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Ethnomodelling as a glocalization process of mathematical practices through cultural dynamism

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Abstract**:** Ethnomodelling can be considered as the association between ethnomathematics and mathematical modelling that enables members of distinct cultural groups to perceive a different reality in relation to the nature of mathematical knowledge. It also provides insights into many diverse forms of mathematics developed locally. Thus, ethnomodelling is defined as the study of mathematical phenomena that adds cultural components to the modelling process. The development of this connection is conducted through three cultural approaches: local, global, and glocal, which are used in the conduction of ethnomodelling investigations that aim to work against colonialism in order to value and respect sociocultural diversity of members of distinct cultural groups. Because ethnomodelling seeks to promote the development of understanding of differences through dialogue; it is important to argue for its inclusion as a translational process for systems taken from the reality of the members of diverse cultures. In this article we argue that ethnomodelling creates a firm foundation that allows for the integration of these three cultural approaches in exploring mathematical knowledge developed in distinct cultural groups through cultural dynamism.

*Keywords***:** Cultural Dynamism, Ethnomathematics, Ethnomodelling, Glocal Approach, Mathematical Modelling.

Initial considerations

Brazilian literature in the area of mathematics education is composed of a broad bibliographic production that discusses trends in the teaching and learning process in mathematics, such as problem-solving, technologies in mathematics education, the history of mathematics, ethnomathematics, and mathematical modelling. These investigations show that there is growing amount of research involving pedagogical action in the classsrooms that propitiates the development of discussions related to the connections between theoretical bases and practices that have involved two or more of these trends.

Among all of the possible connections between these trends, we highlight the connection between ethnomathematics and mathematical modelling, which provides the development of pedagogical actions in the classroom aimed at raising awareness about the cultural aspects of mathematics. For example, D'Ambrosio (1990) argues that ethnomathematics is characterized as a way of understanding

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mathematical thinking of cultural the members of distinct cultural groups while mathematical modelling functions as a tool that becomes important for individuals to act and interact in the world.

Similarly, Rosa (2000) conducted a study entitled: *From reality to mathematical modelling*: *a proposal for using ethnomathematical knowledge* in which he proposed the development of a mathematical curriculum based on ethnomathematics and mathematical modelling for immigrant students in California in order to achieve their educational objectives in this pedagogical action. The results obtained in this study showed that it is necessary that the modelling process is conducted by using the mathematizations developed by the students in their cultural groups in order to respect and value their own traditions and cultures.

Two years later, Scandiuzzi (2002) reported in the article entitled: *Água e óleo: modelagem e etnomatemática?* (Water and oil: modelling and ethnomathematics?), that there are philosophical and epistemological aspects that would make impossible the development of the relation between ethnomathematics and mathematical modelling by pointing out that these trends apply distinct research methods. On the other hand, Rosa and Orey (2003) published the article entitled: *Vinho e queijo: Etnomatemática e Modelagem Matemática!* (Wine and cheese: ethnomathematics and mathematical modelling!) in which they compare the relation between these two trends by affirming that this combination has the potential to be developed in the pedagogical action in the classrooms.

At the same time, Bassanezi (2002) addressed the relation between ethnomathematics and mathematical modelling in the book entitled: *Ensino-aprendizagem com modelagem matemática* (Teaching-learning with mathematical modelling) by coining the term *ethno/modeling* in which its meaning is related to the position of assuming mathematics as a knowledge field present in people's daily lives that enables them to considere it as strategies and techniques for action and interpretation of their own realities.

In this direction, Caldeira (2007) defined *ethno/modelling* as the mathematical knowledge constructed and rooted in the cultural practices of distinct communites, as well by considering its influences in the educational process through the use of assumptions of mathematical modelling as means to achieve the proposed objectives of this pedagogical action. In another study, Sonego (2009) defined ethnomodelling as a set of pedagogical actions that uses methodological tools applied in mathematical modelling in the social and economic contexts of the students, who witness and explore mathematics by acknowledging and respecting cultural values acquired in their daily lives.

In this same decade, Klüber (2007) analyzed these two trends in mathematics education by concluding that there is a tendency for an explicit approximation between mathematical modelling and ethnomathematics in accordance to the philosophies and epistemologies of social sciences while at the same time it shows an implicit approximation when mathematical modelling is oriented by the philosophical and epistemological assumptions of exact and natural sciences.

Subsequently, Rosa and Orey (2010a) wrote an article entitled: *Ethnomodeling: an ethnomathematical holistic tool*, in which they developed the theoretical and methodological bases of ethnomodelling. Meantime they wrote several articles in English on the theoretical basis for the development of investigations in ethnomodelling. Later, Rosa and Orey (2012) wrote their first article in Portuguese on ethnomodelling by defining it as a pedagogical/methodological approach that considers it as a practical application of ethnomathematics, which adds cultural perspectives to the mathematical modelling processes.

This context led us to discuss the biases that exist against local or non-academic orientations in relation to the presence and type of mathematical knowledge found in many places and contexts (Rosa, 2010). By acknowledging the importance of local mathematical knowledge, experiences, ethnomodelling encourages connections, debates, discussions, and sense of mindfulness of the nature of mathematics as it relates to the ongoing support to the curriculum development as it relates to changes and transformation of culture and society.

It also creates a democratic environment for the discussion of curricular decolonization of mathematical thought and its uses in the context of how learners perceive mathematics in their daily lives (D'Ambrosio, 2001). This is similar to forms of language; academic formal forms of language (mathematics) vs local accents, jargon, and colloquial forms of language that convey meaning and connections between people in local/regional dialects. Neither are right or wrong, both convey power and information and are valuable.

By promoting cultural diversity developed in non-Western perspectives allows us to become more mindful of the perspective and world view of the others, which is one of the ways to concretely decolonize mathematical knowledge. For example, Battiste (2011) states that decolonization of education is the examination of assumptions inherent in western knowledge, mathematics, science, and modern educational theory in order to make visible and dispel the assumption that local knowledge is primitive and in binary opposition to scientific, western Eurocentric or modern knowledge. In addition, the objective of a decolonized education is related to the resurgence and empowerment of local mathematical knowledge.

This colonial strategy is related to the devaluation and disqualification of mathematical knowledge developed by the conquered, as well in science and technology. Thus, mathematical ideas, procedures, and practices developed by members of distinct cultural groups in different regions of the world have been disregarded. Part of this method is related to the belief, still predominant in society, that western mathematics is the privileged manifestation of the rationality of humanity, hence universal and culture-free of influences from the sociocultural contexts (D'Ambrosio, 2006).

According to this assertion, developing techniques and strategies to counter Eurocentric discourse that positions western knowledge as superior and local mathematical knowledge systems as inferior or at best as exotic curiosities is a valuable exercise. As a part of a decolonized research program, ethnomodelling recognizes that there is a myriad of non-western forms of knowledge, which offer a response to the ups and downs of survival and transcendence among the members of distinct cultural groups.

This perspective has enabled us to identify three approaches: global (etic), local (emic) and glocal (dialogical) actions that have assisted us in investigating, studying, and discussing issues related to decolonization and culture, while helping us to understand the mathematical ideas, procedures, and practices developed by, and useful to members of distinct cultural groups.

A global (etic, outsider) approach is related to an outsiders' view on the beliefs, customs, and scientific and mathematical knowledge developed by the members of diverse cultures by comparing by describing differences among cultures through the use of accounts, descriptions, and analyses of mathematical ideas, procedures, and practices expressed in terms of the conceptual schemes and categories that are regarded as meaningful by the community of scientific observers (Lett, 1996).

Globalization emphasizes utilitarian approaches to school mathematics by aiming to reinforce the ongoing western biases and dominant value systems prevailing in most mathematical curricula. In particular, school mathematics can be criticized as participating in a cultural homogenizing action or force to the society formatting. It is in danger of providing a critical filter for status and/or becomes a perpetuator of mistaken illusions of certainty, and as an instrument of power that helps to globalize dominant mathematical practices and ideologies (Rosa & Orey, 2019).

For example, Pike (1967) argues that more influential trends in cross-cultural investigations privileges etic approaches that are based on an outsider account of other cultures. Therefore, global descriptions generate scientific theories about the causes of sociocultural differences and similarities. These constructs are associated with many structures and criteria developed by external observers as a framework for studying members of distinct cultural groups. According to Sue and Sue (2003), this approach is known as culturally-universal.

A local (emic, insider) approach is related to the insiders' view on how members of distinct cultural groups have come to develop mathematical ideas, and procedures, and solve mathematical tasks. It respects cultural practices, social understandings, customs, religion, gender, and beliefs by enabling them to describe their culture in their own terms. It seeks an understanding of daily phenomena through the eyes of members of a culture being studied in order to capture meanings of mathematical daily life activities.

This approach represents the accounts, descriptions, and analyses expressed in terms of conceptual schemes and categories that are regarded as meaningful to the members of distinct cultural groups. Emic approach values and recognizes the contributions of local people to the development of scientific and mathematical knowledge because it has been validated within local contexts (Lett, 1996).

A local voice or approach both values and recognizes the contributions of emic approaches to the development of scientific and mathematical knowledge because it has been tested and validated within local contexts. It creates a framework from which members are able to better understand, translate, and interpret the world around them (Rosa & Orey, 2019). In this regard, it is important to emphasize that emic approach matches shared perceptions that portray the features of a specific cultural group, which are in accordance with understandings deemed appropriate by way of the culture of the participant insiders. In this context, Sue and Sue (2003) affirm that this approach is known as culturally specific (Lett, 1996).

The glocal (emic-etic, dialogical) approach represents a continuous interaction between the globalization (etic) and localization (emic) that offer perspectives in which both approaches develop elements of valuable outlooks related to the same phenomenon. For Rosa and Orey (2019), it is a blending, mixing, a certain give and take by all participants, and adaptation of the two approaches in which one component addresses, indeed, involves the voices of the members of local cultures, systems of values, and daily mathematical practices. All of this occurs with the hope that members from non-dominant cultures have more power over revelations of their own mathematics and science. That is, a perspective of this is how we do it, vs the traditional this is how they do it.

In the context of ethnomodelling, Rosa and Orey (2017) proposes a dialogue between local and academic (global) approaches to the construction of mathematical knowledge through cultural dynamism (glocalization[3]). In this regard, D'Ambrosio (2006) states that intense cultural dynamics caused by interactions between localization and globalization may produce innovative ways of thinking, reasoning, and solve societal issues and problems concerning politic, economic, health, and environment. This means that in the:

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[3]*Glocalization* is a concept, originally coined by business circles that means to create products for the global market, but customized to suit local cultures and tastes. It is a term coined by Japanese marketing professionals as *dochakuka*, which is composed by three ideographs *do* (land), *chaku* (arrive at), and *ka* (process of). This neologism is composed of the terms *globalization* and *localization*, which have emerged as the new standard in reinforcing positive aspects of worldwide interaction, be it in textual translations, localized marketing communication, and sociopolitical considerations. It serves as a negotiated process whereby local customers considerations are coalesced from the onset into market offerings via bottom-up collaborative efforts. The concept of glocalization follows a sociological/historical approach regarding society and its dynamic social transformations (Khondker, 2004). For example, it is possible to refer to a glocalized product if it meets most of the needs of an international community as well as customized for the members of distinct cultural groups (Robertson, 1995).

> (...) process of ethnomodelling, global mathematical knowledge must be reinvented and adapted to the local reality. In addition, effective localization requires global mathematical knowledge just as localization, paradoxically, also helps to promote globalization. This process is essentially about accessibility, namely making things easy to be accepted on local terms by the local while rendering selves subject to change and transformation (Rosa & Orey, 2016a, p. 203).

According to Latour (1993), it is necessary to focus on the process of glocalization whereby practices undergo local transformation at the same time as hey are diffused globally. In this regard, Rosa and Orey (2016a) state that:

> Ethnomodelling yields a number of insights into glocalized research, including the interplay of political, cultural and technical dimensions of institutional work in the process of internationalizing new practices, and in particular, the interaction of symbolic transformation of mathematical practices during the glocalization process (p. 203).

Similarly, Rosa and Orey (2016a) affirm that vivid encounters between cultures provokes the emergence of *glocalized societies*4 in which members of distinct cultural groups develop active interactional processes in an ongoing negotiation between the local and the global mathematical, scientific, and technological knowledge in a dialogical manner through the development of cultural dynamism.

In these societies, focusing on local (emic) approach and then building on it to integrate global (etic) influences, it is possible to develop mathematical ideas, procedures, and practices rooted in local traditions and contexts, but also equipped with a global knowledge that creates a sort of localized globalization. However, Pike (1967) argues that a more influential trend in cross-cultural investigations privileges the etic approach based on outsiders' accounts of other cultures. According to Rosa and Orey (2017), this dialogical construct shows us the importance of emic knowledge that is related to the insiders' perspective that provides insights into cultural nuances and complexities.

Hence, it is necessary to incorporate emic knowledge into the existing etic framework of the mathematics curriculum to develop a holistic understanding of cultures. In this process, the elaboration of curricular activities emerges from creative and dynamic encounters of local and global knowledge through cultural dynamism. Thus, it is important that both educators and investigators incorporate and acknowledge the importance of diverse cultural traditions and linguistic backgrounds that students bring to schools.

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[4]In glocalized societies, glocalization refers the ability of members of distinct cultures when encounter members of other cultural groups who subsequently absorb influences that naturally fit into and enrich that specific culture. At the same time, they resist those features that are truly alien, and to compartmentalize those characteristics that, while different can nevertheless be enjoyed and celebrated as different. In a glocalized society, members of distinct cultural groups must be empowered to act globally on their own terms. Hence, it is necessary to work with different cultural environments to describe mathematical ideas, procedures, and practices developed by other peoples (Rosa & Orey 2016b).

The aim of this theoretical article is to share our understandings of ethnomodelling by discussing our concern for voices that have been silenced by colonialism, and at the same time as we learn to adapt the three cultural approaches when developing investigations that seek to connect ethnomathematics and mathematical modelling. Our primary argument is that ethnomodelling has been shown to create a firm foundation that allows for the integration of local (emic), global (etic), and glocal (dialogical) approaches in exploring mathematical knowledge developed by the members of distinct cultural groups.

It is important to emphasize here that this process allows us to unpack ways in which mathematical practices are used across context of place and time by showing that ethnomodelling is not simplistic, folkloristic, nor primitivist translations to other mathematical knowledge systems. This can only occur when the voice of those who do the mathematics uncovered are heard and respected.

The need for a more culturally bound perspective on mathematical modelling

When researchers investigate the knowledge and traditions possessed by members of diverse cultural groups, they may be able to find distinctive mathematical ideas, procedures, and practices. In this regard, D'Ambrosio (2006) has affirmed that the description of non-academic or non-western mathematical systems have been retained and further developed by these members is the major focus, indeed basis, of cultural anthropology. As well, we argue that an outsiders' understanding of cultural traits are in danger of misinterpretation and/or ignored because of bias that may overemphasize inessential features of cultures and create misconceptions in relation to the mathematical knowledge developed by its members. It is also in danger of delegating the others to curiosities that are considered primitive, exotic, or less powerful forms of mathematical knowledge.

The challenge arising from this approach is related to how we determine and understand culturally-bound mathematical procedures, and practices without allowing the cultural and academic backgrounds of educators and investigators to influence the cultural background of the members of a cultural group under study. We point out that this may happen when members of distinct cultural groups share the interpretation of their own culture (local, emic) opposed to an outsiders' interpretation (global, etic). This is why it becomes necessary for researchers to become aware of cultural nuances of the phenomenon they are investigating by developing the identification of a diversity of perspectives in which members experience, interpret, understand, perceive or conceptualize mathematical ideas, procedures, and practices from the perspectives of the members of that specific culture.

The mathematical knowledge many students think of as mathematics is only one form of it, which is a dominant academic western construct. There is nothing wrong with it, to say the least, but focusing on just one form of knowledge, which ignores the beauty of progressive discoveries and inventions from cultures around the world through history that form a mosaic of cultural contributions, denies learners a true access to what truly forms mathematics and mathematical thinking. It is important to recognize that in this process the contributions of other cultures and the importance of the dynamics of cultural encounters have equal validity (D'Ambrosio, 2006).

It is important to emphasize that western mathematics and sciences are invaluable and contribute to the search for solutions to specific problems. There can be no confusion here about the importance of modern academic mathematics and science. At the same time, a more local perspective help in the development of mathematical ideas that are imbedded in cultural contexts. Thus, the identification of specific problems rather than focusing solely on mathematical content, enables interactions between cultural perspectives. In order to understand how mathematics (tics) is created, it is necessary to comprehend problems (mathema) that precipitate it by considering its cultural contexts (ethnos) that drives them. In this perspective, ethnomodelling is the process of formulation of problems that grow from real situations, which form an image or sense of an idealized version of the *mathema* (D'Ambrosio, 2001). Consequently, mathematics if conceived as a universal language whose principles and foundations are not always the same everywhere around the world, so it maybe a language, but one with valuable accents and unique perspectives.

This is related to notions that members of diverse cultures have developed different ways of knowing/doing mathematics in order to increase understanding and comprehension in their own cultural, social, political, economic, and natural contexts is often controversial within the mathematics community. Thus, it is important to show how non-western populations developed their own unique and distinct ways to mathematize their surroundings and realities (Rosa & Orey, 2007). People have a far older history than modern, scientific, academic mathematics that is linked to aspects of power and domination, colonization, capitalism, and materialism. This interpretation is at risk because of our known or unknown biases.

Mathematization is the process by which members of distinct cultural groups think and act mathematically in order to translate or interpret their own surroundings by applying mathematical knowledge, most notably universal abilities that all cultures and peoples use: that of counting, measuring, classifying, patterning, gaming, quantifying, and modelling (Rosa & Orey, 2017). These members have developed successful and specific mathematical tools that have allowed them to organize, analyse, comprehend, understand, and solve problems faced in their daily life and their unique historical/political contexts (Bishop, 1988; D'Ambrosio, 1985, Rosa & Orey, 2007).

These tools enable them to identify and describe the beautiful and often very unique mathematical ideas, procedures, and practices developed in diverse cultural contexts by the processes of schematizing, formulating, and visualizing problems in truly different ways, as well as by discovering relations and regularities, and translating real-world problems through processes of mathematization. For example, Lewis (2018) argued that:

> This [approach] yields the discovery of relations and regularities transferring real- world problems to academic mathematics through the process of mathematization. This cultural view of mathematization informs the current study, as cultural components play an integral part in local modelling processes and, in unpacking local conceptions of mathematical modelling (p. 6).

All human beings have developed successful and specific mathematical activities that have allowed them to organize, analyse, comprehend, understand, and solve problems faced in their unique historical, economic, social, cultural, political, and environmental contexts. These activities enable members of distinct cultures to apply unique procedures and techniques developed in diverse cultural contexts in order to schematize, formulate, and visualize problems in distinct ways, as well as to discover relations and regularities to translate real-world problems through mathematization.

In this process, mathematical ideas, procedures, and practices developed by the members of distinct cultural groups are the results of experience that use sophisticated schemes of observation, experimentation, visualization, and formulation of mental ethnomodels5 that help them to conceptualize patterns and create artifacts. For example, D'Ambrosio (2006) affirms that ethnomathematics deals with the concepts of reality and action as part of the advancement of schematizing, formulating, and visualizing processes, which are the bases of the development of different forms of knowledge developed in distinct contexts. This context enabled Lewis (2018) to state that:

> Ethnomathematics affords consideration of the intersection of multiple cultures in the course of evolving group understandings. These multiple cultures include, but may not be limited to, the academic classroom culture negotiated and established by the teacher and school learners, the learners own espoused culture, as well as the culture associated with learning mathematics (p. 4).

In this regard, ethnomodels provide us with consistent representations of the mathematical knowledge socially constructed and shared by the members of distinct cultural groups. Thus, ethnomodels help to link the development of mathematical practices developed by members of different cultural groups with their cultural heritage (Rosa & Orey 2016b). This context enabled us to define that in the ethnomodelling process, ethnomodels can be classified as emic, etic, and dialogic.

Local or emic ethnomodels are representations developed by the members of distinct cultural groups taken from their own reality as they are based on mathematical ideas, procedures, and practices rooted in their own cultural contexts, such as religion, clothing, ornaments, architecture, and lifestyles.

Global or etic ethnomodels are elaborated according to the view of the external observers in relation to the systems taken from reality. In this regard, ethnomodellers use techniques to study mathematical practices developed by members of different cultural groups by using common definitions and metric categories.

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[5]Ethnomodels are small units of information rooted in sociocultural contexts. They also are representations of reality that help members of distinct cultural groups to interpret and understand daily phenomena (Rosa and Orey 2017).

Glocal or dialogical ethnomodels are based on the shared understanding that complexity of mathematical phenomena is only verified in the context of cultural groups in which they are developed. In these ethnomodels, the emic approach seeks to understand a particular mathematical procedures based on the observation of the local internal dynamics while the etic approach provides a cross-cultural understanding of these practices.

These ethnomodels often dependent on unique conceptions of space and time that are contextualized and culturally bound. This perspective has allowed us to justify the need for a culturally bound perspective on mathematical modelling process coupled with sources rooted on the theoretical basis of ethnomathematics through ethnomodelling, which is defined as the study of mathematical phenomena that adds cultural components to the mathematical modelling process.

Discussing ethnomodelling

Investigators, philosophers, and anthropologists such as Ascher and Ascher (1997), D'Ambrosio (1985), Gerdes (1991), Orey (2000), Urton (1997), and Zaslavsky (1973) have revealed in their investigations sophisticated mathematical ideas and procedures that include geometric principles in craft work, architectural concepts, and practices often unique in the activities developed by the members of local cultures. These mathematical practices are related to numeric relations found in measuring, calculations, games, divination, navigation, astronomy, and modelling, as well in a wide variety of mathematical strategies and techniques used in the confection of cultural artifacts (Eglash et al., 2006).

Consequently, D'Ambrosio (2006) discusses how cultural artifacts provide necessary material tools that help in the development of geometric ideas, procedures, and practices related to clothing, shelters, navigation, defense, and transportation, which have come to assist members of distinct cultural groups to solve daily problems by using their own mathematical techniques and strategies. These artifacts can be considered as tools, devices, and *instruments of observation*. In addition to these cultural artifacts, several manifestations were discovered that also take advantage of geometric concepts in order to satisfy the daily needs of members of distinct cultural groups.

In this regard, ethnomodelling is a research area that responds to its surroundings and is culturally dependent because it is socially bounded. One of the goals of ethnomodelling is not to provide mathematical ideas, procedures, and practices developed in other cultures a western stamp of approval, but to value and recognize that they are, and always have been, just as valid in the overall human endowment of mathematics, sciences, and technologies (Rosa & Orey, 2017). Therefore, ethnomodelling privileges the organization and presentation of mathematical ideas and procedures developed by the members of distinct cultural groups in order to enable its communication and transmission through generations.

The elaboration of ethnomodels that describes these systems are representations that help the members of these groups to understand and comprehend the world around them. Ethnomodels link cultural heritage of these members with the development of their mathematical ideas, procedures, and practices. Similarly, Lewis (2018) emphasized that:

> This construct of ethnomodelling becomes powerful since it does not disregard prior exposure to western mathematical conventions, but, instead, accounts for their enactment in problem solving contexts. It looks at not only how individuals employ those ideas, but also how they navigate through the problems while drawing on various types of knowledge (p. 26).

In accordance to this context, Rosa and Orey (2010b) state that ethnomodelling is described as the intersection between cultural anthropology, ethnomathematics, and mathematical modelling (figure 1).

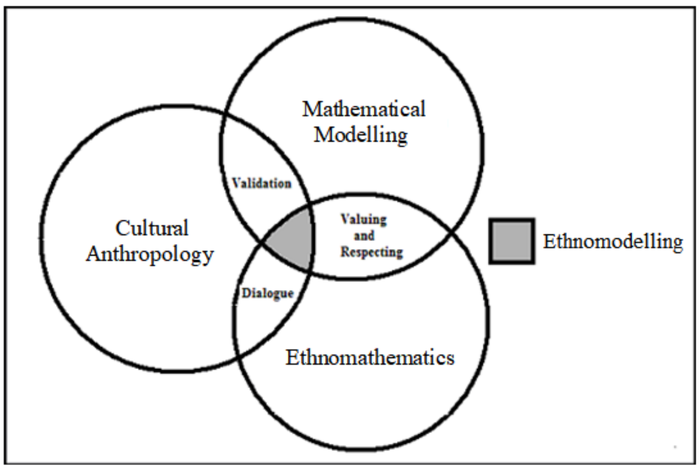

Figure 1: The intersection of three research fields described by ethnomodelling
Source: Rosa and Orey (2010b)

In the ethnomodelling process, the intersection between mathematical modelling and ethnomathematics relates to the respect and the valorization of tacit knowledge6 acquired by the members of distinct cultural groups, and which enables us to access, translate, and assess problem-situations faced daily as we elaborate ethnomodels in different contexts. From this perspective, Knijnik (1996) has stated that ethnomathematics is not considered merely as folklore, but as knowledge that deserves to be rescued so that members of distinct cultural groups and their perspective is valued.

Local mathematical practices are interpreted and decoded in order to understand their internal coherence and their close connection with the practical world. Thus, ethnomodelling is a socioculturally bound construct that forms a basis for significant contributions of an ethnomathematical perspective in re-conceiving mathematics through innovative perspective for the modelling processes.

[6] This knowledge is related to the ways in which members of distinct cultural groups appropriate mathematical knowledge relating them to their own experiences, beliefs, and cultural values.

When we look at how members of distinct cultural groups use their own (mathematical) knowledge and traditions to translate and solve problems faced in their own environments, local (emic) knowledge serves as an intersection between ethnomathematics and cultural anthropology. Hence, Eglash et al. (2006) stated that cultural anthropology has always depended on acts of translation between local (emic) and global (etic) knowledges addressed to help the members of distinct cultural groups to understand specific mathematical practices developed in diverse contexts. For example, Lewis (2018) describes this process by applying an ethnomodelling cycle (Figure 2).

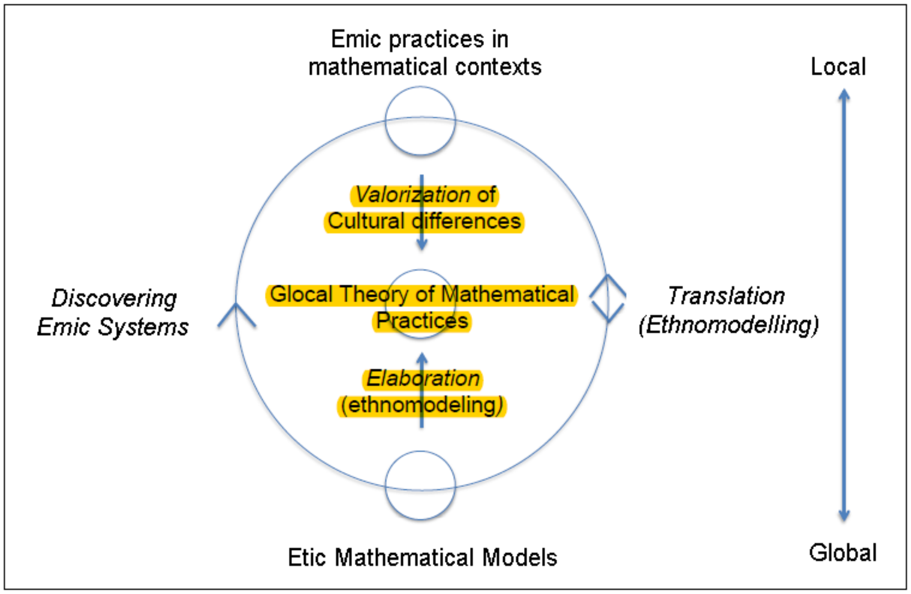

Figure 2: Ethnomodelling cycle
Source: Adapted from Lewis (2018, p. 58)

It is important to state here that Lewis (2018) affirms that, in the ethnomodelling cycle, the interactions between members of the cultural unit as they engage in solving relevant daily problems. The right side of this cycle conveys the process of translation or the interaction between the local (emic) and the global (etic) views, which is mediated through the glocal (dialogical) process in the cultural dynamism of ethnomodelling. Consequently, Rosa and Orey (2017) argue that this translational process is conducted with the elaboration of local (emic), global (etic), and glocal (dialogical) ethnomodels.

According to Rosa and Orey (2017), this translational process is used to describe the development of modelling local (emic) cultural systems that may have western (etic) mathematical representations. This means that, ethnomodelling translates from local mathematical knowledge into the analogous knowledge forms found in distinct contexts. This is one way to approach ethnomodelling by applying emic and etic perspectives to express it and to represent it. In this case, this process is a matter of translation. Thus, ethnomodelling is used to help members of distinct cultural groups to translate mathematical ideas, procedures, and practices found in their own communities among diverse mathematical knowledge systems.

It is reasonable to expect that an ethnomathematical perspective applies modelling procedures that can establish relations between local conceptual frameworks and the mathematical ideas embedded in global designs through translations (Eglash et al., 2006). For example, four-fold symmetry is a design theme used in many Native American cultures as an organizing principle for religion, society, and technology. It has emerged through native structures analogous to the Cartesian coordinate system that helps researchers to translate this mathematical practice among distinct cultural systems. In this context, Orey and Rosa (2016b) affirm that:

> (…) mathematical thinking is influenced by a variety of factors acting on the environment in which cultural groups exist, including language, religion, economics, as well as social and political activities. The authors advocate the use of ethnomodelling, a methodological tool designed to help in the conceptualization of mathematical activity. They characterize ethnomodelling as a means for people to engage in humanistic mathematics (p. 11) or introducing members of a particular cultural group to the mathematical ideas of their culture and, as such, calling for an examination of problems relevant to their particular community (p. 12).

However, it is important to emphasize that the epistemological basis of ethnomodelling is not restricted to methods of direct and/or literal translations of non-western mathematical practices to the western traditions because it is necessary to understand local (emic) and glocal (etic) perspectives as a way to explain the validity of mathematical ideas, procedures, and practices from the insiders and/or outsiders' points of view. In this case it is necessary to point out that western mathematics is not the only reference, but valid explications can also come from the insiders' mathematical knowledge. For example, ethnomodelling consists of studies that highlight historic, cultural and mathematical procedures, strategies, techniques, and practices like those found in First Nations peoples, Chinese, Hindu, and Islamic contexts. In this regard, the Chinese Chu Shih-Chieh triangle can be mapped onto Pascal's triangle by a rotation of ninety degrees.

Methodological procedures of ethnomodelling

An emic approach focuses on the perspective found within cultural groups, in which investigators attempt to develop research criteria relative to internal or logic characteristics without disclosing their own biases or questioning their own paradigms. However, D'Ambrosio (2006) stated that every culture is subject to both inter and intra-cultural encounters. Rearing and conquering these biases are common facets of these encounters.

In this regard, Raza et al. (2001) stated that this concern may be an issue related to the variable of cultural distance that is the degree to which cultural norms, worldviews, attitudes, perceptions, and mathematical and scientific ideas differ among distinct cultures. This perspective would make the dual emic-etic (glocal, dialogical) approach also varies and sometimes is challenging to interact. It is necessary to state that complete cultural relativism is never one-hundred percent achievable. For example, Ascher and Ascher (1997) stated that:

> Ethnomathematics is not a part of the history of Western mathematics although we will, of necessity, need to use Western terminology in discussing it. As Western, we are confined in what we can see and what we can express to ideas in some ways analogous to own (p. 43-44).

In our point of view, these assertions addressed a similar issue in relation to the ethnomodelling process that was not formulated in terms of local (emic) and global (etic) approaches. Thus, in order to avoid the predominance of western scientific ways of thinking, there are some methodological procedures researchers can adopt in conducting investigations in ethnomodelling that acknowledge the tensions that may arise between local and global) approaches. One of the challenges researchers must address is to learn how to manage these tensions. One way to reduce this tension is to develop participatory investigations in which participants act as co-researchers in the design of a study, as well in the data collection and analysis. This means that while the findings from such a participatory process may be useful, a supplementary agenda is often to increase participants' sense of being in control of, deliberative about, and reflective on their own lives and situations (Patton, 2015).

The notion of *thick description* (Geertz, 1973) is another methodological procedure that can be applied to lessen the gap between local (emic) and global (etic) approaches. The use of thick, rich, and deep descriptions, as well the use of participants' own words serves to reduce a researchers' selectivity by heightening their awareness of preconceived categories, as well it limits the level of subjectivity that they may introduce into the data analysis process (Patton, 2015). During ethnomodelling investigations from an etic perspective, assumptions related to motivations and influences based on our own cultural lens, which are related to the spectrum of ethnocentricity and cultural relativism may develop cultural biases. Ethnocentrism is a form of bias that refers to judging another culture or its peoples, solely by the values and standards of one's own culture (McCornack & Ortiz, 2017).

Cultural relativism is the principle that individuals' beliefs and activities should be understood by others in terms of their own culture (Tilley, 2000). However, cultural biases can be avoided or, at very least, minimized when researchers move toward cultural relativism by showing positive regard to the participants and being cognizant of their own cultural assumptions in order to recognize when the global (etic) approach is being imposed. In this context, researchers need to be mindful of their own biases and prejudices.

When researchers observe members of distinct cultural groups, they usually come to deal with tacit knowledge and subjective feelings about these participants. In other words, they may develop their research with conscious or unconscious bias and/or prejudices. A sense of **o**bservational objectivity can be reduced or eliminated by ensuring that observers are well trained and screened for potential biases and prejudices by developing clear rules and procedures. Therefore, researchers also need to make sure that behaviors are clearly defined and the results from observations describe them in context through thick description of the methodological procedures adopted in their investigations. In this regard, meanings are gained in relation to a specific environment and, therefore, is difficult to generalize or transfer to other contextual settings.

When communicating about *others* and their behavior in investigations, our language echoes the unique relation between our own and the existing stereotypic expectancies we have with categorized individuals often without our conscious awareness. It is a constant battle, and something that requires constant mindfulness. Therefore, writing in the first person can be used to make writing more concise when providing personal reflection, stating a position, and outlining the structure of the proposed research protocol.

In this regard, investigators, if not blinded by, or very best aware of their own biases and prior experiences, worldviews, and ideologies (biases), should eventually come to an informed sense of understanding that demonstrates a difference from the point of view of the mathematical knowledge of the people and the phenomenon being modelled. Therefore, we should be able to describe mathematical ideas and procedures in a way that matters to the insiders as well as how they infer about their own mathematical practices.

However, Yin (2014) argued that, regardless of the steps taken to address the complementarity between the local (emic) and global (etic) approaches, researchers “cannot in the final analysis avoid their own research lens in rendering reality. Thus, the goal is to acknowledge that multiple interpretations may exist and to be sure that as much as possible is done to prevent a researcher from inadvertently imposing her or his own (etic) interpretation onto a participant’s (emic) interpretation" (p. 12).

There is a diversity of methodological procedures that researchers can apply to avoid bias, such as: triangulation, collaboration, peer review, member checking, peer debriefing, respondent validation, persistent observation, and prolonged involvement. These procedures seek a balance between emic and etic approaches in the conduction of ethnomodelling research through the development of a mindful and dialogical approach. Therefore, the mindful glocal (dialogic) approach with global (etic) perspectives becomes more akin to local (emic) perspectives that would happen if the research team consists of both members of the community and investigators.

Cultural components of ethnomodels

In the context of mathematics education and traditional development of mathematical modelling, we argue that the implications of many unique and diverse cultural aspects of human social systems are not always considered. The cultural component in this process is critical because it accounts for, and emphasizes a wide diversity of culture composed by a myriad of diverse and unique mathematical ideas, procedures, practices, and values that are incompatible with traditional and universal one size fits all[7] (global) dimension of mathematical curriculum development and ethnomodelling process.

For example, the results of the study conducted by Cortes (2017) showed that the development of local mathematical procedures and practices during the functioning of a daily farmer's market in Brazil and then modeled them by elaborating ethnomodels in order to interpret and understand the reality of the mathematical thinking produced by the members of this distinct cultural group. Thus, by using his emic knowledge the farmer explained that:

> A product whose cost is 50 reais will have a sale price between 5 and 6 reais a kilo. Other product, whose cost price is 80 reais will have a sale price between 8 and 10 reais a kilo. And a third product whose cost price is 100 reais will have a selling price between 10 and 12.00 reais and so on. However, it is important to note that this selling price may be added to other expenses related to the market expenditures.

Based on the emic information given by the farmer, students developed a possible etic ethnomodel to represent this phenomenon in which the determination of the sale price, besides being related to the quantity of products purchased, is also linked to an emic construct developed by the labor experience of the farmer. In accordance with an etic point of view, figure 3 shows the elaboration of a possible etic ethnomodel representing the sale process developed by the farmer.

$$\text{If } CP = 40, \text{ then } SP(m) = v \cdot m \quad \text{where } 5 = v = 6$$
$$\text{If } CP = 80, \text{ then } SP(m) = v \cdot m \quad \text{where } 12 = v = 16$$
$$\text{If } CP = 100, \text{ then } SP(m) = v \cdot m \quad \text{where } 16 = v = 20$$

CP = Cost Price
SP = Sale Price
m = Mass (kg) of the product
v = Variation of price including expenses and charges

Figure 3: Etic ethnomodel that represents the price of the products determined by the farmer
Source: Cortes (2017)

The interpretation of these emic and etic ethnomodels allows us to infer that the sale price is comprised in an interval whose parameter of variation is approximately 10% of the cost price, since 10% of 50 equals 5, 10% of 80 is equal to 8, and 10% of 100 is equal to 10, which is a good approximation for the parameters highlighted above. However, in addition to this parameter, the farmer also adds an increase of costs related to the expenses and charges that are included in the final price of the products.

Another aspect to be considered in this process is related to the daily needs of the members of distinct cultural groups and how this aspect reflects in the development of their own mathematical

[7] *One size fits all* is a flawed teaching approach because it assumes that all students learn in the same ways. In this context, school curricula should be differentiated to suit students' educational needs in order to help them to receive the best possible education and be prepared for their success in the future.

knowledge that support the solutions of problems they face in their everyday life. In this regard, Gerdes (2014) affirms that geometric knowledge can also be considered as an important *cultural trait*[8] of these members because they use geometric figures and drawings to represent sacred entities, places, and animals, as well as to make utensils and/or to organize their settlements.

For example, in conducting ethnomodelling research, Sharma and Orey (2017) and Pradhan (2017) applied ethnomodelling in their investigations to demonstrate how they came to respect and to truly listen to alternative views and perspectives on the cultural artifacts they were mathematizing during a study of the construction of the local drum and processes evolved. When developing ethnomodels, the researchers realized how these artifacts were produced, how the builders calculated, measured, and decorated them. These processes helped the researchers to show how the forms of local mathematical thinking are used in alternative contexts in Nepal.

In a study developed in the Brazilian context, Madruga (2017) demonstrated the relation between the expressions of choreographers, who developed dance choreographies for samba schools, and their connections to the modelling processes from the perspective of ethnomodelling. The results of this study showed that these choreographers use procedures and strategies to create dance choreographies that apply similar techniques used in the mathematical modelling processes. In this perspective, the use of emic knowledge during the development of choreographic process converges to the dialogic approach of ethnomodelling, which are consistent to the surroundings and cultural context of the members of this distinct cultural groups.

According to D'Ambrosio (2001), mathematical techniques, strategies, and practices are socially learned and historically diffused across generations between the members of diverse cultures. These contexts enabled us to recognize that mathematical knowledge, especially in regard to what is meant by the cultural component, varies widely. This is, to our mind, how counting, measuring, classifying, patterning, gaming, quantifying, and indeed, modelling came to be across diverse contexts in all its amazing diversity.

Ethnomodelling allows for a range of views that allow researchers to see mathematical practices as socially learned and transmitted by members of distinct cultural groups all the way to academic mathematical practices viewed as a set of universally recognized abstract symbolic

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[8]According to Samovar and Porter (2000), cultural traits are socially learned system of beliefs, values, traditions, symbols, and meanings that the members of a specific cultural group acquire throughout history. It identifies and coalesces a cultural group because traits express the cohesiveness of the members of the group. It is a deposit of knowledge, experiences, actions, attitudes, hierarchies, religion, notions of time, roles, spatial relations, concepts of the universe, and artifacts developed by the members of distinct cultural groups in the course of generations through individual and group strivings.

systems with an internal logic that provides for a mutually agreed upon and previously defined structure.

Brazilian wood cubing: ethnomodelling of landless peoples' movement

The wood cubing method involves the calculation of the volume of a tree trunk, thus, *cubing* means to determine the volume of a given object by measuring it in cubic units. Performing calculations for wood cubing involves popular and scientific methods. In this context, Knijnik (2006) states that the *cubagem de madeir*a (wood cubing) is a process associated with the sociocultural environment of the members of *Movimento dos Sem Terra – MST* (Landless Peoples' Movement - LPM). Therefore, cubing wood is a traditional mathematical practice used by members of this cultural group to determine how many cubic meters of wood are needed in the construction of sheds, houses, and animal shelters. For example, Knijnik (1996) studied the elaboration of mathematical activities related to the determination of the volume of tree trunks with participants of this movement.

It is important to state here that the emic knowledge related to the development of this method to determine the volume of a tree trunk was orally transmitted and shared by *MST* family members across generations. Thus, mathematical knowledge involved in these local methods is also related to productive activities that members of this cultural group performed in their daily routines. According to Knijnik (2006) *cubing wood* has the features of the landless peasant culture. In this context, D'Ambrosio (1999) argue that the self-validation of these methods within agricultural communities and settlements results from the development of local agreements of signification that results from a long cumulative process of generation, intellectual and social organization, and diffusion of this knowledge.

Ethnomodels of wood cubing

Member of the *MST* use their own practices to estimate the volume of a tree trunk, which is called cubing. This practice was verbally diffused from generation to generation by the members of this distinct cultural group. The results of the interviews conducted in the study developed by Knijnik (2006) show that the members of this cultural group consider wood cubing as an important daily practice because it consists of estimating/calculating how many cubic meters of wood there is in in a particular tree, a forest, or in a truck load of lumber.

For example, one of the members of this cultural group stated the he used a tree trunk found in the forest to explain the cubing process used to determine its volume by the following emic ethnomodel:

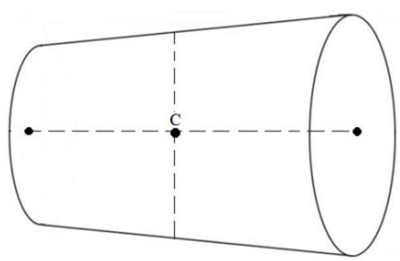
> To begin this process, I chose this point here in the middle of the log, because there it is thicker and here it is thinner [he was pointing out to the extremities of the tree trunk]. So, the point in the middle of the log gave us, more or less, its average. Now, I took this string and I turned it around this point. So, I folded it into four parts and then I measured it to see how many centimeters were

> there. There were 42 centimeters. Now, I took 42 and multiplied it by itself. Thus, 42 by 42 gave me 1764. Hence, I measured the length of the log, which is 1 meter and 50 centimeters. Now, I multiplied that length of the log by the number I had before, which is 1764. So, I multiplied 1764 by 1 and 50, which gave me 264600 cubic centimeters of wood. It is the same as doing side times side times length (Knijnik, 1996, p. 32-33).

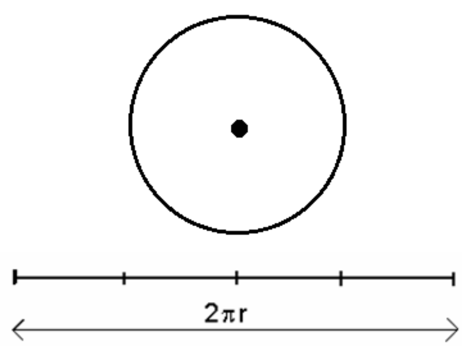
Another group member explained his method of determining the volume of a tree trunk by stating that "The measurement process I know is almost the same, except that, I make the measure at the thin end of the tree trunk because at its thick end we will square the wood in the sawmill, and if you lose some wood, it will not disappear" (Knijnik, 1996, p. 32-33).

According to this context, the results of the study conducted by Amorim et al. (2007) show that cubing procedure used to calculate the volume of tree trunk is given by the following emic ethnomodel used by the members of this cultural group although it is presented in mathematical terms:

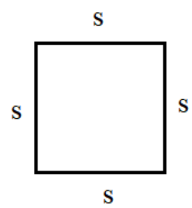
a) First, it is necessary to estimate the center point of the tree trunk, that is, the diameter is taken at half the length of the log.



b) From this point, by using a string, the perimeter of the trunk (circumference) is determined.



c) Then, the string that is related to the perimeter that was previously determined is folded into four equal parts, which gives: $2\pi r = 4$ *sides* or $2\pi r = 4s$.

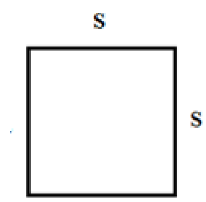


$$2\pi r = 4s$$

$$s = \left(\frac{2\pi r}{4}\right)$$

$$s = \left(\frac{\pi r}{2}\right)$$

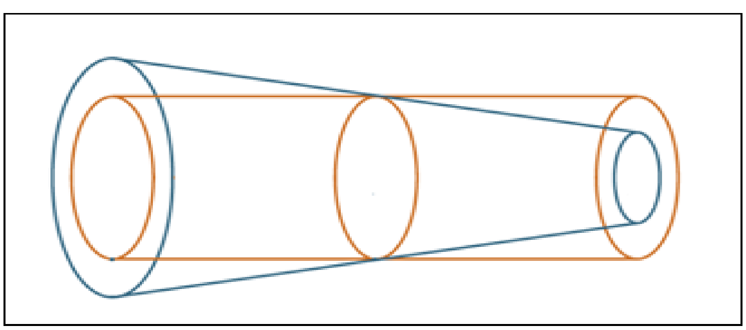
d) Then, the measure of the quarter of the string (circumference) is squared.



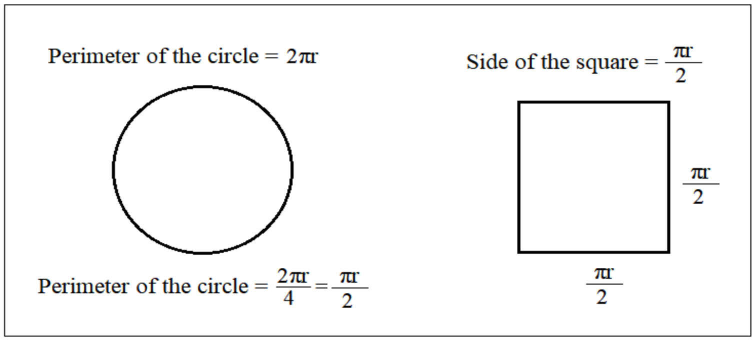
$$A = \left(\frac{\pi}{2}\right)^2$$

e) And the value of the quarter of the string (circumference) is multiplied by the height of the tree trunk in order to obtain the volume in cubic meters (m^3) of the wood. The volume is calculated as if the log was a cylinder.

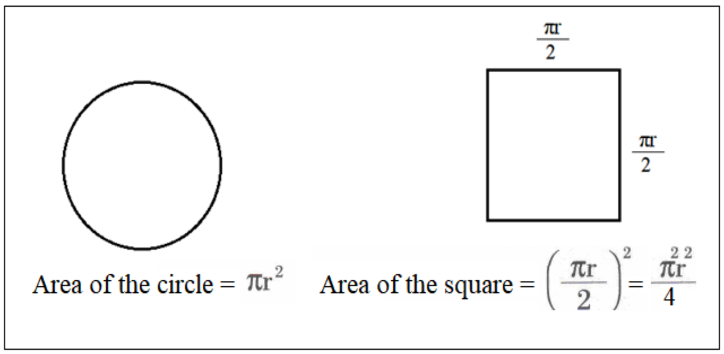
In the etic ethnomodel below, the members of this cultural group approximate the truncated cone (tree trunk) to a cylinder. This approximation is given as perimeter by determining the average between the perimeters of the smallest and the largest bases of the tree trunk.



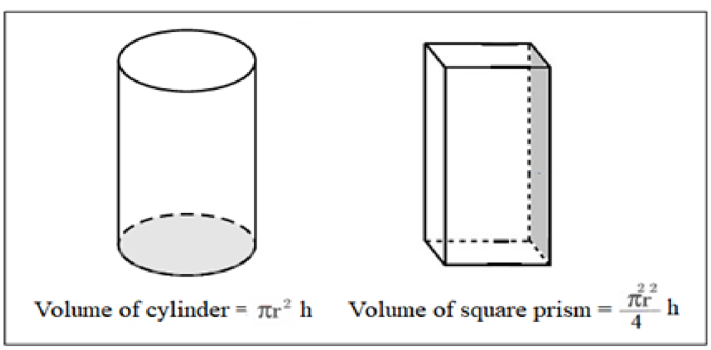
The minor difference at the top of the tree trunk is compensated by the major difference at its bottom. By dividing the string into four parts and raising it to the square, the members of this distinct cultural group calculate the area of a square by transforming the circle into a square.



Although the perimeters are the same, the areas are different. Subsequently, the volume of a square prism is calculated by multiplying its area of the base by its height.



The development of the dialogical ethnomodel shows that the volume calculated in this way is relatively accurate if the shape of the tree trunk approaches a cylinder. This context enables the members of this specific cultural group to develop a comparison between the volume of wood in the prism and in the truncated cone. This ethnomodel shows that this mathematical practice combines western mathematics (measurement and basic operations such as multiplication) and mathematical techniques developed locally. It also shows that there is no a great gap between local (emic) procedures and global western mathematics (etic) because they interact glocally (dialogically).



This method used to determine the volume of a tree trunk basically consists of two steps. In the first step, a tree trunk (essentially a cylinder) was identified through a mathematization process in which its circumference coincides with the middle part of the tree trunk. In the second step, a tree trunk (again a cylinder) was identified as a square prism whose side measurement is equal to a quarter of the perimeter of the cylinder base in this mathematization process.

This method of cubing wood (*cubagem*) finds the volume of the trunk as the volume of a square prism whose side of the base was obtained by determining the fourth part of its circumference, which corresponds to the base of the cylinder, and was obtained through an ethnomodelling process, that is, as part of the elaboration of a dialogical ethnomodel of the tree trunk.

In the dialogical approach of this particular mathematical practice, the emic observation sought to understand the mathematical practice of *cubing wood* from the perspective of the internal cultural dynamics of the members of this group and their relation to the environment in which they live. In the etic approach we explain this mathematical practice through the understanding of more than one feature of this local knowledge.

This particular type of mathematical knowledge developed by *MST* members consists of socially learned and transmitted mathematical practices, which are represented in the elaboration of ethnomodels taken from sociocultural systems. This process aims to translate procedures used in this mathematical practice for the understanding of those who have different cultural backgrounds, so that a comprehension and an explanation of this practice from the perspective of outsiders can be developed. Tacit procedures (emic knowledge) used in this particular mathematical practice have been shared to the members of *MST* through generations. Hence, D'Ambrosio (1985) stated that mathematical practices can be seen as socially learned and historically diffused from one generation to another between the members of groups.

In the examples presented in this article, dialogic (glocal, emic/etic) approaches focus on mathematical features applied in the school and academic (etic) settings and its connections to mathematical procedures and practices developed locally (emic) by the members of distinct cultural groups. This pedagogical action looks at, explores, and describes systems of tacit mathematical knowledge developed by these members on their own sociocultural terms. For example, Lewis (2018) emphasizes that it is important to examine mathematical activities within the cultural context of the students in order to engage them in acts of dialogical translation between local (emic) and global (etic) mathematical practices.

This approach helps students to understand the development of the of perspectives both insiders and outsiders regarding the process of elaborating ethnomodels that represent local mathematical practices. Thus, the:

> (…) elaboration of these cultural mathematical practices can support their development as well as offer pedagogically a connection between the emic practices and the etic mathematical knowledge which a school tries to instill in student learners. As such, ethnomodelling can be a seemingly powerful pedagogical tool in the presence of relevant cultural mathematical activities (Lewis, 2018, p. 63).

In this regard, Rosa and Orey (2010b) state that the dialogical (glocal) approach translates procedures used in these mathematical practices for the understanding of those who have different backgrounds so that both are able to understand and explain mathematical practices from the cultural dynamism perspective. This context reveals that these three cultural approaches can be used as a pedagogical action in the mathematics classrooms in order to help students to (re)discover mathematical relations by developing ethnomodels through ethnomodelling.

Glocal approach in ethnomodelling research

To recap, the relation between local (emic) and global (etic) is dynamic and neither is more significant than the other. An emic approach is developed when members of distinct cultural groups develop their own interpretation of their cultural group (*local*, *emic*) opposed to an outsider's interpretation (*global*, *etic*) of this specific culture, and share it. We can make a similar analogy to ethnomodelling because it is possible to state that the emic approach is about differences that make mathematical practices unique from an *insider's* point of view. We argue that *emic* ethnomodels are grounded in what matters in the world of the members of distinct cultural groups in which that their mathematical reasoning is being modelled by investigating mathematical phenomena by means of their interrelationships and structures through the eyes of the people native to a specific cultural group.

Global or etic ethnomodels represent how the modeler thinks the world works in the context of the members of distinct cultural groups under study through systems taken from the modelers' reality while emic ethnomodels represent how people who live in such contexts think these systems work in their own realities. A glocal approach plays an important role in ethnomodelling research because emic approaches are also taken into consideration in this process and possesses the objective of sharpening issues regarding what the ethnomodels include in order to serve cultural and practical goals in diverse modeling investigations.

In this context, ethnomodels represent mathematical ideas and procedures that are global (etic) if they can be compared across cultures by using common definitions and metrics while the focus of the local (emic) analysis of these features are emic if mathematical techniques and practices are unique to a subset of cultures that are rooted on the diverse ways in which etic activities are carried out in specific cultural settings. Usually, in ethnomodelling investigations, a local (emic) analysis focuses on a single culture and employs descriptive and qualitative methods to study mathematical ideas, procedures, and practices of interest. It also focuses on the study within a cultural group context in which the investigators attempt to develop research criteria relative to internal or logic characteristics of a given cultural system or context.

A glocal (emic-etic, dialogical) approach includes the recognition of other epistemologies and the holistic nature of mathematical knowledge by combining ethnomathematics and mathematical modelling through ethnomodelling. Thus, it is necessary to state that:

> From the perspective of research, elaboration serves as a means with which to foster further dialogue between the observer/investigator and the cultural unit. In this way, these cultural perceptions merge into a shared glocal view of the mathematical practices which occur (Lewis, 2018, p. 63).

In this context, Rosa and Orey (2019) argue that the "dialogic ethnomodels enable a translational process between emic and etic knowledge systems. In this cultural dynamism, these systems are used to describe, explain, understand, and comprehend knowledge generated, accumulated, transmitted, diffused, and internationalized by people from other cultures (p. 16). Hence, an important goal of ethnomodelling investigations is the acknowledgment of the development of both local (emic) and global (etic) knowledge by the members of distinct cultural groups. Thus, we may invoke a notion of local vitality, which releases an unexpected and astonishing cultural power, reinforced by the advantage supplied by the continual full participation in the community simultaneously with the action in the global world in a cultural dynamism. In accordance to Lewis (2018), it is important to use ethnomodelling as a means for valorizing the mathematical activity of the students and to further showcase the particular components of the local knowledge as well as their connection to the mathematical modelling process through ethnomodelling.

Final considerations

Like all human beings, researchers have been enculturated to some particular worldview. In an increasingly *glocalized world*, it is necessary that mindful distinctions of phenomena derived by insiders and external observers be shared. Defining the emics and etics of a given phenomenon, while using epistemological terms, provides a reliable means towards a deeper understanding of their complementarity. This must, and can be done, carefully and by relating to and respecting local contexts in order to support the usefulness in discussions in relation to emic (local) and etic (global) mathematical practices.

Researchers who come from an emic perspective believe that factors such as cultural and linguistic backgrounds, social and moral values, and lifestyles come into play when they respectfully incorporate mathematical ideas, procedures, and practices developed by members of distinct cultural groups. This context enables the recognition of emic knowledge that is not interpretable in mathematical representations or to understand that global (etic) knowledge has no priority over other mathematical ideas, procedures, and practices.

Thus, it is necessary to create innovative collaborations between academics (global, etic) and communities (local, emic) and articulate of the principles and priorities emanating from both sides. In this context, an emic approach provides diverse perceptions and alternative conceptions of common mathematical ideas and procedures. While, global (etic) approaches propitiate frameworks for determining the predominance of Eurocentric beliefs on the development of mathematical knowledge. On the other hand, the rationale behind the emic-etic dialogue is the argument that mathematical phenomena in their full complexity can only be understood within the context of the culture in which they occur.

A combined glocal (emic-etic, dialogical) approach requires researchers to attain the local (emic) knowledge developed by members of cultural groups under study. This encourages them to put aside any perceived or unperceived cultural biases so that they may be able to become familiar with the cultural differences that are relevant to the members of these groups in diverse sociocultural contexts (Rosa and Orey 2015). This perspective represents a continuous interaction between globalization (etic) and localization (emic) approaches, which offers a perspective that they are both elements of the same phenomenon through dialogue.

Ethnomodelling process is inherent to the development of dialogic (glocal, emic-etic) approaches related to dynamic modifications of the modelling process that strengthens an understanding of the ethnomodelling investigation for both the local (emic) and global (etic) communities through glocalization. It increases the potential for the continual growth of the debate related to the nature of ethnomodelling and how it links to culture. It offers a basis for decision-making processes in the elaboration of ethnomodels by analyzing the mathematization procedures in the production of cultural artifacts. Ethnomodelling supports the development of advanced mathematical ideas and procedures that show how powerful mathematical knowledge originated in diverse cultural contexts.

This approach can help us to understand how we can decolonize mathematical knowledge, and most importantly, allows us to unpack ways in which sophisticated mathematical practices have been used across time and place by showing that ethnomathematics is not simplistic, folkloristic, nor primitivist translations to other mathematical knowledge systems. This means that ethnomodelling opposes a dominant and Eurocentric discourse in mathematics education, which emphasizes the school curricula developed by colonizing countries and imposed on local communities during the process of colonization. It also challenges the notion that members of local and/or distinct cultural groups develop only exotic and/or simplistic mathematical ideas, procedures, and techniques (Rosa & Orey, 2019).

According to Eziefe (2011), an essential issue to discuss is attributed to a lack of cultural relevance in mathematics or its incompatible cultural learning styles in the classrooms while it is also relevant to acknowledge ongoing colonization as the fundamental cause of inequalities. Consequently, Battiste (2013) affirms that there is a disconnection between themes embraced in mathematics education as decolonizing, anti-oppressive, and social justice discourse and the mathematical discourse produced by educators and researchers in these investigation fields.

By reflecting on our own work, the complementarity between local (emic) and global (etic) approaches must be present when conducting ethnomodelling investigations because both approaches are essential for a better understanding of human behaviors, especially those related to the development of mathematical knowledge due to the glocal (dialogical) approach is related to the stability of the relations between local (emic) and global (etic) approaches through cultural dynamism. As researchers, we need to strive for a sense of mindfulness between the insiders and outsiders' worldview, perspective and paradigms, which is best accomplished through dialogical approaches to ethnomodelling.

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