The Mathematics Enthusiast

Volume 17
Number 1 *Number 1*

Article 10

1-2020

Use Mathematical Writing as a Practical Approach to Increase Students' Problem Solving Skills: A Case Study

Guat Poh Betsy Lee

Chap Sam Lim

Lai Mei Leong

Recommended Citation

Lee, Guat Poh Betsy; Lim, Chap Sam; and Leong, Lai Mei (2020) "Use Mathematical Writing as a Practical Approach to Increase Students' Problem Solving Skills: A Case Study," *The Mathematics Enthusiast*: Vol. 17 : No. 1 , Article 10.

DOI: https://doi.org/10.54870/1551-3440.1486

Available at: https://scholarworks.umt.edu/tme/vol17/iss1/10

Use Mathematical Writing as a Practical Approach to Increase Students' Problem Solving Skills: A Case Study

Guat Poh Betsy LEE[1]
University of Nottingham Malaysia Campus
Chap Sam LIM
Lai Mei LEONG
Universiti Sains Malaysia

Abstract: *In mathematical problem solving, students' written work mostly reveals their mathematical algorithm skills and has very little information about their reasoning skills of the problem solving process. This study extends the features of mathematical writing that integrate the language and mathematical thinking to increase students' mathematical problem solving skills. The main feature of this study is the use of mathematics writing workbook as a practical approach to guide the students in the problem solving process. Thirty Foundation students in Engineering participated in a six weeks of writing to solve mathematical problems. An exploratory case study analysis was used to examine the written contents of the participants' mathematical writing workbook, the performance of their formal test as well as their perceptions of mathematical writing. The trace of work in the workbooks showed that mathematical writing has somehow given some impact on these students to visualize, aware and recognize their problem solving behaviors in words.*

Keywords: mathematical writing, problem solving skills

Introduction

The meaning of mathematical problem solving varies ranging from working rote exercises to doing mathematics as a professional (Schoenfeld, 2016). It can be referred as a hierarchy of skills associate with a sequence of problem solving activities (Mcguire, 2001; Stanic & Kilpatrick 1989, as cited in Schoenfeld, 2016). Nevertheless, the activity of problem solving only happens when an individual must complete a task but does not possess sufficient knowledge or experience to reach an appropriate solution (Dougherty & Fantaske, 1996). In other words, a

[1] Betsy.Lee@nottingham.edu.my

specific task may not be a problem to an expert who can routinely solve the problem but may become a problem solving task to a novice who does not have immediate access to the solution (Zawojewski, 2010). The theme of problem solving also involves a series of effective mechanisms that cope with the problem situation in order to achieve an ultimate solution (Dougherty & Fantaske, 1996; Mcguire 2001; Zawojewski, 2010) In fact, writing is a form of mechanism of learning where students from time to time rely on pencils and papers intervention to perform activities such as taking notes, performing calculation and solving mathematics problems.

Mathematical writing is multifaceted in mathematical context, and according to Morgan (1998), there's no absolute definition of mathematical writing. It could be viewed as a thematic condensation of terms, symbols and images to channel meaningful context for mathematical learning (Seo, 2015). Hence, mathematical writing is not a static form but can work in many dynamic ways, from a relaxed and casual feature of writing to an intellectual and creative type of writing. Nevertheless, many students spontaneously use writing to present their computation techniques when they solve a mathematics problem. Their problem solving work reveals their strength in algorithm skills rather than recording their mathematical reasoning. This scenario has been stated by Ball (1993) that "it is difficult to discern what some students know or believe—either because they cannot put into words what they are thinking or because I cannot track what they are saying" (p. 387). Thus, the capacity of mathematical writing should be extended to integrate language and thoughts besides formulae and equations. Writing is actually an act of problem thinking. Writing and the problem solving share the same mental procedures to process information in order to achieve a set of goals. Writing as a problem solving activities aims to make students self-conscious about the way they conceptualize (Berkenkotter, 1982), while a problem solving approach to writing formulize the disorderly dynamics of thinking and ideas into a heuristic set of sub-writing process (Flower & Hayes, 1977). This shows an explicit linkage between writing and mathematical problem solving skills. Hence, the purpose of this study is to extend the hidden potential of mathematical writing by adopting it as a constructive tool to increase students' problem solving skills.

Problem Statement

Since the mid of 80's, the Mathematics Curriculum at school in Malaysia has undergone significant changes, attempting to make problem solving as the center of attention in the teaching and learning of mathematics (Zanzali, 2000). To instigate problem solving as a practical approach in a mathematics classroom, the Polya's problem solving model served as a general strategy for problem solving, whereby it guided the students on how to solve various mathematical problems in a systematic way by going through a sequence of cognitive activities (Malaysian New Integrated Mathematics Curriculum, 2003). Nevertheless, the question remains as to what extent the school teachers in Malaysia have adopted the problem solving approach into their teaching. Saleh (2009) investigated the problem solving teaching strategy among the Form Two mathematics teachers. Her findings revealed that the mathematics teachers did not apply problem-solving strategies in their teaching. The teachers deduced that the implementation of problem-solving strategies in the classroom teaching unnecessary since it was not evaluated in the examination. Three years later, Saleh and Aziz (2012) further investigated the teaching practices in Malaysian secondary schools, whether teachers have shifted to newer alternative teaching strategies rather than conventional teaching methods. They concluded that majority of the existing teachers were still attached to the traditional teacher centered approach. As a matter of fact, they reported that their findings were consistent to previous findings for the past 20 years which showed that the quality of teaching has remained unchanged i.e. traditional teaching method. Various trainings and workshops were conducted by Ministry of Education to enhance the teaching profession for the past two decades (Ghazali, 2017), nevertheless, some teachers seemed to be contented with their repetitious teaching style and have ceased trying new teaching strategies.

Communication in mathematics is another important feature in the teaching and learning of the Mathematics Curriculum in Malaysia (Malaysian New Integrated Mathematics Curriculum, 2003). The curriculum strongly emphasizes effective communication to drive students' problem solving abilities and writing appeared to be one of the influential mediums to communicate mathematically in a comprehensive form. In other words, mathematical writing has become an integral part of the mathematics curriculum in Malaysia since the last few decades but despite its posited importance, there is little or no research concerning the practices of mathematical writing from preschool to tertiary education in Malaysia. There remains a paucity of evidence on good practices of mathematical writing in the Malaysian education. Hence, it is hoped that this study would stimulate needed research to further develop the potential of mathematical writing as a problem solving approach to foster the students' problem solving skills in all educational levels.

Literature Review

Problem solving in general links together two elements: a problem task which is generally defined with respect to the problem solver and a series of actions taken in finding an explicit way to attain a solution (Zawojewski, 2010). A task can be defined as an assignment that one needs to be accomplished within a time frame. Nevertheless, the level of efforts and struggles to execute a task depends on the strength and ability of a problem solver as what

Schoenfeld (1985, as cited in Mayer 2002 p. 70) has observed: “The same tasks that call for significant efforts from some students may well be routine exercise for others.” Thus, a task is characterized to be a problem depends on the individual’s knowledge and experience (Yeo, 2007; Xenofontos & Andrew, 2014).

The problem solving model is normally served as a general strategy for problem solving and Polya (1945, as cited in Zollman, 2010) was credited as the key figure that began the investigation for assisting students to mathematical problem solving. The problem solving model guides the students on how to solve various mathematical problems in a systematic way by going through a sequence of cognitive activities, for example, reading and understanding the problem, planning, performing the planning, getting the answer and confirming the answer. This process forces students to assess their understanding, rather than just getting the final answer (Parker Siburt, Bissell & Macphail, 2011). Over the last few decades, there were a few problem solving models adapted in working on a mathematical problem (Tamychik, Meerah & Aziz, 2010), however, the role of a problem solving model, whether a model from the 1980’s or those developed in the later years served the same purpose, that is to assist each student to comprehend and engage in the problem solving process.

The use of problem solving strategy for writing was first attempted by Flower and Hayes in 1977. According to their report, the act of writing is a complex task that involves highly complex cognitive processes. The conventional teaching in writing often failed to inspire novice writers to possess the ability to write well. However, they discovered that the mental process of writing can be treated as a form of problem solving (Flower & Hayes, 1977). A problem solving approach to writing helped the writer to experience one mode of thinking to the other and construct meaningful ideas to produce a good composition. In fact, writing and problem solving are intertwined actions where they hold the same continuous mental process. The significance of writing in some way inspire a group of mathematics educators to explore the use of writing approaches to mathematical problem solving (Berkenkotter, 1982; Bell & Bell, 1985; Lester, Garofalo & Kroll, 1989; Pugalee, 2001, Martin, 2015; Seo, 2015; Kosko & Zimmerman, 2015; Kosko, 2016). They saw the potential of writing that make students self-conscious about the way they conceptualize.

Research regarding the distinguishing features of mathematical writing on the specific cognitive development is rather extensive. Different modes of mathematical writing appear to have its own specific purpose and convention (Martin, 2015; Seo, 2015; Cohen, Casa, Miller & Firmender, 2015; Kosko & Zimmerman, 2015; Kosko, 2016). Nevertheless, writing to problem solving or problem solving to writing is a powerful mechanism that guides a writer or a problem solver to experience a series of cognitive process and strategic actions or plans to compose a good article or a solution. With regard to the relationship between writing and problem solving, many researchers have discovered that the use of writing approach through problem solving model helped students to visualize their mathematical thinking in words and to describe their action at each phase of problem solving (Berkenkotter, 1982; Bell & Bell, 1985; Lester, Garofalo & Kroll, 1989; Pugalee, 2001). For example, Hensberry and Jacobbe (2012) conducted a four-day intervention study about the effect of Polya's model and diary writing on students' problem solving. The diary was in the form of worksheet with prompts and space for students to write their responses before and after each problem solving exercise Although the time of the study was very short but the diaries managed to picture the students' thinking and actions before and after the problem solving exercise. Hence, this study intends to use the mathematics writing workbook as an approach to extend the feature of mathematical writing through problem solving process that encourages students to vocalize their thinking process and increase their problem solving skills.

Theoretical Framework

The theoretical framework of this study is shown in Figure 1. The framework is guided and modified based on the Hayes and Flower's model (1981) and Hayes's revised model (1996, as cited in Alamargot & Chanquoy, 2001) of writing process. It contains two main parts i.e. the problem task environment and individual. The problem task environment is defined as the external representation of the writers that influence their performance. The individual dimension comprises of the writer's cognition in mathematical problem solving.

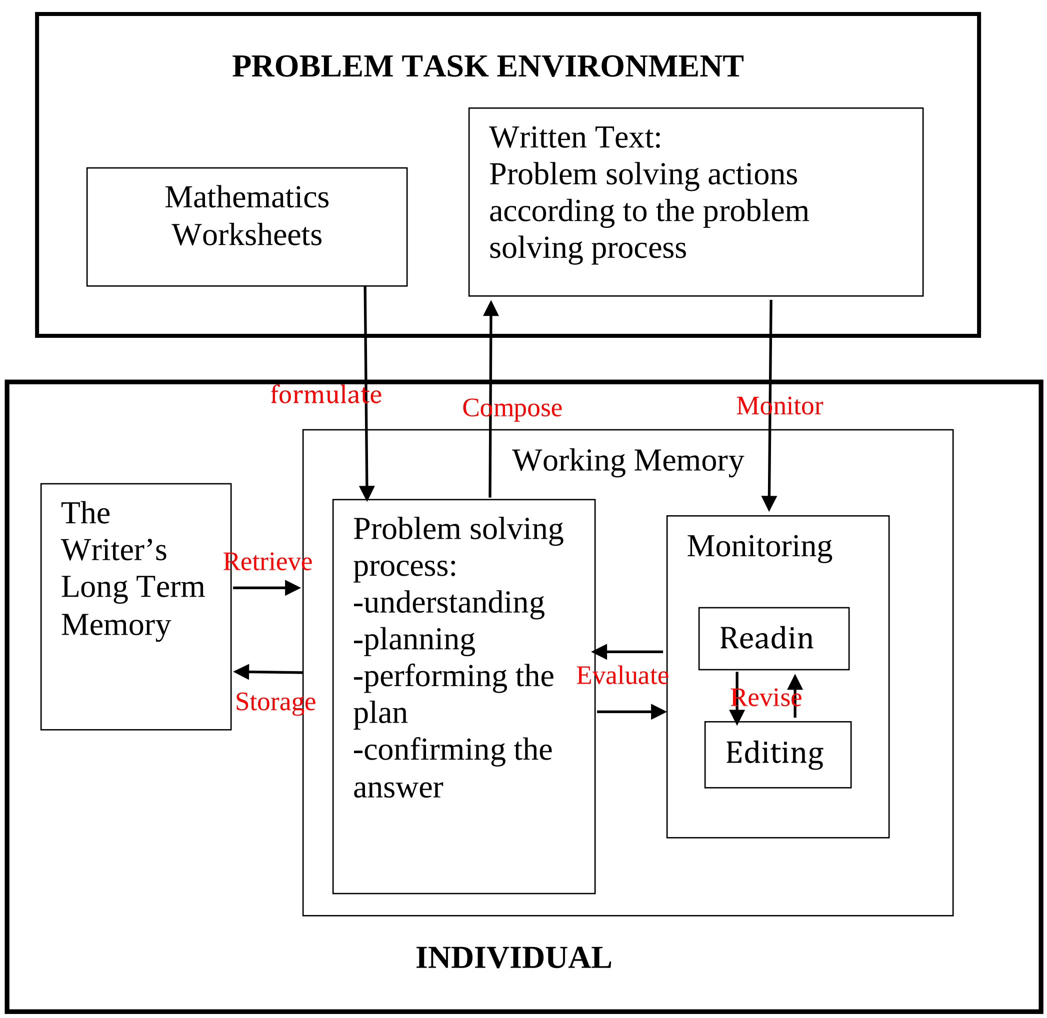


Figure 1. Theoretical framework.

The task environment serves two distinct functions:

- access external information such as related text material, sample problems or diagrams that constitute to the mathematics worksheets, and
- the written text which is used as a reference to writer, in order to read and revise the already written text.

The external information made available for a problem solver to formulate an initial mental representation of the task environment, i.e. a problem space (Silver, 1987). As a problem solving activity proceeds, a problem solver may alter sources of external information and may re-evaluate the mental representation that plays an essential role in problem solver's understanding of the problem.

On the other hand, three components constitute to the individual dimension i.e. the writer's long term memory, problem solving process and monitoring. Nevertheless, the initial mental processing of capturing information begins with the sensory stimuli either through visual, auditory or tactile. Sensory information retains temporary in the sensory buffer before being transfers to working memory or loses it. Working memory or short-term memory is the cognitive activation zone where all the information processing takes place. It is here that the sensory information is either processed and kept in long term memory or interacted with elements retrieved from long term memory. Long term memory is a boundless storage capacity that accumulates all the knowledge and skills that a person has.

The approach of writing to problem solving process plays as a substitute to keep information processing consciously in mind. The problem solving process employed is Polya's problem solving model (Polya, 2014) that consists of four phases i.e. understanding the problem, planning, performing the plan and confirmation of the answer. Writing is used as a responsive instrument to answer a series of questions that is connected to each phase of problem solving as shown in Table 1. These questioning techniques encourage students to investigate, analyse and demonstrate knowledge of the underlying concepts in order to reach the final solution. The written responses in term of words, symbols and images reveal the problem solving actions taken to answer each guided question. Problem solving action is the set of problem solving approaches as individual progresses from the initial state to final state of a problem. The approach of writing to problem solving process is stringer writers to make their problem solving actions to be concrete and visible for the component of monitoring. Monitoring plays the role of reading and editing. Reading process allows regular reread and verify the written text while editing process re-evaluates the problem solving process which creates a new version of written text.

Table 1

A Set of Questions Directed toward the Problem Solving Process

Four Phases of Problem Solving	Questions
Understanding the problem	What is the unknown? What are the data? What is the condition? Is the condition sufficient to determine the unknown? Etc.
Planning	Do you know the related problem? Could you imagine a more accessible related problem? Did you see all the data? Have you taken into account all essential notions involved in the problem? etc.
Performing the plan	Do you check each step? Can you see clearly that the step is correct?
Confirmation of the answer	Can you check the result? Can you check the argument? Can you use the result, or any other method, for some problem?

Note. “From How to solve it: A new aspect of mathematical method“. By G. Polya, 2014, p. xvi-xvii

Research Questions

This study was designed to answer two research questions related to the use of mathematical writing in a Pre-calculus course: 1) How does mathematical writing exercise help to increase the students' mathematical problem solving skills? 2) What are the perceptions of students towards the mathematical writing as an approach to improve their problem solving skills?

Methodology

This study employed case study as the prioritized approach to explore the use of mathematical writing as a stimulant approach in developing students' mathematical problem solving skills. Case study is appropriate as it involves detailed investigation of a single individual or a single group and provides an in-depth understanding of the real context with multiple sources of data collection (Yin, 2014). The data from this study was obtained through students' written responses in the mathematics writing workbook, a formal test and individual interviews.

Thirty participants involved in this study were Foundation in Engineering students at one of the higher institutions in Malaysia. They were Malaysian students who have just completed their high school education and newly enrolled into the Autumn semester 2015 of the Foundation in Engineering programme. The intervention process was centered at the mathematics writing workbook. The workbook is a homework practical-worksheets that demanded clear and precise description of the Polya's (2014) problem solving model. The students were given one worksheet at the end of each week of teaching lesson and they were requested to submit the given task on the following week. A written comment on students' work was given before the next worksheet in order to help the students understand their strengths and weaknesses of their work. After six weeks of experiencing the mathematical writing worksheets, the students took a formal test which was part of the assessments of the pre-calculus course that consisted of 40% of the final grade. Subsequently, an interview session was carried out to gain the students' perception about the mathematical writing experience.

The vital section of this study is a detailed review of the writing features in the mathematics writing workbook. The students' written responses in the mathematics writing workbook contained not only text but also geometrical drawing and representation of equations which were also regarded as part of the problem solving approaches or actions. To measure the students' problem solving actions, the students' work in the mathematics writing workbook were coded according to a set of guided steps of problem solving procedures adapted from Polya's checklists (see Appendix A). In other word, the students' problem solving work were analyzed based on their actions that are responding to each guided question. The coding scheme was derived in relation to Lucas et al. (1979) scheme where they established a dictionary of problem solving descriptions that reflect the use of Polya's heuristic approaches during problem solving. To resolve various interpretation that might appear in the form of text or diagram, the scheme was further illustrated using the coding strategy employed by Glogger, Holzäpfel, Schwonke, Nückles and Renkl (2009) whereby the grain size of problem solving actions determine the segment size. Glogger et al. (2009) claimed that the grain size of the units may differ in scope between and within categories which sometimes make no sense of segmenting and then coding the segment. For instance, the understanding phase under problem solving such as identifying key ideas could consist of just two words, a formula or a paragraph with clear description. However, these examples would be rated differently in term of the description level of written responses although there were segmented and coded in the same category. The quality of the written text was mainly focused on the students' efforts of writing where their responses are readable and understandable by a reader. The writtten text was rated using a 4-points scale developed by Lim and Pugalee (2004), ranging from 1 (low level of desctiption) to 4 (high level of description).

Two independent raters were assisting in the process of reliability and validity in this study. Both the raters are mathematic lecturers who have more than ten years of experience in teaching engineering mathematics at the tertiary level. At the beginning, they coded 9 students' mathematical writing workbook including the entries from the 6 weeks of the study and rated the students' written text based on journal rubric, as described above. They also took part in the coding scheme of the problem solving actions where the coding was discussed and some of code definitions were further illustrated to achieve rating consistency. The overall inter-rater percentage of agreement was rather good i.e. 80.9%.

Analysis and Findings

After six weeks of writing intervention, the participants took a Pre-calculus test and their average test score was 73. Figure 2 shows the average test score of Pre-calculus from the year 2014 to 2015, where the module is assessed namely Summer and Autumn semester for each year. Looking at the achievement score over the two years, there was a tremendous improvement of average score from 55 to 73. According to the achievement standard at the university, a score of 70 and above is considered high achievement rate while a score between 50 and 70 assess as moderate performance. There were 20 out of 30 students who obtained a score above 70 (see Table 3). Eight students showed moderate achievement while only two students obtained a score below 50.

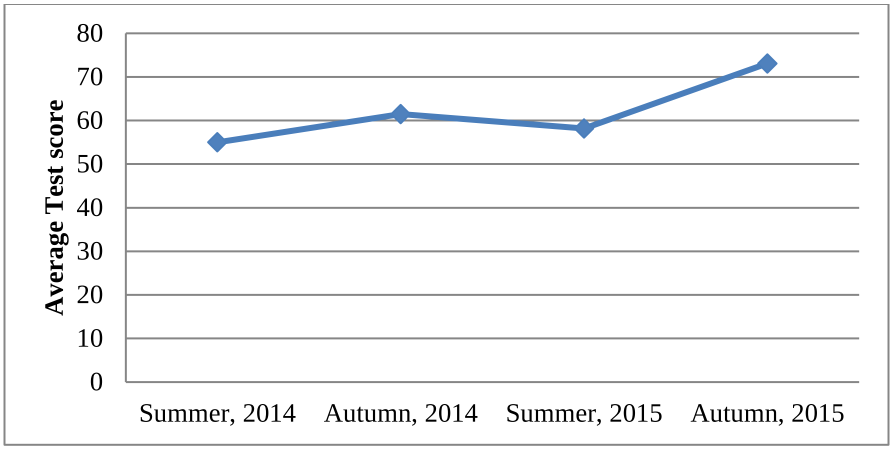


Figure 2. The average test score of pre-calculus from the year 2014 to 2015.

In the mathematics writing workbook, there were six worksheets and each worksheet consisted of three problem tasks. Based on the 4-points scoring criteria of clear explanation, adapted from Lim and Pugalee's (2004) journal rubric, the students' written work on each problem tasks were examined according to the rubric. By summing the score for each problem task, an average score was computed. Students who scored 2.5 and above were categorized as high written responses, while students who scored between 1.5 and 2.5 were categorized as moderate written responses and students who scored lower than 1.5 were categorized as low written responses. Table 2 illustrates the degree of students' written responses in the mathematics writing workbook against their test score. Twenty students showed high achievement in their test and only six of them demonstrated extensive written responses in their mathematics writing workbook. Nine high achieving students' written work provided descriptions that were moderately comprehensible and the remaining five high achieving students only wrote a brief desription of the problem solving process. Four students showed moderate performance in their test that also reflected their moderate efforts in mathematical writing. Nevertheless, there was one student who put the same efforts in mathematical writing but his achievement in the test was low.

Table 2

The Students' Test Score and the Degree of Written Responses in the Mathematical Writing Workbook

Formal Test score	Number of Students	Demonstrate High Written Responses	Demonstrate Moderate Written Responses	Demonstrate Low Written Responses
Score above 70	20	6	9	5
Score of 50-70	8	-	4	4
Score below 50	2	-	1	1

In order to further explore the impact of mathematical writing on the students' mathematical problem solving skills, the number of problem solving actions at each problem solving phases was counted from each students' mathematics writing workbook. There were four major phases in problem solving. At the understanding phase, four actions were identified: (a) restates problem in other words or other ways, (b) highlight/identify key terms or key ideas, (c) represent problem information in visual form and (d) introduce suitable notation such as gives name/symbol to an object. The planning phase of problem solving involved generating possible ways of solutions where the problem solvers attempt to search for a path through the representation of the problem. The corresponding actions included (a) organizing data into smaller sections, (b) stating the plan for intermediate goals and (c) making informed decision about strategies such as making a table, a diagram or writing an equation.

Since planning provides the necessary goals to achieve, hence performing the plan is an important phase to put plans into action. It requires efforts to validate the plans. In this third phase, three actions were examined in the students' written responses, comprised of (a) implementing strategies according to the plan, (b) performing calculation or techniques and (c) organizing the work so that it is easy to understand. The final phase is the process of making sure a solution is correct. It involves re-checking activities and re-evaluating the solution for accuracy. The actions at the final phase included (a) engaged in checking the logic/accuracy of the computations, (b) reflecting on the answers and (c) reflecting on the learning experience.

The number of problem solving actions at each problem solving phases was counted from each student mathematical writing workbook and by summing up these actions, the means of actions taken by the students at each problem phases were tabulated against the students written responses rates (see Table 3).

Table 3

Means (standard deviation) of Actions at each Problem Solving Phases

Degree of Written responses	Action used to respond to Problem Solving Phases			
	Understanding	Planning	Performing the plan	Confirming the answers
High	2.47(0.46)	1.98(0.38)	2.27(0.47)	1.11(0.48)
Moderate	2.42(0.63)	2.11(0.08)	2.05(0.21)	1.08(0.38)
Low	0.94(0.54)	1.11(0.13)	1.17(0.15)	0.15(0.06)

Students who perceived high and moderate written responses adapted at least two problem solving actions at the first three problem solving phases and applied one approach to confirm their answer. Majority of them restated the problem by using their own words to decode the key terms and connected to other mathematical representations such as diagram or equation which revealed their internal mental representation. They described about their plan in a systematic manner and stressed on their sub-goals. Concurrently, they performed the plan accordingly and their solutions showed a well ordered working steps. Some of them took the initiative to work with other numbers or reconstruct the graph to verify their answers. On the other hand, students who perceived low level in written responses only employed one problem solving action during the first three problem solving phases and hardly used any strategy to verify their answers. Somehow, they were fondly attached to use one or two words to highlight the key terms or apply equation or diagram to illustrate their understanding. They hardly described their plan but they showed enthusiasm to use calculation as part of strategy to search for a solution. In other words, their work revolved mainly around numbers and formulas.

To provide more fruitful evidence about using the mathematical writing as an approach to increase the students' problem solving skills, it is interesting to compare two students who scored equivalent high score but showed different efforts of written responses in the mathematical writing workbook. The following discussion is about Amelie and Brandon who scored a grade of 90 and above in their Pre-calculus test but exhibited incompatible writing efforts in their mathematical writing workbook. To show how the students' writing exhibit their problem solving actions, discussion is centered at the mathematics writing worksheet one that consists of three graphing problems using vertical/horizontal shifting techniques, as shown below.

Problem task 1
Given the function $f(x) = \left|x - \frac{1}{2}\right| + \frac{3}{2}$. *Apply transformations to the graph of a standard function and state the domain and range of the transformed graph. Please sketch all the graphs in the same coordinate plane.*

Problem task 2
Given the function $g(x) = \frac{1}{(x-1)^2} - 2$. *Apply transformations to the graph of a standard function and state the domain and range of the transformed graph. Please sketch all the graphs in the same coordinate plane.*

Problem task 3
Given the function $h(x) = \sqrt[3]{(x+2)} + 1$. *Apply transformations to the graph of a standard function and state the domain and range of the transformed graph. Please*

sketch all the graphs in the same coordinate plane.

Amelie. Amelie was a 18 years old girl who stayed in Miri, East Malaysia. She joined the university in West Malaysia because she would like to pursue degree in Chemical with Environmental Engineering that is not offered in the universities at East Malaysia. In a way, she appeared to be aware of her own strength and willing to take challenges. This can be seen in her seriousness attitude in participating in this study

Amelie's work involved a prominent level of written responses with an average score of 3.18 that showed an excellent use of her writing to explore and review the mathematics she is learning. She highlighted each phase by using appropriate language to record a sequence of ideas. In other word, her work showed her problem solving actions explicitly through writing at each phase. For example, she wrote about her understanding of problem task 1 as follows.

Understanding the problem:

The function f(x) involves modulus function and it can be obtained by sketching the standard modulus function i.e. g(x) =$|x|$*, the transformations of the graph should be done after sketching the standard modulus function. We can identify the domain and range from the graph ate the end of the working.*

As part of her understanding to the problem, she was trying to make sense of the problem to a standard modulus function and used appropriate symbol to define it. She restated the problem by using her own words to extend her understanding. Her subsequent writing provided information about the types of plans to be carried out.

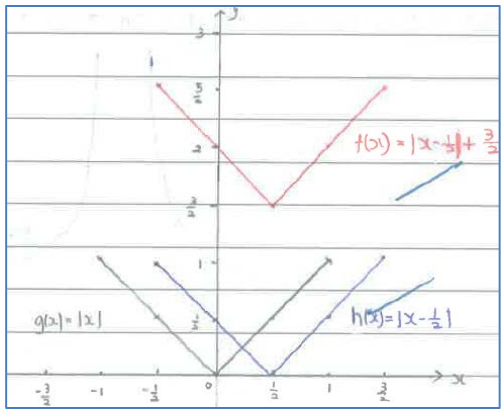
Planning:

I need to sketch g(x) and f(x) can be obtained by shifting the graph of g(x) horizontally to the right by ½ unit followed by shifting vertically up by 3/2 units.

She indicated clearly about her sub-goals and informed the decision strategies to transform the graph one to the other. Through writing, her plans that guided her execution was evident in her following statement.

Performing the plan:

Sketch the graph of g(x) and apply transformation to obtain the graph f(x).



The graph of g(x) has been shifted horizontally to the right by ½ units followed by shifting vertically up by 3/2 units

Her last statement revealed her awareness of her planning where she executed the plans accordingly. Her graph sketching was well organized where she used distinct colors and symbols to indicate the shifting procedures. When she arrived at the phase of verifying her answers, she informed about the strategies used.

Confirming answer:
Construct a table of plot points to confirm the graph that has been drawn.

x	$y=\lvert x-\frac{1}{2}\rvert+\frac{3}{2}$
$-\frac{1}{2}$	$\frac{5}{2}$
0	2
$\frac{1}{2}$	$\frac{3}{2}$
1	2
$\frac{3}{2}$	$\frac{5}{2}$

∴ *The domain of the graph,* $D_f = (-\infty, \infty)$
The range of the graph, $R_f = [\frac{3}{2}, \infty)$

Her responses showed that she engaged in checking her answers through computing the output values from the value of x =-1/2 to 3/2. This has been reflected at her self-evaluation section i.e. "*confirming answer is a good way to make sure we obtain the correct graph*".

Amelie has maintained regular entries of a series of problem solving actions and Table 4 shows her mean scores of problem solving actions in the mathematics writing workbook. She applied two to three problem solving actions to constitute her understanding of the problem and planned her problem solving techniques either organized the data into smaller sections or employed some strategies such as construct a table, write an equation etc. or both. Subsequently, she implemented her plans accordingly and always engaged in re-checking her solution. Her experience in writing has somehow increased her awareness in problem solving which was evident in her self-evaluation section where she wrote "*Planning is crucial in sketching graph. However, I need more help with identifying the basic function or standard function of a given function*". Her writing revealed that she was monitoring her internal representation of the information as well as identifying appropriate sub-goals for solving problems.

Table 4

Problem Solving Actions taken by Amelie when attempting the Mathematical Writing Practical Worksheet One

	Amelie's Problem Solving actions to each Phases			
	Understanding	Planning	Performing the plan	Confirming the answers
Problem task 1	3	2	3	2
Problem task 2	4	2	3	2
Problem task 3	3	2	3	2

Her experience in mathematical writing was further explored through an interview session to discover about her perceptions toward the use of mathematical writing workbook.

Amelie loved mathematics and she obtained "A+" grade for modern mathematics and additional mathematics in her Malaysian Certificate of Education (SPM). Her experience in learning mathematics in school was drill and practice method as she claimed:

> "*For teaching, she (teacher) taught about the concepts, examples, explanations, after that give us exercises… she photo-stated a lot of worksheets for us to do*"

Because of her learninng experience in school, she felt uncomfortable in the initial introduction of the mathematical writing workbook. When she was asked about the first thing came to her mind about the writing part, she smiled and said:

> *I don't know how to write I refer to the examples given. I tried to think and write.*

Although she relied on the examples to write but she started to be conscious of what she wrote. The six weeks of mathematical writing intervention somehow has changed her mind set about using writing in mathematical problem solving, as she continued:

> *"I think this (writing) is quite useful because it helps us think, it guides us to the actual concepts and answers"*

The four phases of problem solving has actually activated her ability to think where she spent some time to organize her thought in the flow of the problem solving processes, especially the planning phase, as she said:

> *I think planning takes quite a long time but I still can manage but for the confirming answer normally I don't have any idea to confirm my answer"*

Her low retention of mathematical knowledge and skills has induced her inabilities to apply other approaches to re-checking the answer. Nevertheless, she learned how to organize her thinking in a systematic way as she said:

> *I think my problem solving improved because you have to breakdown all your thinking into steps that's why you can really can think about each phase what it is really about. Best part of writing is we can really break down all of our questions into smaller parts and try to solve it one by one. It will be more systematic"*

Hence, the composing process has increased her awareness to reorganizing the problem in a systematic way.

Brandon. The first impression about Brandon was his cool and calm personality. He is aiming for degree in Civil Engineering and has the intention to pursue untill Master Degree. Nevertheless, he believed that “doing mathematics” is about getting the correct answer using number and formula, as he claimed during the interview.

Brandon performed equally well as Amelie in the formal test but his average score in writing was 1.5 that showed his low contribution in mathematical writing. Nevertheless, when he first atttempted the problem task 1, he employed the mathematical writing to undergo the problem solving phases. In his writing, he demonstrated his initial attempt to understand the information provided by identifying a standard modulus function i.e. $g(x) = \left|x - \frac{1}{2}\right|$ and used the keyword “piecewise function“ to illustrate his understanding about $g(x)$. He recapped the problem as follows.

> *f(x) involves a modulus function and it can be obtained by sketching the standard modulus finction $g(x) = \left|x - \frac{1}{2}\right|$which is a piecewise function can be defined as ...*

Subsequently, he informed about his sub-goals where he revealed his planning and performed the plan accordingly. He constructed a table to compute the output values of $g(x)$ and used the data to sketch its graph. The interesting part is he wrote in detailed about the shifting method.

> *Sketch the graph of f(x) by moving the graph of g(x) upwards by the value of 3/2 where (0.-1/2) becomes (0,1), (2, 3/2) becomes (2,3) and (-2,3/2) becomes (-2,3).*

He used the three coordinate points of $g(x)$ to demonstrate a vertical shifting that produced the three new coordinate points. He was aware that only the y-coordinates will undergo the vertical movement. Nevertheless, he skipped the verification phase and failed to check the accuracy of his work. He made a few carelesss mistakes when he implemented the computation sections, otherwise he would have obtained the correct answer. Table 5 showed his problem solving actions when he attempted the three problem tasks. There was a decline in number of problem solving actions especially at the understanding phase when he attempted problem task 2 and 3.

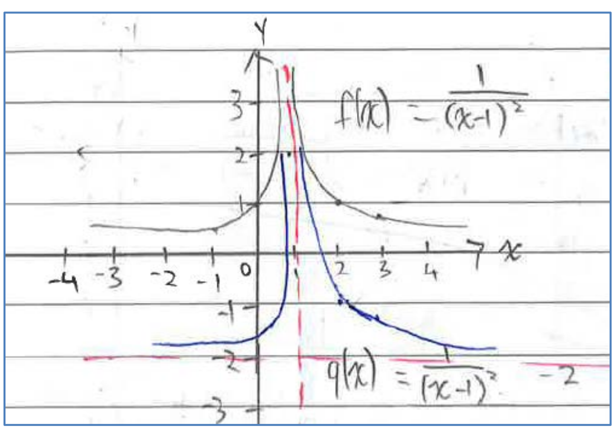
Table 5

Problem Solving Actions taken by Brandon when attempting the Mathematical Writing Practical Worksheet One

	Brandon's Problem Solving actions to each Phases			
	Understanding	Planning	Performing the plan	Confirming the answers
Problem task 1	3	2	2	0
Problem task 2	2	1	1	0
Problem task 3	0	1	1	0

The first writing activity seemed to enlighten his skilll in shifting technique where he explored the mechanism of shifting techniques through coordinate points translation. This can be showed in his subsequent problem solving for problem task 2, as he wrote

Let f(x)=$\frac{1}{(x-1)^2}$ *and we can obtain the graph of g(x) by moving the graph downward by 2 units.*



When he attempted the problem task 3, again his work showed a purely cognitive action that only involved standard calculation work and sketching the graph. He only wrote briefly about his plan i.e. "*can obtain h(x) by drawing* $g(x) = \sqrt[3]{x+1}$ *and move upwards by one unit.* Although his work was entirely presenting his computation techniques, he managed to obtain the correct answers. It is kind of interesting to note that he seemed to learn through the first writing task where he discovered his own way of solving the problem that he continuously applied to the other two problem tasks.

Communicating with Brandon about his learning experience is another channel to explore the impact of mathematical writing on his mathematical problem solving skills. He performed excellently in his mathematics subjects at the Malaysian Certificate of Education where he scored a A+ grade and A grade for his modern mathematics and additional mathematics respectively. His experience in learning mathematics during his secondary school i.e. for the past eleven years, is the same as Amelie i.e. drill and practice method. He was never exposed to any new learning strategies until he participated in this research. Initially, he felt uncomfortable with mathematical writing as he claimed *"I find it troublesome to write"*, which is because he learned mathematics through practice without thinking. His learning behavior is molded since young as he stated: *"Teacher asked us do a lot of practices"*. In other words, his belief about mathematical problem solving was circulated around numbers and formulas but not words. Nevertheless, his exposure to mathematical writing has somehow impacted him in a positive way when he shared his experience about mathematical writing:

> *It is actually much better because you do the questions more carefully and you understand the questions more. It is better so you won't skip any important details.*

His remark showed his awareness about his problem solving process during writing, as he continued:

> *I felt planning is the most difficult to write. Sometimes I understand the questions but I don't know how to use which formula or how to solve it. When I do the writing, it makes me a little bit more confused.*

That showed that he is engaged cognitively in his learning and is slowly driven away from his habitual practices i.e. doing without thinking. He learned how to organize his thinking in a systematic way as he said

> *Because when you do the writing you break down into parts, you understand more clearly when you do questions like that in the future, you won't get jumble up your mind with the questions you know, you do the questions more systematically*

Nevertheless, although he felt positively about the mathematical writing but this did not change his way of approaching a mathematical problem during the test as he said

> *No, not really. That is because previously i didn't really do all these writing type, used to straight away do it. Maybe I used to look at the questions and directly write down the answers*

This showed that the learning culture at school have somewhat engendered his perception about mathematical problem solving that created his aversion in mathematical writing.

Both the students came from the same learning environment i.e. teacher-centered approach where they were rote learners and with intensive guidance from teachers. Nevertheless, regardless of their differences level in the mathematical writing responses, the writing mechanism has assisted these two students to think critically about their own problem solving.

The Challenges of Mathematical Writing

Writing seems to be a difficult task for these Foundation in Engineering students especially to express ideas in language. Some students show an unenthusiastic attitude toward writing activities which were noticeable in their mathematical writing workbook. The students' indolent behaviour in writing might have been due to their previous mathematical learning experiences at school. The examination orientated nature of the curriculum in Malaysia education somehow or another groom teachers and students to focus only on contents and skills that are evaluated in the public examination (Lim, 2010; Salleh & Aziz, 2012). Students prefer to "regurgitate information" according to the examination format and disregard intentionally what is being put down in their answers (Nordin, 2009, as cited in Lim, 2010). In other words, students simply memorize the method without understanding the concepts and perform the solving steps by rote. With a minimum of eleven years of exposure to rote learning environment at school, the students have developed strong memorization mindset that they constantly apply this technique to solve any type of mathematics problems. As a result, some of them refused to perform beyond their comfort zone that they participated passively in the mathematical writing.

The learning culture at school may also have engendered the students' perception about solving a mathematical problem that create another possible factor on students' engagement in mathematical writing. According to Callejo and Vija (2009, as cited in Stylianides & Stylianides, 2011), majority of the students believe that mathematical problem solving circulates around numbers and formulas, not words. In other words, using formula or number is the only problem solving strategy that students learn and use to find answers to a mathematics problem. This can be seen in Ahmad, Salim and Zainuddin (2004) research work where they surveyed on students' methods and strategies of solving word problem that involved fraction. Their findings showed that many students did not write the steps of working properly and disregarded the use of correct mathematical syntax and grammar. Hence, students seem to have the impression that "doing mathematics" requires an accessible calculation method that only involves formulas and numbers. Another possible issue is students' writing practices only take place during language lessons at school. Several researchers on language studies claimed that most students struggled to develop their writing skills due to their inability to think critically and thus, most students perceived that writing was difficult (Bakar, Awal & Jalaludin, 2011; Shah, Mahmud & Din, 2011). Owing to their insufficient writing experience at school, students may find it difficult to demonstrate their understanding in a written form.

Nevertheless, the result from the study provides some useful implications of mathematical writing. One significant implication is that the act of mathematical writing eventually assisted some students to engage in their problem solving process i.e. experienced the transformation of thoughts and ideas about mathematics into visible mathematical expression. Even though some students felt mathematical writing was inconvenient but yet they generated new knowledge and skills during the process of writing. In a way, when they started to write, the act of writing stimulated their thinking process and somehow assisted them to develop the ability to understand.

Conclusion

Problem solving can be described as a journey of self-discovery that involve various of internal information processing behaviours in order to find possible way of getting the ultimate solution. Nevertheless, problem solving is a learning experience that require instruction and practise to trigger the activation of one cognitive process (Lester, 1987). This study reveals the potential of mathematical writng approach to problem solving which allows students to experience in each mathematical problem solving process and activate their mind to plan, organize, execute and reflect. Beside the great improvement of the students‘ performance in the formal test, the mechanisme has assisted the students to think critically about their own problem solving such as Amelie who is able to consolidate her skills at each problem solving phase through mathematical writing. In the case of Brandon, his initial efforts of writing has somehow driven him to perceive an effective approach in the graph shiffing and he has confidently applied it to subsequent problem tasks without further communicating his problem solving actions through writing.

As a whole, mathematical writing is a conscious and interactive process that allows the transmission of knowledge obtained through action. The process of introducing mathematical writing in mathematical problem solving is rather challenging because it takes time for students to move away from rote learning behaviour and getting them to understanding the essential of mathematical writing. However, once a student starts to write, he/she starts to think. It no longer allows the students' mind to be stagnated at the dimension of recalling and remembering but increase their mental power to higher forms of thinking about the problem solving. With the effort to write in place, students feel the impact of a series of cognitive processes that stimulate them to think and take strategic actions or plans in order to compose a good solution.

Reference

Ahmad, A., Salim, S. S. & Zainuddin, R. (2004). A study on students' performance in solving word problems, involving fractions: toward the development of a cognitive tool, *Proceedings of International Conference of Computers in Education (ICCE)*, 833-838.

Alamargot, D. and Chanquoy, L. (2001). General introduction: A definition of writing and a presentation of the main models. *Studies in writing: Through the models of writing* (pp 1-29). Netherlands, Kluwer Academic Publishers.

Bakar, N. A., Awal, N. M., & Jalaluddin, N. H. (2011). Investigating Malay language writing proficiency level among upper secondary school students. *GEMA: Online Journal of Language Studies*, 11(2), 39-51.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, 373-397.

Bell, E. S., & Bell, R. N. (1985). Writing and mathematical problem solving: Arguments in favor of synthesis. *School Science and Mathematics*, *85*(3), 210-221.

Berkenkotter, C. (1982). Writing and problem solving. *Language connection: Writing and reading across the curriculum*. Retrieved from: https://wac.colostate.edu/books/language_connections/chapter3.pdf

Bean, J. C., & Peterson, D. (1998). Grading classroom participation. *New directions for teaching and learning*, *1998(74)*, 33-40.

Carson, J. (2007). A Problem with problem solving: Teaching thinking without teaching knowledge. *The Mathematics Educator*, *17*(2), 7-14.

Clement, J. & Konold, C. (1989). Fostering basic problem-solving skills in mathematics. *For the Learning of Mathematics*, *9*(3), 26-30.

Cohen, J. A., Casa, T. M., Miller, H. C., & Firmender, J. M. (2015). Characteristics of second graders' mathematical writing. *School Science and Mathematics*, *115*(7), 344-355.

Dougherty, B. C., & Fantaske, P. (1996). Defining expectations for problem-solving skills. *New Directions for Higher Education*, *1996*(96), 55-66.

Flower, L. S., & Hayes, J. R. (1977). Problem-solving strategies and the writing process. *College English*, *39*(4), 449-461.

Flower, L., & Hayes, J. R. (1981). A cognitive process theory of writing. *College composition and communication*, 365-387.

Freitag, M. (1997). Reading and writing in the mathematics classroom. *The Mathematics Educator*, *8*(1), 16-21.

Fung M. G. (2010). Writing in a mathematics class? A quick report on classroom practices at the collegiate level. *Currents in Teaching and Learning, Teaching Report*, *2*(2), 22-34.

Ghazali, N. H. C. M. (2017). The implementation of School-Based Assessment System in Malaysia: A study of teacher perceptions. *Geografia-Malaysian Journal of Society and Space*, *12*(9).

Glogger, I., Holzäpfel, L., Schwonke, R., Nückles, M., & Renkl, A. (2009). Activation of learning strategies in writing learning journals: The specificity of prompts matters. *Zeitschrift für pädagogische Psychologie*, *23*(2), 95-104.

Hensberry, K. K., & Jacobbe, T. (2012). The effects of Polya's heuristic and diary writing on children's problem solving. *Mathematics Education Research Journal*, *24*(1), 59-85.

Kramarski, B., & Mizrachi, N. (2006). Online discussion and self-regulated learning: Effects of instructional methods on mathematical literacy. *The Journal of Educational Research*, *99*(4), 218-231.

Kosko, K. W., & Zimmerman, B. S. (2015). Emergence of argument in children's mathematical writing. *Journal of Early Childhood Literacy*, *0*(0), 1-25. https://doi.org/10.1177/1468798417712065

Kosko, K. W. (2016). Writing in mathematics: A survey of K–12 teachers' reported frequency in the classroom. *School Science and Mathematics*, *116*(5), 276-285.

Lazakidou, G. & Retalis, S. (2010). Using computer supported collaborative learning strategies for helping students acquire self-regulated problem solving skills in mathematics. *Computers & Education*, 54, 3-13.

Lester, F. (1987). *Teaching mathematical problem solving*. Retrieved from: http://ncm.gu.se/pdf/namnaren/3243_88_3.pdf

Lester, F., Garofalo, J. & Kroll, D. L. (1989). *The role of metacognition in problem solving: A study of two grade seven classes. Final report.* Retrieved from:

http://files.eric.ed.gov/fulltext/ED314255.pdf

Lim, L., & Pugalee, D. K. (2004). Using journal writing to explore they communicate to learn mathematics and they learn to communicate mathematically. *Ontario Action Researcher*, *7*(2), 17-24.

Lim, C. S. (2010). Assessment in Malaysian School Mathematics: Issues and Concerns. In *Meeting of the APEC-Tsukuba International Conference, Japan*. Retrieved from: http://www.criced.tsukuba.ac.jp/math/apec/apec2009/doc/pdf_20-21/LimChapSam-paper.pdf

Lucas, J. F., Branca, N., Goldberg, D., Kantowski, M. G., Kellogg, H. & Smith, J. P. (1979). A process-sequence coding system for behavioural analysis of mathematical problem solving. In G. A. Goldin & C. E. McClintock(Eds), *Mathematics education report: Task variables in mathematical problem solving* (pp. 353–378). Columbus, Ohio: ERIC. Retrieved from: http://files.eric.ed.gov/fulltext/ED178366.pdf

Mayer, R. (2002). Mathematical problem solving. *Mathematical Cognition: A Volume in Current Perspectives on Cognition, Learning, and Instruction*, 69-92.

Martin, C. L. (2015). Writing as a tool to demonstrate mathematical understanding. *School Science and Mathematics*, *115*(6), 302-313.

Mcguire, J. (2001). What is problem solving? A review of theory, research and applications. *Criminal Behaviour and Mental Health*, 11, 210-235.

Ministry of Education Malaysia [MOE] (2003a). *Mathematics Syllabus for Integrated Curriculum for Primary School*. Curriculum Development Centre.

Ministry of Education Malaysia [MOE] (2003b). *Mathematics Syllabus for Integrated Curriculum for Secondary School*. Curriculum Development Centre.

Morgan, C. (1998). *Writing mathematically: The discourse of investigation* (Vol. 9). Psychology Press.

Parker Siburt, C. J., Bissell, A. N., & Macphail, R. A. (2011). Developing metacognitive and problem-solving skills through problem manipulation. *Journal of Chemical Education*, *88*(11), 1489-1495.

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton University Press.

Pugalee, D.K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics*, *101*(5), 236-245.

Pugalee, D. K. (2004). A comparison of verbal and written descriptions of students' problem solving processes. *Educational Studies in Mathematics*, *55*(1), 27-47.

Saleh, F. (2009). Problem solving schemes of secondary school mathematics teachers. *Colleccion Digital Eudoxus, Resolucion de Problemas*, 116-123.

Saleh, S., & Aziz, A. (2012). Teaching practices among secondary school teachers in Malaysia. *International Proceedings of Economics Development and Research*, *47*, 63-67. Retrieved from: http://www.ipedr.com/vol47/014-ICERI2012-S00023.pdf

Schoenfeld, A. H. (2016). Learning to think mathematically: problem Solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education*, *196*(2), 1-38.

Schoenfeld, A. H. (2007). What is mathematical proficiency and how can it be assessed? *Assessing Mathematical Proficiency*, *53*, 59-73.

Seo, B. I. (2015). Mathematical writing: What is it and how do we teach it? *Journal of Humanistic Mathematics*, *5*(2), 133-145.

Shah, P. M., Mahmud, W. H. W., Din, R., Yusof, A., & Pardi, K. M. (2011). Self-efficacy in the writing of Malaysian ESL learners. *World Applied Sciences Journal 15 (Innovation and Pedagogy for Lifelong Learning*, 8-11.

Stylianides, G. J., & Stylianides, A. J. (2011). An intervention of students' problem-solving beliefs. In *Proceedings of the 7th congress of the European Society for Research in Mathematics Education*, 1209-1218.

Tambychik, T., Meerah, T. S. M. & Aziz, Z. Mathematics skills difficulties: A mixture of intricacies. *Procedia Social and Behavioral Sciences*, *7*, 171-180.

Xenofontos, C. & Andrews, P.(2014). Defining mathematical problems and problem solving: prospective primary teachers' beliefs in Cyprus and England. *Mathematical Education Research Journal*, *26,* 279-299. https://doi.org/10.1007/s13394-013-0098-z

Yeo, J. B. (2007). Mathematical tasks: Clarification, classification and choice of suitable tasks for different types of learning and assessment. *Mathematics and Mathematics Education technical report series.* Retrieved from https://repository.nie.edu.sg/bitstream/10497/949/3/MathematicalTasks.pdf

Zanzali, N. A. A. (2000). *Designing the mathematics curriculum in Malaysia: Making mathematics more meaningful*. Universiti Teknologi Malaysia. Retrieved from: http://math.unipa.it/~grim/Jzanzali.PDF

Zawojewski J. (2010). Problem solving versus modeling. In Lesh R., Galbraith P., Haines C. & Hurford A. (eds), *Modeling Students' Mathematical Modeling Competencies* (pp. 237-243). Springer, Boston, MA.

Zollman A. (2010). Commentary 2 on Problem Solving for the 21st Century. In Sriraman B. & English L. (eds). *Theories of Mathematics Education. Advances in Mathematics Education* (pp. 297-301). Springer, Berlin, Heidelberg.

Appendix A

Coding scheme based on a Set of Guided steps of the Polya's Problem Solving Model

Phases	Guided steps	Actions
Understanding	• Can you state the problem in your own words? • What are you trying to find or do? • What are the unknowns? • What information do you obtain from the problem?	Making sense of the problems; Identify key ideas; Highlight key terms; Introduce suitable notation i.e. give names/symbol to object; Relating it to a certain mathematics domain – defining/drawing a visual representation of the problem
Planning	• Break the problem down into smaller parts; • Use some strategies such as: make a table; make a diagram; write an equation; etc. • Identify sub-goal.	Organizing data into smaller sections; Indicating appropriate sub-goals; Making informed decisions about strategies such as make a diagram, write an equation
Performing the plan	• Implement the strategy in step II and perform the necessary mathematics computation; • Check each step of the plan as you do it; • Keep an accurate record of your work; • Organize your work into easy to understand visuals	Implementing strategies according to planning; Work is organized and easy to understand; Performing tasks such as doing calculation, solving equation.
Confirming the answers	• Check the results in the original problem; • Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable? • If possible, determine whether there is another method of finding the solution.	Apply other methods to verify the answers; Reflecting on the answers.