric relations compared with the other textbooks with lesser use of capital letters. Besides, these textbook series had more positional relations of lines, angles and shapes with points that would ease the identification of geometric elements and make the diagram to be remembered (Winn, 1991).

Besides, the Lines and Shapes relation likewise Angle and Shape were found only in the Malaysian and Singaporean Series One textbook. However, the percentage of positional relations with shapes were higher in the Series One of the Singaporean textbooks. Probably the textbook had emphasized more geometric relations such as sum of angles in a polygon and this might help writers to construct more challenging questions. According to Winn (1991), spatial relations can make abstract ideas more concrete with diagrams. Hence, this signifies that the worked examples from the Singaporean textbook have more concrete ideas that were represented in geometrical diagrams. For example, a question on an isosceles triangle could be represented in a geometric diagram rather than mentioning it in the verbally stated question.

Furthermore, comparison based size relationships like the Angle and Angle are familiar on the topic of Lines and Angles. For example, the relationship could help students to identify the vertically opposite angles or corresponding angles (Alshwaikh, 2011). However, not all the diagrams provide an opportunity for students to compare the size of angles in the Malaysian Series Three and Series Two textbook compared with a 100% of diagrams with this relation in all the other series of textbooks in both countries. Meanwhile, the Angle and Shape representing measurement based size relationship would help students to identify the sum of angles in a polygon (Alshwaikh, 2011). This relation was found only in the Malaysian and the Singaporean Series I textbooks.

Besides, this study also found that labels such as parallelism and quadrant for 90° representing the size relations were more dominant in the Singaporean series of textbooks compared within the Malaysian series of textbooks. Meanwhile, none of the analysed diagrams from the Malaysian and Singaporean textbook series shows colours for the difference in size of angles. Hence, the use of colours representing size relations was not emphasized in the analysed textbooks for Lines and Angles. This might be related to what Winn (1991) mentioned that using colours in diagrams would affect the focus of the learner.

Conclusion

The deficiency of narrative diagrams that explains the less use of geometrical diagrams presenting real life situations or diagrams involving human actions could be a possible hindrance for students in realizing the importance of geometry in real life situations. However, the conceptual diagrams are more mathematical and symbolic, a richer image of diagrams in worked examples with a combination of narrative and conceptual diagrams would engage students and teachers with different kinds of mathematical meanings (Alshwaikh, 2011). Meanwhile, most of the worked examples in the Singaporean textbooks had not given much importance to letters to represent the identifying of objects. However, according to Halliday (1995), the identifying processes is a more general relation compared with the attributive process in a verbal language. Translating it into geometric diagrams, the capital letters to identify objects are more general than the small letters that represent attributes. However, it may not be the same to identify attributes in the Malaysian context. To identify attributes with small case letters, students and teachers are required to read and understand the verbally stated question before lettering to identify attributes in the geometric diagrams. Hence, the inclusion of small case letters in the Malaysian textbooks by the textbook writers would be more appropriate and organised as it resembles the problem that students need to solve.

The differences in spatial relations in textbooks from both countries for the geometric concepts reflects deeper connections shown in the Singaporean textbooks with ‘shapes’. However, the Malaysian textbooks are keen to represent relations with ‘points’. Future research is needed to look into the relationship between spatial relations with types of questions in constructing reliable geometric elements in the diagrams.

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