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How to engender learning in the learning process? Mathematics, events and the invention of a mathematical education

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Abstract**:** This article proposes an exercise of problematization and denaturalization of a math classroom that is born from research carried out with a cartographic approach, in which, when occupying the elementary school math classroom, questions are asked: what happens in a math classroom? Which mathematics happens in a classroom? The event is here in affinity with the philosophy of the event of Gilles Deleuze, in which event is that which cuts time off, drags it, breaks it. Events are possible worlds that erupt *in* the things that happen. To do so, the writing of this paper takes place in relation to an episode in a classroom, evoking events that involve mathematical production, "mistakes" and learning, experiencing engendering of learning in the learning process. This writing involves two problematizing movements: the first one unfolds the problem of connection between learning and teaching in mathematical education; the second one in which, by the force of the event in the math classroom, the connection between learning and teaching as something necessary is under suspicion since it suffers an inflection in tradition in and of a mathematical education.

*Keywords***:** Base Math classroom, Mistake, Teaching, Learning, Cartography.

Introduction

"So there was something you could learn... what? Little by little, she would know, certainly. Lóri wanted to learn, did not know where to begin and felt ashamed" [3]
(Clarice Lispector, An apprenticeship or the book of pleasures).

Math classroom: a teacher, some students, a mathematics, a syllabus, some rules, a discipline, a teaching for a learning process... In a math classroom: what happens? Math classroom: a space-time already known for long: a didactic and a methodology of teaching mathematics lead us to think: how to teach mathematics? How to propose activities and procedures that can reach more and more students? A kind of psychology and sociology of mathematical education are engaged in studying: why do students

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[3] All quotes have been translated from Portuguese into English by the authors. Therefore, all papers, articles and books in the references have their Portuguese titles.

not learn? What are the reasons or factors that prevent or hinder learning? A certain way of understanding the curriculum asks: what mathematical contents should be taught? What curriculum for a basic education? Many other areas and knowledges focus on the classroom. Scanted and overcoded by knowledge and powers and disciplining, a math classroom emerges as the place of teaching and learning mathematics. Made natural both as an object of investigation and as an experiential time-space, “everyone knows” what a math classroom is and what happens there. It is just that the “everyone knows” “[...] is the form of representation and the discourse of the representative. When Philosophy ensures its beginning with implicit or subjective suppositions [the 'everyone knows that'], it can therefore play the innocent, because nothing has kept, except, it is true, the essential, that is, the form of this discourse". (Deleuze, 2006, pp. 129-130). The “everyone knows” drags mathematical education and research with the classroom: learning that happens takes place as a consequence of teaching. Therein lies a mathematical education occupied with the production of improvement of the quality of teaching so that a more effective, or significant, or operational, or efficient learning... may happen. “Everyone knows” that effective learning and therefore a good quality of mathematical education depend on a well-trained teacher with a solid background in mathematical content and with good didactics. Effective learning happens because of efficient teaching.

This article proposes an exercise of problematization and denaturalization of a math classroom that is born from research carried out since 2006, with a cartographic approach (Deleuze & Guattari, 2007; Rolnik, 1993; Guattari & Rolnik, 1996; Kastrup, 2008), in which, when occupying the elementary school math classroom, questions are asked: what happens in a math classroom? Which mathematics happens in a classroom? The event is here in affinity with the philosophy of the event of Gilles Deleuze, in which event is that which cuts time off, drags it, breaks it... Events are possible worlds that erupt *in* the things that happen. The event opens a field of the possible[4] and resists the ideality of the generic fact, which is

[4] The expression “opens a field of the possible” points to two directions. In the first, the event breaks the causal relationship, that is, "[...] the event itself is in disconnection or in rupture with causalities: it is a fork, a meander in relation to the laws, an unstable state that opens a new field of the possible." (DELEUZE; GUATTARI, 2016, p. 245). In the second, as an effect of the first, opening a new field of the possible is imposed as a movement of

non-relational and effectively departed from the movement of variation of life. The generic fact: generic mathematics, ideal, the one that composes the ideal curriculum: the mathematics that must be taught and that drags with it the entire educational process of schools– methodologies and didactics, times and spaces – away from the classroom, from relationships, from events, and so on. Which mathematics happens in the classroom, beyond and below this ideality – mathematics, curriculum, methodology, didactics...? Which learning happens?

By problematizing this classroom, this mathematics and this teaching-learning a field of research establishes an existence with the classroom that places its attention on the unusual, on the unpredictability, the inconstancy: the classroom as an event. This problematization drags to other places a naturalized understanding of mathematics, the classroom, learning and teaching. Thus, the act of learning is not to retain from the world elements that allow us to recognize objects, the act of learning mathematics is not to retain from this subject that which allows us to reproduce its objects. On the contrary, it always involves problematization, contact with the outside that forces you to think, to learn. It is by the force of this problematization, of this contact with the outside, that learning is engendered in the learning process. Initially, learning calls into question the layers that are structured as a substance or an essence. By problematizing the substance, the act of learning engenders action, it is a verb, pure force that, once put in motion, begets creation.

The writing of this paper takes place in relation to an episode in a classroom, evoking events that involve mathematical production, "mistakes" and learning, experiencing engendering of learning in the learning process. This writing involves two problematizing movements that have a close relationship with each other, not of cause, effect or overcoming, but of composition with an episode of a math classroom that was observed in its processes in the research developed by the Travessia Research Group. The first movement unfolds the problem of connection between learning and teaching in mathematical education,

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creation: "Deleuze reverses the usual relationship between the possible and the event. The possible is what can occur, effectively or logically. [...] In Bergson's wake, Deleuze says otherwise: as far as possible, you do not have it before, you do not have it before you created it. What is possible is to create what is possible." (ZOURABICHVILI, 2000, p. 335). What is then the field of the possible? One cannot answer a priori, only in the event.

which leads to the discussion of the notion of "mistake" in teaching and learning mathematics. A second movement in which, by the force of the event in the math classroom, the connection between learning and teaching as something necessary is under suspicion since it suffers an inflection in tradition in and of a mathematical education. These two problematizing movements constitute an investigative field, in a cartographic observation of the processes in math classrooms: existences, lives that take place in a classroom, together with mathematics and mathematical education.

A Milieu

"In sudden rebellion she did not want to learn what he patiently seemed to want to teach and she herself want to learn." (Clarice Lispector, An apprenticeship, or the book of pleasures)

The inspiration from Clarice Lispector's beautiful excerpt leads us to think about teaching and learning, learning and pleasures, pleasures and desire. In the universe of research in mathematical education, teaching and learning appear as almost mandatory discussions, folded and unfolded by their connections with mathematics, with development, with culture, with history, with the curriculum, with didactics; in short, with so many other elements of tradition. It is as if, depending on what connections are prioritized, mathematical education produces a unique way of understanding the space in which teaching and learning take place: the classroom. That is, the classroom appears as a space in which a whole mathematical education is built, using strategies and ways of understanding learning as an immediate result of these teaching strategies.

However, the quote also leads exactly to the problematization of that necessary link between teaching and learning, so characteristic of a certain way of handling mathematical education. That is to say: what happens when, in *sudden rebellion*[i], one does not want to learn what someone patiently wants to teach and what, at the same time, one seems to want to learn? It is by the problematizing force of this issue that this paper is constituted.

Since 2006, the research in mathematical education carried out within the scope of the Travessia Research Group [ii]has been dedicated to problematizing the classroom as a naturalized space in mathematical education. One of the effects of this research can be translated into a question: *which mathematics happens in a classroom?* In this paper, we analyze an episode of a math lesson in a seventh grade class of Elementary School in order to propose a discussion that is entangled by the question that names this paper: how to engender learning in the learning process?

Mathematical education of the link teaching-learning: 17x – x = 16x

Math teacher, seventh grade class[5]. Content: 1st degree equations. Discussions around a new element for Elementary School students: it is at that point that something starts to appear which, once present in school mathematics, will dominate much of the time and the language used in the solving of activities, modeling of problems and introduction of other contents: the *x*. Attentively, the teacher knows that students need to understand the simplification of like terms to learn how to solve 1st degree equations. That is why she begins the lesson with the explanation, *always nice and properly*, surrounding herself with care in the use of the language and of the examples, so that students learn how to operate with like terms:

Guys, first we have to know that: we can only add and deduct expressions with the same literal part. For example, we can add 5x and 3x, because in both the literal part is x, do you understand? Now, if we have 9x minus 4y, we can't deduct it. When we solve a 1st degree equation, we almost always find the variable x, then we can operate with the variable. Let's go back to the example I gave: 5x + 3x. How do we solve this? Well, to solve it, we can think like this: what is 5x? It's five time the variable x, right? When we say five times something, we're talking about that thing added to itself five times, right? So 5x is the same things as x + x + x + x + x. Has everybody understood that? Understanding this is very important! Now, let's move on to 3x. What is 3x? It's three times the variable x, isn't it? If we do to 3x the same thing we did to 5x, we can write that 3x equals x + x + x. Now see what happens: Didn't we want to add 5x to 3x? 5x is x + x + x + x + x and 3x is x + x + x. So if we add everything to the same expression.,

[5] The episode that comes into play this time comes from a math classroom, the result of an internship action in Elementary School. The Travessia Research Group has been, in its research, addressing the problem of learning. With this, the classroom is taken, therefore, as privileged investigative space. These investigations have been using cartography as a methodological device, as we will discuss later. Finally, we highlight the disruptive strength of this episode, which is also discussed in Clareto (2015) and in Camarota, Rotondo and Clareto (2019).

we get $5x + 3x = x + x + x + x + x + x + x + x$*. Now let's go to the final question: how many times is x being added here? Eight times, right? How do we write eight times x? 8x. So, 5x + 3x = 8x.*

The teacher finishes the explanation by writing on the board.

Miss, so every time that we have some x plus some other x we have to do all of that? What's the result? 8? What is this x again? When we add, don't we get a result? Like a number? 4 plus 4 is eight.

The question makes the teacher realize that she must take one step at a time, in order for the students not to get lost. She uses the student's question to say the following.

Guys, we've talked about this: x is a value we still don't know. When can only find the value of x when we start to solve the equations. What we're doing today is a step before we start to solve, ok? So take it easy, we'll get there. X is a number, we just don't know which. Now about the other question you asked, if we need to do all of this process every time, the answer is no. I did it like that to show you how it works. Now, let's see: we did 5x + 3x, wasn't it? And the result of that was 8x. Now look: if we add 5 to 3 we get 8. And 8 is there in the result. We just repeat the x. So to do 5x + 3, we can add 5 to 3 and repeat the x. Now another example: if we try 4x + 7x. We add 4 to 7 and repeat the x. So 4 plus 7 equals 11 and we repeat the x. So it's 11x. Do you understand? This way you don't have to do all of that long process I showed in the beginning. This works when we're adding, but also when we're deducting. For example, if it was 9x − 4x. We do 9 minus 5 and repeat the x. So we get 5x.

In order to be sure that the students had understood all of that, the teacher shows the longer process of deduction as she had done to addition. In the end, she writes on the board a list of operations. The students, with their pencils and notebooks, start to solve the operations proposed by the teacher. While walking among the desks, the teacher tries to pay attention to the answers the students give to each exercise. Until she sees, in a girl's notebook, the following: $17x - x = 17$.

* * *

A mathematical education comes into play. On one hand, it is necessary to assert that $17x - x = 17$ is a mistake and to understand why the girl makes that mistake. On the other, it is also necessary to create a way of correcting the reasoning that leads her to assert that $17x - x = 17$. Two mechanisms come forth: first, an image of the thinking in which knowledge works as a result or the "generality of the concept or the calm possession of a rule of the solutions." (Deleuze, 2006, p. 236); second, because of the first, a mathematical education that asserts the connection between teaching and learning as a relationship of cause and consequence. This is why claiming that a student makes a mistake is not only an evaluation that is the result of mathematical knowledge, but also the triggering of an entire pedagogical mechanism that intends to fix that mistake, restitute to the thought process the image of the possession of rules and of solutions, or to give the girl who has not learned the image of thought that is capable to lead her to assert that $17x - x = 16x$.

However, what is a mistake? How does it make a mathematical education work?

* * *

How can mistakes exist? How and why does a student make a mistake in a math classroom? How to evoke the formation of the mistake in the mathematic thinking of the student? Between the possibilities of correcting mistakes and a process of looking at the mistake in its potential to improve learning, a mathematical education gains form. By understanding mistakes as something to be corrected, be it by looking at the mistake as part of the learning process of the student, be it by understanding it as a motivational aspect of learning, the problem of analyzing mistakes maintains itself as constitutive of the learning process and evokes the link between teaching and learning.

Mistake as that which

> [...] does not correspond to the production expected of a student (or teacher) who must have already encountered the subjects presented in that particular question or the strategies to solve problems in Mathematics. Thus, it is a referential that takes for the supposed truth the institutional knowledge, which means what the institution "School" expects to be displayed by students (or teachers) of a certain level in their written productions in Mathematics (Cury, 2010, p. 2).

Mistake as the non-fulfillment of a teaching expectation offered by the teacher. Teaching and learning in a tight relationship of cause inconsequence. Mistake as the lack of syntony between the teaching of the teacher and the learning of the student.

The expectation of the teacher, which might be broken by the mistake, has by principle three theses: mathematics is, in its nature, intelligible *(good nature of thought*); the student is, by nature, intelligent (*willingness of the thinker*); mathematical knowledge has, *a priori,* affinity with truth (*representation of a true world*). Once these three theses are assured, the issue of learning mathematic subjects is then surrounded by methodological or didactic questions, psychological of cognitive questions, questions of curricular adequacy or hierarchic allocation of contents, social or economic questions… The three theses assure from the start the connection between teaching and learning. An irreducible and hierarchical connection. A cause to a consequence. Teaching leads the methodological passage between the truth that is still unknown, yet potentially possible to be known: $17x - x = 16x$. This passage occurs, on one hand, due to the intelligible nature of mathematical knowledge, that is in tune with true knowledge, and on the other, due to the student that is apt to learn, once he or she is intelligent. The production of the true image of mathematical knowledge ($17x - x = 16x$) lies within the realm of the mediation of the teacher and his or her teaching.

Thus, how are mistakes possible? How to conjure away the formation of mistakes? Mistakes are the effect of external forces – passions, the body... – that stand against thought; consequently, against the truth. The mistake is, therefore, external to thought, causing it to devise from the truth. The problem of mistakes becomes, then, a methodological problem: "the method is a device by which we recreate the nature of thought, adhere to this nature and conjure away the effects of the strange forces that alter it and distract us. With the method, we conjure away the mistake" (Deleuze, 2018, p. 133).

Mistakes, with their external causes, are now pointed out as misunderstanding, as a category of the negative. A mathematical education leads to discussion, therefore, of the not understood, of the not comprehended. Mistakes are then what is lacking. The teacher lacks the language for proper explanation, the updated methodology, the relevant training, the didactic resources and materials... The student lacks:

understanding, attention, content mastery and prerequisites... Mistakes as what divert the good thinker – intelligent and willing to think correctly – and good thinking – intelligible and with affinity with the truth.

What is certain is that, after these deviations that cause issues in mathematical thinking are overcome, the well-applied didactic-methodological strategies turn $17x - x = 17$ into $17x - x = 16x$. Studying and researching mistakes in mathematical learning produces a scan of the "path" trodden between the "not yet known" and the "true knowledge", mapping the deviations and the elements that induce error, in the expectation of proposing didactic-methodological strategies that make the connection between teaching and learning exist with fewer deviations. A mathematical education moves in this environment for this purpose.

It is an entire discourse around effectiveness that is at stake, though not explicitly. The three theses that guarantee an image of thought that is attuned to the representation of the true world erect a teleology: *one* mathematics becomes *the* mathematics. Taken as a parameter, their results become goals to be achieved. Effectiveness: one must learn that $17x - x = 16x$, and not anything else; efficiency: it is necessary to learn that $17x - x = 16x$ with the smallest possible number of deviations or mistakes; performance: one must make everyone learn that $17x - x = 16x$. It is not surprising that effectiveness, efficiency and performance are in such direct consonance with the economic discourse itself. They are a punctual application in mathematical education of a principle of general functioning of the capitalist social machine. It is as if, by effectiveness, efficiency and performance, the learning-teaching connection is fulfilled in such a way that teaching functions as an a priori *prevention* of mistakes, so that one can reduce time and investment in education while ensuring a result. An economy of desire comes into play: an exchange of libidinal flow is constituted to create subjectivity, considering effectiveness, efficiency and performance. Once subjective, such parameters begin to make the subjectivity their own control.

A mathematical education that relies on the dogmatic image of thought and the pair of correct/incorrect as correlated to this image operates then a double machine: of interpretation and subjectivation[iii]. As an interpretation machine, mathematical education takes the dogmatic image to unload on it all the mathematical sentences produced by the students. This unloading is expressed in terms of being correct whenever a student says that $17x - x = 16x$, but expresses itself as a mistake every time a student says anything else, for example, that $17x - x = 17$. Still as an interpretation machine, this mathematical education shows that the student who makes mistake in fact strives to say something else, to say what is expected of him or her. As a machine of subjectivation, mathematical education enters a space of legitimacy that, in its right is, at the limit, the space of legitimacy of the teacher's own discourse. That is, what the student learns in the course of learning is the image and model of the teacher, and the students understand as subjective the whole discourse of effectiveness/efficiency/performance as being the teacher's own discourse.

Mathematical education in the breaking of the teaching-learning connection: 17x – x = 17

"In sudden rebellion she did not want to learn what he patiently seemed to want to teach and she herself want to learn." (Clarice Lispector, An apprenticeship, or the book of pleasures)

Sudden rebellion: not wanting to learn what someone is patiently trying to teach her and she herself want to learn. The wanting of an assertive thinker who wishes to learn, because she is willing to learn: *willingness of the thinker*. However, desire permeates this wanting. A desire [iv]and its agencies: "Desire builds agency, it establishes itself in agency" (Deleuze, 1988).

What happens, then, when this connection between teaching and learning is broken? When a sudden rebellion prevents that which is patiently intended to be taught and learned from being taught or learned? What happens on the threshold of this rupture? Agency: the threshold brings forth new agency[v]: learning disconnected from teaching... How does that happen?

A new agency established by the desire not to learn what the willingness of the thinker claims as true. A desire, from the realm of the unconscious machinery, breaking apart from the willingness to seek truth. Rupture between thought and the willingness to seek truth. Two severe consequences: first, the truth is not the element of thought. That means there is no affinity between thought and truth.

A pause: how come there is no affinity between thought and truth? Perhaps it is a mistrust that comes from the affinity of truth with Good in the dogmatic image of thought: there is only affinity of thought with truth in a moral image of thought, "[...] for only Morality is able to persuade us that thought has a good nature, the thinker, willingness, and only Good can found the supposed affinity of thought with the Truth." (Deleuze, 2006, p. 193). Another question comes from the forces that act in thought: it is through the dogmatic image that the truth is imposed as a universal and abstract concept. By being this way, "one never refers to real forces that *create* thought, one never relates one's own thought to the real forces that they assume *as thought*. One never relates that which is true to what it presupposes." (Deleuze, 2018, p. 133, author highlights). But what real forces are these that create thought? Deleuze brings from Nietzsche's philosophy the answer: truth is the effecting of a meaning and the creation of a value, not a universal logical operator. This universal logical operator, the truth, has no affinity with thought, because it hides the work of the forces that overtake thought. In another way of thinking, thought has affinity with evaluation. The question is no longer whether thought produces truths, but what meanings thought creates, what values it finds.

What unfolds from this is the second consequence: mistakes no longer stand as such: they are not the negative of thought. No longer an exteriority that diverts the *thinker* from good thinking: foolishness, wickedness and madness are in the very structure of thought. They are not exteriors that produce deception, but express the non-sense of thought (Deleuze, 2006). Thought brings, withing itself, the nonsense... Thought is no longer in tune with the truth: "thought is creation, not a willingness to seek truth, as Nietzsche knew how to show." (Deleuze & Guattari, 2010). Similarly, "Thinking is creating, there is no other creation, but creating is, first of all, engendering, 'thinking' in thought" (Deleuze, 2006, p. 213). Thinking is creating and the truth is a creation of thought. Mistakes no longer make any sense...

What unfolds to a mathematical education when $17x - x = 17$ becomes something other than a mistake, when it becomes a creation?

* * *

How does a mathematical education in which 17x – x = 17 is a creation take place? Is it possible to say that a new problem emerges here? A new problem replacing the relationship between learning and teaching, re-taking "teaching" and "learning"...

What to teach? What to learn?

Learning that is not of the order of wanting – rational and conscious, willing and subjectivist – but of the order of desire – bodily and unconscious, experimental and inventive. Likewise, teaching that is not of the order of wanting – willful and teleological – but of the order of desire – experimental and that works in or due to agencies. Learning and teaching that do not have an irreducible, inseparable and teleological connection, in which teaching is irrevocably directed to learning. What relationship, then, exists between teaching and learning? What mathematical education for such learning, for such teaching?

A mathematical education that is not concerned with filling the space between teaching and learning with a methodology, with a necessary path. A mathematical education that focuses on the event of learning.

How come, event? In Deleuze, the event does not concern an effect on the state of things, a reception of these states taken indiscriminately. Rather, it is about affirming the externality of relationships and the encounter with that which forces us to think. That is, the event happens in the existence of a problem, in the connection of the world with the outside, not the outside of the world, but the outside in this world here, the exteriority of one's own thoughts (Zourabichvili, 2016). At the same time, by the force of this exterior which fosters thought, something is in the process of being constituted, of being formed.

> It is always an event that leads us to ask ourselves: what happened? Now what is going to happen? What happened to get us into this? So that we could become capable or incapable of...? Those are que questions relating to every event. [...] Something happens that changes everything, that displaces the powers and the capabilities. The event in Deleuze is the first redistribution of powers [...]. Through the event, everything begins again, but in another way; we are redistributed, sometimes re-engendered, even in an unrecognizable way. Everything is repeated, but distributed in another way, our powers being incessantly turned, resumed, according to new dimensions. In this sense, "repetition is the power of difference". (Lapoujade, 2015, p. 67-68).

Mathematical education as an event then finds its potency in the redistributions it manages to operate, breaking the causalities. A new field of the possible is opened: "The possible does not preexist, it is created by the event. It is a matter of life. The event creates a new existence, produces a new subjectivity (new relationships with the body, time, sexuality, the environment, culture, work...)" (Deleuze, Guattari, 2016, p. 246). Learning as an event opens a new field of the possible alongside tradition – the school mathematics – by varying this tradition, forcing it to start over in another way, redistributed. Varying mathematics. Opening other possible school mathematics. Other possible classrooms: a new existence, a new subjectivity... Mathematical learning as an event. How does an event vary a tradition? With experimentations: a mathematical tradition of schools takes place in experiments in and with the math classroom. Experimentation, a contact with the *ex*[vi], with the outside. In the math classroom, a contact with the outside *in* mathematics itself: the movement of redistribution of powers, which resumes tradition in other ways, makes it vary.

> Thinking is experimenting, but experimentation is always what one is doing – the new, the remarkable, the interesting, which replace the appearance of truth and are more demanding than it. What one does is not what ends, less so what starts. History [or tradition] is not experimentation, it is only the set of almost negative conditions that make it possible to experience something that escapes history [or tradition]. Without history [or tradition], experimentation would remain undetermined, unconditioned, but experimentation is not historical, it is philosophical. (Deleuze & Guattari, 2010, p. 133).

The event is not apart from tradition. On the contrary, it takes place *within* tradition: tradition – the mathematical knowledge that states that $17x - x = 16x$ – is the set of conditions that enable the *experimentation* of other ways of producing mathematics. Thus, teaching, as an element of tradition – analogous to history, to curricula, etc. – is only *a set of almost negative conditions* that enable experimentation. Is that not the case when a student in a classroom claims that $17x - x = 17$? It is still of numbers and operations, algebra and algebraic expressions that $17x - x = 17$ concerns. These elements the student seeks in the explanation of the teacher, in the conditions of teaching, in tradition, in school mathematics... Yet, that is not all: something has changed. The direction and value of tradition changes, the conditions of experimentation are not determinations of experimentation. It is a whole tradition that reaches its limit: *something has changed*.

It is by the force of this something that changes that teaching no longer implies a relationship with learning. What changes is that which can no longer be taught, that which, in tradition, produces an inflection, implies an invention, a rupture from the assumptions of an image of thought that identifies thinking to be recognized. Without being able to be taught, since it is not the object of a recognition, that which changes *can only be learned*. It is an engendering of learning in the learning process that implies an inflection in tradition by the very movement of facing problems in and to this tradition. $17x - x = 17$, what mathematics does it entail? What problems does it create in the relationship with numbers and operations, algebra and algebraic expressions? What variations, what power redistributions do such engenderings carry out?

To learn, the umpteenth power of learning[vii], reaffirms that which can only be learned, that which, in a field of experimentation and contact with the outside, forces the existence of other problems. It is a whole tradition, a whole mathematics, which is in *periri*[viii], in danger: the engendering of learning in the learning process is confused with what is being done, with the new, the remarkable, the interesting, not the object of recognition and no longer depending on the good nature of thought and the willingness of the thinker. The act of learning within the learning process is an effect of external forces taken as something external that triggers, at the same time, passions and the body, triggers one's own desire that, free from the lacking imposed by the ghost of mistakes, can engender something new in thought.

> The crisis of the dogmatic image drags with it the subject, the willing thinker. It so happens that creating does not concern an arbitrary decision or a decree. To make the truth depend on an act of creation is not to confine it in subjectivism, to subject it to the whim of an individual will [...]. Deleuze shows that, on the contrary, the act of thinking necessarily sends subjectivity into crisis, and that the need, far from meeting the notions of a constituted thinking subject, is only conquered when thought is out of itself, a thought that is only absolutely potent at the extreme end of its impotence. (Zourabichvili, 2016, p. 47).

To learn, at the extreme end of the impotence of a subject constituted in and by the dogmatic image, implies a desubjectivation, a withdrawal from the logic of identity that would allow, a priori, to understand what it means to think, what it means to teach and learn mathematics, what is a mathematical truth etc. To learn, at the extreme end of the impotence of the subject and of the truth, implies the need for invention, the redistribution of the powers of tradition: a subjectivation. Not a return to the modified subject, but a subjectivity that is the effect of engendering learning in the learning process, so without pretensions to unity and universality. "The subject, as well as the object, are effects, results of the process of invention." (Kastrup, 2005, p. 1275). A subjectivity that is adherent to movements, experiments and problematizations, porous to events to come.

Therefore, what mathematical education does all of this entail? What to learn and what learning?

Learning: a noun, the inside of the act of learning, an invagination that unravels stratifications and codifications of the movements of inventing oneself and the world in math classrooms. Learning as an object of research in mathematical education: a certain path, a certain crossing, certain confrontations become the correct path, the correct crossing, the correct confrontation: if everyone knows how to learn mathematics, then everyone knows how to teach mathematics. An if-then that constitutes a link between teaching and learning. Teaching that $ax - bx = (a - b)x$, for $a, b, x \in \mathbb{R}$ implies learning $17x - x = 16x$. From the mapping of paths, crossings and the confrontations with learning, teaching proposals are made that can be based on these paths. So, how is it still possible to state that $17x - x = 17$? It is that the internalization of learning in teaching hides what is part of the folding movement. The very inside is an operation from the outside, it is constituted with the outside, with what forces to us think, with the problematic (Deleuze, 2013). That is why to learn is the operation of learning. To learn: a verb that expresses the very invention of oneself and of the world, the umpteenth power of learning. Movement, action that engenders the classroom and mathematics in one mutual movement. Learning as the pure flow of problematization that acts in favor of the non-crystallization and non-universalization of the ways of thinking and producing. One mathematics happens in the classroom: an operation from the outside that shuffles codes from a scanted learning, engenders learning in the learning process.

An investigation field: research in *Travessia*[6]

[6] Travessia is a Portuguese word that denotes a passage, a crossing, a journey. It has been notably used in Guimarães Rosa’s *Grande Sertão: Veredas* (translated into English as *The Devil to Pay in the Backlands*). Due to the lack of an exact word to translate it, we have decided to keep it in the original language.

And constituting a problem?
It is not about truth or falsehood, it is about meaning!
(Gilles Deleuze, Difference and Repetition)

A classroom situation becomes a problem when an intern, a Mathematics undergraduate, who observes a teacher in a seventh-grade class in elementary school becomes restless before the answer given by a student: $17x - x = 17$. Restlessness that moves: how can the student make this mistake after such a good explanation? What is happening here? What was the student thinking? These initial questions give way to questions that, in relation to that event, set in action a problematization: what happens when what is taught patiently and with conceptual and methodological accuracy is not answered as expected, frustrating the expectations of the teacher (and of the intern)? What occurs? What meanings do these questions propose? What meanings do they inaugurate? What meanings does $17x - x = 17$ inaugurate?

In an exercise of problematization and denaturalization of the math classroom, a banality – often witnessed in the classroom – generates research: what if it were true? This first movement seeks to unfold the sentence $17x - x = 17$, taken as true. Truth and falsehood dispute the attention of a tradition, of a school mathematics. An already constituted truth persists and everything else is taken as an unfolding from this truth: how does one confuse the true with the false? How does one mistake them?

This exercise of problematization unfolds itself as one begins to ask no more for the negative of thought, the mistake, but for the power of the sentence given by the student: $17x - x = 17$. Here a twist happens: to take the sentence as true and to produce something with it displaces the logical operator who says: it is a false sentence, therefore, a mistake. *Something has changed*: truth is no longer the abstract and general concept that surrounds a dogmatic image of thought. On the contrary, to take $17x - x = 17$ and to operate a production of something is to radically pursue the issues: what meanings does $17x - x = 17$ have? What values does $17x - x = 17$ perform? This second movement problematizes: how does this thought occur? Or how does one come to think? Unfolding: how does teaching relate to learning? How to engender learning in the learning process?

These problematization exercises happen as the observation of processes in the math classroom. To track processes is to map, to create maps and "the map is open, it is connectable in all its dimensions, dismountable, reversible, likely to receive modifications constantly. It can be torn, reversed, it can adapt to assemblies of any nature, be prepared by an individual, a group." (Deleuze & Guattari, 2011, p. 30). Tracking processes and producing maps is what cartography does. Before it is constituted as a way of research, cartography is a way of existing in the world, a way of producing the world... A world in which everything is in process, in which all constituted forms have a constituent, a strand of virtuality. Everything is procedural. All relational. Thus, cartography is placed as a field of the possible for research, an investigative approach that is placed in process monitoring the production of maps of the geographies of thoughts and ways of existing[ix]. In our case, the geographies of the classrooms.

Thus, by observing students in elementary school in their processes of production of mathematical thoughts, research has been developed with the Travessia Research Group since 2006, following movements around the tradition of school mathematics.

The math classroom: something from within the school and the educational process opening itself to something that is outside the already thought-out and naturalized in education and mathematical education. A problematizing investigative field opens possibilities in mathematical education, making the classroom its focus of research, interest and study. Questions are raised: how does a problematizing investigative field arise and hold itself in the field of mathematical education, which puts its focus on the classroom[x]? How do everyday situations in a classroom, already trivialized or naturalized, become an investigative problem[xi]? How to generate (oneself) in research in a math classroom that has always been trivialized, naturalized in its spaces and times, in its procedures and relationships, in its propositions and paths[xii]? A classroom – which is further inside a school and a formal educational process – facing the outside, the exteriority – that which has not yet been thought out, not naturalized in this space and in this process... What happens in this encounter? Or what forces from outside a classroom and mathematics come into play? A space time that, when constituted in a field of research, clashes with the outside, its external thinking (Foucault, 2009; Deleuze, 2013). In this friction, ex-plosions and in-plosions of a classroom: they detonate the classroom from the outside in and from the inside out: what is left? What can resist this friction, underneath the math classroom? How much unusuality can resist underneath a classroom? (Clareto & Silva, 2016). A mathematical production attentive to the event. A political aesthetic ethical resistance engendering learning in the learning process.

References

Cammarota, G. (2013). *Fabulações e modelos ou como políticas cognitivas operam em educação matemática*. [Master's Thesis, Universidade Federal de Juiz de Fora].

Cammarota, G., Rotondo, M., & Clareto, S. (2019). Formação docente: exercício ético estético político com matemáticas. *Perspectivas da Educação Matemática*, *12*(30), 679-694.

Clareto, S., & Cammarota, G. (2015). Professores de matemática em formação: invenções e (sub)versões. In B. S. D'Ambrósio, & C. E. Lopes (Eds.), *Ousadia criativa na prática de educadores matemáticos*. (pp. 65-86). Mercado das Letras.

Clareto, S., & Silva, A. A. (2016). Quanto de inusitado guarda uma sala de aula de matemática?. *Bolema*, *30*(56), 926-938.

Cury, H. N. (2010). *Análise de erros: O que podemos aprender com as respostas dos alunos*. Autêntica.

Deleuze, G. (1988). O abecedário de Gilles Deleuze. Entrevistado por Claire Parnet. TV Escola.

Deleuze, G. (2006). *Diferença e repetição*. Graal.

Deleuze, G. (2013). *Foucault*. Brasiliense.

Deleuze, G. (2016). Quatro proposições sobre a psicanálise. In D. Lapoujade (Ed.), *Dois regimes de loucos*. (pp. 82-90). Editora 34.

Deleuze, G. (2018). *Nietzsche e a filosofia*. n – 1 edições.

Deleuze, G., & Guattari, F. (2010). *O que é a filosofia?*. Editora 34.

Deleuze, G., & Guattari, F. (2011). *Mil platôs. Capitalismo e esquizofrenia 2*. (vol. 1). Editora 34.

Deleuze, G., & Guattari, F. (2012). *Mil platôs. Capitalismo e esquizofrenia 2*. (vol. 5). Editora 34.

Deleuze, G., & Guattari, F. (2016). Maio de 68 não ocorreu. In D. Lapoujade (Ed.), *Dois regimes de loucos*. (pp. 245-248). Editora 34.

Dore, L. (2018). *Que diz o muro da escola? Aprendizagens e deslizamentos e matemática*. [Master's Thesis, Universidade Federal de Juiz de Fora].

Foucault, M. (2009). O pensamento do exterior. In M. B. da Motta (Ed.), *Ditos e escritos III: Estética: literatura e pintura, música e cinema*. (pp. 219-242). Forense Universitária.

Guattari, F., & Rolnik, S. (1996). *Micropolítica: Cartografias do desejo*. (4th ed.). Vozes.

Kastrup, V. (2005). Políticas cognitivas na formação do professor e o problema do devir-mestre. *Educação & Sociedade*, *26*(93), 1273-1288.

Kastrup, V. (2008). *Cartografias da invenção: pistas e políticas de um método de pesquisa*. [Conference presentation]. VI Seminário de Pesquisa do Programa de Pós-Graduação em Educação da Universidade Federal de Juiz de Fora, Juiz de Fora, MG, Brazil.

Lapoujade, D. (2015). *Deleuze, os movimentos aberrantes*. n – 1 edições.

Larrosa, J. (2002). Notas sobre a experiência e o saber de experiência. *Revista Brasileira de Educação*, (19), 20-28.

Lispector, C. (1998). *Uma aprendizagem ou o livro dos prazeres*. Rocco.

Oliveira, M. E. (2018). *Aprender enquanto travessia: entre banalidades e formações e matemáticas e línguas e peles e escritas... uma vida.* [Doctoral Dissertation, Universidade Federal de Juiz de Fora].

Rolnik, S. (1993). *Cartografia sentimental: Transformações contemporâneas do desejo*. Sulina.

Silva, A. A. (2016). *Aprendizagens em uma sala de aula de matemática*. [Master's Thesis, Universidade Federal de Juiz de Fora].

Zourabichvili, F. (2016). *Deleuze: Uma filosofia do acontecimento.* Editora 34.

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[i] A sudden rebellion does not concern the decision-making condition of a conscious subject who *wants* to rebel. On the contrary, a sudden rebellion is disruptive: it problematizes the willingness of thought and summons an inventive movement; it acts at the level of unconscious desire, and not at the level of a subject's desire.

[ii] Travessia is a research group allocated to the Faculty of Education of the Federal University of Juiz de Fora, certified by the National Council for Scientific and Technological Development - CNPq, a Brazilian public foundation whose attribution is the formulation and execution of Brazilian public policies to encourage and promote research. The group's record in the National Directory of Research Groups in Brazil can be accessed at: http://dgp.cnpq.br/dgp/espelhogrupo/585946. The group welcomes researchers with different interests around Education, having as conceptual intercessors authors of the philosophy of difference, especially Gilles Deleuze, Félix Guattari, Friedrich Nietzsche and Michel Foucault.

[iii] The inspiration for this discussion comes from *Quatro proposições sobre a psicanálise* (Deleuze, 2016). From it, we use the double machine to think about how mathematical education operates with interpretation and subjectivation.

[iv] Desire as a machine of production and not as the lack of something, as psychoanalysis wants: "Desire is the system of a-significant signs from which unconscious flows are produced in a historical social field. There is no blooming of desire, whatever the location, the small family or neighborhood school, that does not shake the device or does not question the social field. Desire is revolutionary because it always wants more connections. (Deleuze, 2016, p. 84)

[v] "We can then establish a conceptual difference between the "limit" and the "threshold", with the limit designating the penultimate, which marks a necessary restart, and the threshold the last, which marks an inevitable change. [...] beyond that limit, there is still a threshold that would change its agency." (Deleuze & Guattari, 2012, p. 140). The limit, therefore, can do nothing but repeat the same, let itself be captured and represented. It can only insist on the same agency, on the same connection of forces, on the same tradition. The threshold, on the other hand, points beyond the limit, summoning an invention, a change in the connections of forces, a change in tradition, a change, then, of agency. Despite this, there is no evolution between the limit and the threshold: "There are collective mechanisms that, at the same time, conjure and anticipate the formation of a central power. They occur, then, depending on such a threshold or a degree that what is anticipated becomes consistent or not, what is conjured ceases to be so and happens. And this threshold of consistency, or embarrassment, is not evolutionary, it coexists with its inferior" (Deleuze & Guattari, 2012, pp. 130-1).

[vi] The prefix *ex-* means "movement towards the outside" or "to be taken from". These two meanings are connected to the idea of experimentation. On one hand, the experimentation points to a connection with the outside, with the forces outside that make one think; on the other hand, experimentation is the contact with the consistency of this outside and it points to an ability to be affected, to be taken from the recognition schemes that we are accustomed to relate to the act of thinking.

[vii] This paraphrases Nietzsche and philosophy: "Thinking is the umpteenth power of thought." (Deleuze, 2018, p. 139).

[viii] The root *periri* that is present in experimentation is the same as that the root of *periculum*, danger (Larrosa, 2002). Experimenting always implies, therefore, a risk: it endangers the stratifications that constitute tradition.

[ix] A geography of thought and of the ways of existing approaches a way of existing with thought and life, placing it in relationships, taking its functioning in surface movements, in migrations, in currents, in flows. Deleuze considers geophilosophy more than a history of philosophy. Guattari states: "The subject and the object offer a bad approximation of thought. Thinking is neither an extended thread between a subject and an object, nor a revolution of one around the other. Thinking is first constituted on the relationship between the territory and the earth." (Deleuze & Guattari, 2010, p. 103).

[x] The work of Cammarota (2013) discusses this issue, by turning mathematical education and classroom research into a political problem, a problem of a cognitive policy. Cammarota, Rotondo and Clareto (2019) also discuss the problematic field of the classroom through teacher training. This emerges as an ethical political aesthetic process with mathematics.

[xi] The works of Silva (2016), Dore (2018) Oliveira (2018) surround this issue by bringing everyday situations of classrooms, the banalities, to the center of the discussion. Learning comes as the main issue and causes the diversion of an entire didactic-methodological tradition of mathematical education.

[xii] Clareto e Silva (2016) discuss this through the problematization of the notion of mistake in the math classroom, while Clareto and Cammarota (2015) explore what happens in a teacher training classroom when the banalities of a school mathematics invade this space.