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Beauty is in the blind spot of the beholder

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Beauty is in the blind spot of the beholder

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Abstract: The paper addresses the time-old question of what is beauty. A rather ambitious, if not to say presumptuous, endeavour. But I do not aim high – I do not claim to get anywhere near unearthing the secret. Rather, I will use examples from mathematics, poetry, music and chess to substantiate one thesis: that the elusory character of beauty is not incidental. Its defiance of definition is part of its essence. The aesthetic sensation requires unawareness of its precise origin. Beauty is felt when some order is perceived, that is not fully comprehended. The order is too complex, or well hidden, or too novel, to surface in its entirety. This is the reason for our ability to enjoy a piece of art for the hundredth time – we never fully fathom its inner order. This is also the reason for the feeling of awe that the beauty inspires: mystery and magic are at its heart. I will compare mathematical techniques and features with those of poetry - like compression, summoning patterns from one field to solve problems in another, or self-reference, and show how beauty is generated in the two domains in a similar way. I will also comment on the beauty-generating effect of unexpectedness in both domains. That novelty generates beauty is a trite observation ("the most expected feature of a poem is its unexpectedness", as somebody put it), but the question why this is so is not often addressed – I will connect it with the "blind spot" idea. In a final section I try to answer the question that is at least as difficult as "what is beauty" – "why beauty?". The fact that it pervades our lives indicates that it has an important role – what is it? To arouse the reader's curiosity, let me summarize the attempted answer in one word – 'change'. That aim that is so coveted and so hard to achieve – a change in the pattern of our actions, aims and perceptions. The style of the paper is non-scientific, and non-erudite, reflecting my belief that scientific pretensions in the humanities deflect from "softer", more genuine, understanding.

Keywords: beauty, order, economy in energy, compression, self-reference, humor

INTRODUCTION

Some fifteen years ago, I taught in elementary schools. Returning home one day, my daughter asked me what I taught. I told her – why $2 \times 3 = 3 \times 2$. "It is the same thing", she said. I asked her to raise three fingers on her right hand, and three on her left. This is 2×3. And how about 3×2? She raised three pairs of fingers. "Now show that they are the same", I said. To my astound,

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she raised three fingers in each hand - 2×3, put her hands one against the other, and spread the three pairs of fingers - 3×2. A proof, using fingers, of the commutativity of multiplication.

A few months later, I showed this trick in Grade 2 in a school in Migdal Ha'emek, a social-economically not very advanced town. A student in the first row, sitting just below my podium, mumbled to himself "This is beautiful". The boy probably had no clue before, that mathematics could be beautiful. And yet, he had real aesthetic pleasure. Why? What makes an argument or an example beautiful?

Here is a clue: grownups, when shown this trick, laugh. Why do people laugh is another famous riddle, not much easier than "what is beauty". But one problem can help solve the other - cracking with bare hands two nuts is easier than cracking one. I will return to this example in a later section.

Language has its wisdom. Similarity in names indicates similarity in structure. If the same word is used in two situations, the two probably have a common feature. We use the same word for beauty in chess, in mathematics, in poetry, in music, and even for human beings – there must be something common to all. The same process occurs in our brains when we perceive them. What is the common denominator? This is a notoriously difficult question. I will not be so pretentious as to claim a solution – but I will point at one important feature of beauty, which also explains its elusiveness: its subconscious recognition.

MAIN IDEAS IN THE PAPER

Here is a little roadmap, pointing out what is possibly new in the paper and what is definitely not. There is a vast literature on features that make mathematical arguments and ideas beautiful. A condensed list: unexpectedness, clarity, brevity, explanatory power, combining distant ideas, importing ideas from distant areas, finding an underlying pattern in a seemingly chaotic picture. My aim is not to add any items to this list, but to find a common denominator to all. Even more ambitiously – connect this to beauty in other realms, especially in other arts. So, the paper is not a descriptive one, or a catalogue type, but trying to reflect on a psychological riddle – what is it that turns on the lightbulb of "beauty" in our brains.

In particular, I will dwell on poetic techniques that achieve the effect of subconscious absorption of the message. An obvious one is the metaphor, with its well-known effect of condensation – transmitting many ideas in one brush stroke. Another is sudden twists at the end of the poem, that forces the reader to re-interpret all at once all that happened before – a shrewd trick for condensation without brevity.

I will compare the use of self-reference in poetry, humour and mathematics, and link it with the "blind spot" idea.

The last section is on the benefits of beauty. Why Natural Selection implanted in us the capability of its perception. Here, again, a link is established with humour, and with the role of arts in our lives.

THE RIDDLE

Definitions are hard. We may use a concept freely and unhesitatingly, yet have no idea how it works. But among even the most notoriously hard nuts to crack, the notion of 'beauty' stands out. Generations of thinkers have tried their hand, with little to show for their efforts.
This, I believe, is no coincidence. Beauty's elusory character is part and parcel of its essence. Simply, for something to be beautiful we must not know why it is so. Or at least not fully understand. A woman is truly beautiful if you cannot explain why she is so – the difference between plain beauty and magical beauty is hard to put into words.
Think for example of admiration for a beautiful scene. It evokes 'awe', meaning that it is hard to comprehend. We are accustomed to relating to our surroundings in a functional, practical way: how can we use what we see, in what way can we negotiate it? Admiration for a magnificent mountain stems from inability to comprehend the dimensions, that are beyond our ordinary dealings.

Beauty is subjective, so it is claimed. It is in the eye of the beholder, which is thought to explain its elusiveness. But this is far from true. There are areas of consensus on what is beautiful and what is not. One such area is mathematics. All mathematicians share the same aesthetic values. Beauty is their main criterion for quality: 'there is no place in the world for ugly mathematics', said the English mathematician Hardy. Beauty is almost synonymous with depth. An idea that sheds new light on a problem is beautiful, while a proof that uses old ideas – no matter how complex – will not be considered beautiful. Of course, this is not a sufficient criterion. There is more to beauty than novelty.

In music things are similar. Nobody would say about a Salieri opera what Bernhard Shaw claimed about the final scene of Don Giovanni: that it was the first time God trod on earth. Somewhere over the rainbow there is an absolute criterion for beauty. It may be hard to unearth, but it is there. This paper does not claim to solve the riddle, but it points out one necessary condition: blindness to the origin of beauty. Beauty is not in the eyes of beholders, but in their blind spots. Beauty requires magic.

To demonstrate my claim, I will use mainly two fields – mathematics and poetry. I will start with poetry, since it is accessible to a wider audience.

MAGIC

The less you understand a poem, the more impressed you are by the magic.
(Samuel Butler, Hubridas)

Would you pay to be deceived? Of course not. But just think, isn't this precisely the magician's art? We like to be fooled. Poetry does precisely this and has up its sleeve many magician's tricks that all aim to convey subliminal messages. A poem, every poem, is a reverse pickpocket. Instead of stealing something from our pocket, it furtively puts something in. Magicians deceive to

entertain; poetry uses the deception to transmit truths, without our noticing. ‘Poetry is always a search for the truth’ (Franz Kafka, 1880-1924).

Poets’ best-known magician’s trick is the metaphor. The reason for its tremendous success is that it simultaneously serves two functions: it transmits information and yet hides it. On the one hand, it is an extremely efficient way to convey information: it takes a pattern familiar from one domain, and transfers it to another. As if transporting a building in its entirety, instead of brick-by-brick. A feat of economy. Describing somebody as ‘similar to Charlie Chaplin’ is much more effective than listing details of his physique. Here is an example from a poem by the national Hebrew poet, Bialik.

We were left with no friend or fellow

like two flowers in the desert.

(Hayyim Nahman Bialik, ‘In Twilight’)

An entire world is transmitted by a single brushstroke (I use another metaphor). The sense of loneliness; the thirst for love; the lovers seeing no one besides one another; their vulnerability; the contrast between their attitude to the world and their feelings for each other – and I certainly missed many more levels of meaning. The visual image – the metaphor’s most efficient weapon – serves as a conduit of information.
But the real power of the metaphor is in its other function – camouflage. You know what the poet means, but you can pretend that there is only the overt meaning. Here is real magic, Emily Dickinson’s ‘Adrift’.

Adrift! A little boat adrift!

And night is coming down!

Will no one guide a little boat

Unto the nearest town?

So Sailors say – on yesterday –

Just as the dusk was brown

One little boat gave up its strife

And gurgled down and down.

So angels say – on yesterday –

Just as the dawn was red

One little boat – o'erspent with gales –

Retrimmed its masts – redecked its sails –

And shot – exultant one!

Dickinson divulges more about her life than she would dare explicitly acknowledge, even to herself. 'Adrift,' 'o'erspent with gales', 'gave up its strife,' – these are hard things to concede. Although we know the real meaning, the metaphor allows us to pretend that it is indeed about a boat. It is a brush of a feather on our skin, felt but not fully acknowledged.
The message of 'Adrift' is that there is depth to life, something beyond external appearance. Beneath the little boat overwhelmed by the tempest there is a very brave vessel; and even if it appears about to sink, in another dimension it spreads its sails and takes off. Those who know Dickinson's story can appreciate how accurate is this description of her life.

COMPRESSION

Say it all, but say it slant.

(Emily Dickinson)

One merit of poetry few will deny: it says more and in fewer words than prose.

(Voltaire, 1694-1778)

The German word for poetry 'dichtung', translates as 'compression'. The American poet Ezra Pound said, 'Great literature is simply language charged with meaning to the utmost possible degree'. This is another device used by poetry to layer its messages. When this happens, we do not follow all that is happening. Things happen too fast to be consciously registered, as metaphors compress so much into one image.

Some like conciseness, while others prefer lengthy. For the latter, a devious variant of compression was invented: the twist. Things are said slowly, elaborately and explicitly. But then a twist occurs that changes the meaning of it all. Everything said so far should be re-interpreted. Since the change is abrupt and compressed into one line, there is no time for conscious digestion of the new information. As a result, most of the meaning is conveyed only subliminally. A clever way to achieve the effect of knowing-without-knowing.

The life of the medieval Spanish-Jewish poet Shlomo Ibn Gabirol (1021-1070) was as hard as any. Orphaned at a young age, he was tormented by a terrible skin disease accompanied by digestive illnesses. His only comfort was in his patron Yekutiel. The following is an elegy on Yekutiel, which you realise only in the last line.

See the Sun
Translated by A.Z. Foreman

See the sun gone red toward evening now

As though it covered itself in a shroud of crimson,

Stripping the edges of dead north and south,

Putting a violet pall on the west wind.

And earth - left in her nakedness below -

Seeks shelter in the shadow of night, and rests;

And then the skies go silent black as though

Covered in sackcloth for Yequthiel's death.

The last line calls for re-interpretation of every previous piece of information. You now understand that Yekutiel was like the sun for the poet, that the night is the darkness that will prevail as from now in his life; that the day is a symbol of human life, which ends – and more and yet more.

ORDER

The most beautiful thing we can experience is the mysterious. It is the source of all art and science. (Albert Einstein)

Why do we enjoy music? One of the more convincing theories follows ideas of Herbert Spencer, who described human actions in mechanical terms, in particular ‘minimisation of energy expenditure’. You hear sounds that seem chaotic, muster energy to cope with the chaos, but then discover a hidden order – in the timing, namely the rhythm, and in pitch – the harmony. The order in the latter was discovered by Pythagoras, who found that tones that sound pleasing when played together have simple frequency ratios. He made this a cornerstone of his philosophy of nature, claiming that numbers govern all.

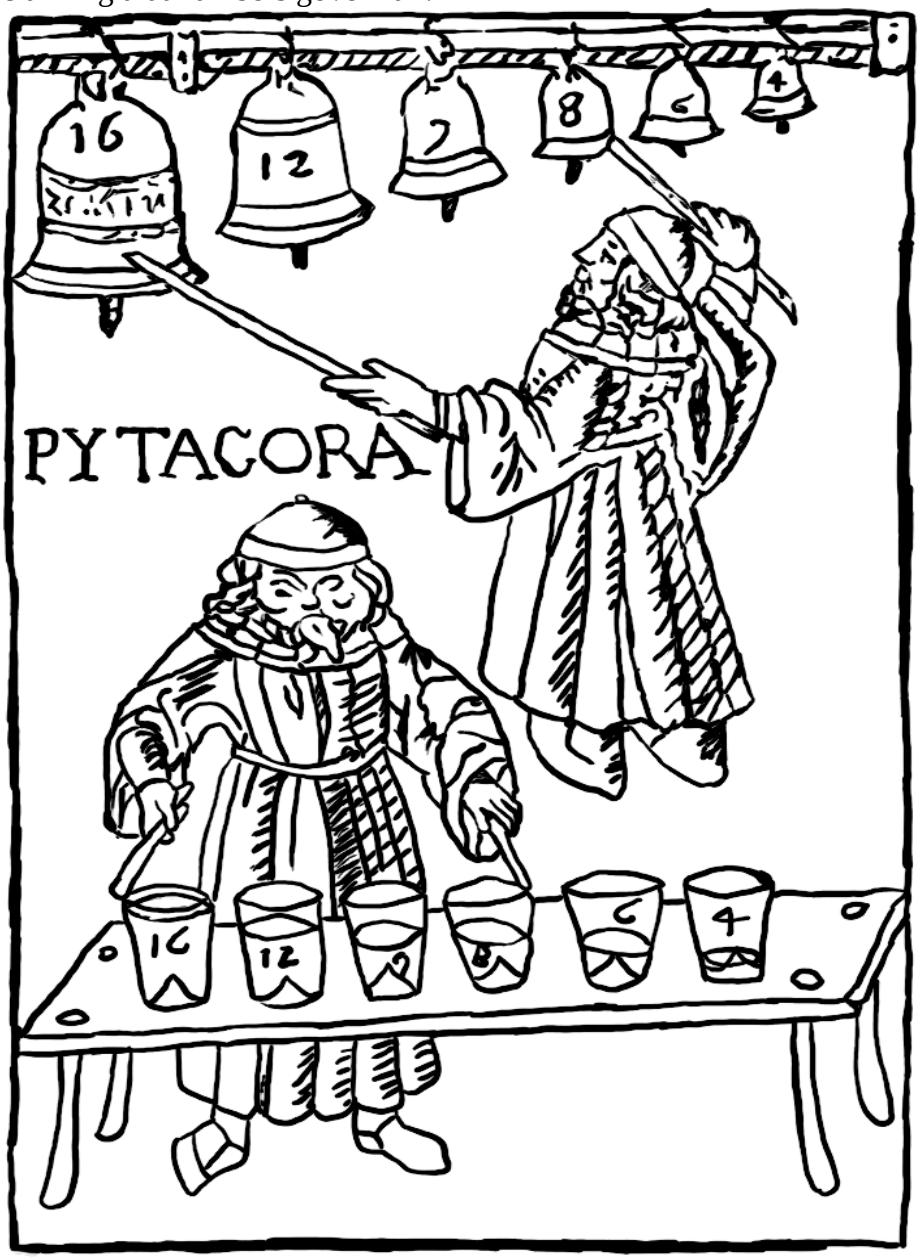

Fig. 1: Pythagoras discovered that musical harmony is governed by mathematical rules. The secret is order: different pitches sound harmonious when there is order in their frequencies. The sensation of beauty arises when we sense this order subconsciously.

Some two thousand and four hundred years later, Helmholtz explained how our ear recognizes the order in pitch – two tones of simple frequencies ratio have not too distant common overtones.

Once order is perceived, Spencer deduced, the energy generated to deal with the stimuli can be saved. The energy saved (this is also a Freudian idea) turns into pleasure, similar to the glee you feel upon victory over an enemy, when the energy for the fight is no longer needed.
This fits well the 'knowing-without-knowing' pattern. To generate the effect of beauty, the order must be hidden, or else the energy preparation stage will be skipped. Mozart told his father that 'there is something in my music for the amateurs as well as for the experts' – there is overt order, and at the same time covert one. The deeper order (that is aimed at the experts) remains un-deciphered even in the hundredth hearing. 'If a poem hides half of its ideas, we shall never tire of it', said the Japanese poet Basho.
The same is true of mathematics. An idea is beautiful when it is too surprising or too condensed to be fully understood. Famous in this respect are concise formulas, that express in a few symbols deep truths. Euler's formula:

$$e^{\pi i} = -1$$

is so mysterious (how can these numbers, which come from such different mathematical realms, be joined together?) and so useful, that it will never reveal to us even 50% of its depth. It will probably continue to mystify mathematicians 1000 years from now. And so will Einstein's $E = mc^2$. The beauty of it testifies that we formed the concepts of 'energy' and 'mass' correctly. Beauty is the ultimate touchstone of truth. Paul Dirac, the great British physicist who developed Quantum Mechanics, said that the beauty of a formula is more important than its compliance with experiment.

Perhaps the closest in the literature to the idea of `knowing without knowing` is the approach using information theory. Schmidhuber (1990)[2] related beauty to compressed description. Beautiful objects, so he and his followers found, have low Kolmogorov complexity, meaning that they can be described simply. An order is found, where disorder seems to prevail.

ECONOMY OF ENERGY

As a child, I found a story in an elementary school textbook. A king hired two artists to paint the walls of a hall in his palace. Each had to paint half of the hall. One artist toiled for months, and produced marvellous images. The other rested all this period. When the day came to show their work to the king, the lazy painter glazed his side with mirroring paint. 'You see', he told the king, 'my side is just as beautiful as the other'. The king summoned the first painter to his side and gave him his pay. To the other painter he said, 'Go take your pay in the mirror'.

[2] J. Schmidhuber. Curious model-building control systems. International Joint Conference on Neural Networks, Singapore, vol 2, 1458–1463. IEEE press, 1991

The moral of this story should have been that leeching has its penalties. But mathematicians will appreciate the ruse of the second painter: he saved energy, using symmetry. 'Mathematics is for the lazy', said the Hungarian mathematician George Polya. 'It means letting the principles work for you.' In this case, the principle of symmetry. Here is a famous case in point. How to prove that light is reflected at the same angle at which it reaches the mirror? The principle here is that light travels the shortest way possible (like mathematicians, light is lazy). Mirror the target point, the one the light reaches, and note that the reflected line is equal in length to the line itself. And the shortest line between the source of light and the mirror image of the target point is of course straight. You can see it in the following diagrams:

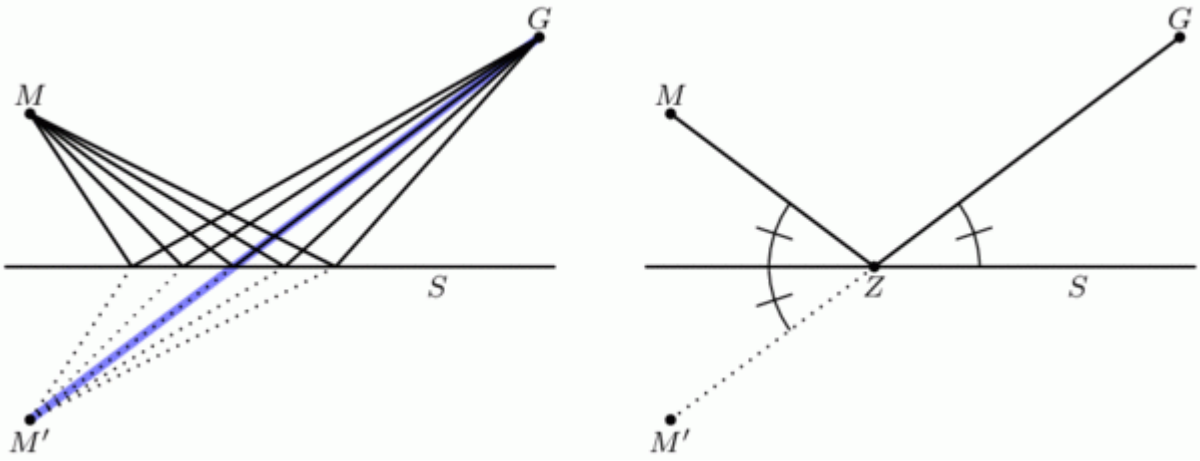

Fig. 2: Economy of thought is one of the secrets of beauty. A new element introduced into the picture – here mirroring the target point – suddenly makes things simple.

NON-CONSTRUCTIVE EXISTENCE PROOFS

There are many contenders for the title of 'the most beautiful mathematical proof'. But by many criteria - simplicity, unexpectedness, sophistication relatively to its period - there are few challengers to Euclid's proof that there are infinitely many primes. (A number is 'prime' if it has no divisors other than 1 and itself.)

First, establish that every integer has a prime factor – just keep factoring the number and its factors, until you can no longer factor.
Suppose now, by negation, that there are only finitely many primes. We want to show that this is impossible, namely it produces a contradiction. Multiply all prime numbers (we are assuming that there are only finitely many of them, so we can multiply them). The product you obtained, plus 1, is not divisible by any of the mulitplicands (if a number is divisible, say, by 3, then adding 1 results in a number not divisible by 3). By our assumption that these multiplicands exhaust the list of all

primes, we obtained this way a number not divisible by any prime, contradicting the previous statement (that every number is divisible by some prime).

The beauty of this proof is mainly in that it is non-constructive. It does not produce a new prime – it only shows that such a prime must exist. There is beauty in that, because there is mystery. The new prime is ethereal, elusive, yet we know it is there.

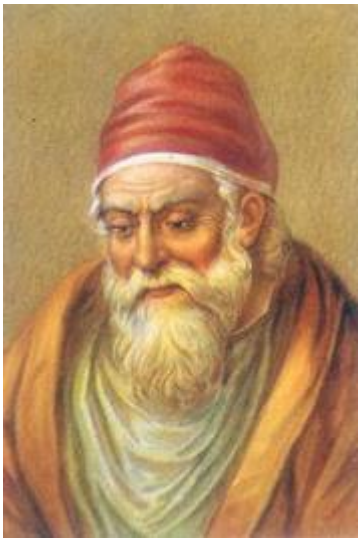

Fig. 3: Euclid, the great Alexandrian mathematician, who lived in the 4th century BC. His books, those that survived, served the mathematical world for centuries.

Existence proofs that do not point at the object, but just show it must exist, are always beautiful. A famous technique used by such proofs is the pigeonhole principle. For example, there are two people in Prague (the place where these lines are being written) having the same number of hairs on their heads – isn't it surprising? The proof – there are more inhabitants of Prague than possible numbers of hairs on a person's head. With the surprise comes aesthetic pleasure, due to the economy in energy embodied in the fact that I know there exist two such people, without having to search the city and count hairs.

UNEXPECTED TOOLS

In 1955 Martin Kneser, an algebraist, proposed a conjecture in pure combinatorics. The question he asked is this: suppose you want to partition the k-subsets of an n-set into classes P_i, so that the k-sets in each P_i pairwise intersect . How many classes would you need?

There is a simple way of partitioning. Let the ground set be $\{1, \dots, n\}$. For every $i \leq n - 2k + 1$ let P_i be the set of k-tuples that contain i, but no $j < i$. Of course, every two sets in P_i meet – both contain i . Namely P_{n-2k+1}, is defined as all the rest of the k-sets, namely all k-tuples that do not contain any $i \leq n - 2k + 1$. All k-tuples in this last set are contained in the set $\{n - 2k + 1, \dots, n\}$

, and since this set is of size $2k-1$ there is no space in it for two disjoint k-tuples. So, its elements also intersect, pairwise.

So, we have partitioned the k-tuples into $n-2k+2$ intersecting parts. Can we do better, namely partition into fewer intersecting sets? Kneser conjectured that no, this is optimal. This was open for 23 years, and was eventually proved by Lovasz, in 1978. It is acknowledged by all to be one of the most beautiful proofs of all time. The reason is that very unexpectedly, the proof uses topology. It is based on a well-known theorem of Borsuk, belonging to the "fixed points" family: a continuous function from the n-dimensional sphere to the n-dimensional ball necessarily sends some two antipodal points to the same point. Forty years have gone since this breakthrough, and no truly combinatorial proof is known. Some order governs this problem, that for some mysterious reason is geometric in nature. Hidden order – have I mentioned it already?

MIND OVER MATTER

It is always beautiful, when mind wins over matter. For example, in Dickinson's `Adrift' the inner forces beating external appearance is mind over matter. A short mathematical proof that capsules a lot of wisdom in a few lines is mind over matter. The invisible gaining over the visible is just the phenomenon we are pointing out – the "blind spot", the deep currents that are only indirectly expressed by external appearance.

One domain in which this is easy to observe is chess. Easier than in mathematics, since it is not as deep. Like in the case of mathematics, people play chess and watch chess for the aesthetic pleasure. In fact, the entire game is based on "mind over matter" – you can sacrifice everything, as long as you capture your opponent's king. The quest for the king is a rather abstract notion, through most of the game. In the first stages the players try to gain space, or material – the visible and the tangible. But all along they have in mind a final goal.

There are "beauty contests" in chess. Every big tournament has, besides the usual prizes for the winners, a beauty prize. It is not hard to pinpoint the two ingredients that make an idea beautiful. One is unexpectedness, and the other superiority of mind over matter. More often than not, a beautiful "combination" (ruse) involves sacrificing matter to obtain a long-range advantage.
Here is an example that even people only rudimentarily acquainted with chess may appreciate. It is a move widely regarded as one of the most beautiful ever played. It is taken from a game between Topalov and Shirov, two prominent present-day players. There are not many pieces left on the board. Besides the few pawns, each player has only one piece left. One would think that he would guard his piece like the apple of his eye. But black gives up his piece.

47...Bh3

The black bishop moves to the place where it can be taken by White's pawn (pawns take opponents' pieces diagonally). This would probably be most people's answer to the question "what is the most unlikely move to be played here" – giving your piece with no apparent good purpose. In the aftermath, you realize the purpose – freeing the square for the king, who wants to advance and support the advance of the black pawns. This can be done in many ways, but the point is that Black wants to make white waste a move (just one move!) on capturing the bishop. Time is more than material. As we said - beauty is in what you do not see. Something is going on that is not on the surface. Chess teaches us non-materialistism.

SELF-REFERENCE, BEAUTY AND HUMOR

A serious contender for the title of "the most beautiful mathematical proof" is Cantor's proof that the reals are uncountable. If you could count all reals, you could certainly count all numbers of the form 0.01101… - the numbers between 0 and 1, that have only 0's and 1's in their decimal expression. Say, $r_1, r_2, r_3, \ldots$ is a counting of all these numbers. But then, says Cantor, I will construct for you a number α of that form that you have not counted. The first digit of α is the first digit of r_1, reversed (0 if the first digit of r_1 is 1, 1 if it is 0). The second digit of α is the second digit of r_2, reversed, and so forth. Then α is different from each of the numbers r_i – it is different from r_1 in the first digit, from r_2 in the second digit, and so forth.

What makes this argument so beautiful? First, it is incredible that you can prove such a profound fact, having impact on all of mathematics, in one paragraph. Importance is usually associated with difficulty. But it is not only that – it is the turning of the construction (in this case, of the assumption of countability of the reals) against itself. A number (in this case, α) can encode infinite information, more precisely – information of power $\aleph_0$, and thus can rebel against a countable "contestant".

The generalization of this theorem is that it is impossible to "earmark" every subset B of a given set A by a different element $x(A)$ of A. There are just too many subsets. The proof is again surprisingly short – if such an earmarking existed, then the set C of elements earmarking a subset

not containing them could not be earmarked: $x(C)$ being in C would mean that $x(C)$ marks a set not containing it, so it cannot be in C (by the definition of C). On the other hand, $x(C)$ not being in C would mean that $x(C)$ marks a set containing it, so it is in C. This has an even clearer tinge of self-reference, and indeed, it was used to form the famous paradox of Russell, of the set of sets not containing themselves. Russel didn't really invent his paradox – he just analysed a paradox found by Cantor, resulting from the assumption that there is a "set of all sets". Paradoxes are usually formed by forbidden self-reference.

Why does self-reference generate a sense of beauty? Because it confuses us. Is the object in the world, or in me? We shall return to it later. In the meantime, here is a poem with self-reference, and a joke. As I told you, jokes draw their funniness and poems their poetic charm from a similar source.

- *What is worse, ignorance or apathy?*
- *I don't know. But who cares?*

In poetry, famous are the poems on the art of poetry: "ars poetica". Here is the last stanza from the poem "Ars poetica" by Archibald Mcleish

A poem should be equal to:
Not true.

For all the history of grief
An empty doorway and a maple leaf.

For love
The leaning grasses and two lights above the sea—

A poem should not mean
But be.

Let me close this section with a story that is funny, and also has some tinge of beauty to it.

> *Two friends walk in the wood. "You know", boasts one of them, "I can telly you precisely how many needle-leaves there are on this pine tree". "Yes?" said his friend, "how many?" – "675,312" said the braggard. "Indeed?" asked his friend, seized a fist of needle-leaves, and asked – "and now how many?"*

Both beauty and funniness are born from turning the bragging against itself – I will use your two pretensions (of knowing the number before and after) against each other.

SYMMETRY

Tyger! Tyger burning bright

In the forests of the night

What immortal hand or eye

Could frame thy fearful symmetry.

(William Blake, 'Songs of Innocence and Experience')

Studies[3] show that people with symmetrical faces are perceived as more attractive. This is part of the general rule, that signs of health and youthfulness are promoted by natural selection. But there is more to it than that. There is beauty in symmetry, beyond plain health.
Does this fit the idea of 'knowing-without-knowing explicitly'? Indeed. Symmetry is order, and a mysterious one at that. The symmetry of the Taj Mahal is refined. You find more and more symmetrical details, that you didn't perceive at first sight. Children will be awed by simple symmetry. The experienced eyes of grownups need more delicate symmetry to sense beauty.

Like in the story of the "painter" who used a mirror instead of painting his side of the palace's hall, symmetry also is economical in perception. You see half of the picture, and know it all. As Spencer pointed out, saved effort turns into aesthetic pleasure.

3

Jones, B. C., Little, A. C., Tiddeman, B. P., Burt, D. M., & Perrett, D. I. (2001). Facial symmetry and judgements of apparent health Support for a '' good genes '' explanation of the attractiveness – symmetry relationship, 22, 417–429.

Fig. 4: The Taj Mahal, a 17C mausoleum in Agra, is considered one of the most beautiful buildings ever built. Its symmetries are too subtle to be consciously grasped in full.

LAWS OF CONSERVATION

Let me return to the beginning of the paper: why did the child of Grade 2 sense beauty? The secret is "hidden order". Both words are crucial: an order should be discovered that you (almost) knew beforehand. There is a jump that does not cost you energy, since it is there, in your mind. The child discovered an underlying structure, that was there before. We all know the law of conservation of number, regardless of the organization of the objects. If you reshuffle stones on the table, their number does not change. If you re-arrange the fingers, their number remains the same.

"Poetry is always search for the truth", said Kafka. And we can add – truth that you know, deep inside. It is "knowing without knowing". An ancient Jewish belief is that before it is born, a baby knows all the "Torah" (religious books), but when it is born an angel comes and slaps him (I guess this refers only to male babies) on his lip, and he forgets all. Something similar happens here – you feel that even if you did not know this, you could know.

So, it is revealing hidden order, effortlessly. It was always there. And this is also why people laugh when they see the trick. Two things connect, but so effortlessly that the connection remains subconscious.

WHY DON'T WE TIRE

A poem that hides 50 percent of itself is never going to tire us.
Basho, Japanese Haiku poet

Music lovers can listen to a great musical piece hundreds of times, without tiring. The same goes for reading great poetry, and I personally can teach the proof of Sperner's multicolored simplex theorem many times, each time enjoying it together with the students.

Why is this so? The (well-known) answer is that even after the hundredth hearing, we do not fully understand the order. Even the performer, who knows every note and has discussed each line a hundred times with his teachers, students and colleagues, has not grasped it all. The reason for that is that the mind of the composer is infinitely intricate, and every piece is a projection of part of it on the world. Every piece of Mozart is an expression of his entire mind. We only see glimpses of it, but each such glimpse is a gift. "Unity in variety", so put it Felix Mendelson. The piece is extremely complex, but there is a guiding line. And it is important that this line is not fully comprehended.

WHY BEAUTY?

An even harder question than "what is beauty" is "why beauty" – why has evolution imprinted in us the capability of experiencing its joy? Does it have survival value, or benefit? The fact that it so ubiquitous, and such a central part of our lives, indicates that the answer is "yes". The ability to appreciate beauty is not a luxury. It must have some essential function. To see this, just imagine life without arts and without beauty.

To find this function, let me follow the footsteps of the Chinese-American writer Lin Yutang, who spoke about the function of humour in our lives. Humour is intended, so he claimed, to change our way of thinking.

Before going deeper into this claim, let me dwell on how important is the ability to change. It is an extremely difficult task. I know it from my personal life: whenever I have some insight about my relationship with people, I later realize that it comes from the very same place (motives) that this insight concerns. Meaning that it is not really new, and not so useful. Our grips on the world – conceptual images on one side and desires and wishes on the other – are so strong that they are not easily changed. To make the change, we must severe some links connecting us to the world. "I renounced an axiom", was Einstein's answer to the question how he made his discoveries.

Humour is based on severing connections – be they conceptual or intentions (an intention connects an action to part of the world – the aim). Take for example the dialogue between two octogenarians:

- *Do you remember how we used to chase girls?*

- *Yes. But I don't remember why.*

The connection between the action (the chasing) and its aim (in this case the connection is the drive) is severed. When a link is severed, others can replace it.

In short, we want things, mostly unconsciously, and would not let go. The wisdom that comes with age should not be ascribed to experience, but to the weakening of drives – old people want less, and hence understand more. The wishes do not stand in understanding's way. Beauty, humour and art in general, are a way to attain wisdom without paying the price of old age.

All arts (and dreams, and play) are based on severing functionality. They are not functional. Responses are enacted that are detached from actual action. "The function of music", said the British conductor Thomas Beecham, "is to release us from the tyranny of conscious thought". This is true of all beautiful things – the "blind spot" effect means that consciousness is being circumvented. Which enables change.

Let us return to the beauty of a stupendous mountain. It is beautiful because it is too big for us to even imagine how to tackle it. Most objects that we meet in our everyday life we consider from a functional point of view. What use can we make of it, how to tackle it, or avoid it if it poses a threat. The mountain leaves us speechless. Of course, a plain huge bulk would not have the same effect. It needs also to be complex, beyond easy perception.

Beecham was a great believer in the usefulness of beauty. "Every citizen should be forced to listen to Mozart for a quarter of an hour a day", he said. Britain would have been different if they followed his advice. In fact, this is true for the entire world. The reverence evoked by beauty could replace religion. Beauty is the best way to convey that there is something beyond the here and now.