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It is necessary to know a specific mathematics to teach mathematics

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Abstract: In this theoretical essay we problematize the statement "To teach mathematics one must know a specific mathematics for teaching" of the discourse of specific mathematical knowledge to teach. The study was inspired by Foucaultian concepts and the analysis focused on studies in the area of Mathematical Education that mobilize discourses regarding Mathematical Knowledge for Teaching, Mathematics Teacher's Specialized Knowledge and Mathematics for Teaching. The analysis showed that the discourses operate under two truth regimes about the mathematical knowledge needed to teach. Analyzing the enunciative intertwining, we observe an approximation with: i) the educational discourse, which deals with the inseparability between theory and practice; and ii) the capitalist economic discourse, which associates of economic development with the mathematical proficiency of a country.

Keywords: Statement. Mathematics. Teaching. Discourse.

Introduction

In this study, we developed a theoretical essay that is characterized by presenting a logical, rigorous, coherent, and critical argumentation about a given topic without resorting to an explicit methodological path and the previous delimitation of a corpus of literature (Barbosa, 2018). Our aim was to problematize the statement: "To teach Mathematics it is necessary to know a specific Mathematics for teaching" that emerges from discourses that circulate in research in the area of Mathematics Education and deal with the existence of a specific Mathematics to teach. Thus, the construction of our argument was inspired by Foucauldian concepts and focused on Mathematics Education discourses that characterize discursive formations that delineate them around the existence of a mathematical knowledge that would be specific to teach Mathematics.

Despite a better presentation in the next section of this text, we should clarify how Foucauldian studies take the concepts of discourse, statement, and discursive formation. However, we emphasize, as Fischer (1995; 2001) points out, that this is not a simple task, since these concepts are not closed in

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themselves and, "in almost all formulations on discourse [and discursive formation], Foucault refers to the utterance" (Fischer, 2001, p. 202).

In Foucauldian studies, the use of the word discourse goes beyond the use of signs to designate the things they speak of; it constitutes the thing itself, although it does not deny that discourses are made of signs. An enunciation, in turn, can be exemplified as a large headline to describe a discourse[2], that is, one does not need to read the whole story to know what a journalistic text is about and, although it can be represented by a proposition, a sentence or a discourse act, it should not be confused with them (Foucault, 2016).

Magnus, Caldeira and Duarte (2016), for example, when analyzing the answers given by male and female teachers of Basic Education to the question, "What do you understand by Mathematical Modeling?", identified that an enunciation that runs through the discourse of Mathematical Modeling is: "Mathematical Modeling is the construction of models". The authors argue that this utterance "goes back to an old and enduring scenario in mathematics (...) that nature is written in mathematical language" (Magnus; Caldeira; Duarte, 2016, p. 1059). According to Foucault (2016, p. 142), "an utterance belongs to a discursive formation, as a sentence belongs to a text." Thus, discursive formations "are linked at the level of utterances" (Foucault, 2016, p. 141); they do not constitute the final stage of discourse, they do not gather everything that can appear on a given topic, they constitute the systems that makes it possible, delineates it, giving rise to new discourses. According to Araújo (2007, p. 8), in "each formation, concepts are arranged in a certain way and used according to the field of knowledge and the way it relates, differentiates itself, associates itself or not with other fields of knowledge".

The study of Montecino and Valero (2016) argues that, since the 1990s, research in the field of Mathematics Education would be shaping the constitution of various discursive formations around the mathematics teacher, with the predominance of a cognitivist perspective. According to the authors, these formations are favoring the homogenization of mathematics teachers based on an unattainable ideal

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[2] Notes from the lecture on "The cultural politics of textbooks" delivered by Professor Dr. Márcio Antonio Silva, via Skype, to the Research Group on Teaching Science and Mathematics (ENCIMA) at the Faculty of Education, UFBA, on April 11, 2019.

image, ignoring aspects of the complexity of teaching and establishing an epistemology of knowledge deficit. On the other hand, still according to the authors, by thinking about this teacher from a social perspective, these trainings "establish the knowledge of a useful subject that seeks to form part of the desired truth and lead others toward the desired truth" (Montecino; Valero, 2016, p. 1622 - our translation), developing an epistemology of desired and feared ideas.

Neubrand's (2018) study, for example, discusses three such discursive formations[3]: "Mathematics-for-Teaching" (Canada), Rowland's "Knowledge Quartet" (UK), and Lindmeier's "Structure Model" (Germany), which the author calls theoretical approaches about the components of knowledge that teachers need to teach mathematics with professional awareness and that researchers need to describe and evaluate. For the author, different conceptualizations would respond to different needs and could further foster the development about the field of mathematical knowledge related to teachers for teaching purposes.

Following this same line, considering that discursive formations are "a law of coexistence of the utterance" (Foucault, 2016, p. 143), we developed this study focusing our analysis on three discursive formations identified as: Mathematical Knowledge for Teaching (MKT), Mathematics Teacher's Specialized Knowledge (MTSK) and Mathematics for Teaching (MfT). These discursive formations were chosen because they have established a research agenda in the field of Mathematics Education (Hoover *et al.*, 2016) and because they shape much of the research around the teacher who teaches Mathematics in Brazil.

Foucauldian tools mobilized

In order to problematize the statement "To teach Mathematics one needs to know a specific Mathematics for teaching", we mobilize from Michel Foucault's toolbox some concepts that help us to understand its intersections with other statements and the productivity of the discourse of specific Mathematics to teach Mathematics. In this sense, the discourse of the specific Mathematics to teach

[3] The author's choice of theoretical approaches was based on three projects of great repercussion: the "Michigan Project" (Canada), "COACTIV" (Germany), and "TEDS-M" (international in scope).

Mathematics is not restricted to a set of signs that designate things concerning the teaching of Mathematics; they are "practices that systematically form the objects of which they speak" which make them "irreducible to language and the discourse act" (Foucault, 2016, p. 60).

Taking discourse as a "set of utterances that rest on the same system of formation" (Foucault, 2016, p. 131), Foucault dealt with psychiatric discourse (Foucault, 1978), clinical discourse (Foucault, 1977), among others. Using his theoretical tools, there is a growing number of studies that problematize statements that circulate in the discourse of Mathematics Education. These studies have problematized statements in the discourse of Mathematics textbooks (Santos; Silva, 2019; Souza; Silva, 2017); of Mathematical Modeling (Magnus; Caldeira; Duarte, 2016); of playful practices in Mathematics teaching (Sartorei; Duarte, 2015); of the approximation of Mathematics teaching with the student's reality (Knijnik; Duarte, 2010); of male superiority in Mathematics (Souza; Fonseca, 2009); of academic mathematics and school mathematics (Wanderer; Knijnik, 2008); of the difficulty in learning Mathematics (Knijnik; Silva, 2008).

The enunciate, according to Foucault (2016, p. 96), "appears as the ultimate element, indecomposable, susceptible of being isolated in itself and capable of entering into a game of relations with other elements similar to it; (...) as an atom of discourse." The statement Man is better at mathematics than woman, described by Souza and Fonseca (2009), appears as indecomposable in the discourse that deals with male superiority in mathematics. However, an enunciation is not closed in itself, as it maintains a relation with other enunciations. In this sense, Souza and Fonseca (2009) show that this statement enters the game of discursive relations with other statements, such as those produced in the discursive field of performance evaluation policies of the Ministry of Education (MEC) in Brazil, when it establishes as universal categories for the analysis of performance the male and female genders.

The enunciates, therefore, go through different discourses, and there is no possibility of a formulation capable of homogenizing or unifying them, without considering their dispersions, which makes them unique at a given moment. Thus, M. Foucault proposes the concept of discursive formation as a set of verbal performances that are not linked together at the level of sentences, propositions or formulations,

but at the level of enunciations (Foucault, 2016). Being different as to form, time and space, discursive formations are a description of the dispersion or regularity of the object; its non-identity, the rupture that is produced in it, the discontinuity that suspends its permanence (Foucault, 2005).

Considering the discursive formations under which our analysis falls MKT, MTSK and MfT, based on Fischer (1995), we understand that they are not homogeneous, nor are they completely opposed to each other; but that there are mutual incorporations that do not allow the identification of borders between their discourses. In this sense, they were analyzed around the notion of discourse of the specific mathematics to teach.

Discourses constitute and are constituents of "truths". The use of quotation marks in the word truths is done in order to highlight, according to Foucault (1989), that there is no "truth", but regimes of truth. That is, "truth" is historically produced within discourses that are not in themselves true or false; therefore, what exist are discourses that, by being accepted in a certain society, come to function as true. In the same way, there are mechanisms and instances enabled/recognized as capable of distinguishing and sanctioning true or false discourses; there are techniques and procedures valued for obtaining the truth as, for example, the scientific method; and there are those who have the function of saying what works as true (Foucault, 1989).

In the following section, we present how the discourse of Mathematics specific to teaching has circulated in the studies of Mathematics Education, which, as an instance that sanctions discourses, assumes the role of establishing regimes of truth about Mathematics specific to teaching. We have chosen to present it by means of enunciations. For Foucault (2016), enunciations occur whenever a set of signs is emitted. Thus, a spoken sentence constitutes an enunciation. In this way, we consider that the written texts that put into circulation the discourse of specific mathematics for teaching are enunciations that constitute the statement "To teach mathematics it is necessary to know a specific mathematics for teaching".

The discourse of specific mathematics to teach

In order to present how the discourse of specific Mathematics for teaching has circulated in the area of Mathematics Education, we focus our look on the discursive formations that it helps to compose and we highlight enunciations that refer us to the idea that to teach Mathematics it is necessary to know a specific Mathematics for teaching.

The Mathematical Knowledge for Teaching (MKT) discursive formation dates back to the early 2000s, given the contribution of research led by Deborah Ball and collaborators (Ball; Bass, 2003; Stylianides; Ball, 2004; Ball; Thames; Phelps, 2008; Hoover *et al.*, 2016), inspired by Lee Shulman's studies on teachers' professional knowledge (Shulman, 1987). This research developed the notion of MKT with the expectation that it would include the full range of mathematics used in teaching a given subject by teachers. MKT, therefore, would be different from the mathematical knowledge needed by other professionals who use mathematics for purposes other than teaching as noted in the following enunciations.

> The underlying epistemological assumption of this body of research is that **teachers need to understand and use mathematics in ways that are specific to the work of teaching and that often differ from the ways in which mathematics is attuned to the needs of other workplaces** such as nursing and engineering physics (Ball & Bass, 2003b; for analyses of the mathematical needs and demands of such other workplaces, see Hoyles, Noss, & Pozzi, 2001; Noss, Healy, & Hoyles, 1997). (Stylianides; Ball, 2008, p. 308 – emphasis added)

> Secondly, and this is the more difficult area to conceptualize and understand, **the mathematics that is used in teaching the curriculum is not synonymous with doing mathematics in other domains of practice (e.g. engineering, nursing, business).** (Adler, 2005, p. 3 – emphasis added)

> This kind of exploration will only be possible if **the teacher has a solid and grounded mathematical knowledge for teaching (MKT).** (...) [in the sense of a] **specific knowledge needed to teach, which is broader than any other type of knowledge that allows simply providing answers to the various mathematical situations proposed.** (Ribeiro, 2011, p. 410 - translation and emphasis added)

In admitting that knowledge of mathematics for teaching purposes differs from that used by other professionals, the enunciations turn to require efforts by other researchers to expand investigations on the topic. The expectation is that further research would provide a consistent validation result of MKT, as well as enhance the MKT construct proposed by Ball, Thames, and Phelps (2008). This construct

describes subdomains for Content Knowledge (Common Content Knowledge, Horizon Content Knowledge, and Specialized Content Knowledge) and for Pedagogical Content Knowledge (Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum) describing characteristics that would determine each.

> **Substantial areas of mathematical demands in different practices of teaching still call for investigations from many researchers.** Like the collaborative work on the Human Genome Project (International Human Genome Sequencing Consortium, 2001; Naidoo, Pawitan, Soong, Cooper, & Ku, 2011), **research on MKT would need cooperative work for elaborate and systematic conceptualization.** A major prerequisite for such collective work is the identification of a method to research MKT. **If relevant methods of studying MKT are specified, research of MKT will be powerfully advanced.** To systemically research MKT, a comprehensive method needs to be specified. (Kim, 2016, p. 72 – emphasis added)

> **That there is a domain of content knowledge unique to the work of teaching is a hypothesis that has already developed**. However, **the notion of specialized content knowledge is in need of further work** in order to understand the most important dimensions of teachers' professional knowledge. Doing so with care promises to have significant implications for understanding teaching and for improving the content preparation of teachers. (Ball; Thames; Phelps, 2008, p. 405 – emphasis added)

The pointing out of insufficiencies arising from the search for refinement of MKT has led researchers to establish new research agendas, such as Carrillo, Climent, Contreras, and Muñoz-Catalán (2013) who suggest a reformulation of MKT. The authors' proposal is that the specialized nature would define all knowledge in respect to mathematics teaching, eliminating the reference to Common Content Knowledge and establishing an understanding of Mathematics Teacher's Specialized Knowledge (MTSK) as it is possible to observe in the enunciations that follow.

> **Tackling these shortcomings** [in relation of MKT] **by viewing all mathematics teachers' knowledge as specialized has led us to reinterpret and rename these subdomains in what can be considered a reformulation of MKT.** (Carrillo; Climent; Contreras; Muñoz-Catalán, 2013, p. 1 – emphasis added)

> Among the diversity of ways to consider teacher's knowledge **(...) we assume as theoretical perspective the conceptualization of Mathematic Teachers Specialized Knowledge - MTSK** (CARRILLO et al., 2013). This conceptualization (...) *considers all the specialized teacher's knowledge*, including in this specificity both aspects related to mathematical knowledge and pedagogical content knowledge. (Ribeiro; Correa; Almeida, 2016, p. 3 - translation and emphasis added)

> **The MTSK (...) addresses different aspects and dimensions of teacher knowledge presenting itself, on the one hand, as a theoretical lens that makes it possible to model the core knowledge of the professional knowledge of the teacher who teaches mathematics.** On the other hand, it presents itself as **a powerful methodological and analytical tool to investigate the different practices of teachers who**

> **teach mathematics** from the dimensions of their mathematical and pedagogical knowledge. Note that these dimensions correspond to a **deepening of the specificities of the teacher's knowledge in various mathematical topics and contents** (...). (Policastro; Almeida; Ribeiro, 2017, p. 128 - translation and emphasis added)

The MTSK is also organized into sub-domains associated with Mathematical Knowledge (Knowledge of Topics; Knowledge of the Structure of Mathematics; Knowledge about Mathematicas) and Pedagogical Content Knowledge (Knowledge of Features of Learning Mathematicas; Knowledge of Mathematics Teaching; Knowledge of Mathematicas Learning Standards). As in the MKT, each of these subdomains is described by listing characteristics that would determine each of them.

Within the discursive formations of MKT and MTSK, we corroborate the claims of Barwell (2013) and Montecino and Valero (2016) that the cognitive perspective has dominated most research on mathematics teacher education in the expectation that it would be possible to identify what these teachers know about mathematics and how they teach what they know about mathematics in terms of domains and subdomains of knowledge. We will say that in these researches, discourses circulate that consider mathematical knowledge as part of cognitive processes and, therefore, would be of the individual scope. Moreover, they operate with a regime of truth that says of the existence of a specific and necessary mathematical knowledge to teach, to which teachers can have access through the increasing specialization of domains and subdomains of this knowledge. This knowledge being available to all teachers, those who do not demonstrate it would be outside the order of the discourse, and could not be recognized as good mathematics teachers.

However, other discourses dispute space in the discourse of specific mathematics for teaching and present themselves as a counterpoint to the idea that a teacher's mathematical knowledge can be categorized and described in domains and subdomains, as observed in the following enunciations.

> **Contemporary research emphases on identifying and measuring what individual teachers can explicitly articulate are, in our view, simply inadequate –** both as tools to assess what teachers really know and as means to support the development of the vibrant body of M4T knowledge. (Davis; Rennert, 2014, p. 116 – emphasis added)

> We concur, and would add **that much of teachers' mathematics for-teaching is tacit. Hence we work**

> **from a positive rather than a deficit perspective of teacher knowledge as we focus on the mathematics that teachers actually enact.** (Davis; Smmit, 2006, p. 295 – emphasis added)

> At worst, **narrow conceptions of mathematics-for teaching may ignore the complex and sophisticated sets of competencies teachers bring to the profession and promote (even unknowingly) deficit views of teacher knowledge.** (Oslund, 2012, p. 307 – emphasis added)

Within the discursive formation of MfT these discourses circulating the understanding that the specific mathematics to teach to be captured should take into account the context and the social interactions established in it. In these interactions, mathematical knowledge would be analyzed from the situations that make it emerge when teachers and students/teachers and researchers operate with mathematics for teaching purposes, as observed in the enunciations below.

> As researchers **we attempt to identify and represent the knowledge they bring to bear on novel problems and the interpretation of well-known concepts and new ideas.** Pivotal to this work, we deliberately **problematize distinctions "established/dynamic" and "collective/individual"** – a move that is prompted by complexity science. (Davis; Smmit, 2006, p. 295 – emphasis added)

> **MfT is a way of being with mathematics knowledge** that enables a teacher to structure learning situations, interpret student actions mindfully, and respond flexibly, **in ways that enable learners to extend understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice**. (Davis; Rennert, 2014, p. 11-12 – emphasis added)

> (...) **teachers' mathematical knowledge is tied to context**, so it would **be useful to explore how specific contextual factors influenced both teachers' understandings of mathematics and their approaches to teaching tasks**. Contextual factors include the curricula and textbooks that were used, the local and regional school structure, and the student populations, as presumably teachers' attunements are specific to, and honed by, these factors. (Rhoads; Weber, 2016, p. 10 – emphasis added)

In the researches that are anchored in this perspective, the enunciations deal with a collective and participative commitment guided by assumptions that state that individual and collective knowledge could not be dichotomized, because it would be emergent and tacit. They believe that teachers participate in the creation of mathematics as they select and emphasize certain interpretations over others.

Based on Barwell (2013), we argue that the discourses that make up the discursive formation of MfT are close to the perspective that the knowledge of mathematics teachers would be situated in teaching

contexts, embedded in classroom practice, therefore, it would not be located in the teacher's "head". These discourses operate with a regime of truth that treats mathematical knowledge as emerging from a social practice in which discursive interactions thematize mathematics for teaching purposes. They deny the possibility of a deficit of knowledge because, since it is contextual, it is mobilized as it is required.

Once established the discourses that have worked as true, in the given historical moment, we problematize next the statement: "To teach Mathematics one needs to know a specific Mathematics for teaching".

The description of the statement

As Foucault (2016) has well exemplified, the statements "The Earth is round" or "Species evolve" do not constitute the same statements before and after Copernicus or before and after Darwin. Similarly, we can think that Euclid's Postulate of Parallels does not constitute the same statements before and after Bolyai and Lobachewsky. Before Bolyai and Lobachewsky, the statements surrounding the postulate of parallels guaranteed, for example, that the sum of the measures of the internal angles of any triangle is equal to 180°. After Bolyai and Lobachewsky it is known that the sum of the measures of the internal angles of a triangle can be equal to, greater than, or less than 180° depending on the axiomatic adopted. Foucault (2016, p. 126) explains these examples by stating that "what has changed is the relation of these statements to other propositions (...) the field of experience, of possible verifications, of problems to be solved, to which we can refer them."

The above examples aim to show how an utterance only makes sense if analyzed within the discursive formation that determines its conditions of existence and establishes its correlations with other utterances, in a regularity proper of temporal processes (Foucault, 2016). According to Foucault (2016), analyzing discursive formations does not mean finding their place of birth or origin, but staying within the dimension of discourse with its relations and its variants. It is these relations that make it possible to describe utterances as an elementary part of a discourse, "a function that crosses a domain of structures

and possible units and makes them appear, with concrete contents, in time and space" (Foucault, 2016, p. 105).

According to Foucault (2016), the enunciation relies on sets of signs, which is not to be confused with a sentence, a proposition or discourse act, but is related to four basic elements that determine its conditions of existence, they are: a referential, a subject, an associated field and a materiality. Considering these elements, we pay attention to the following statement: "To teach Mathematics, it is necessary to know a specific Mathematics for teaching", which, as an event, "bursts in a certain time, in a certain place" (Fischer, 2001, p. 202), which we will describe by resorting to enunciations present in studies of the area in which discourses that compose the discursive formations of MKT, MTSK and MfT circulate.

Reference

We begin our description by the referential which, according to Foucault (2016, p. 141), "is not exactly a fact, a state of affairs, or even an object, but a principle of differentiation" and, to this end, we consider the following enunciations.

> **Because the work of Shulman and his colleagues is foundational, we begin by reviewing the problem they framed, the progress they made, and the questions that remained unanswered.** We use this discussion to clarify the problems of definition, empirical basis, and practical utility that our work addresses. **We then turn to mathematics in particular, describe work on the problem of identifying mathematical knowledge for teaching, and report on refinements to the categories of mathematical knowledge for teaching.** (Ball; Thames; Phelps, 2008, p. 390 – emphasis added)

> **The specialization of MTSK should allow it to be differentiated from general pedagogical knowledge** (knowledge of pedagogy and general psychology, which also forms part of mathematics teachers' professional knowledge), **from the specialized knowledge of teachers of other disciplines, and the specialized knowledge of other mathematics professionals. In other words, it is specialized in respect of mathematics teaching.** (Carrillo; Climent; Contreras; Muñoz-Catalán, 2013, p. 4 – emphasis added)

The statements above illustrate how discourses associated with MKT and MTSK, while considering L. Shulman's work as foundational, circulate the idea that it has not been sufficient to account for the specific demands of mathematics education. In this sense, Mathematics gains prominence when the domains of teacher knowledge are questioned, requiring domains of knowledge of their own, suggesting a revision of Shulman's work (1987) turning to Mathematics in particular. In this sense, a universal

mathematical knowledge, proper of the teacher, is taken as a reference, which can be identified through a categorization of mathematical knowledge that deals with its specificity for teaching purposes.

On the other hand, discourses that are associated with MfT deny the idea of a mathematical knowledge that can be categorized in advance, through individual teachers' actions, because they refer to it as being emergent, constantly evolving, and distributed among teachers.

> In brief, **we argue that the knowledge needed by teachers is not simply a clear-cut and well-connected set of basics, but a sophisticated, emergent, and largely enactive mix of realizations of mathematical concepts** coupled to an awareness of the complex processes through which mathematics is produced. (...) **we intend to flag the coherent-but-never-fixed character of the complex form of teachers' knowledge.**
> (...) **toward an understanding of teachers' disciplinary knowledge as a responsive and evolving, autopoietic – that is, living and emergent – system of realizations that is distributed across a body of educators.** (Davis; Renert, 2014, p. 118 – emphasis added).

> Their knowledge is perhaps best understood as **an attitude toward mathematical engagement, and not as mastery of a domain of mathematics**. Teachers' mathematics **can be seen as a mode of being that is enacted when teachers approach a new topic, make sense of a student's error, or reconcile idiosyncratic interpretations.** In quite different terms, **mathematics for teaching entails awareness that personal mathematical knowing and collective mathematics knowledge are co-implicated, self-similar forms**. (Davis; Renert, 2009, p. 42 – emphasis added)

In this case, the referent is a distributed mathematical knowledge, because it would be distributed in the category of teachers, which is identified as an attitude, a system of realizations of mathematical concepts. It is a knowledge that is not fixed, because it evolves with each realization; it is not accumulative, nor exclusive of a teacher, because it is emergent and feeds back on the collective mathematical knowledge.

Subject

The second element to be observed in our description is the subject. According to Veiga-Neto (2016, p. 107), the Foucauldian notion of subject does not take it as "an entity that pre-exists the social world", as if it were the representation of a consciousness or the author of a given formulation. The subject is "a position that can be occupied, under certain conditions, by different individuals" (FOUCAULT, 2016, p. 141), so there is no subject a priori, but subject positions that are made available by discourses. The enunciation in question provides the subject position of expert mathematics teacher. The enunciations

below illustrate that this subject position is provided by the constituent discourses of the three discursive formations analyzed.

> Also involved here is a mathematical knowledge specific to the mathematics teacher (**note that we consider that right from the early years the teacher is a mathematics teacher, since early childhood education).** (Ribeiro; Correa; Almeida, 2016, p. 410 - translation and emphasis added).

> Reflecting a growing interest in mathematics education at all levels, many in the mathematics community have turned their attention to the **mathematical preparation of prospective precollege teachers**. (Cuoco, 2001, p. 168 – emphasis added)

> There is growing support for the **notion that there is specificity to the way teachers need to hold and use mathematics in order to teach mathematics** – and that this way of knowing and using mathematics differs from the way mathematicians hold and use mathematics. Both **mathematics and teaching are implicated in how mathematics needs to be held so that it can be used effectively to teach**. (Adler, 2005, p. 4-5, emphasis added)

This position, historically, has been occupied by different subjects as long as they either demonstrate to possess what is considered as a notorious knowledge in relation to Mathematics (in the case of the various bachelors[4], especially in Engineering) or are teaching Mathematics in the initial years of basic schooling (in the case of Pedagogues[5]) or are teaching Mathematics at any stage of schooling (in the case of licensed or lay teachers[6]). For the discourse of teaching specific Mathematics, this is a position that will only be occupied by those who not only know Mathematics, but know a Mathematics that is specific for teaching, therefore it is specialist. In a previous study (Grilo; Barbosa; Maknamara, 2020), we identified the variability of subject positions being made available by these discourses.

Associated field

[4] The option for the masculine gender to refer to bachelors aims to highlight a period in the history of mathematics education in Brazil such that this position was practically reserved for men. For more details, see Valente (2008).

[5] The option for the female gender, in this case, points to the fact that the teaching positions at this stage of Basic Education in Brazil are mostly occupied by women.

[6] This is a person who works as a teacher without the required minimum qualification. The term "Lay Teacher" is used to refer to those who work in the early years of elementary school and who do not have a high school degree, either in the normal modality or in Pedagogy. As for teachers who work in the final years of primary school and high school without a degree course in the specific area of work, although it is not common to use the term to designate them, these professionals are also not qualified, even if they have a degree course and have knowledge in the area they teach (for example, engineers who teach mathematics), in accordance with art. 62 of Law 9.394/96, they are lay teachers. (Augusto, 2010)

According to Foucault (2016), an utterance does not exist in isolation with "a domain of coexistence for other utterances" (Foucault, 2016, p. 141). The author called this domain of coexistence the associated field. Thus, the associated field does not refer to the actual context of the formulation or a particular situation in which it was articulated, but the possibilities of correlating it to other utterances of the same or other discourses. In this sense, as an example, we will show the interlacements and dispersions that the statement "To teach Mathematics one needs to know a specific Mathematics for teaching" establishes with others that make up the discourse of Mathematics Education, the educational discourse and the capitalist economic discourse.

In the scope of the discursive formations analyzed here, we identify that this statement maintains approximations with the statement "To teach Mathematics one needs to know Mathematics", as observed in the following statements.

> **That teaching demands content knowledge is obvious**; policy makers are eager to set requirements based on commonsense notions of content knowledge. **Scholars can help to specify the nature of content knowledge needed**, but providing this specification demands that we use greater precision about the concepts and methods involved. (Ball; Thames; Phelps, 2008, p. 394 – emphasis added)

> However, **there was no apparent decline in the near-universal conviction that knowledge of advanced mathematics was a vital part of teacher preparation**. (p. 7)
> **We believe that a vital characteristic of highly effective teachers is strong disciplinary knowledge** but, as research has proven, this knowledge cannot be construed using simplistic means, such as a count of college mathematics credits or a written inventory. (Davis; Renert, 2014, p. 120 – emphasis added)

> **It is widely accepted that a good mathematics teacher needs a deep and sound mathematical background** (e.g., Shulman 1986; Cooney and Wiegel 2003). (Prediger, 2010, p. 74 – emphasis added)

However, other enunciations point to a rupture in relation to the idea that knowing Mathematics would be enough to teach Mathematics. This dispersion among the statements was identified when, despite admitting as a sine qua non condition the need for teachers to know Mathematics, as we saw earlier, they emphasize that knowing Mathematics is not enough to teach it, opposing the perspective of a self-sufficient knowledge that would be enough by itself.

> (...) effective subject matter knowledge of mathematics teachers (mathematics-for-teaching, or MfT, in short) **is much more than a readily catalogued or objectively tested set of concepts. M4T comprises a complex network of understandings, dispositions, and competencies that are not easily named or measured.** The embodied complexity of M4T must be experienced – seen, heard, and felt. (Davis; Renert, 2014, p. 3 – emphasis added)

> **Overall then, what was observed across these ranging ACE [Advanced Certificates of Education] is the persistence and dominance of compressed mathematics in formal assessment**. Yet, the courses of which they are part were specifically designed for teachers. The courses are not part of mainstream mathematics courses, and so are not bound by mathematical goals, say, for undergraduate mathematics students. **Moreover, ACE programs, typically, are managed by mathematics teacher educators, most of whom would assert that to teach mathematics well, it is not enough to be able to do pieces of mathematics.** (Adler, 2005, p. 9 – emphasis added)

The recognition of a specificity in relation to teaching, which would involve a complex network of competencies that is not restricted only to the ability to do mathematics, brings the statement "To teach mathematics one needs to know a specific mathematics to teach" closer to theoretical perspectives in the educational field. These theoretical perspectives, especially based on the works of Shulman (1987), argue for a non-dissociation between theory and practice, so that teachers' professional knowledge would be constituted by domains that articulate and feedback on content knowledge and pedagogical knowledge.

We observe that in the MKT and MTSK discursive formations the emphasis given to the development of teachers' professional knowledge stems from the need to find ways that would be appropriate to present mathematics to students in a way that would facilitate their learning. In this sense, it would be necessary to update, refine, improve teachers' mathematical knowledge to consequently improve students' performance as seen in the enunciations.

> First, **in studying the relationships between teachers' content knowledge and their students' achievement, it would be useful to ascertain whether there are aspects of teacher's content knowledge that predict student achievement more than others**. (Ball; Thames; Phelps, 2008, p. 405 – emphasis added)

> **They found that teachers' mathematical knowledge was significantly related to student achievement gains (...)** This result, while consonant with findings from the educational production function literature, was obtained via a measure focusing on the specialized mathematical knowledge and skills used in teaching mathematics. **This finding provides support for policy initiatives designed to improve**

> **students' mathematics achievement by improving teachers' mathematical knowledge.** (Hill; Rowan; Ball, 2005, p. 371 – emphasis added)

> (...) **from the perspective of promoting an improvement in practice and student learning, a more central focus on teacher knowledge and on the situations that can be configured as more critical in that knowledge**, taking into account their specificities, is essential. (Ribeiro; Policastro; Marmoré; Bernardo, 2018 - translation and emphasis added)

On the other hand, since the MfT discursive formation considers teacher knowledge to be emergent, tacit, and distributed in the teaching category, the way it sees the association with student performance differs from MKT and MTSK as shown in the utterance below.

> Because of its dynamic and nested character, mathematics-for-teaching cannot be considered a domain of knowledge to be mastered by individuals. **It always occurs in contexts that involve others – and, hence, an awareness of how others might be engaged in productive collectivity is an important aspect.** It is thus that our research into teacher's mathematics-for-teaching is oriented by an assumption that we also take into our teaching of mathematics: **The 'learning system' that the teacher can most directly influence is not the individual student, but the classroom collective** (Davis and Simmt, 2003). (Davis; Smmit, 2006, p. 309 – emphasis added)

This concern with student performance, either individually or collectively, brings the statement in description closer to the statement that "Without Mathematics a country does not develop". This statement supports the capitalist economic discourse, for which the role of the school is to form a generation of young people qualified for the labor market. According to Saraiva and Veiga-Neto (2009, p. 199), the labor market is currently "focused on the cooperation between brains" and on the ability "to produce the innovations that mobilize cognitive capitalism. According to the authors, in cognitive capitalism "the multiplication of capital is much more related to the creation, to the generation of ideas" than "by the expropriation of material labor of its employees" (Saraiva; Veiga-Neto, 2009, p. 192).

In this capitalist production model, the mathematical knowledge gains prominence, as pointed out in the following statement.

> **In the new economy, mathematics has emerged as the gateway discipline** – that is, the numerate have supplanted the literati in access and influence (Baker, 2008). **Yet the sorts of mathematical competence that are of value to current and future society bear ever-diminishing resemblance to the emphases**

seen in contemporary classrooms. (p.13) Schools have traditionally emphasized the development of technical competence, which was an obvious need in an industrial economy. **But in a knowledge-based economy, the development of conceptual fluency is of greater importance and has been the focus of major initiatives in school mathematics.** (Davis; Renert, 2014, p. 127 – emphasis added)

We believe that a deeper study of the associated field may point to other enunciative entanglements beyond those exemplified here.

Materiality

The utterances find support in distinct media. These media shape what M. Foucault calls *materiality* - *"a status*, rules of transcription, possibilities of use or reuse" (Foucault, 2016, p. 141). That is, materiality can be identified in the ability of the enunciation to appear in pedagogical or scientific texts, official documents, in media reports, or in various enunciations that show us concrete ways in which it appears. In the case of the research on which we focus our look, the materiality of the enunciation in description occurs in the format of articles published in journals recognized by peers as authorized spaces for the production of truths regarding Mathematics Education.

Final Considerations

This essay aimed to problematize the statement "To teach mathematics one needs to know a specific mathematics for teaching", which allowed us to define the type of productivity of the discourse of specific mathematics to teach mathematics. With the help of Foucauldian theoretical tools, we present how this discourse has circulated in the studies of Mathematics Education focusing on the discourse formations of MKT, MTSK and MfT.

We argue that the discursive formations of MKT and MTSK are composed of discourses that operate with a regime of truth that says that it is possible to access a specific mathematical knowledge for teaching through an increasing specialization of domains and subdomains of this knowledge. A knowledge, which even showing itself in practical situations, can be measured. On the other hand, the discursive formation of MfT is composed of discourses that operate with the regime of truth that

mathematical knowledge for teaching is not of individual order, but collective; therefore, it would be distributed among teachers. It is dynamic, complex, which would not be consistent with the possibility of being captured in categories.

Analyzing the discourse of specific mathematics for teaching around the discursive formations MKT, MTSK and MfT we described the statement "To teach mathematics one needs to know a specific mathematics for teaching". We saw that this statement is associated with a specialist teacher subject, capable of occupying different positions that show his/her mathematical specialty. Observing its interlacements and ruptures with other statements, we cite as an example its approximation to the educational discourse that deals with the inseparability between theory and practice, mainly supported by Lee Shulman's studies. We also point out how this statement is intertwined with statements of the capitalist economic discourse, which associates economic development to the mathematical proficiency of a country.

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