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Billy Jackson

Shandy Hauk

Jenq Jong Tsay

Alma Ramirez

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Professional Development for Mathematics Teacher Educators: Need and Design

Billy Jackson
University of Louisville, USA

Shandy Hauk
San Francisco State University, USA

Jenq Jong Tsay
University of Texas Rio Grande Valley, USA

Alma Ramirez
WestEd, USA

Abstract: *The purpose of this report is to share a conceptual model useful in the design of professional learning about teaching for university mathematics faculty. The model is illustrated by examples from a particular design effort: the development of an online short-course for faculty new to teaching mathematics courses for prospective primary school teachers. How novice mathematics teacher educators grow as instructors is an emerging area of research and development in the United States. At the same time, it is well established that effective instructional design of any course, including a course for faculty, requires breadth first: understanding and anticipating the needs of the learner. Therefore, given the sparse knowledge base in the new arena of mathematics teacher educator professional growth, effective design requires leveraging the scant existing research while also exploring and iteratively refining broad goals and objectives for faculty learning. Only after a conceptual foundation is articulated for what is to be learned and what will constitute evidence of learning, can cycles of design productively examine and test-bed particular course features such as lesson content, structures (like scope and sequence), and processes (like communication and evaluation). In the example used in this report, several research-based perspectives on learning in/for/about teaching guided design goals and short-course objectives. These valued perspectives informed creation and prioritization of principles for short-course design which, in turn, informed evaluation of faculty learning. With these conceptual foundations in place, design of lessons to realize the goals, principles, and objectives rapidly followed. The work reported here contributes to the knowledge base in two ways: (1) it addresses faculty professional development directly by describing and illustrating a model for supporting instructional improvement and (2) it provides meta-narrative to scaffold the professional growth of those who design professional learning opportunities for post-secondary mathematics faculty.*

Keywords: professional learning about teaching, post-secondary faculty, equitable instruction, mathematical knowledge for teaching, mathematical knowledge for teaching future teachers

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Introduction

Many mathematics faculty members in North American universities are fluent in more than one natural language (e.g., English, Mandarin, Russian, Arabic, Spanish, French) as well as one or more dialects of advanced mathematics. These are also people who value the Western academic cultural norms of the transmission and product models for college instruction (Davis, Hauk, & Latiolais, 2009; Hora & Ferrare, 2013). Place a person with these multiple fluencies, views, and areas of expertise in a room with 20 undergraduates whose life goal is to become a primary school teacher and tell the instructor: "Teach them math." Three words: Teach. Them. Math. Each word has a cacophony of meaning. The layers of meaning are large in number and the likelihood of shared definitions for "teach," "them," and "math" are small. What does it mean to teach? What distinguishes "them" from "me" or "us" (if anything)? And which mathematics does "math" mean? And, with what depth and breadth and connectedness to other mathematics?

Research and policy have addressed these questions, particularly in the preparation of future teachers (e.g., Association of Mathematics Teacher Educators [AMTE], 2017; Bakhtin, 1981; Conference Board of the Mathematical Sciences, 2012; Daniels, 2001; Gutiérrez, 2009; Halliday, 2003; Hauk Toney, Jackson, Nair, & Tsay, 2014). Explicit in them is guidance for what "Teach them math" might mean, along with a clear call for research and development of professional learning among college instructors about teaching future teachers (Castro Superfine and Li, 2014a; Konuk, 2018; Zaslavsky and Leiten, 2004).

Mathematics faculty who do not have much experience in teacher education may not know about the “cognitive and epistemological subtleties of elementary mathematics instruction.” (Bass, 2005, p. 419). These same faculty often struggle in teaching mathematics that is relevant and useful to prospective teachers (Flahive & Kasman, 2013). Yet, more than 75% of all 2- and 4-year post-secondary institutions in North America offer mathematics courses for prospective elementary school teachers (i.e., those who are studying to obtain certification to teach pupils of ages 4 to 14), with almost 90% of all U.S. institutions offering such courses within mathematics departments (Masingila, Olanoff, & Kwaka, 2012). Indeed, researchers have noted that mathematics faculty seek professional support for the work of teaching prospective teachers (Greenberg & Walsh, 2008; Masingila et al., 2012). The instructor pool for such courses is varied in the U.S. It includes both tenured/tenure track and non-tenure track faculty (e.g., contingent faculty, lecturers, adjunct instructors, and, in some places, graduate students) and fewer than half of those who teach mathematics courses for prospective teachers have any primary or secondary teaching experience themselves (Masingila et al., 2012).

This report is a response to the calls in the literature for details about the design and use of professional development for college mathematics instructors who teach prospective teachers (AMTE, 2017; Castro Superfine and Li, 2014a; Smith, 2003; Zaslavsky and Leiten, 2004). In particular, the authors share what we have learned from recent experiences in designing and piloting online professional learning experiences for mathematics faculty in the U.S. and Canada. The *Professional Resources and Inquiry into Mathematics Education (PRIMED) for K-8 Teacher Education* project is a grant-funded effort to develop and research the impact of a short-course to support mathematics faculty to build their *mathematical knowledge for teaching future teachers* (more on this below).

In general, the term *mathematics teacher educator* (MTE) describes someone who provides guidance, mentoring, or learning opportunities to prospective or in-service teachers at any grade or level including primary, secondary, or tertiary (i.e., up to and through university). In this report, MTE refers specifically to a subset of the larger group: those who have an advanced degree in mathematics and work in post-secondary mathematics departments.

The existing literature on professional learning design for faculty in mathematics departments who teach prospective teachers is limited (e.g., see the recent literature review of professional learning by teacher educators, across all disciplines, by Ping, Schellings, & Beijaard, 2018; Stylianides & Stylianides, 2014). Given the current sparsity in the research literature on the needs of mathematics faculty who are becoming mathematics teacher educators, a definitive guide to the construction of their professional development is not possible. Thus, this report offers description and illustration of the use of a principled approach to such construction.

The work reported here contributes to the knowledge base in two ways. First, it addresses MTE professional development directly by describing and illustrating a model for supporting post-secondary instructional improvement. Second, it provides meta-narrative to scaffold the professional growth of those who design professional development for MTEs.

It is well established that in a new arena (e.g., the professional growth of MTEs), effective instructional design requires breadth first: significant dwell time on exploring and iteratively refining goals and objectives (Anderson, 1983; Perez, Johnson, & Emery, 1995; York & Ertmer, 2016). Then, cycles of design examine and test-bed particular features of course depth: lesson content, structures (like scope and sequence), and processes (like communication and evaluation).

For those new to designing professional learning for MTEs, building design expertise requires a conceptual model, "a framework ...initially that would lead them through a series of questions pertaining to front-end analysis" (Perez et al., p. 345). Broadly, a conceptual model is a representation of the relationships among ideas abstracted or generalized from human experience and its purpose it to communicate fundamental principles and functions of a system in ways that support understanding of the system. A conceptual model is an anchor for conversation by designers and a point of reference for future efforts. Concept foundations are important in mathematics education development and research at all levels:

> Whether it is tacit or explicit, one's conceptual model of a situation, including one's view of what counts as a relevant variable in that situation, shapes data-gathering – and it shapes the nature of the conclusions that can be drawn from the data that are gathered. Whether and how those factors are taken into account in formulating a study and gathering data for it will shape how that study's findings can be interpreted (Schoenfeld, 2007, p. 71-72).

Hence, what one might be able to conclude about the effects of a particular effort, like a professional short-course, depends *a priori* on the conceptual framework underpinning what is valued and measured about the course and, *a posteriori*, on what types of data will constitute evidence of the effect. Thus, for the system that is professional learning about teaching by faculty, a conceptual model must include not only targets of instruction (e.g., particular lesson content, structure, or processes) but also goals and measurable objectives. This assertion will be familiar to those steeped in educational theory but may be new to others. What follows aims to assist designers of faculty professional learning in considering their own conceptual models, in part by making explicit the authors' design choices and reasoning for those choices.

Section 2 gives a summary of the framework for building a conceptual model of instructional design; it includes example actions taken in the PRIMED project. With this orientation to the concept foundations and PRIMED example in hand, Section 3 examines the "breadth first" imperative, with details of theory, research, and pragmatic experience that informed conceptual model development. Section 4 moves into the "depth" of the course by telling the story of the major components of a conceptual model for course design and development -- goals, objectives, structures, and processes -- in the context of the PRIMED effort. In particular, these components were refined through a cycle of attention to *values*, to carefully articulated *principles* for professional learning, and to what constitutes evidence of impact on the instructional *practices* of those who complete the short course. Finally, Section 5 describes the landscape of next steps in design and development of professional curriculum and instruction for mathematics faculty who teach prospective teachers.

Model Overview

Engaging in the construction of conceptual framing for curriculum design is a nonlinear, cyclic, and iterative process (Perez, Johnson, & Emery, 1995). As an advanced organizer, actions from the PRIMED model accompany the framework summarized in Table 1. Notice that the table is organized by key design components. The components are in the categories commonly found in any development effort: goals, objectives, structures, processes (York & Ertmer, 2016).

Table 1.

Framework for Building a Conceptual Model for Design of Faculty Learning

Component	Questions	Actions
Goals: Overarching	What do designers want their MTEs to learn? Why those things?	• Make a list of goals. • Prioritize the list.
Goals: Values	What do designers take as foundational values in the work of faculty professional learning?	• Make a list of values assertions. • Prioritize the list.
Goals: Principles	What principles for course design are called for given the high priority values assertions and the targets for program content, structures, and instructional approaches?	• Generate a list of principles. • Continue to refine and link to evaluation goals.
Objective: Practices	What will mathematics college instructors be able to do in their own classrooms as a result of the particular opportunities in the professional learning program?	• Make a list of target classroom practices. • Brainstorm what needs to be in the program to support the development of those practices.
Objective: Content	What will faculty learn in the program that will allow them to attempt the target practices?	• Make a list of target content. • Brainstorm what emphasis for each target is needed (and why).
Structure: Contexts	What contexts and aims intersect? How will the program offer learning opportunities to faculty related to these intersecting aims?	• Make a list of target structures (for an example, see Castro Superfine & Li, 2014a).
Processes: Program Approaches	How will the program facilitate the learning of faculty using the given structures?	• Make a list of instructional approaches for the program (for an example, see Castro Superfine & Li, 2014a).
Processes: Program Evaluation	What does evidence of faculty learning of content and productive adaptation of target practices look like? How can it be captured in order to measure progress in faculty development?	• Generate a list of practices and other forms of evidence that professional learning goals have been achieved. • State how each can be measured

Theoretical Perspectives Informing the Model

Mathematical Knowledge for Teaching (MKT) – Which "Math" to Teach

The types of mathematical knowledge required of prospective teachers for their future work have emerged from several decades of research rooted in Lee Shulman's (1986) efforts. There are particular understandings and skills associated with effective instruction, a sociological synergy of mathematics and mathematics education called *mathematical knowledge for teaching* (MKT; Ball, Thames, & Phelps, 2008). MKT for elementary grades as modeled by Ball and colleagues is made up of six kinds of knowledge.

Three of the components of MKT are types of *subject matter knowledge* for teaching. *Horizon content knowledge* is about how topics are related across the span of curriculum. *Common content knowledge* is used in everyday activities by teachers and others, including mathematicians, engineers, and homemakers. *Specialized content knowledge* is specialized in the sense that it is mathematics specific to the task of teaching.

Specifically, specialized content knowledge includes ways to represent mathematical ideas, provide mathematical explanations for rules and procedures, and examine and understand innovative solution strategies. Specialized knowledge for teaching primary grades is sparse or absent for many with advanced mathematics expertise but little school teaching experience (e.g., mathematics professors; Bass, 2005). As an example, consider fraction division. Most mathematics faculty can readily use the invert-and-multiply algorithm to divide fractions. Thus, this piece of knowledge is *common content*. Yet, absent concentrated effort and considerable time, few mathematicians can explain in a way meaningful to a 10-year-old *why* the algorithm is justified in some problem situations and not in others, thereby making knowing the grade-level-appropriate "whys" *specialized*. Lest the reader be skeptical of such a claim, consider the "expert blind spot" reported by Gros, Sander, and Thibaut (2019, p. 5). They found that one out of every four mathematicians surveyed could not solve a subtraction problem commonly given to many 10-year-olds that required thinking of objects as referents in sets (many incorrectly told interviewers that the problem had no solution).

The other three categories in MKT are types of *pedagogical content knowledge* (PCK) and are neither purely pedagogical nor exclusively mathematical. *Knowledge of curriculum* includes awareness of the content and connections across standards and texts (i.e., of the intended curriculum; Herbel-Eisenmann, 2007). *Knowledge of content and students* is "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (Hill, Ball, & Schilling, 2008, p. 375). *Knowledge of content and teaching* is about teaching actions or moves, such as productive ways to respond in-the-moment to students to support learning. Consider a fraction example: teachers who are aware that students often invert the dividend instead of the divisor are demonstrating knowledge of content and students and, if they have appropriate knowledge of content and teaching, might use fraction diagrams to scaffold understanding.

All six of the components of MKT are situated in a seventh kind of knowing called *knowledge of discourses*. This kind of knowing is about the various ways of communicating about mathematics that happen in classrooms among students, across students and teachers and others, among teachers, and across teachers and others outside of the classroom (Hauk, Toney, Jackson, Nair, & Tsay, 2014).

Mathematical Knowledge for Teaching Future Teachers (MKT-FT)

Like the MKT used by school teachers, there is a related idea at the tertiary level for teaching mathematics to prospective teachers: *mathematical knowledge for teaching future* *teachers* (MKT-FT; Hauk, Jackson, & Tsay, 2017). A rich and textured MKT-FT is especially vital in the inquiry-oriented and activity-based approaches to post-secondary teaching shown to improve student learning, increase persistence, and reduce inequities (Bressoud, Mesa & Rasmussen, 2015; Freeman et al., 2014; Holdren & Lander, 2012; Laursen, Hassi, Kogan, & Weston, 2014). Instructors acquire MKT-FT in many ways: grading, examining their own learning, observing and interacting with students or colleagues, reflecting on and discussing their own practice *and* the practices known to be effective in teaching (Kung, 2010; Speer & Hald, 2009; Speer & Wagner, 2009; Yackel, Underwood, & Elias, 2007).

The model of MKT summarized in §3.1 is well suited for the primary and elementary setting but has limited generalizability beyond middle school grades (Speer, King, & Howell, 2015). In the MKT model, the “content knowledge” in "specialized content knowledge" and "pedagogical content knowledge" is *mathematics*. However, in MKT-FT, the “content knowledge” includes both MKT itself as well as mathematics not necessarily included in the MKT of primary and elementary teachers (such as algebraic structures like groups and rings).

To be clear, an asterisk (*) is used below to indicate when a term is referring to MKT-FT of a mathematics teacher educator rather than the MKT of a primary school teacher:

*Subject matter knowledge** in MKT-FT includes a compendium of common, specialized, and horizon knowledge of mathematics *and* of MKT. Note that subject matter knowledge* is distinct from knowledge of prospective teachers as learners (which is included in MKT-FT pedagogical content knowledge*).

*Common content knowledge** is the body of mathematical knowledge and mathematical knowledge for teaching that is *shared* between pre- and in-service teachers *and* those who teach them. Olanoff (2011) reported that Ball herself has noted that the common content knowledge* in MKT-FT includes MKT itself.

*Specialized content knowledge** for those who teach teachers is specific to the professional work of mathematics teacher educators and embodies those non-pedagogical aspects of MKT-FT that are required in the teaching of MKT. This includes (perhaps implicit) knowledge of educational theory and K-12 practice. Smith (2005) argues these are necessary to prepare prospective teachers to engage with the multitudinous curricula they will encounter as teachers.

*Horizon content knowledge** includes an awareness of historical and current trends in local, state, and national policy. In the U.S. this would include the standards for teacher preparation (AMTE, 2017) and the Common Core State Standards (National Governors Association Center for Best Practices, 2010).

Notice that the above descriptions are expansive from MKT yet also contain MKT. Similar relationships exist for the MKT-FT components of *pedagogical content knowledge** which involve knowledge of mathematics *and* MKT *and* the teaching of each of these. Figure 2 is an attempt to represent the self-similar structure of pedagogical content knowledge*. Like fractals, the self-similar structure refers to the embedded nature of MKT within MKT-FT: every piece of MKT pedagogical content knowledge maps in a four-to-one way to pedagogical content knowledge* for working with prospective teachers. For example, Figure 1 makes visible that a mathematics teacher educator's *knowledge of content and students** includes knowing what prospective teachers understand about PCK as well as what prospective teachers know (in the context of working with children) about discourse, curriculum, content and students, and content and teaching.

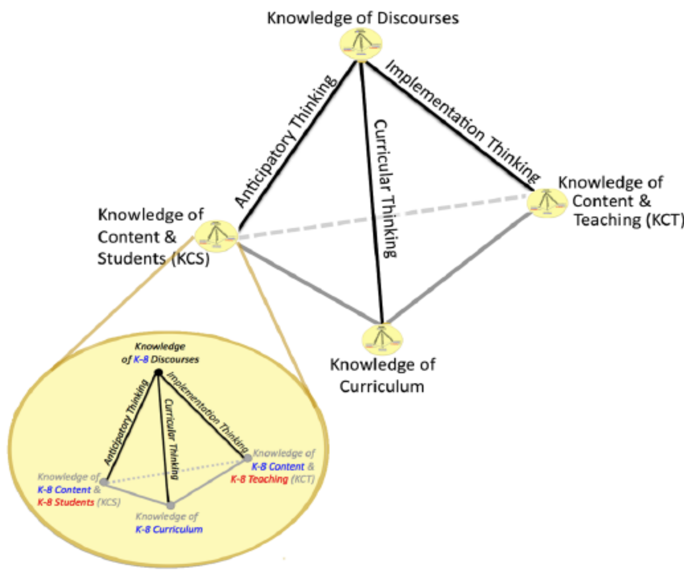


Figure 1. Illustration of the self-similar nature of *pedagogical content knowledge** in MKT-FT (from Hauk et al., 2017).

In MKT-FT, *knowledge of content and students** includes awareness of learners as adults (as opposed to the children inherent in MKT) and responsiveness to the ways in which prospective teachers create, use, and interact with both mathematical ideas and MKT, as well as conceptions they hold about MKT, mistakes they make, and their beliefs about the nature of mathematics and MKT in general (Zopf, 2010; Sztajn, Ball & McMahon, 2006). That is, for mathematics teacher educators, it includes knowing and being able to anticipate the needs of prospective teachers as learners of MKT.

*Knowledge of content and teaching** is a knowledge of what to do in response to the situations that arise in the post-secondary classroom with adult learners who have well-established routines for interacting with mathematics. It also includes how those are similar to and different from the MKT needed when teaching children who are first establishing their mathematical habits (Rider & Lynch-Davis, 2006; Smith, 2005, Wilson & Ball, 1996).

For MKT-FT, *knowledge of curriculum** includes awareness of how aspects of MKT are presented in common texts for the course. It also includes the ways mathematics is offered to the adult learners who are prospective teachers.

Finally, *knowledge of discourses** includes an awareness of, and responsiveness to, the differing communities and communication practices prospective teachers bring to class and are learning in other classes. In the U.S. this includes what prospective teachers experience in teacher education and teaching methods courses which occur in environments where professional cultural values often differ from those in a department of mathematics (e.g., in a school or department of education or curriculum and instruction).

MKT-FT depends on context, including discourse contexts and cross-cultural or "intercultural" sense-making. Growth of such knowledge hinges on unpacking classroom, mathematical, professional, and personal discourse and connecting it to the other aspects of pedagogical content knowledge in Figure 1. It is worth noting that our model of MKT-FT presumes a highly nonlinear interaction among all subcomponents. For example, researchers have long known that task design and revision is certainly a part of MKT-FT (Jeppsen, 2010; Olanoff, 2011; Zaslavsky, Watson, & Mason, 2007). In fact, task design is a highly variable component of MKT-FT in that it requires an instructor to use multiple components of MKT-FT (i.e., aspects of pedagogical content knowledge* and subject matter knowledge*) simultaneously in order to produce a task that learners can complete successfully and from which prospective teachers can learn as intended.

Teaching for Robust Understanding - Noticing and Responding to "Them"

Teaching for Robust Understanding (TRU, Figure 2) is a research-based framework designed to attend to particular aspects of instruction in an effort to answer the following question (Schoenfeld, 2016):

> What are the attributes of equitable and robust learning environments in which all students are supported in becoming knowledgeable, flexible, and resourceful disciplinary thinkers?

The Mathematics	Cognitve Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which students grapple with and make sense of central mathematical ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with tasks that are not low level but not impossible. The level of cognitive demand should be conducive to what has been called productive struggle.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core content being addressed by the class. Classrooms in which a small number of students get most of the "air time" are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.*	*The extent to which students are provided opportunities to "walk the walk and talk the talk" – to contribute to conversations, to build on others' ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.*	*The extent to which classroom activities elicit student thinking and subsequent instruction responds to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen their understandings.*

Figure 2: Teaching for robust understanding (TRU) framework, Schoenfeld (2016).

The TRU framework is a powerful tool because it provides a working definition of *effective mathematics instruction* that includes attention to equity along with language for describing and measuring characteristics of classroom activity. Given the centrality of educational equity and inclusion in the PRIMED project, the TRU framework served a dual purpose: as a resource to guide MTEs in examining their own and their peers' responsiveness to prospective teachers in curricular and instructional choices, and as a resource for short-course design, development, implementation, and evaluation.

Martin (2012) underscores the importance of paying attention to issues of equity, access, and agency in content courses for prospective teachers, noting that presumptions by mathematics faculty about the primacy and sufficiency of mathematical skill are not a warrant for ignoring all other aspects of mathematical knowledge for equitable teaching:

> Despite the tensions, I am convinced that a focus on mathematics content knowledge alone is not in the best interest of the students or of the children they will teach. "We'll focus on the math, you'll get that other stuff in education" is insufficient. Such a compartmentalized approach to educating and developing elementary school teachers whose responsibility it will be to educate the whole child seems contradictory. *Moreover, there exist very few examples of highly skilled, human services, professional work where knowledge of those who are served and the knowledge needed to serve them are artificially separated* [emphasis added]**.** To the degree that math departments perpetuate such separation, they reinforce to preservice teachers the idea that teaching mathematics to children is mostly about teaching *mathematics* and less about teaching *children*. Yet, this is not confined to the preservice context, as my own experiences in mathematics departments have shown that some of the most gifted mathematicians are ineffective in teaching students, partly because they often lack deep understanding of who they teach. Hence content knowledge is necessary but not sufficient. (p. 19)

We agree with Martin. To properly serve the prospective teacher as a whole, it is necessary for MTEs in content courses to pay attention to both knowledge of the content and knowledge of the contexts in which that content knowledge will be used. In fact, as Hauk et al. (2014; 2017) and Felton-Koestler (2020) have pointed out, such knowledge is a part of MKT for school teachers, and thus is also part of a mathematics teacher educator's MKT-FT: both as common content knowledge* and as a part of pedagogical content knowledge*.

TRU serves as a learning tool for novice MTEs who may have unwittingly promoted inequities in their classrooms, as Martin suggests. TRU uses language that is accessible to the novice MTE with little to no previous exposure to the specialized professional terminologies of education and equity.

Another advantage of the TRU framework is that post-secondary classes for prospective teachers serve as models of instruction. College classrooms in which equity, access, and agency are central themes can have a significant impact in helping to ensure that learners' future primary and elementary classrooms attend to these issues. Such attention in classes for prospective teachers goes a long way in ensuring that the AMTE's (2017) recent *Standards for Preparing Teachers of Mathematics* are being fulfilled. Thus, the TRU framework was a structural cornerstone of the PRIMED short-course. It supports novice mathematics teacher educators in becoming self-aware about, and attending to, these issues in their own courses. When MTEs also introduce TRU to their prospective teachers, prospective teachers have a tool for thinking critically about these issues as well.

Constructing a Conceptual Model for the PRIMED Short-Course

This section explains how the PRIMED design team engaged with the framework described in Table 1. It includes critical considerations of breadth (values, goals, principles) and illustrations from the depth of course content, structure, and process components.

Create (and Refine) the Goals – Define "Teach"

When first proposed, the outline for the short-course was based on both (a) the literature and (b) our own experiences of the things we – as faculty working in mathematics departments – wished we had known or had available to us when we first started teaching mathematics courses for prospective teachers. In early experiences of teaching mathematics to prospective teachers, each of us had asked ourselves some version of these driving questions:

Question 1: What is it that is supposed to be taught in these content courses? It *must* be more than teaching primary and elementary mathematics to adults! What (types of) mathematics do college students who will be school teachers need to know in order to teach mathematics to children, facilitate active learning by children, and choose and revise tasks, curricula, and assessments effectively?

Question 2: How does one take what is learned from the actual teaching of prospective teachers and use it to respond to their needs and revise instruction appropriately?

Given these two driving questions, the research literature most pertinent to conceptual model development was the knowledge base already reviewed in Section 2: mathematical knowledge for teaching (Question 1) along with mathematical knowledge for teaching future teachers and instructional noticing and responding using the TRU framework (Question 2). A more in-depth view of Question 1 is provided by Zhang, Brown, Joseph, and He (2020).

Articulate Values and Refine Principles

Here, "values" are the designers' judgements, based on research and experience, about what is important when teaching courses for prospective teachers. The values are used in determining and prioritizing the guiding principles for design. Then, from the instructional targets embodied in the principles, measurable objectives of learning are identified (i.e., the observable practices the participating MTEs will engage in if the target principles are achieved). The three steps of articulating values, determining target principles, and identifying evidence in MTE practice are all needed to take design from goals to measurable objectives (see Figure 3)

Designer Values

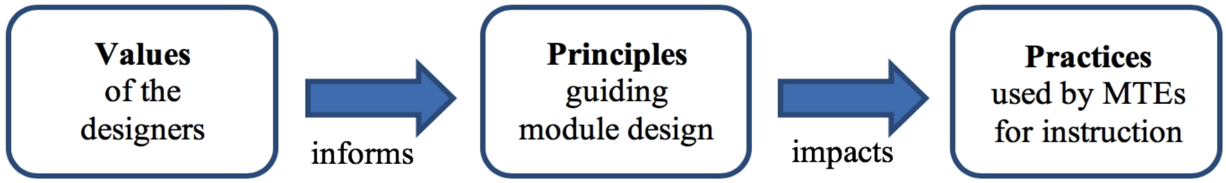


Figure 3. Pathway from values, to principles, to short-course measurable objectives (in the case of PRIMED, the objectives were MTE instructional practices).

The fundamental values behind the short-course were rooted in the research and frameworks discussed in Section 3 (MKT, MKT-FT, and TRU). The description of each valued idea is accompanied by examples of warrants from related research.

Valued Idea 1: Mathematics teacher educators need to know something about the special mathematics knowledge, beyond common mathematics knowledge, that is the "math" involved in "Teach them math" when "them" are prospective teachers. (Castro Superfine and Li, 2014b; Olanoff, 2011; Zopf, 2010)

Valued Idea 2: Noticing and using the relationships among mathematical ideas, mathematical knowledge for teaching children, and mathematical knowledge for teaching prospective teachers is essential for MTEs to have opportunities to learn both how different people do mathematics and how to unpack that mathematics for teaching it. (Bergsten and Grevholm, 2008; Castro Superfine and Li, 2014b; Martin, 2012; Thames, 2008)

Valued Idea 3: The goal of teaching mathematics to prospective teachers is twofold: prospective teachers leave the course equipped to do the mathematics their future students will do *and* are prepared to anticipate and respond to their students' thinking to facilitate connected mathematical learning (Appova and Taylor, 2019; Li and Castro Superfine, 2018; Taylor and Appova, 2015).

Valued Idea 4: What counts as mathematics is shaped by MTE values and beliefs. Mathematics – from its concepts to its educational manifestation in curriculum and instruction – is *not* culturally neutral. Faculty need to notice how learners think and learn – either differently or the same as they themselves and/or other students do – in order to develop a sense of "other" from "self" and be aware of what differentiates *them* from *us* when figuring out how to "Teach them math." (Davis, et al., 2009; Felton-Koestler, in press; Gutiérrez, 2009; Martin, 2012)

Valued Idea 5: MTE knowledge for teaching prospective teachers depends on *context*, including discourse contexts. Effectively teaching prospective primary and elementary teachers requires attending to and orchestrating interactions among classroom, mathematical, professional, and personal language use and valued forms of communication (i.e., discourses). (Hauk et al., 2014, 2017; Jackson, Dimmel, & Mueller, 2016)

Valued Idea 6: Good teaching meets students where they are and offers them opportunities to learn and to demonstrate that learning. MTE eliciting of, responding to, and reflecting on how their prospective teachers are experiencing the course through formative assessment is crucial to MTEs professional development (Martin, 2012; Schoenfeld, 2014, 2016, 2019).

PRIMED Principles

With designer values set and a firmly established basis for each from the teacher education research and practice literatures, the next action in was to translate the values into instructional principles for short-course design. The principles, in turn, set the tone for the selection and revision of short-course content. The six principles follow, along with illustrations of how they are explained and justified to the MTEs in the short course.

Principle 1: Knowledge of MKT is essential for teaching prospective teachers. Essentially, MKT is, itself, the core of the common content knowledge* of experienced mathematics teacher educators. But, given the limited (or absent) elementary and/or secondary teaching experience of university mathematics faculty, many new to the teaching of prospective teachers do not have knowledge or experience of several components of MKT. It is vital that instructors have a working foundational exposure to MKT.

Principle 2: Mathematics, MKT and MKT-FT are interdependent. Noticing and using the relationships among mathematical ideas, mathematical knowledge for teaching children, and mathematical knowledge for teaching future teachers provides MTEs opportunities to learn. Vignette 1 gives an example of a short-course activity that uses the intertwining of mathematics, MKT, and MKT-FT.

Vignette 1 - Video-based Task for Noticing MKT and MKT-FT

Early in the short-course there is a video-based task where participants are prompted to reflect on specific questions (see below) while watching two short videos. Each participant watched two videos. The first is of the well-known teacher educator and textbook author Sybilla Beckmann teaching a course for pre-service teachers on number and operations as they consider the challenges of word problems and the phrase “4 times as much as.” The second video is of a 4th grade teacher named Mrs. Carr as her students work on the problem: “Maria saved $24. She saved 3 times as much as Wayne. How much did Wayne save?”

As you watch the videos, consider the following:

Prompt 1. What does the teacher do to promote students sharing their thinking about the task?

Prompt 2. How does the teacher use student productions to move the discourse forward?

Prompt 3. Beckmann is teaching future teachers. Carr is teaching 4th graders. In what ways does their “knowledge of student thinking” need to be different (given the difference in the two student populations)?

Earlier in Beckman’s class (before the first video starts), she has had her PSET students create word problems that include the language “4 times as much as” and that require division rather than multiplication to solve. In the short video, Beckmann uses “4 times as much as” to illustrate a common pitfall for elementary grades learners: use of a keywords strategy in solving word problems. In class before the second short video starts, Ms. Carr has drawn a diagram of 24 dots circled in groups of 3 on the board. The second video starts with Ms. Carr polling her 4th grade students about their thinking. She then asks a student to show his solution to the problem by using the diagram she drew. He writes 24 x 3 = 72 on the board. When she asks him to “show where that is in the picture” (i.e., the 8 groups of 3 she drew), he points to the statement of the problem and says 24 is there and the "times 3" comes from the statement “3 times as much as.”

This task was used in an initial pilot of the short-course during the Fall of 2017. Perhaps most interesting (for us as course designers) were MTE responses to Prompt 3. For example, one person noted that the difference is rooted in understanding “why do you think this” (what Carr said to her Grade 4 students) versus “why might a student think this” (what Beckmann said to her college students). Another participant gave the following response:

> This is such a good question. Beckmann needs to have some knowledge about both student populations... what will the 4th graders be thinking? so that she can help prepare her students [prospective teachers] to handle it as Carr does. How do we help our students to be able to come up with that diagram representing multiplication "in the moment" that they are dealing with student confusion?

Notice that the task calls for an awareness of the layers of MKT and MKT-FT: the third prompt directly addresses the self-similar nature of the two constructs. Both of the MTE responses above provide evidence that the MTEs are becoming aware of this self-similarity and are noticing how each of MKT and MKT-FT are used in practice (even if they do not formally know what MKT and MKT-FT are at this point, which they did not; it was discussed later).

Principle 3: Teaching mathematics to prospective elementary school teachers is more than teaching elementary school mathematics. Vignette 2 exemplifies two overlapping views common among mathematics faculty who teach prospective teachers (culled from our own research): (1) the goal of mathematics courses for prospective elementary school teachers is that the learner leaves equipped to *do the mathematics* that a child must learn to do (Professor Macy) and (2) there are two goals in teaching mathematics to prospective teachers: learners leave the course equipped to do the mathematics their future students will do *and* are prepared to anticipate and respond to students' doing of mathematics in order to facilitate its learning (Professor Jameson).

Vignette 2 – Teaching from 10,000 Feet

Jameson: What are you going to do for the fraction division assignments?

Macy: The odd ones, assign the odd numbers at the end of the chapter in the book.

Jameson: Oh. And how will it look on the test, testing the concept?

Macy: Concept? We're not talking about concepts! The question is whether these future teachers can do the math they will be teaching.

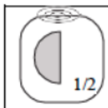
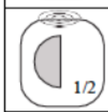
Jameson: I agree it is about that AND I think it is more than just whether they can DO it. I mean, my sister teaches fifth grade and it is clear from our conversations that she has to be able to *do* it and be able to *see the ways students are thinking* and the *ways students are confused*. She has a kind of higher-altitude, 10,000-foot view of fifth grade math. She can *predict* what students might do and can get students doing and talking about math problems that help clarify their confusion instead of making it worse.

Macy's view is certainly common among many novice mathematics teacher educators. Indeed, some participants in our initial pilot held this view. So, it is not surprising that MTEs with such a view about the knowledge needed for teaching mathematics would focus mainly on having students *do* primary and elementary mathematics in their courses. This stands in stark contrast with experienced mathematics teacher educators, who stress specialized content knowledge and elements of primary and elementary pedagogical content knowledge in their mathematics courses for prospective teachers (Li & Castro Superfine, 2018; Taylor & Appova, 2015). The design team spent a great deal of time pondering ways to have MTEs reflect on their beliefs and values while also exposing them to views of experienced mathematics teacher educators. We were fortunate to have the Li and Castro Superfine (2018) article *Mathematics teacher educators' perspectives on their design of content courses for elementary preservice teachers*. It includes many quotes from experienced instructors and is a piece of research literature in mathematics education that we felt would be accessible to MTEs (it avoids language that mathematics faculty often identify, dismissively, as "jargon"). Also, because we believe that MTEs are obligated to ensure that courses for prospective teachers meet established policy, such as in AMTE's (2017) *Standards for Preparing Teachers of Mathematics*, the short-course includes a brief dive into the AMTE document as part of Lesson Experiment 3.

Principle 4: Mathematics is a human endeavor, as is its teaching. Powerful instruction as defined in the TRU framework requires acknowledging mathematics as culturally-rich rather than culturally neutral. That is, teaching effectively requires intercultural competence (Bennett, 1993, 2004; Kramsch, 1998; Leininger, 2002). MTEs must be able to navigate differences and similarities in the forms of communication and activity valued in academic mathematics and those valued in prospective teachers' own personal and professional worlds. At the heart of MKT-FT is establishing and maintaining relationships in, and exercising judgement relative to, cross-cultural situations. For example, knowledge of content and students*, as a component of MKT-FT, is intertwined with intercultural competence as it requires MTEs to notice how learners think and learn either differently or the same as they themselves do and/or as others they know do. Examining instruction through the lens of components of TRU such as equitable access and agency are ways for MTEs to build awareness of and responsiveness to prospective teachers and their needs as learners. That awareness then necessitates MTEs to become facilitators and immerse themselves in how prospective teachers (and their future students) think and learn. Vignette 3 below illustrates a short-course task that targets such MTE development.

Vignette 3 - Example Task

Consider the set of questions below.

	Suppose a whole serving is 1/2 of a cookie. How many servings (whole or fractional) can I make from 3/4 of a cookie?	
	Suppose a whole serving is 1/2 of a cookie. How many servings (whole or fractional) can I make from 1/4 of a cookie?	

Prompt 1. Suppose you are a 6th grader who is answering the questions for the first time. You have never been exposed to an algorithm for fraction division, and so you do not have that knowledge going into the activity. Do the activity accordingly.

Prompt 2. What algorithm for fraction division does the activity suggest would be appropriate? HINT: To answer this, think about HOW you got your answers to each of the questions.

Prompt 3. Imagine you are a pre-service teacher and have been given this task, what is challenging? Why?

Next, read the transcript of pre-service teachers completing Prompts 1 and 2. Notice what students struggle with and how they talk about their thinking. Compare that with your anticipations from Prompt 3.

Finally, read the samples of pre-service teacher responses to TRU-based prompts about their experience with the activity [see appendix]. Reflect on what students said and discuss what the instructor might do to improve the task or use of it. Post team thoughts to the discussion board by [date, time]. Within 48 hours after [date, time], review and respond to at least one other team's discussion board post.

Notice that the task requires MTEs to think about the questions from a prospective teachers' perspective, anticipating their struggles. This is followed up by discovering actual struggles from a real prospective teacher mathematics class in which the same mathematics questions and Prompt 1 were used. The task also engages MTEs' common content (i.e., MKT) knowledge development as they complete the questions themselves from a 6th grader's perspective.

Principle 5: Skill in multiple mathematical discourses is necessary for teaching prospective teachers. As with MKT, in MKT-FT building knowledge of discourses* entails cross-context or intercultural sense-making. Using knowledge of discourses* hinges on unpacking classroom, mathematical, professional, and personal discourse and connecting it to other aspects of communication and learning. In particular, the connections among knowledge of discourses*, knowledge of content and students*, and knowledge of content and teaching* are foregrounded in the TRU framework in the Equitable Access and Agency, Ownership, and Identity dimensions. Indeed, equitable opportunities to participate in discourse depend on attending to issues of culture, language, and status (Cohen & Intili, 1982; Echevarria, Vogt, & Short, 2004; Khisty & Chval, 2002).

Principle 6: Effective teaching of prospective teachers requires formative assessment. As in the TRU framework, the basis of powerful instruction is meeting prospective teachers where they are while offering opportunities to learn and to demonstrate that learning. Eliciting, responding to, and reflecting on how prospective teachers are experiencing the course through formative assessment is crucial to MKT-FT development (Patterson, Parrott, and Belnap, 2020).

Objectives: Determine what is Evidence of Learning

With the values and principles in hand, we turned to considering learning goals for course participants. This included discussion of what MTEs might do in their instructional decision-making and classroom teaching that would serve as evidence that course goals had been achieved. That is, we developed and prioritized learning objectives and what constituted evidence for goal attainment *in parallel*. For the PRIMED project, specific professional practices for planning, instructing, and reflecting on teaching constituted the primary objectives. Table 2 gives principles and associated practices.

Table 2

Measurable Objectives for PRIMED: Instructional Practices of MTEs

Principle	Practices
Knowledge of MKT is Essential for Teaching Prospective Elementary Teachers	• Locate/use resources from research and practice literatures in planning for and doing instruction • Notice and be explicit with prospective teachers about differences between their learning and children's learning (Yackel, Underwood, & Elias, 2007)
Mathematics, MKT and MKT-FT are Interdependent	• Instruction engages future teachers in making connections between knowing mathematics to do it and knowing mathematics to teach it (Ball, Hill, & Bass, 2005; MET II,2016; AMTE, 2017).
Teaching Mathematics to Prospective Teachers is More Than Teaching School Mathematics	• Lessons designed/revised by MTEs will be informed by the research and practice literature and policy: such literature may be direct or indirect in that faculty may use things from an article or a text that is informed by the literature (e.g., Beckmann, 2018 or Boaler, 2016).
Mathematics is a Human Endeavor, as is its Teaching	• Engagement in purposeful gathering of information from prospective teachers about their experiences • Direct and tacit response to that information that may include adjustments in practice • Seek out resources in the research and practice literature in a mindful effort to learn and understand ways in which prospective teachers engage with and think about mathematics and its teaching
Skill in Multiple Mathematical Discourses is Necessary for Teaching Prospective Teachers	• Use of activities to engage in communication and representation that are mathematically accurate, effective in reaching the intended audience (e.g., peers or children), and level-appropriate in the rhetorical devices used (Jackson, Dimmel, & Mueller, 2016) • Use of class discussions in which prospective teachers are mathematically agentic, empowered participants (Davis & Martin, 2018; Ernest, 2002; Martin, 2012)
Effective Teaching of Prospective Teachers Requires Formative Assessment	• Regular reflection by MTEs on their experiences as learners • MTE directed/guided reflection by prospective teachers on the process and outcomes of tasks in the post-secondary learning environment. • MTE uses classroom artifacts and prospective teacher utterances to make informed instructional decisions that are consonant with effective instruction as outlined in the TRU framework

Identify Course Content

Content was selected and refined in order to support novice mathematics teacher educators in shaping their own ways to address Questions 1 and 2. We used our answers and those offered in current research and policy documents as the starting point. Rooted in instructional practice that helps to address the two driving questions, the short-course content has MTEs explore:

- theories of how people learn (adults and children),
- examples of activity-based instruction in classes for prospective teachers,
- examples of activity-based instruction in primary and elementary classes for children,
- finding and refining activities and tasks for use in classes for prospective teachers,
- various linguistic/discourse needs of primary, elementary, and tertiary students in mathematics,
- different kinds of knowledge needed to teach mathematics to primary and elementary students,
- different kinds of knowledge needed to teach mathematics to prospective teachers,
- roles of cognitive demand, agency, equity, and assessment in mathematics teaching,
- being a consumer of research, practice, and policy documents to inform instruction.

Because the focus in this report is on a conceptual model for short-course design rather than the course content, the list is brief (some additional details about short-course content are in Appendix A).

Organize the Structure

Decisions about structure took into account the connections across three learning contexts (short-course, college classroom, future primary and elementary classrooms). For each of the three environments, there were related aims for faculty learning, college classroom practices, and learning by prospective teachers.

Essentially, the theory of change underpinning short-course structure follows the logic that if *A: MTEs engage in short cycles of instructional change aimed at valued practices*, then *B: their teaching becomes more equitable and effective*. If B happens, then *C: Prospective teachers learn about mathematics, MKT, and equitable instruction*. If C happens, then *D: Prospective teachers are equipped to "teach them math" when faced with a classroom full of children*. In other words, the short-course must have MTEs engage in purposeful tasks and activities, reflect on their learning and teaching experiences, and consider and test those reflections during instruction by trying to enact engaging tasks with their prospective teachers in equitable ways. When prospective teachers engage with tasks through meaningful discourse, they construct their own MKT and consider the usefulness of having MKT in their future work with children. Also, when MTEs foster challenging discussions about what counts as mathematics, who children (“they”) are, and what teaching is, prospective teachers engage with issues of equity, access, and empowerment and enrich their MKT.

Table 3

Theory of Change for PRIMED Short-Course Design

Layer	PRIMED Goals	MTE Learning Outcomes	MTE Practice Outcomes	Prospective Teacher Learning Outcomes
Primary/ Elementary Teaching and Learning	MTEs understand children's ways of engaging with mathematics	Construct MKT	Effectively address and assess prospective teachers' MKT during instruction	Formulation of rich and textured MKT
University Teaching and Learning	MTEs understand prospective teachers' ways of engaging with mathematics	Construct MKT-FT	Orchestrate rich conversations and model effective learning environments	Awareness of models for effective teaching and learning
Professional Development of MTEs	MTEs build a habit of reflective discussion with one or more colleagues	Awareness and use of resources (including TRU)	Provide feedback on instruction, allow for change in practice	More effective and enjoyable learning experiences

As a result of this attention to contexts and aims, and a review of the literature on what has proven impactful for both teacher change and student success (e.g., Blank & de las Alas, 2009), we made decisions about short-course structure. The short-course for MTEs consists of five online modules, completed across ten weeks (about one module every two weeks). So that MTEs can use their own classrooms as laboratories for small lesson experiments, they must be teaching a mathematics course for prospective teachers *while* they are completing the PRIMED short-course.

Each of the five modules involves about three hours of commitment above and beyond usual planning and instruction time for a total of 15 hours of work across the 10 weeks. Three modules (1, 3, and 4) are completed asynchronously and two (2 and 5) have 90-minute sessions that are completed synchronously (i.e., in real-time with web-based video and audio meeting tools). The online environment was chosen in part to support MTEs to work in pairs. A professional thought partner was a required structure for the course. The pairing of faculty to complete the course addresses the concern raised in the literature by faculty who report isolation in their instructional development (Olanoff, 2011; Zopf, 2010). Some MTEs signed up with partners, others were assigned a partner by the developers. In both cases, MTEs met together online outside of the two synchronous meetings. See Appendix A for more on duration and sequencing.

Choose a Short-Course Approach to Learning

Research has demonstrated that effective professional development opportunities model the kinds of instruction faculty are being asked to learn to do (Connolly & Millar, 2006). Among the myriad options emerging from the research literature on effective learner-centered and inquiry-based approaches to instruction, the PRIMED experience is rooted in *task-based activity* (for more, see Jackson, Hauk & Tsay, 2018). The current literature regarding online professional learning (e.g., Bigatel, Ragan, Kennan, May, & Redmond, 2012; Dixon, 2010; Morris & Finnegan, 2009; Shattuck & Anderson, 2013; Smith, 2005; Southern Regional Education Board, 2006), our own lived experiences as online learners and instructors, and the expertise of advisors and critical friends in the collegiate mathematics education community contributed to defining a lean set of course formats. And, as indicated above, the course format has the structure of MTE teams to ensure a thought partner who is also a partner in accountability. To date, most short-course participating faculty have been teams of two people who work at different institutions.

An overarching PRIMED project goal is to create a version of the course that is self-sustaining – a free, asynchronous, short-course of activities and resources that any pair of MTEs could complete. This meant that design began with 12 of the 15 hours of the course designed as asynchronous engagement, including the course launch (Module 1). Also, the approach establishes a reliable (to the MTE) pattern of interaction based on a recurring learning cycle (see Figure 4). Activities in the cycle are represented in the online materials by icons that become familiar (see Appendix B).

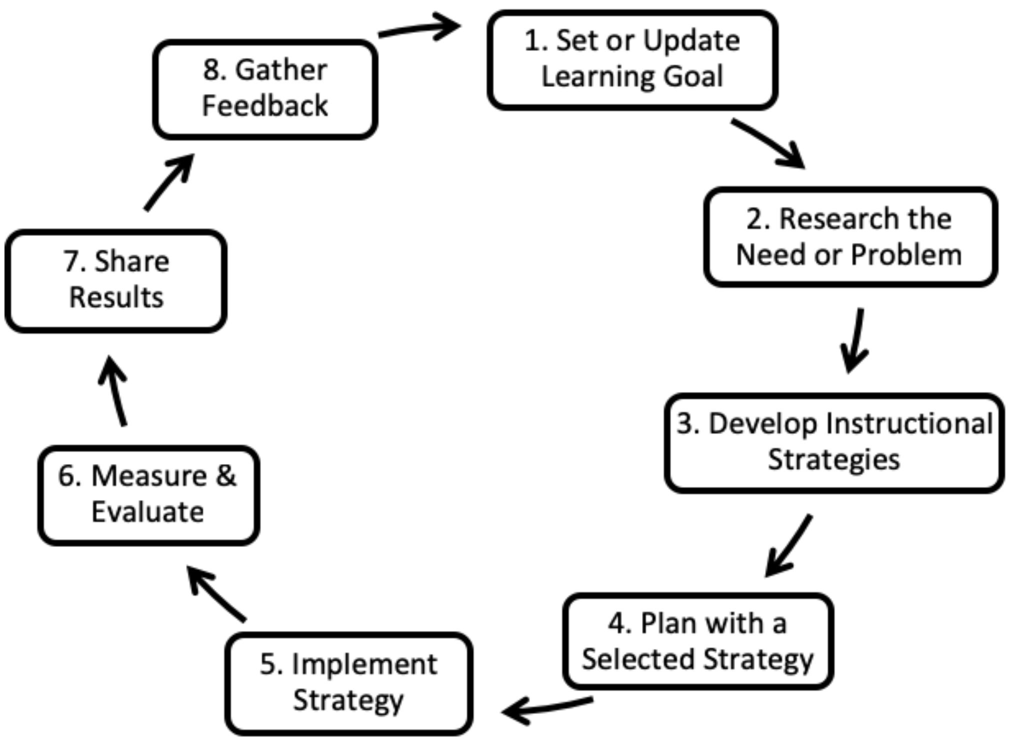


Figure 4: Learning cycle for the PRIMED short-course.

MTEs complete three learning cycles during the short-course. The modules include exploring and responding to prospective teacher and children's mathematical work, examining video of college classes populated by prospective teachers and video of primary and elementary classrooms, and "lesson experiments" aimed at low-stakes trying out of ideas in the MTE's local classroom (the experiments are instantiations of steps 5 through 8 in the learning cycle; a bit more detail on the current version of the short-course is in the Appendix A).

Conclusion and Next Steps

There is certainly a documented need for professional learning opportunities for MTEs who are new to teaching mathematics courses for prospective primary and elementary teachers. The PRIMED project has begun to address this need with the design and development of an online short-course.

Noting Schoenfeld's (2007) remarks on the need for a conceptual model that frames an educational project, we have shared the PRIMED design team's framework and conceptual model in the hopes that it may be of use to other groups as they develop professional learning opportunities and materials for mathematics faculty. As Schoenfeld indicated, being cognizant of such things is important because they determine what types of data might be collected and what types of conclusions about the impacts of professional learning can be made. The design and evaluation process is a cyclic one. As of this writing, conducting evaluation and principle-specific research to measure the effects of PRIMED is part of future project activity. Data collection includes MTE responses to directed reflections in the modules, pre-post assessment of prospective teacher MKT development by means of the Learning Mathematics for Teaching assessment, and documentation of MTE enacted practices in their planning, doing, and reflecting on instruction.

The set of principles developed for PRIMED can provide a template for the recursive process of creating a conceptual model for any faculty professional learning opportunity. Toward that end, we claim that Table 1 offers a transferable framing with steps for conceptual model development.

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References

Anderson, J. R. (1983). *The architecture of cognition*. Cambridge, MA: Harvard University Press.

Appova, A. and Taylor, C. (2019). Expert mathematics teacher educators' purposes and practices for providing prospective teachers with opportunities to develop pedagogical content knowledge in content courses. *Journal of Mathematics Teacher Education,* 22, 179-204.

Association of Mathematics Teacher Educators [AMTE]. (2017). *Standards for Preparing Teachers of Mathematics*. Available online at amte.net/standards.

Bakhtin, M. M. (1981) *The dialogic imagination*. Austin, TX, University of Texas.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*, 14-22.

Ball, D. L., Thames, M H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Bass, H. (2005). Mathematics, mathematicians, and mathematics education. *Bulletin of the American Mathematical Society,* 42(4), 417-430.

Beckmann, S. (2018). *Mathematics for elementary teachers with activities* (5th ed.). New York: Pearson.

Bennett, M. J. (1993). Towards ethnorelativism: A developmental model of intercultural sensitivity. In R. M. Paige (Ed.), *Education for the intercultural experience* (2nd ed., pp. 21-71) Yarmouth, ME: Intercultural Press.

Bennett, M. J. (2004). Becoming interculturally competent. In J. Wurzel (Ed.), *Towards multiculturalism: A reader in multicultural education* (2nd ed., pp. 62–77). Newton, MA: Intercultural Resource Corporation.

Bergsten, C., & Grevholm, B. (2008). Knowledgeable teacher educators and linking practices. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics teacher education: The mathematics teacher educator as a developing professional*, Vol. 4 (pp. 223-246). Rotterdam, The Netherlands: Sense Publishers.

Bigatel, P. M., Ragan, L. C., Kennan, S., May, J., & Redmond, B. F. (2012). The identification of competencies for online teaching success. *Journal of Asynchronous Learning Networks*, *16*(1), 59–77.

Blank, R. K., & De Las Alas, N. (2009). *The effects of teacher professional development on gains in student achievement: How meta-analysis provides scientific evidence useful to education leaders*. Technical Report. Washington, DC: Council of Chief State School Officers. Available online at https://files.eric.ed.gov/fulltext/ED544700.pdf

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco: Josey-Bass.

Bressoud, D., V. Mesa, C. Rasmussen (2015). *Insights and recommendations from the MAA national study of college calculus*. Washington DC: Mathematical Association of America.

Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers II* [METII] (2012). Issues in Mathematics Education, Vol. 17. Providence, RI: American Mathematical Society.

Castro Superfine, A. & Li, W. (2014a). Developing mathematical knowledge for teaching teachers: A model for the professional development of teacher educators. *Issues in Teacher Education, 23*(1), 113-132.

Castro Superfine, A. & Li, W. (2014b). Exploring the mathematical knowledge needed for teaching teachers. *Journal of Teacher Education,*65(4), 303-314.

Cohen, E. G. & Intili, J. K., (1982). *Interdependence and management in bilingual classrooms*, Final report II (NIE contract #NIE-G-80-0217), Stanford: Stanford University Center for Education Research.

Connolly M. R. & Millar S. B. (2006). Using workshops to improve instruction in STEM courses. *Metropolitan Universities*, 17, 53–65.

Daniels, H. (2001) *Vygotsky and Pedagogy*. London, Routledge/Falmer.

Davis, J. & Martin, D. B. (2018), Racism, assessment, and instructional practices: Implications for mathematics teachers of African American students. *Journal of Urban Mathematics Education*, 11(1&2), 45-68.

Davis, M. K., Hauk, S., & Latiolais, M. P. (2009). Culturally responsive college level mathematics. In B. Greer, S. Nelson-Barber, A. Powell, & S. Mukhopadhyay (Eds.), *Culturally responsive mathematics education* (pp. 345-372). Mahwah, NJ: Erlbaum

Dixson, M. D. (2010). Creating effective student engagement in online courses: What do students find engaging? *Journal of the Scholarship of Teaching and Learning, 10*(2), 1-13.

Echevarria, J., Vogt, M. E., & Short, D. (2004). *Making content comprehensible for English learners: The SIOP model*. Boston, MA: Allyn and Bacon.

Ernest, P. (2002). Empowerment in mathematics education, *The Philosophy of Mathematics Education Journal, 15*. Retrieved from http://www.ex.ac.uk/~PErnest/pome15/empowerment.htm

Felton-Koestler, M. D. (2020). Towards a framework of knowledge for equitable mathematics teacher education. In A. Appova, R. M. Welder, and Z. Feldman, (Eds.), *Supporting Mathematics Teacher Educators' Knowledge and Practices for Teaching Content to Prospective (Grades K-8) Teachers*. *Special Issue: The Mathematics Enthusiast*, ISSN 1551-3440, vol. 17, nos. 2 & 3, pp. 435-468. ScholarWorks: University of Montana. Retrieve (open access) from: https://scholarworks.umt.edu/tme

Flahive, M. & Kasman, R. (2013). Guided discovery in a discrete mathematics course for middle school teachers. In C Beaver, L. Burton, M. Fung, K. Kruczek (Eds.), *Resources for preparing middle school mathematics teachers* (pp. 85-96). Washington, DC: Mathematical Association of America.

Freeman, S., Eddy, S., McDonough, M., Smith, M., Okoroafor, N., Jordt, H., & Wenderoth, M. P. (2014). Active learning increases student performance in science, engineering, and mathematics. *Proceedings of the National Academy of Sciences, 111*(23) 8410–8415.

Greenberg, J., & Walsh, K. (2008). *No common denominator: The preparation of elementary teachers in mathematics by America's education schools*. Washington, DC: National Council on Teacher Quality.

Gros, H., Sander, E., & Thibaut, J. P (2019). When masters of abstraction run into a concrete wall: Experts failing arithmetic word problems. *Psychonomic Bulletin & Review*. Retrieved from https://doi.org/10.3758/s13423-019-01628-3

Gutiérrez, R. (2009). Framing equity: Helping students "play the game" and "change the game." *Teaching for Excellence and Equity in Mathematics*, *1*(1), 4-8.

Halliday, M.A.K. (2003) On language in relation to the evolution of human consciousness. In: J. Webster (Ed.) *On language and linguistics* (Vol. 3 in the collected works of M.A.K Halliday). London: Continuum.

Hauk, S., Toney, A., Jackson, B., Nair, R., & Tsay, J.J. (2014). Developing a model of pedagogical content knowledge for secondary and post-secondary mathematics instruction. *Dialogic Pedagogy: An International Online Journal*, *2*, 16-40.

Hauk, S., Jackson, B., & Tsay, J.J. (2017). Those who teach the teachers: Knowledge growth in teaching for mathematics teacher educators. In A. Weinberg, C. Rasmussen, J. Rabin, M. Wawro, S. Brown (Eds.), *Proceedings of the 20th Conference on Research in Undergraduate Mathematics Education* (San Diego, CA).

Herbel-Eisenmann, B. A. (2007). From intended curriculum to written curriculum: Examining the "voice" of a mathematics textbook. *Journal for Research in Mathematics Education*, *38*(4), 344-369.

Hill, H. C., Ball, D. L., & Schilling S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, *39*(4), 372-400.

Holdren, J. P., & Lander, E. (2012). *Report to the President—Engage to excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics*. Washington, DC: President's Council of Advisors on Science and Technology.

Hora, M. T., & Ferrare, J. J. (2013). Instructional systems of practice: A multidimensional analysis of math and science undergraduate course planning and classroom teaching. *Journal of the Learning Sciences*, *22*(2), 212-257.

Jackson, B., Dimmel, J., & Mueller, M. (2016). Developing mathematical knowledge for teaching in content courses for pre-service elementary teachers. In T. Fukawa-Connelly, N. Infante, M. Wawro, and S. Brown (Eds.), *Proceedings of the 19th Annual Conference on Research in Undergraduate Mathematics Education*, Pittsburgh, Pennsylvania.

Jackson, B., Hauk, S., and Tsay, J. J. (2018). A model of task-based learning for research on instructor professional development. *Proceedings of the 21st Annual Conference on Research in Undergraduate Mathematics Education*, San Diego, CA.

Jeppsen, A. (2010). *Curricular decision-making in community college mathematics courses for elementary teachers* (Doctoral dissertation), University of Michigan, Ann Arbor, MI. Retrieved from https://deepblue.lib.umich.edu/bitstream/handle/2027.42/78825/ajeppsen_1.pdf?sequence=1

Konuk, N. (2018). *Mathematics teacher educators' roles, talks, and knowledge in collaborative planning practice: Opportunities for professional development* (Doctoral dissertation), The Pennsylvania State University, State College, PA. Retrieved from https://search.proquest.com/docview/2205942899/fulltextPDF/9FA8410C4E3D4E9DPQ/1?accountid=14665

Khisty, L. L., & Chval, K. B., (2002). Pedagogic discourse and equity in math: When teachers' talk matters. *Mathematics Education Research Journal, 14*(3), 154-168.

Kramsch, C. (1998). *Language and culture*. Oxford University Press.

Kung, D. T. (2010). Teaching assistants learning how students think. In F. Hitt, D. Holton, & P. Thompson (Eds.) *Research in collegiate mathematics education VII* (pp. 143–169). Providence, RI: American Mathematical Society.

Laursen, S. L., Hassi, M. L., Kogan, M., & Weston, T. J. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, *45*(4), 406-418.

Leininger, M. (2002). Culture care theory: A major contribution to advance transcultural nursing knowledge and practices. *Journal of Transcultural Nursing*, *13*(3), 189-192.

Li, W., & Castro Superfine, A. (2018). Mathematics teacher educators' perspectives on their design of content courses for elementary preservice teachers. *Journal of Mathematics Teacher Education*, *21*(2), 179-201.

Martin, D.B. (2012). Teaching other people's children to teach other people's children: Reflections on integrating equity issues into a mathematics content course for elementary teachers. In L. Jacobsen, J. M. Mistele, & B. Sriraman (Eds.), *Mathematics teacher education in the public interest* (pp. 3-23). Charlotte: IAP.

Masingila, J. O., Olanoff, D. E., Kwaka, D. K. (2012). Who teaches mathematics content courses for prospective elementary teachers in the United States? Results of a national survey. *Journal of Mathematics Teacher Education*, 15, 347-358.

Morris, L. V., & Finnegan, C. L. (2009). Best practices in predicting and encouraging student persistence and achievement online. *Journal of College Student Retention, 10*, 55–64.

Moschkovich, J. (2016). Hearing mathematical competence expressed in emergent language. In D.Y. White, S. Crespo, & M. Civil (Eds.), *Cases for mathematics teacher educators: Facilitating conversations about inequities in mathematics classrooms in the public interest* (pp. 161-169). Charlotte, NC: IAP.

National Governors Association Center for Best Practices, Council of Chief State School Officers (2010). *Common Core State Standards for Mathematics*. Washington DC: Author.

Olanoff, D.E. (2011). *Mathematical knowledge for teaching teachers: The case of multiplication and division of fractions* (Doctoral dissertation). Syracuse University, Syracuse, NY. Retrieved from http://surface.syr.edu/cgi/viewcontent.cgi?article=1063&context=mat_etd

Patterson, C. L., Parrott, A., & Belnap, J. (2020). Strategies for Assessing Mathematical Knowledge for Teaching in Mathematics Content Courses. In A. Appova, R. M. Welder, and Z. Feldman, (Eds.), *Supporting Mathematics Teacher Educators' Knowledge and Practices for Teaching Content to Prospective (Grades K-8) Teachers*. *Special Issue: The Mathematics Enthusiast*, ISSN 1551-3440, vol. 17, nos. 2 & 3, pp. 807-842.

ScholarWorks: University of Montana. Retrieve (open access) from: https://scholarworks.umt.edu/tme

Perez, R. S., Johnson, J. F., & Emery, C. D. (1995). Instructional design expertise: A cognitive model of design. *Instructional Science, 23*(5-6), 321-349.

Rider, R. L., & Lynch-Davis, K. (2006). Continuing the conversation on mathematics teacher educators. In K. Lynch-Davis, & R. L. Rider (Eds.), *The work of mathematics teacher educators: Continuing the conversation* (pp. 1-7). Association of Mathematics Teacher Educators Monograph, v.3. San Diego, CA: AMTE.

Ping, C., Schellings, G., & Beijaard, D. (2018). Teacher educators' professional learning: A literature review. *Teaching and Teacher Education*, *75*, 93-104.

Schoenfeld, A. H. (2007). Method. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 69–110). New York: MacMillan.

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational Researcher*, 43(8), 404-412.

Schoenfeld, A. H., [and the Teaching for Robust Understanding Project]. (2016). *An Introduction to the Teaching for Robust Understanding (TRU) Framework*. Berkeley, CA: Graduate School of Education. Retrieved from http://map.mathshell.org/trumath.php or http://tru.berkeley.edu.

Schoenfeld, A. H. (2019). Reframing teacher knowledge: a research and development agenda. *ZDM*, 1-18.

Shattuck, J. & Anderson, T. (2013). Using a design-based research study to identify principles for training instructors to teach online. *The International Review of Research in Open and*

Distance Learning, *14*(5). Retrieved November 28, 2018, from http://www.irrodl.org/index.php/irrodl/issue/archive

Shulman, L. S. (1986). Those who understand: A conception of teacher knowledge. *American Educator, 10*(1), 9-15, 43-44.

Smith, K. (2003). So, what about the professional development of teacher educators? *European Journal of Teacher Education, 26*(2), 201-215

Smith, K. (2005). Teacher educators' expertise: What do novice teachers and teacher educators say? *Teaching and Teacher Education: An International Journal of Research and Studies*, *21*(2), 177-192.

Smith, T. C. (2005, July). Fifty-one competencies for online instruction. *The Journal of Educators Online*. Retrieved February 22, 2015, from www.thejeo.co

Southern Regional Education Board. (2006). *Guidelines for quality online teaching*. Atlanta, GA: Author.

Speer, N. M., & Hald, O. (2009). How do mathematicians learn to teach? Implications from research on teachers and teaching for graduate student professional development. In M. Carlson and C. Rasmussen (Eds.), *Making the connection: Research and practice in undergraduate mathematics education* (pp. 305–317). Washington, DC: Mathematical Association of America.

Speer, N. M., King, K. D., & Howell, H. (2015). Definitions of mathematical knowledge for teaching: using these constructs in research on secondary and college mathematics teachers. *Journal of Mathematics Teacher Education, 18*, 105-122.

Speer, N. M., & Wagner, J. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education, 40*(5), 530–565.

Stylianides, A., & Stylianides, G. (2014). Viewing "mathematics for teaching" as a form of applied mathematics: Implications for the mathematical preparation of teachers. *Notices of the AMS, 61*(3), 266-276.

Sztajn, P., Ball, D. L. & McMahon, T. A. (2006). Designing learning opportunities for mathematics teacher developers. In K. Lynch-Davis and R. L. Rider (Eds.). *The work of mathematics teacher educators* (pp.149-162). San Diego, CA: Association of Mathematics Teacher Educators.

Taylor, C.E., & Appova, A. (2015). Mathematics teacher educators' purposes for K-8 content courses. In Beswick, K., Muir, T., & Wells, J. (Eds.). *Proceedings of the 39th Psychology of Mathematics Education conference,* vol 4, (pp. 241-248). Hobart, Australia: PME.

Thames, M. (2008). *A study of practice-based approaches determining the mathematics that (K-8) teachers need to know*. Unpublished manuscript.

Wilson, S. M., & Ball, D. L. (1996). Helping teachers meet the standards: New challenges for teacher educators. *The Elementary School Journal*, 97(2, Special Issue: Teacher Assessment), 121-138.

Yackel, E., Underwood, D., & Elias, N. (2007). Mathematical tasks designed to foster a reconceptualized view of early arithmetic. *Journal of Mathematics Teacher Education,* 10, 351-367.

York, C. S., & Ertmer, P. A. (2016). Examining instructional design principles applied by experienced designers in practice. *Performance Improvement Quarterly, 29*(2), 169-192.

Zaslavsky, O. & Leikin, R. (2004). Professional development of mathematics teacher educators. *Journal of Mathematics Teacher Education*, *7*(5), 5-32.

Zaslavsky, O., Watson, A., and Mason, J. (Eds.) (2007). Special Issue on Mathematical Tasks. *Journal of Mathematics Teacher Education*, *10*(4-6).

Zhang, P., Brown, A., Joseph, M., & He, J. (2020). What topics are taught in elementary mathematics content courses and why. In A. Appova, R. M. Welder, and Z. Feldman, (Eds.), *Supporting Mathematics Teacher Educators' Knowledge and Practices for Teaching Content to Prospective (Grades K-8) Teachers*. *Special Issue: The Mathematics Enthusiast*, ISSN 1551-3440, vol. 17, nos. 2 & 3, pp. 403-434. ScholarWorks: University of Montana. Retrieve (open access) from: https://scholarworks.umt.edu/tme

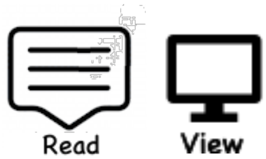
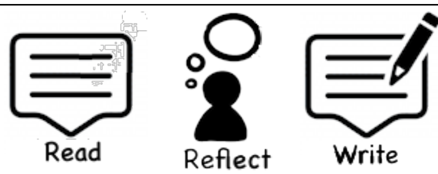
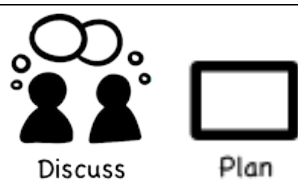
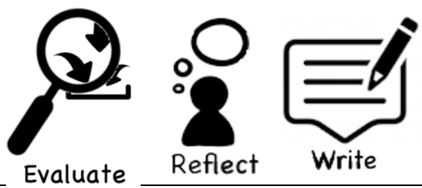
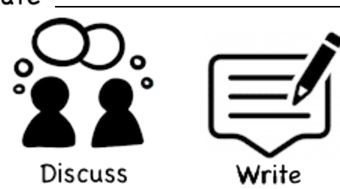
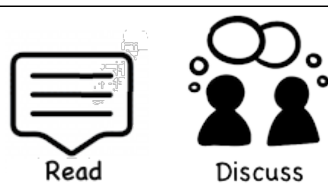
Zopf, D. (2010). *Mathematical knowledge for teaching teachers: The mathematical work of and knowledge entailed by teacher education* (Unpublished doctoral dissertation). University of Michigan, Ann Arbor, MI. Retrieved from http://deepblue.lib.umich.edu/bitstream/2027.42/77702/1/dzopf_1.pdf

APPENDIX A: Overview of Content, Sequence, and Pacing (~1 module every 2 weeks)

	Topic	*Time**
	Preparation: Read short essay (5pp) on constructivism; Read short report on the mathematical autobiographies of pre-service K-8 teachers (6pp).	0.5 hrs
Module 1 ASYNC	Encouraging active learning using well-structured tasks. The nature of task-based learning, what it looks like in a classroom for future teachers; how aspects of mathematical knowledge for teaching for elementary school teachers can be mathematically productive layers in a task.	1.5 hrs
	Homework: read Li & Castro Superfine (2018; 20pp) and short intro to Teaching for Robust Understanding (TRU) framework (4pp); watch two video clips from courses for pre-service elementary teachers.	1 hour
Module 2 SYNC	Understanding instructional choices and their equitable implementation in courses for prospective teachers. Examine the goals of experienced METs (discussion of article and video); intro to TRU; discuss and practice monitoring progress towards intended goals using TRU tools.	1.5 hrs
	Homework: Lesson Experiment #1 - create and implement a 5-15 minute task-based activity in-class. Partner observations using a focal TRU dimension, reflect, and suggest revisions. Read short intro to MKT (4pp).	1.5 hrs
Module 3 ASYNC	Mathematical knowledge for teaching (MKT): Expanding the definition of "mathematical content" in courses for prospective teachers. Read about, view video clips from elementary and tertiary settings, discuss the nature of MKT and the version of MKT for teaching prospective teachers; focus on specialized content knowledge* and knowledge of content and students*.	1.5 hrs
	Homework: Lesson Experiment #2 – find, tune, and implement a group-worthy task for an existing lesson. Partner observations using a focal TRU dimension (in person or Skype), review Common Core standards (all practices and for the domain of experiment task), reflect, and suggest revisions for next use.	2.5 hrs
Module 4 ASYNC	Building and assessing discourse knowledge in courses for future teachers. Explore the many ways pre-service teachers think about and communicate mathematical ideas; examine how expert mathematics teacher educators bring attention to equity to assessment, particularly formative assessment. View video clips, read and discuss text-based case (Moschkovich, 2016), review, compare, and score student response on test items developed by expert MTEs.	1.5 hrs
	Homework: Lesson Experiment #3 - design and revise with partner at least two assessment <u>items</u> for one concept (e.g., lesson topic from Module 2 or 3). Use on a quiz/test, develop/use rubric with partner to grade, discuss, revise.	2 hrs
Module 5 SYNC	Closure and next steps: Focus on MKT and MKT for college instruction of future teachers, what was learned, what more does each instructor want to know, where to find research and practice resources.	1.5 hrs
	Total	15 hrs

**Estimated time above and beyond a faculty-participant's usual planning and instruction time.*

APPENDIX B: Online Course Representations for Learning Cycle Components

	**Goal (Set or Update)** Learning goal stated and relevant new information provided. ●Read and (when present) View Video
	***Research*** Given the new information: ●Reflect and make notes ●Discuss with partner ●Partners add thoughts to Discussion Board
	***Develop*** Read, reflect, and respond to others' Discussion Board contributions
	***Plan*** Discuss with Partner and plan for upcoming local classroom lesson experiment.
	***Implement and Gather Data*** Implement planned lesson experiment, collect artifacts/assessment data
	***Evaluate*** Use collected data to compare intention (plan) with reality (what was tried), reflect on (mis)match, make notes.
	***Share Results*** Discuss lesson experiment with partner and post what was learned and what more to learn to Discussion Board
	***Gather Feedback*** Get ideas and insights for lessons learned from Discussion Board and review with partner.