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Marilena Bittar

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A METHODOLOGICAL PROPOSAL FOR TEXTBOOK ANALYSIS

Marilena Bittar (UFMS)[1]

Abstract: Research on textbooks has been carried out in different countries and with different purposes, including a set of studies that seek to know what those books convey in their texts. This type of investigation gives a view of the textbook as something that gives materiality to the proposed curriculum as a way to approach the knowledge taught and/or to identify elements that contribute to understanding reasons for some learning difficulties and persistent errors. Since the early 2000s, I have guided research studies that seeks to understand what kind of mathematics is proposed in textbooks intended for Brazilian basic education and how it is proposed. After a few years of this work, I outlined a model for textbook analysis based on elements of the anthropological theory of the didactic. This model consists of the following, not necessarily linear, steps: constituting the *corpus* for the analysis, the *modellation* of mathematical praxeologies present in the *Course* part of the material, doing the mathematical analysis of the *Activities Proposed*, the *modellation* of *Didactic Praxeologies*, and *Triangulating* data. This text presents this model, illustrating it with research studies I supervised, showing the relevance of the model of analysis developed.

Keywords: Mathematical praxeology. Didactic praxeology. Curriculum. Ostensive. Non-ostensive. Teaching practice.

1. Introduction

This article aims to present a model for textbook analysis that I have outlined from the development and supervision of research studies since the mid-2000s, supported by the anthropological theory of the didactic (Chevallard, 1998, 2003, 2007).

The interest in the role of textbooks is not recent. It indicates a broad and rich plurality of functions, such as: helping the students' formation, only comparable to the teachers' relevance (Usiskin, 2013), helping the teacher's work (Gérard & Roegiers, 1998), and bearing a perspective on the official curriculum (Brazil, 2010). Fan, Zhu, and Miao (2013, p. 635) state that

[1]Marilena Bittar holds a PhD in Mathematics Didactics from the Université Joseph Fourier, Grenoble, is full professor of the Postgraduate Programme in Mathematics Education of the Federal University of Mato Grosso do Sul (UFMS) and Researcher of the National Council for Scientific and Technological Development – CNPq
ORCID: https://orcid.org/0000-0001-9989-7871 E-mail: marilenabittar@gmail.com

"Researchers have generally agreed that textbooks as a major conveyor of the curriculum play a dominant role in modern education scenes across different school subjects."

Mathematics education research on textbooks has intensified in recent years, as evidenced by volume 45 of the ZDM *International Journal on Mathematics Education* and the three editions of the *International Conference on Mathematics Textbook Research and Development* (ICMT). This text does not intend to carry out state of the art on research related to textbooks, however, before addressing the central focus of the article, I highlight some signs of interest that this theme has raised in recent years and the aspects that have been emphasised.

In 2013, ZDM dedicated volume 45, no. 5, to research on mathematic textbooks. Fan, Zhu, and Miao (2013) wrote an article dedicated to a survey they conducted on textbook-related research in six journals published in English. Those authors found 111 articles, which they classified into four categories: textbooks analysis, textbooks comparison, use by teachers and students, and others. Most studies (34%) addressed textbooks analysis. The authors concluded by saying the importance of further research on textbooks and especially on their use. Fan (2013, p.766) corroborates the need for research in this area: "Nevertheless, it appears that, overall, mathematics textbook research as a field of research is still at an early stage of development as compared with many other fields of research in mathematics education."

The year after this publication, the first edition of the *International Conference on Mathematics Textbook Research and Development* (ICMT-2014) was held in the United Kingdom. In this event, most of the lectures and papers presented addressed teachers' and students' use of textbooks, the history of textbooks over time, and new types of textbooks and materials. However, there were not many studies on textbook analysis. In the second MTCI, held in 2017, in Brazil, research studies on textbook analysis was quite significant. I emphasise the study by Steen & Madsen (2017, p. 214), who, guided by the question, "Which potentials and limitations of using textbooks in mathematics teaching can we identify?," and after analysing 382

texts, conclude: "We found that the research has mainly focused on the textbook in relation to the teachers or the students and to a lesser degree on the textbook's relation to the curriculum and concept development within the field" (p. 224).

Some investigations presented proposals for activities for the classroom, taking into account what was found in the LD. Pjanić (2017, p.206) formulated a hypothesis regarding difficulties that students could develop related to the way multiplication and division problems were addressed in the textbook: "A small number or a complete absence of certain types of multiplication and division word problems denies pupils the ability to create new problem-solving schemes and to detect links between opposing problem types." However, no research was found that had as one of the focus to understand the origin of difficulties and persistent errors of students in a given subject, concern that was already present in my doctoral research (Bittar, 1998). On this occasion, the analysis of textbooks[2] allowed me to identify erroneous theorems in action[3] (Vergnaud, 1990) that French secondary school students could build around the concept of vectors. Those results were fundamental for the elaboration of the didactic sequence to be worked with the students, seeking to destabilise knowledge mobilised outside their domain of validity and lead them to the construction of desired knowledge.

Moreover, the textbook can be seen as an intermediate instrument between the official curriculum and the school (students/teachers) and its analysis brings elements that allow for an approximation of the dominant epistemological model (Bosch & Gascón, 2010) with the knowledge taught, as Assude (1996, p. 50) states: "We assume that the text of knowledge is quite representative of a "weighted average to the various constraints" of the institutional relationship

[2]The model of textbook analysis presented in this text began to be developed a few years after the completion of my doctoral thesis. The analysis carried out at the time of my thesis did not consider a specific methodology, except the identification of definitions and properties enunciated, besides invariants (Vergnaud, 1990) likely to be constructed by students.

[3] A theorem in erroneous action is a proposition considered true by the subject, however, it is false or is used outside its domain of validity.

with the objects of mathematical knowledge present in the different didactic systems that effectively perform this text of knowledge."

Thus, considering the role that textbooks play in teaching practice, the functions they perform, and the uses that the teachers make of them in the classroom, knowing in depth the textbook they adopted favours an approximation with the knowledge taught in their classroom. The textbook analysis favours, therefore, inferences about the implemented curriculum, which enables us to design the dominant epistemological model (DEM) for teaching mathematics related to the institution in focus, contributing to the identification of possible points of cognitive imbalance caused by this model:

> a large number of investigations within the ATD have revealed that some gaps and dysfunctions that are observed in the mathematical activity that can be carried out within current educational institutions are clearly linked to the dominant epistemological and didactic models in said institutions. (Bosch & Gascón, 2010, p. 65)

In this perspective, Verbisck (2019) investigated the approach to probability content in Brazilian textbooks over the 13 years of compulsory schooling, which allowed him to outline the dominant epistemological model (DEM) related to this subject in the books studied. The next step, not yet performed, is to think of a reference epistemological model (REM), which consists of an alternative model to the DEM, which aims at emancipating the dominant model. The REM is provisional and can be replaced by another model that is more pertinent or is in greater compliance with the objective of the research.

Thus, the interest in analysing textbooks does not lie in inventorying what books bring: it is important to know "how they are affected by other factors (independent variables), and how they affect other factors (dependent variables)" (Fan, 2013, p. 771).

From the perspective of discussing how textbooks affect other factors, there are studies about their use by teachers (Oliveira, 2010; Pantoja, 2017) and students, and those that relate textbook

analysis and investigations of learning difficulties with the construction/suggestion of didactic sequences (Doering at al, 2017).

Concerning factors that affect textbooks, in the specific case of Brazil, it is important to report that the country has a government programme dedicated to the public policy of textbooks, the *Programa Nacional do Livro e do Material Didático* (National Book and Textbook Programme), responsible for the entire process of selection and purchase of materials for students from Brazilian public schools, which includes a broad assessment of textbooks. This programme has a strong influence on what is conveyed in textbooks (Carvalho, 2018; Bittar, 2020), thus constituting an important source of data for the analysis of Brazilian textbooks. Therefore, I will briefly describe it.

2. Programa Nacional do Livro e do Material Didático (National Book and Textbook Programme)

Since the 1990s, the Brazilian Ministry of Education (MEC) has determined that textbooks to be distributed by the federal government to public schools of all basic education should undergo an assessment process[4]. Since then, the publishers have submitted works for review by the *Programa Nacional do Livro e do Material Didático* - PNLD (National Book and Textbook Programme)[5], which is responsible for the entire management of textbooks distributed to public schools.

In 1996, the first assessment of textbooks for the initial years of elementary school (7 – 10 years of age) was carried out by the PNLD. In this first assessment, 54 collections of 4 volumes were submitted, of which only seven collections were approved by the expert committee,

[4]The obligation of this assessment was established by the Brazilian Federal Constitution of 1988.

[5] Named Programa Nacional do Livro Didático (National Textbook Programme) for more than two decades, it recently incorporated other didactic materials and early childhood education when it was renamed Programa Nacional do Livro e do Material Didático (National Book and Didactic Material Programme), but continues with the acronym PNLD.

indicating the low quality of the textbooks in use. This fact can be confirmed by the report produced from the assessments (Brasil, 1994). However, as the assessments progressed together with the fact that only books approved by PNLD could be acquired by MEC, the quality of the mathematics books gradually improved at all levels of education, as stated by one of the main responsible for the process (Carvalho, 2018, p. 783): "20 years of assessments slowly improved the quality of mathematics textbooks. Of course, they are not perfect, and good textbooks alone do not ensure good teaching and learning."

The objective of the assessment is to improve the quality of the teaching material distributed to students in public schools. For this, there are some excluding criteria. To be approved in the PNLD, a textbook cannot contain conceptual errors, error induction, prejudices, advertisements, disregard for the statute of children and adolescents. Also, in the case of a mathematics book, it must include all the fields of school mathematics. The assessment also considers quality criteria, such as a summary with the contents, the variety of situations proposed, the contextualisation proposed (or not), among others. At the end of the entire assessment process carried out by two reviewers who initially work separately, a report with reviews of the approved works is produced and published, intended for teachers, to assist them with choosing the material to be purchased. This report, called *Guia do Livro Didático* (Textbook Guide), contains an introductory message to teachers that discusses the importance of choosing a book that fits their requirements; presentation of the assessment criteria; description of the structure of reviews and how to use them to select a textbook; a general text on the characteristics of approved collections; reviews of each of the approved collections; and the forms used by reviewers[6].

[6] This is the general format that, with each edition of PNLD, may undergo some minor modifications. Since the creation of the programme, the biggest modification occurred in the organisation of the process that until 2018 was managed by a university, that should compete through public notice opened by the Ministry of Education. This university coordinated the assessment process of an area (in our case, mathematics) and it was its responsibility to compose a coordinating committee that, among other things, chose reviewers to participate in the process. These reviewers were chosen from mathematics teachers and mathematics educators from different Brazilian regions. Since 2018, the coordination of this assessment process is under

I have participated in the PNLD on several occasions since 2002, as a reviewer or member of the coordinating committee, which has increased my interest in studying what is conveyed in the textbook. Although the assessment work was extensive, it was not about analysing what was proposed by the authors. To characterise the difference between assessing and analysing (textbooks), I have used the *microscope metaphor*. The PNLD assessment works with the observation of a material in the light of an optical microscope; it is possible to have access to various aspects and nuances of the material observed. By using an electronic microscope and observing the same material, elements previously invisible, or barely visible, can be better studied with all offal, because, with this tool, it is possible to deepen the study. Even among electron microscopes, some are more performative than others. Depending on what you want to see in the *harvested material*, you do not need a very powerful tool; just an optical microscope. This is the case of the PNLD assessment in which the assessment form is this tool. However, when you want to investigate some aspects of the material more deeply, you need a more powerful tool; in this case, an electronic microscope will allow you to see in-depth the object studied. Thus, the electron microscope is the theoretical framework on which the researcher relies to carry out the investigation. In reading research related to textbooks, indicated in the introduction to this text, I sought to identify which microscopes were used. However, often the theoretical framework and even the methodology of analysis are not clearly explained. Related to the study of the proposed content and, more specifically, to the tasks present in the textbook, I emphasise two studies: Stein, Smith, Henningsen, & Silver (2000 apud Kim 2014) classify tasks in four categories: memorisation tasks, procedures without connections tasks, procedures with connections tasks, and doing mathematics tasks. Mailizar & Fan (2014) classify the tasks in two categories: authentic

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the responsibility of the Secretariat of Basic Education (SEB/MEC) and the reviewers are chosen by SEB on a platform that contains registrations to be a reviewer of the PNLD.

tasks and non-authentic tasks. Also, some texts cite Charalambous et al. (2010) that classify the analysis of textbooks into horizontal, vertical, and contextual:

> the first category included three subcategories of communication: mathematical content, mathematical practices, and attitudes towards the subject. The second category included two criteria of textbook requirements: potential cognitive demands and types of responses, while the third category included three criteria of connections: connections within and between strands, connections between classroom instruction and textbooks, and connections to situations outside of school. (p. 129)

The proposal that I present in this text aims to provide a methodological tool to perform vertical type analyses. The electron microscope I chose to delve into what is proposed in textbooks is the anthropological theory of the didactic (Chevallard, 1998, 2003, 2007), as discussed below.

3. Elements of the anthropological theory of the didactic

For Chevallard (2003), *institution* is a social device that imposes operating rules on *individuals* that become *subjects* of this institution. Students and teachers are subjects of the school institution; parents and children are subjects of the family institution. *Object* is anything that exists to at least one individual or an institution. The *noosphere* designates the sphere responsible for thinking/influencing/deciding what will be defined as knowledge to be taught at each school level. Kaspary (2019, p. 231) considers

> noosphere to be a cluster of institutions. The association of teachers, the textbook publishers, the ministry of education, the community of researchers in education, the church in the case of non-secular or pseudo-laic states, and so many others are institutions that make up the noosphere of a given society.[7]

For Chevallard (1998), all human activity can be described through a task t, which belongs to a type of tasks T, which is defined by a verb and a complement. This task is solved by a techniqueτ, that is justified by a technology θ which, in turn, is justified by a theory Θ. Thus, there

[7] In a study on textbook analysis, we considered the textbook as a representative of the publishing institution.

is the praxeological quartet [T, τ, θ, Θ]; block [T, τ] is called technical-practical, or block of know-how-to-do; and block [θ, Θ] is called theoretical-technological block or block of knowledge.

> Ironing a skirt, for example, is a task; ironing trousers is another task that has similarities with the previous one. We can then talk about tasks that are of the same type: Ironing clothes. Each task of this set demands a technique that depends on the fabric of the clothing and the clothing itself: ironing a pleated skirt is definitely different from ironing jeans, that can be similar to ironing jeans shorts! The type of task is defined (described) by an action verb (ironing) and a complement (clothing); thus, the need for the complement is perceived so that the type of tasks is well defined. The techniques mobilised to solve such tasks can be justified – not always explicitly by the people who mobilise them – by physical laws. (Bittar, 2017, p.367)

A person who irons different types of clothing constantly knows that the iron temperature used for ironing a cotton garment is different from the temperature used for ironing a nylon garment. Although the person does not know the theoretical justification for this required temperature change, she/he does know that if this is not done, the clothing may be damaged or not well pressed.

As all human activity can be described by the praxeological quartet defined by Chevallard, doing mathematics, which is also a human activity, can be described through this model. Let's illustrate with a common activity at the beginning of the algebra study. Consider task t: *solving equation (x+3) (2x-2)=0* that can be described as belonging to task type T: *Solving equations of the form (ax+b) (cx+d)=0* having as a possible technique τ : calculating the values of x for which *(ax+b)=0 or (cx+d)=0*. The technology that justifies this technique is θ: *In the set of real numbers, if a.b=0 then a=0 or b=0* and the theory that justifies this technology is Θ: *The set of real numbers is a domain of integrity.* This is a[8] possible modellation, not the only one, which is done considering the institutional context.

And how do you describe a technique? What are the ingredients that make up a technique? (Chevallard, 1994). The ingredients that make up a technique are called ostensive objects:

[8]In previous texts I used modelling, however, not to confuse it with the theoretical trend of mathematics education, I decided to use the term modellation.

> We will speak of *ostensive object* [...] to refer to every object having a sensitive nature, a certain materiality, and that, therefore, acquires for the human being a perceptible reality. [...] Non-ostensive objects are then all "objects" that, like ideas, intuitions, or concepts, exist institutionally - in the sense that we attribute an existence to them – without, however, being able to be seen, said, heard, perceived, or shown by themselves: they can only be *evoked* or invoked by the proper manipulation of certain *ostensive objects* associated (a word, a phrase, a graphic design, a writing, a gesture, or a long discourse). (Bosch & Chevallard, 1999, p.90)

Thus, to investigate the mathematical activity, we must study the ostensive ones, whose function varies according to the type of activity and the student's cognitive development. In the first two years of PE (6 – 7 years), there is a strong presence of ostensives that represent objects known to children that are gradually replaced by symbols, such as scratches or polka dots. An example of this evolution can be seen in figure 1, with examples taken from the same book intended for the first year, on the left of which there is an illustration found on page 70, and on the right an illustration taken from page 132, when the representations of the objects are replaced by stripes and polka dots.

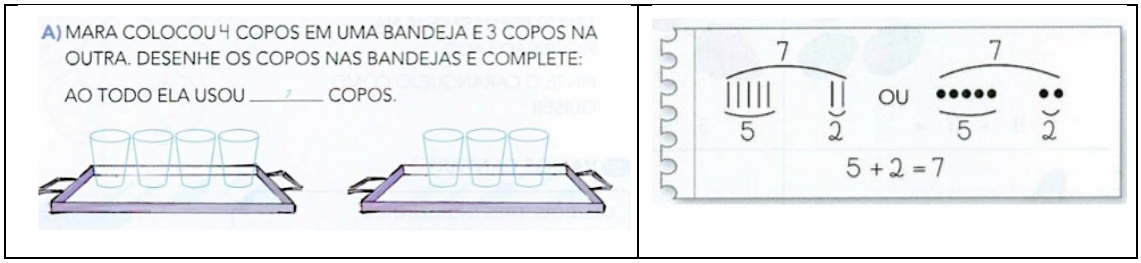


Figure 1: Example of ostensives (adaptation by Kaspary, 2014)

The evolution of ostensives in a same textbook is then perceived, however, they continue to evolve throughout the collection, indicating a praxeological evolution (Kaspary & Bittar, 2018). Besides the techniques being composed of ostensives, they are also tasks. One of the types of tasks identified by Oliveira (2010) was "T_{10}: *Determine the zero or the root of a first-degree polynomial function*" and the technique to be mobilised for its resolution was modelled as follows: "τ_{10}: *Substitute zero in the variable y of the relation y=f (x) and solve the equation obtained*." Thus, we perceived that one of the steps understood by the technique is the type of tasks "*Solving a first degree equation with an unknown*." So, can every techniue be described as a type of task

that itself has a life of its own, and be the object of teaching before it becomes a technique? In general, yes, however, for Chaachoua (2010), the tasks are of two types: extrinsic, which exists as such, and intrinsic, which exists only as part of a technique, to make a technique work. An example of an intrinsic task presented by the author is *Transposing a term from one member to the other member of an equality.* This technique is quite present in the resolution of equations, but it is not object of study in itself, unlike what occurs with the development of algebraic expressions, which is also present in the resolution of some equations, as in equation (x-3)(x+2=(2x-1)(x+1). In this case, the technique *Developing algebraic expression* is an extrinsic task. This classification proposed by Chaachoua can help identify intrinsic techniques often considered "natural" in teaching, generating learning difficulties. Such identification is important for proposing alternative activities.

Once a mathematical praxeology, also called mathematical organisation (MO), has been designated as a work to be studied, this question arises: *How to study this work?*, whose answer is called praxeology or didactic organisation (DO). The DO can be modellated through the praxeological quartet and through the six moments of study defined by Chevallard (1998): first meeting with the organisation *O*, object of study; exploration of the type of tasks T and development of a technique τ; constitution of the technological and theoretical environment [θ/Θ] related to τ; work with the technique; institutionalisation; assessment. The modellation of those moments will allow the identification of the didactic approach proposed by the authors of the textbooks. The overvaluation of the moment dedicated to the study of technique, for example, may indicate a technicist choice of pedagogical approach.

> Theoricist and technicist teaching models share a naive psychologist conception of the didactic process that has in behaviorism its clearest reference. In both cases, the teaching process is conceived as a mechanical and trivial process that the teacher can control entirely: theoreticism tends to conceive the student as an "empty box" that must be filled over a gradual process that starts from the most logically simple concepts until it reaches, step by step, the most complex conceptual systems; technicism, in turn, considers the student as an "automaton" that improves the mastery of techniques

> through simple repetition. For all these reasons we will call "classics" both teaching models as opposed to "modernists" [...], who identify "teaching" and "learning mathematics" as teaching and learning an exploratory activity, free and creative, of non-trivial problems. (Gascón, 2003, p. 136, our translation)

I will not go further in the discussion on elements of the ATD, because I believe that those presented so far are sufficient to understand what will be exposed below, however, it should be noted that the analyses we carry out are not restricted to only these theoretical elements.

4. A model for textbook analysis in the light of the ATD

Since my doctoral thesis, defended in 1998, I have been faced with the need to study how specific contents are presented in textbooks. As previously mentioned, for my thesis, the mathematical object on screen was vectors in basic education. At that time, I did the textbook analysis without a theoretical framework, using only the mathematical object as a reference. Shortly afterwards I started working at PNLD, where I began reviewing textbooks, while, almost simultanueosly studying ATD better. These two situations led me to believe that this theory brought elements that favoured not only the analysis of teachers' institutional practices, but also those practiced by textbook authors in their works. At the time I began, along with my students, developing research involving textbook analysis in the light of the ATD. After just over ten years of this work, I began describing the procedure for those analyses.

Chaachoua and Comiti (2010) also discuss textbook analysis in the light of the ATD, proposing the following elements for the study of this material: when the work is published; the representativeness of the work; the structure of the book; ecological analysis; praxeological analysis. However, one question that arises when starting the analysis of a content proposed in a textbook is *how to perform mathematical and didactic praxeological analysis*? I have been dedicated myself to this theme since 2006, which led me to elaborate the model that I introduce briefly, to then discuss each of its stages with the support of examples:

- constitution of the *corpus* for the analysis – choice of collections or volumes of textbooks to be analysed according to the research objectives;

- elaboration/modellation of mathematical praxeologies present in the *Course* part of the material that is composed of definitions, properties, solved activities, and information provided in various forms, i.e., *Course* is all that is not proposed activity. I assign this nomenclature because I understand that this is what is presented as an explanation for the student;
- mathematical analysis of the *proposed Activities* - this study is done with the support of mathematical praxeology modeled in the *Course* part. At this stage of the work, each proposed activity, which can be an immediate application of a technique or a problem-situation, is carefully analysed, to identify the task and the expected technique to be mobilised, in addition to the theoretical discourse that justifies the technique. For this, the study of the *Course* part is essential, but we are also assisted by the teacher's manual[9], which, besides the responses to the activities, contains resolutions of some of them and various comments so as to assist the teacher in their teaching practice. A task that had not been modelled in the *Course* may appear in the proposed activities, and it is then added to the modelled mathematical praxeology;
- modellation of the didactic praxeology present in the materials analysed. This study is done in parallel to the study of mathematical praxeology (OM), as it is about *modelling* how mathematical praxeologies are presented both in the *Course* and in the *proposed Activities*. It is the analysis of the whole set that will allow the researcher to infer the proposal of the collection for the teaching of some content.

[9] All textbooks acquired by MEC have the Teacher's Manual that contains the answers to the activities proposed, comments on some activities and a pedagogical supplement with various texts on the teaching and learning of mathematics, objectives and organisation of the collection, extra activities, among others.

- data triangulation. Once the the modellation of mathematical and didactic praxeologies has been carried out, the time has come to cross-reference these praxeologies and, with the support of theoretical studies, to answer the research question.

It is important to clarify that these steps are not linear, especially with respect to the modellation of praxeologies or mathematical organisations (MO). It is not about building those praxeologies in the *Course* part and "verifying" how and in what quantity they appear in the *proposed Activities*. There are two main reasons for that: 1) *Proposed activities* and *Course* are interspersed along the same volume; and, 2) the first encounter with a mathematical organisation can occur in a proposed activity, consequently the researcher will complement the OM analysis initiated in the *Course* part. Thus, the praxeology modelled in the *Course* part works, in a way, as an a priori praxeology to be tested and complemented with the analysis of the *proposed Activities*.

We present below each of these steps, with explanations of its operation and from examples, aiming to instrumentalise the reader interested in the use of this methodological tool.

4.1 Constitution of the *corpus* for analysis according to the research objectives

The material that will constitute the *corpus* must be chosen in line with the objective of the research, which is not, textbook analysis. This analysis is done to achieve a certain objective or part of it, that is, this analysis assumes different functions in a study, according to the question that the researcher seeks to answer. If the material to be analysed was published after the implementation of the evaluation process by the PNLD, a first factor of choice is to use the *Guia* (guide) to know the books that can be purchased by public schools, besides having access to other information about the collections. However, both works published after the implementation of the assessment process by PNLD that are not included in this Guide - because they were excluded or did not undergo analysis – and older works may also constitute the *corpus* for the research to be

carried out, always depending on the objective of the investigation. Let's look at some examples of choosing the *corpus* for data production[10].

Oliveira (2010, p. 5) investigated "the relationship between the knowledge acquired in initial education and those mobilised during pedagogical practice by a mathematics teacher at the beginning of his career," related to the concept of affine function. To study knowledge mobilised by the teacher, she relied on the necessary knowledge-based theory for teaching (Shulman, 2001). And to explore the relationships between the knowledge acquired in initial education and those that the teacher taught, the author sought to understand the reasons for the changes that the teacher made in relation to mathematical knowledge proposed in the textbook used by him. For this, it was necessary to analyse the textbook and compare both the mathematical and didactic praxeologies proposed in the textbooks and the teacher's praxeologies. Thus, in the case of Oliveira's research, the *corpus* related to the choice of textbooks to be analysed was the one the teacher adopted.

Seeking to understand the reasons for students' difficulties when they start studying algebra, Nogueira (2008) aimed to characterise the dominant praxeological model (mathematical and didactic) related to the teaching proposal of this content through the analysis of textbooks for the seventh grade of elementary school (12-13 years), when the study of algebra in Brazil begins. The analysis of this model would allow the author to identify possible sources of learning difficulties, because, as discussed at the beginning of this text, the way some content is presented in a textbook can lead students to construct, for example, erroneous theorems in action.

As during the 7th-grade algebra is usually approached in a single chapter of the textbook, the time available for Nogueira's research (2008) allowed her to study different collections. Thus,

[10] I have used data production instead of data collection because I do not consider that the data is ready and it is just, for example, opening the book and collecting it. This is a production of the researcher from the material she has and hrs theoretical and methodological frameworks.

Nogueira chose three volumes of distinct collections, approved by the 2008 PNLD and, with the support of the PNLD Guide, the author chose three collections with different pedagogical trends[11].

Nogueira's choice allowed, on the one hand, to know the mathematical praxeologies proposed in textbooks and, on the other hand, to know different didactic praxeologies proposed around the algebra teaching proposal. We must clarify that, in my proposal here, the analysis of mathematical organisations allows an in-depth study (with the electron microscope) about the textbooks authors' didactic choices, while the evaluation carried out by PNLD allows indicating the overall pedagogical approach of the work.

Kaspary (2014) investigated the proposal for teaching the additive field in the initial years of elementary school (6 – 10 years of age). As this theme is present in each of the five years of this stage of schooling, for the research to be feasible in two years, Kaspary was able to work with only one collection of textbooks, opting for the most adopted one in Brazilian schools. It is important to emphasise that in the FNDE portal, the quantity of each collection purchased by MEC is available, which helps guide the choices of the collections to be analysed.

The research developed by Freitas (2015) aimed to characterise the proposal for teaching the volume of geometric solids in textbooks intended for high school. Unlike the theme addressed by Kaspary, geometric solids occupies part of one of the three volumes intended for high school, so Freitas could work with a wider spectrum of material and his corpus was constituted of the four most adopted collections approved by PNLD 2012, which allowed him to have a more comprehensive look at the knowledge to be taught, i.e., about the dominant epistemological model (DEM).

[11]Although the texts of the Guide do not explicitly bring this data, the reading of the review of each approved collection allowed Nogueira (2008) to identify collections with different pedagogical trends.

The Brazilian official documents state that probability should be taught at all levels of basic education, which led Verbisck (2019) to seek to understand how this theme is proposed in each of the thirteen years of compulsory schooling (6 – 17 years). To make the research feasible in two years, she needed to choose only one collection of each of the three levels of schooling. Assuming as a hypothesis that if the collections were of a same author, there would be greater internal coherence between the collections than in different authors' collections, Verbisck chose to work with a collection of a same author, which is not always possible, since not every author writes for all segments of basic schooling. The constitution of this *corpus* allowed the author to make a longitudinal design of all the dominant praxeology related to probability in basic education.

Through the examples presented in this section, it is possible to observe a diversity of criteria used in the constitution of the *corpus* according to the place that the textbook analysis occupies in the research objectives: most adopted book collections; collection or book used in the institutional context studied; diversity of works in a specific aspect; choose between analysing a book as a whole, a collection of one stage of schooling, multi-stage collections (of the same author), or even excerpts from books or chapters in more than one collection.

After choosing the *corpus*, other dimensions of this analysis are discussed, as follows.

4.2 Modellation of mathematical praxeologies present in the *Course* part

The choice for the term modellation was chosen because it is an elaboration the researcher made from what she found in the material analysed and a reference epistemological model the researcher assumed almost explicitly. Due to its objectives, the researcher models in terms of praxeologies what she sees in textbooks, and she sees according to her theoretical and methodological lenses. Another researcher, with the same objective and different lenses, would probably produce other data.

The modellation of mathematical praxeologies, or mathematical organisations, (MO) begins in the *Course* part and can be done through a solved activity, the classic presentation of the

content, a problem situation, or other forms, depending on the methodological proposal of the work. However, this modellation can begin with an *a priori* model made from previous investigations, as was the case with Kaspary's research (2014). The author began the analysis of the *Course* part with 8 types of tasks modelled *a priori* from the study of situations in the additive field (Vergnaud, 1990,) and, as she found new types of tasks in the *Course* part of the collection analysed, these were incorporated into the MO[12].

I believe I can infer that it is in the *Course* part that information textbook publishers and authors consider important is found, either by conviction, to meet a demand of the noosphere, as we will see in some examples, or to meet a teachers' desire[13]. Moreover, it is in the *Course* part that the techniques to solve the types of tasks presented in the *Course* and the *proposed Activities* are found. It is also in this part that we can find elements of the theoretical-technological block, as we will see in the example illustrated in Figure 2.

The research must respect local realities, besides its own objectives, which means that not all stages of the proposed model are necessarily fulfilled. This was the case with Jolivet's doctoral research (2018)[14], whose objective was to develop a "model of didactic description of resources" for mathematical learning, a model to be implemented in a computerised environment. This author resumes the separation between *Course* and *Activities Proposed* presented in Bittar (2017), and justifies his choice to work with only the second part – *Activities Proposed* - because it represents about 70% or more of French textbooks and teachers in that country mostly use textbooks as a source of proposed activities, producing their own courses. This is a different reality from that of Brazil and many other countries as seen in several texts presented in the ICMT2017 and in Fan,

[12]Situations proposed by Vergnaud do not include situations that are not contextualised, such as: *Calculate 18+34*.

[13] This inference is made with support in readings on the textbook role for the teacher and the experience with PNLD over the years.

[14] To readers interested in analysing statements of activities, I recommend reading Jolivet (2018).

Zhu, and Miao (2013). I emphasise, however, that in the model I present, the analysis of the *Course* part allows modellation praxeologies that will help in the work with the modellation of praxeologies found in the *Activities proposed*. Thus, even if the interest is only to analyse the *Activities proposed*, the analysis of the *Course* part offers tools to better understand the intentions of the textbook author. To illustrate this argument, let us see an example of Nogueira's research (2008) on the introduction of algebra (Figure 2). Given an equation, for the resolution, she makes an analogy with a balanced scale, as the authors explain: "To solve the equation 3x=2x+100+50, we can imagine it as a scale with its pans in equilibrium." Each member of the equation is represented in one of the two pans of the scale, and the movements made in the scale are also made in the equation, on the right side of the page.

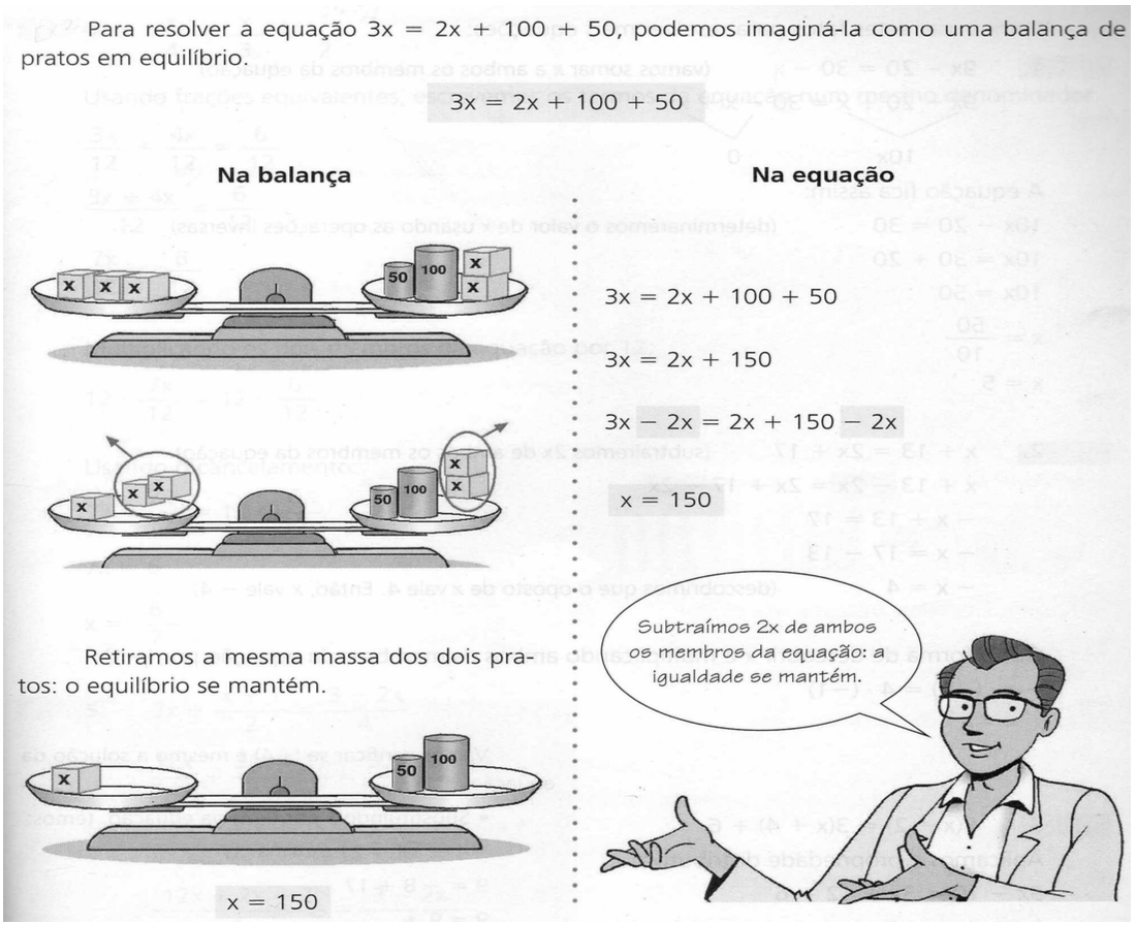


Figure 2: Presentation of the equation resolution technique (Bigode, 2000, p. 183)

In this activity solved, present in the *Course* part, the researcher identified task t: *solving equation 3x=2x+100+50*, belonging to task type T: *solving equations of the form ax+b=cx+d, with a, c not null*. The twin-pan scale functions as a justification for the procedure performed in the equation. The discourse "We remove the same dough from both pans: balance is maintained," justifies the technique "we subtract 2x from both members of the equation: equality is maintained." The discourse, or the twin-pan scale, functions as a technology for technique. At this stage of data production, it is not appropriate to discuss whether the use of this analogy is relevant, since the objective is to identify the dominant praxeological model in the textbook. However, this ostensive plays an important role in the constitution of the proposed technique and, in the future (in the same volume), it will be abandoned, since its instrumental valence or instrumentality[15] (Bosch & Chevallard, 1999) is limited.

4.3 Mathematical analysis of the *Activities proposed*

Each activity proposed is analysed seeking to identify the task to be performed and the technique that the student is expected to mobilise. For this, clues are sought in the *Course*, but also in the teacher's manual. The praxeologies modelled in the *Course* part are resumed in the analysis of the *Activities proposed*, however, new praxeologies can be identified during the analysis of these activities and should be considered.

A central question that arises at this stage of the modellation of mathematical *praxeologies is how to group tasks into types of tasks*? What is the level of granularity in

[15]«The instrumentality [instrumental valence] of an ostensive depends on the number of techniques in which it can intervene and will be greater the more these techniques prove robust in fulfilling the tasks in question." (Bosch & Chevallard, 1999, p. 23)

defining the types of tasks? This is a problem enunciated by Bosch and Chevallard (1999, p. 91):

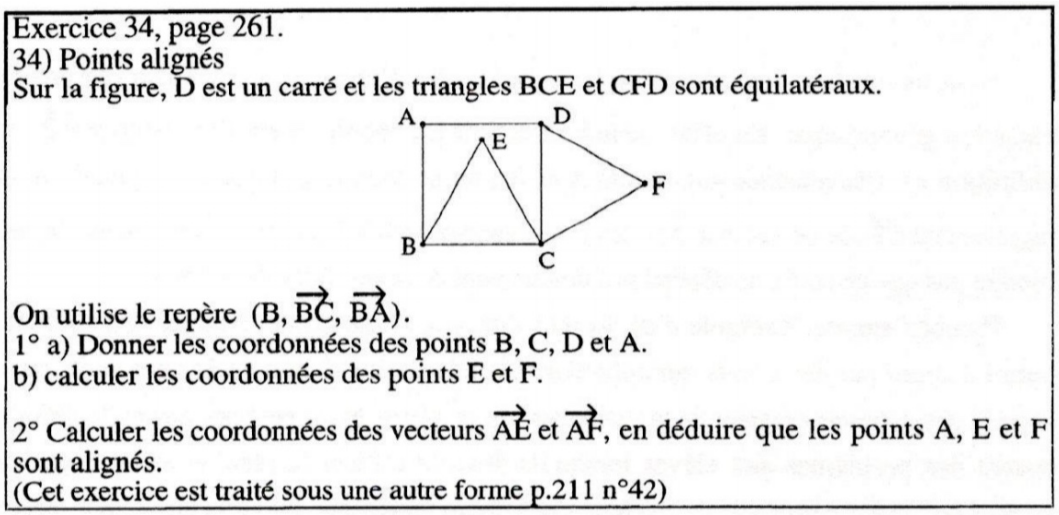
> The problem of delimitation of tasks in a given institutional practice remains open and will vary depending on whether one adopts the point of view of the institution where the practice occurs, or that of an external institution from which we observe the activity and describe it for a specific purpose.

Thus, when performing the mathematical modellation, the researcher should decide on this degree of granularity due to its objectives and also what the data tell him/her. Let us take the research by Freitas (2015) to illustrate this definition. The author analysed 580 (five hundred and eighty) tasks related to the study of solids volume, and initially categorised them into 06 (six) types of tasks, of which 438 (four hundred and thirty-eight) were grouped into the type of tasks T1: *Calculate the volume of a solid*. He noticed that this classification did not say much, so he chose to subdivide T1 into two subtypes of Tasks, T1.1: *Calculate the volume of a known solid*, and T1.2: *Calculate the volume of an irregular solid.* This subdivision aimed to identify the importance attributed to each of them and for this more detailed study was chosen from the four collections studied by Freitas, the most adopted in Brazil, due to its greater representativeness. In this collection, the author found 151 tasks involving solids volume, of which 119 belonged to the type of tasks T1: *Calculate the volume of a known solid* and, of these, 95 referred to prism, pyramid, cone, sphere, cylinder, or parallepiped; 16 referred to pyramid trunk, cone trunk, or spherical wedge; and only eight referred to solids that Freitas called irregular, which were a composition of known solids or solids composed of surfaces that were neither plane nor spherical. Also, when identifying the desired techniques to solve the 119 tasks, he observed that 42 tasks were solved exclusively with the direct application of the formula of the volume of the respective solid. Thus, the author realised that the intended teaching prioritises the use of formulas applied to known solids.

The quantitative study of the data is, therefore, fundamental for the type of book analysis proposed in this text, as it helps to understand the importance given to each praxeology. If, for

example, for two types of tasks in a book, one of them has only three tasks while the other has 20 tasks, this difference is an important element for qualitative study. This aspect of data quantification, which I consider essential, is one of the differentials in relation to other textbook analysis methodologies: we want to know not only what is present, but also what the importance attributed to each element present. As previously mentioned, when a certain type of task is present, but only roughly present, its inclusion may be justified by the search for compliance with what is recommended in curricular guidance documents and/or PNLD recommendations.

For a textbook analysis, we must therefore take into account the institutional context considering official recommendations, and what and where it is presented in the book. As an example, which is of central importance in the proposed model, I bring a task present in French textbooks intended for the 1st grade of high school on three-point alignment.


Exercice 34, page 261.
34) Points alignés
Sur la figure, D est un carré et les triangles BCE et CFD sont équilatéraux.

On utilise le repère $(B, \overrightarrow{BC}, \overrightarrow{BA})$.
1° a) Donner les coordonnées des points B, C, D et A.
b) calculer les coordonnées des points E et F.

2° Calculer les coordonnées des vecteurs $\overrightarrow{AE}$ et $\overrightarrow{AF}$, en déduire que les points A, E et F sont alignés.
(Cet exercice est traité sous une autre forme p.211 n°42)

Figure 3: Three-point alignment (Nakatani apud Bittar, 1998, p.167)[16]

[16]Translation: Points aligned
On the figure ABCD is a square and the ECB and CFD triangles are equilateral.
We used the axis $(B, \overrightarrow{BC}, \overrightarrow{BA})$
1° (a) Give the coordinates of points B, C, D and A.
b) calculate the coordinates of points E and F.
2° Calculate the coordinates of the vectors $\overrightarrow{AE}$ and $\overrightarrow{AF}$ deduce that points A, E, and F are aligned.
(This exercise is approached differently p.211 n°42)

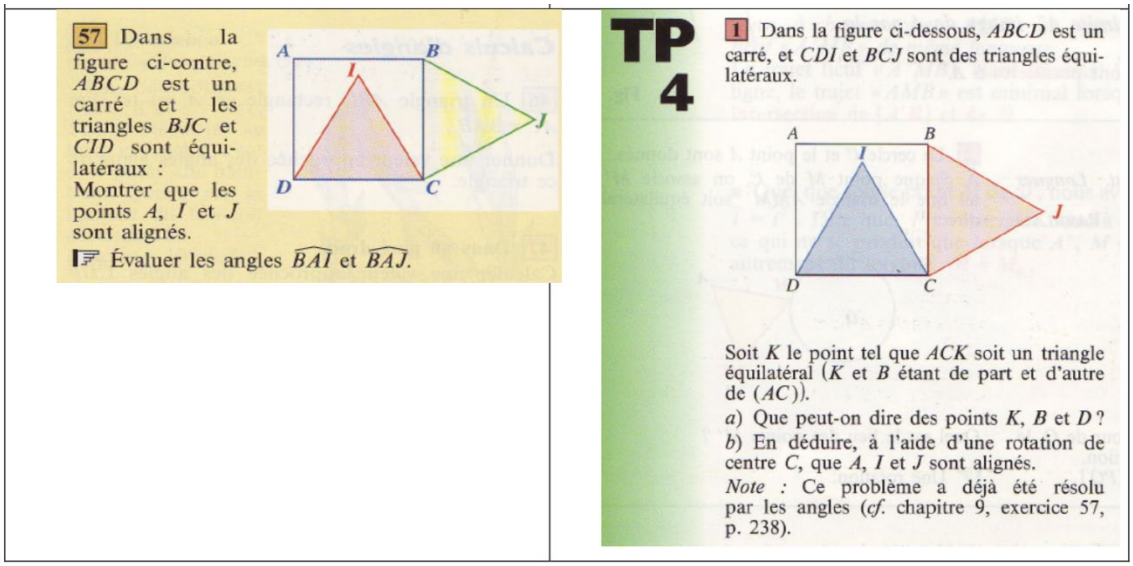


Figure 4: Point alignment (Terracher & Ferachoglou apud Chaachoua, 2010, p. 8)[17]

It is possible to observe that in the three activities, the statements of the tasks carry the technique to be mobilised. The mathematical task (Chaachoua, 2010, p. 9) proposed is the same: "Be ABCD a square, BJC an equilateral triangle outside the square and CID an equilateral triangle inside the square. Show that points A, I, and J are aligned. "However, the technique to be used is different in each of the three examples, thus constituting three specific praxeologies around the same type of tasks. Castela (2008) defines complex point mathematical organisation relative to a type of tasks T as the set of point MOs associated with T. The problem of three-point alignment, which appears on different pages of the same volume to be treated with different techniques, is

[17]Translation
57) In the figure opposite, ABCD is a square and the BJC and CID triangles are equilateral. Show that points A, I, and J are aligned.
Tip: Evaluate BAI and BAJ angles
TP4
In the figure below, ABCD is a square and the CID and BJC triangles are equilateral.
Let K be the point such that ACK is an equilateral triangle, (K and B being on one side and the other side of (AC)).
a) What can we say about points K, B and D?
b) Deduct, with the help of a center rotation C, that A, I, and J are aligned.
Note: This problem has already been solved with angles (cf. chapter 9, exercise 57, p.238)

an example of such complex point mathematical praxeology. Douady (1986) warns of the importance of practising, in the teaching of mathematics, the change of frames, which is favoured in the previous examples. Thus, a textbook analysis may aim to investigate whether and how complex praxeologies are present and, consequently, propose alternative models for teaching.

When analysing the praxeology proposed in a textbook, the interest is to seek to capture, as deeply as possible (microscope metaphor), the intentions contained in this material. Therefore, it is not a matter of indicating the technique that we consider more pertinent, or more economical, but rather to seek the one desired by the institution.

4.4 Modellation of the didactic praxeologies

The analysis of the didactic organisation or praxeology (DO) makes it possible to identify the didactic approach present in textbooks or a teaching activity as a class. Essentially, when seeking to identify the DO of a textbook, we aim to answer the question "How is the teaching of a specific content proposed?"

By studying the didactic moments related to the identified praxeologies, Freitas (2015) concluded that in all chapters related to the study of solids volume, the textbook author follows the same model:

> [...] the volume content begins with the demonstration of its formula, which we believe to be the construction of the technological-theoretical block (the first moment with the mathematical organisation) that underlies the elaboration and application of the technique τ_2. Following the technique institutionalisation, characterising the fifth moment, i.e., the author presents the formula that will be used in the resolutions of the activities proposed in the sequence. Subsequently, we present some examples and exercises solved aiming to work on the technique that had just been constituted and that, in our perspective, characterises the fourth moment, that is, the work with the technique. (Freitas, 2015, p.120)

This analysis allowed the researcher to infer that the didactic choice of the author of the collection studied, regarding the study of the volumes of the solids, resembles a classical didactic organisation (Gascón, 2003).

Pantoja (2017) identified that the first meeting with praxeology related to affine function in the textbook adopted by the teacher was carried out through a contextualised situation, however, in his class the teacher chose to directly present the definition of affine function, without any contextualisation. The identification and characterisation of the teacher's didactic praxeology and that proposed in the textbook will allow the researcher, at the time of data triangulation, to infer the reasons for the teacher's choices.

The meticulous modellation of didactic moments is important not to make *absolute* inferences about the textbook proposal, since, for example, a collection may have a strong technicist trend, but still present activities that are not of this trend. This was Kaspary's conclusion (2014, p. 136), after analysing the moments of study, which indicated a didactic organisation "sometimes more of the empiricist type, sometimes more focused on the classical, and even if sporadically, also of the constructivist type, but always with a strong tendency towards the technicist axis".

4.5 Triangulation of data

After the modellation of the mathematical and didactic praxeologies of the *Course* and *Activities Proposed* parts, the *Data Triangulation* is carried out to look for elements of answers to the research questions.

The triangulation of the quantitative study allows a qualitative analysis of the evolution of praxeologies both within a volume and throughout the volumes of a collection or the entire schooling, as was the case with Verbisck's research (2019) on probability. In total, 23 tasks were proposed in the initial years, 16 in the 5th grade, when the study of concepts related to probability begins in a slightly more formal way. In the final years of elementary school, this number rises to 197, with 136 (69%) belonging to the type of tasks T3: *Determining the probability of occurrence of a specific event*:

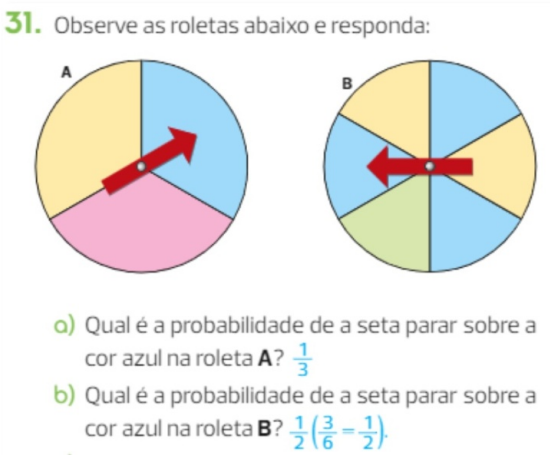
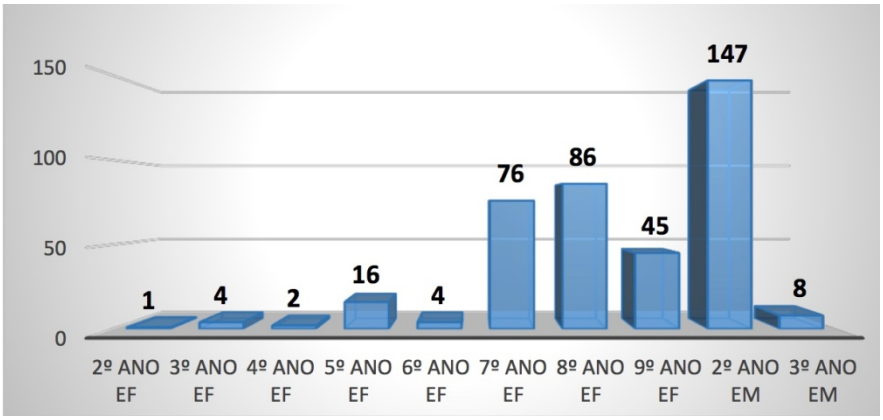


Figure 5: T3-type task example (Verbisck, 2019, p. 103)

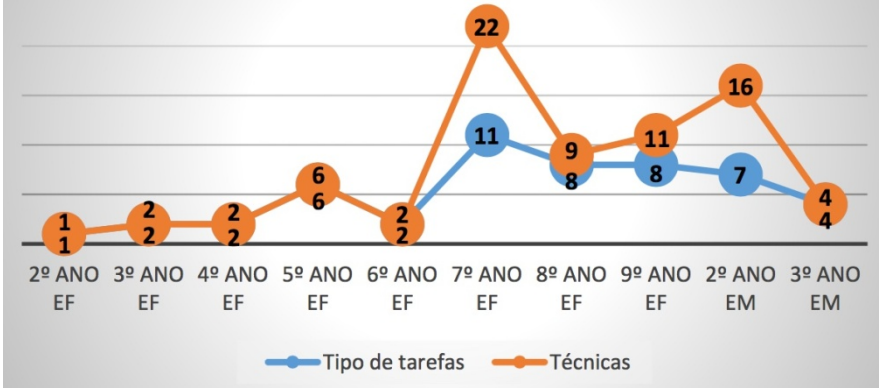
The longitudinal analysis in graph 1 depicts how the probability appears in the 2nd year of the elementary school and is developed in subsequent years until reaching the 3rd grade of high school.



Graph 1: Number of probability-related tasks by volume (Verbisck, 2019, p. 177)

We observe that, except for the 1st grade of elementary school and the 1st grade of high school, in all others, there are tasks related to probability, even when this content does not appear as an object of study, as is the case with the 6th grade, for example. Why is this happening? It is probably due to a demand of the noosphere, and at this time, the intersection with other studies made by the researcher will allow making inferences.

Not all types of tasks and techniques identified throughout the 13 volumes are present in all grades, which is evidenced in graph 2, on the evolution of the quantities of types of tasks and techniques throughout basic schooling.



Graph 2: Types of tasks and techniques in each volume (Verbisck, 2019, p. 178)

In this chart, the 7th grade draws attention, presenting 11 types of tasks and 22 different techniques. These data demand the attention of the researcher for a qualitative analysis that seeks to understand what those techniques are, how they appear, the number of times they are mobilised, what role they have in the praxeology developed,... One technique can be mobilised to give life to another technique that is the teaching objective. The study of the additive field made by Kaspary (2014) shows this: over the years, an evolution of techniques is perceived, evidenced by the ostensives that enable them - counting using images of objects, counting on scratches, use of fingers,... counting from one,... - until reaching, in the 5th grade, the desired algorithms. The instrumental valence of certain ostensives leads to their abandonment.

In her data triangulation, Oliveira (2010) concluded that the teacher mobilised, in the classroom, the same mathematical organisation proposed in the textbook, which did not always occur with the didactic organisation. The first encounter with mathematical praxeology, “affine function” occurred in the textbook through a contextualised activity. The teacher followed this same idea, however, he modified the situation by proposing a problem that involved his students’

daily lives. After an interview with the teacher, the author concluded that this change occurred due to the knowledge acquired in her initial education.

A theoretical tool of the ATD that did not appear in this text, but that has been used at this time of data triangulation is the scale of codetermination levels[18] (Chevallard, 2007).

5. Final considerations

The discussion I presented in this text sought to show that textbook analysis is located in various ways in the research problems. Due to what is known about the place and use of the textbooks in a given institution, this analysis can be a means to outline a modellation of knowledge that lies, in a sense, on the interface between the knowledge to be taught and the knowledge taught. Sometimes we can see the textbook as the realisation of a prescribed curriculum, sometimes as the north for an experienced curriculum, to which access, from a methodological point of view, can be more complex. Part of these analyses aim to seek elements that eventually help to understand possible didactic reasons that justify specific persistent errors and conceptual learning difficulties. Therefore, we observe the scientific and societal interest of analysing textbooks.

Over the years, the research developed under my supervision with the theme of textbook analysis focused on a variety of objects of knowledge (additive field, algebra, mental calculus, contextualisation, the volume of solids, probabilities,...) and different levels of education (from the initial years of elementary school to high school, including teaching practice in the first years of teaching practice). In all these investigations, the anthropological theory of the didactics brought powerful tools to analyse textbooks with a look that problematises the mathematical and

[18]The scale of codetermination levels defined by Chevallard (2007) consists of the following levels: humanity - civilization – society – school – pedagogy – discipline – domain – sector – theme – subject. This theoretical tool allows the study of the ecology of a given didactic system.

didactic activity proposed in textbooks. This is precisely because this is the main sensitivity (Artigue, 2011) of this theory being mathematical activity.

The observation of the regularities that were constituting themselves throughout this process led me to systematise these reflections and propose the model presented in this text, composed of four components: constituting the *corpus* for the analysis, the modellation of mathematical praxeologies present in the *Course* part, mathematical analysis of the *Activities Proposed*, modellation of the didactic praxeologies, and triangulating the data. These steps are not linear, nor is this model static.

One of the possibilities of expansion for the model presented in this text is related to Jolivet's suggestion (2018) about the statements of the activities proposed, because I believe that this study can contribute to the refinement of the data produced. Another perspective is the use of variables to describe mathematical praxeologies, proposed by Chaachoua and Bessot (2019).

Although this model has been designed and used for textbook analysis, parts of it have been mobilised in continuing and initial teacher education, especially the modellation of mathematical praxeologies in the *Course* part and *Activities proposed*. In this type of experience, the object of knowledge is revisited, the dominant models are questioned, and the possibility to reflect on alternative praxeological models is opened.

References

Artigue M. (2011) La théorie anthropologique du didactique : rapports et articulations possibles avec d'autres approches. In A. Bronner, M. Larguier, M. Artaud, M. Bosch, Y. Chevallard, G. Cirade, C. Ladage (Eds.), *Diffuser les mathématiques (et les autres savoirs comme outils de connaissance et d'action. Actes du Second Congrès de la Théorie Anthropologique du Didactique*, Uzès, IUFM de Montpellier

Assude, T. (1996). De l'écologie et de l'économie d'un système didactique : une étude de cas.

Recherches en Didactique des Mathématiques, 16/1, 47-72.

Bigode, A. J. L. (2000) *Matemática hoje é feita assim*. São Paulo: FTD.

Bittar, M. (2020). La politique d'évaluation de manuels au Brésil et son influence sur le curriculum réel. In : *Actes de la 20° École d'été en Didactique des Mathématiques,* Autrans, França, no prelo.

Bittar, M. (2017). A teoria antropológica do didático como ferramenta metodológica para análise de livros didáticos. *Zetétiké*, Campinas, SP. v.25.n.3, (364-387).

Bittar, M. (1998). *Les vecteurs dans l'enseignement secondaire. Aspects outil et objet dans les manuels. Etude de difficultés d'élèves dans deux environnements: papier crayon et Cabri-géomètre II.* Tese de doutorado em Didática da Matemática. Grenoble: Universidade Joseph Fourier

Bosch, M., & Chevallard, Y. (1999). La sensibilité de l'activité mathématique aux ostensives. Objet d'étude et problématique. *Recherches en Didactique des Mathématiques , 19(1)*, pp. 77–124.

Bosch, M. Gascón, J. (2010) Fundamentación antropológica de las organizaciones didáticas: de los "talleres de prácticas matemáticas" a los "recorridos de estudio e investigación". *In*: Bronner, A. at al. Diffuser les mathématiques (et les autres savoirs) comme outils de connaissance et d'action. IUFM de l'Académie de Montpellier, pp. 55-91.

Brasil. (1994). *Definição de critérios para a avaliação de livros didáticos. Português, Matemática, Estudos Sociais e Ciências*. 1ª a 4ª séries. Brasília: FAMEC-UNESCO.

Brasil. Guia de Livros Didáticos. PNLD 2010. Anos finais dos Ensino Fundamental. MEC - Ministério da Educação, Brasília.

Brasil. (1998) Ministério da Educação, Secretaria de Educação Básica. *Parâmetros Curriculares Nacionais. Matemática. 3º e 4º ciclos*. Brasília, DF.

Carvalho, J. B. P. F. (2018) The Brazilian mathematics textbook assessments. *ZDM: International*

Journal on Mathematics Education, v. 50, p. 773-785

Castela, C. (2008). Travailler avec, travailler sur la notion de praxéologie mathématique pour décrire les besoins d'apprentissage ignorés par les institutions d'enseignement. *Recherche en Didactique des Mathématiques , 28 (2)*, pp. 135-182. Retrieved from https://revue-rdm.com/2008/travailler-avec-travailler-sur-la/

Chaachoua, H., Comiti, C. (2010). L'analyse du rôle des manuels dans l'approche anthropologique, *Atas do 2° CITAD – Congrès International sur la Théorie Anthropologique du Didactique*, (771-789).

Chaachoua, H. & A. Bessot, A. (2016): Introduction de la notion de variable dans le modèle praxéologique. In *Educação Matemática Pesquisa,* São Paulo, v.21, n.4, pp. 234-247, 2019.

Charalambous, C. Y., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning, 12*(2), 117-151

Chevallard, Y. (2007) : Passé et présent de la théorie anthropologique du didactique. In *Congrès international sur la théorie anthropologique du didactique*. Baeza, Espanha, pp. 705 – 746,

Chevallard, Y. (2003) Approche anthropologique du rapport au savoir et didactique des mathématiques. *In* Maury, S. & M. Caillot, C. (Éds.). Rapport au savoir et didactiques (pp. 81-104). Paris : Fabert, pp. 81-104.

Chevallard, Y. (1998) Analyse des pratiques enseignantes et didactique des mathématiques: L'approche anthropologique. *Recherches en Didactique des Mathématiques,* 19(2), 221-266.

Doering, L. R., Soppelsa, J. J. C & Ripoll, C. C. (2017). The Euclidean division in the early grades. In: Proceeding of the *Second International Conference on Mathematics Textbook Research and Development*, pp. 86-94.

Douady, R. (1986) Jeux des cadres et dialectique outil-objet. *Recherches en Didactique des Mathématiques,* vol. 7, n°2, pp. 5-31. La Pensée Sauvage.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM: International Journal on Mathematics Education*, *45*: 633–646.

Fan, L. (2013) Textbook research as scientific research: towards a common ground on issues and methods of research on mathematics textbooks; *ZDM: International Journal on Mathematics Education*, *45*: 765-777.

Freitas, M. V. C. (2015). *Um estudo sobre volume de sólidos geométricos em uma coleção de livros didáticos do Ensino Médio.* Dissertação de Mestrado em Educação Matemática. Campo Grande: Universidade Federal do Mato Grosso do Sul.

Gascón, J. (2003) La necessidade de utilizar modelos em didáctica de la matemáticas. *Educação Matemática e Pesquisa*, São Paulo, v. 5, n. 2, pp. 11 – 37.

Gérard, F.-M., & Roegiers, X. (1998). Conceber e Avaliar Manuais Escolares (1 ed.). (J. Ferreira, & H. Peralta, Trads.) Portugal: Porto Editora.

Gonçalves, K. R. (2016). *A teoria antropológica do didático como ferramenta para o estudo de transposições didáticas: o caso das operações de adição e subtração dos números inteiros no 7o ano do ensino fundamental*. Dissertação de Mestrado em Educação Matemática. Campo Grande: Universidade Federal de Mato Grosso do Sul.

Jolivet, S. (2018) *Modèle de description didactique de ressources d'apprentissage en mathématiques, pour l'indexation et des services EIAH*. Recherche d'information [cs.IR]. Université Grenoble Alpes.

Kaspary, D. (2019). Noosfera e assujeitamento, duas noções da teoria antropológica do didático para problematizar o currículo e mudanças curriculares. *Revista Paranaense de Educação Matemática*, v. 8, p. 229-247.

Kaspary, D & Bittar, M. (2018). Ostensivos como ingrediente primário do estudo da evolução praxéologica. In: Ag Almouloud, S., L. M. S. & Henriques, A. (Org.). *A teoria antropológica do didático: princípios e fundamentos*. 1ed.Curitiba: CRV, v. 1, p. 466-482.

Kaspary, D. (2014). *Uma análise praxeológica das operações de adição e subtração de números naturais em uma coleção de livros didáticos dos anos iniciais do Ensino Fundamental*. Dissertação de Mestrado em Educação Matemática. Campo Grande: Universidade Federal do Mato Grosso do Sul.

Mailizar, M. & Fan, L. (2014) Assessing a new Indonesian secondary mathematics textbook: how does it promote authentic learning? In: Proceeding of the *International Conference on Mathematics Textbook Research and Development*, University of Southampton, UK. Pp.319-326.

Nogueira. R. C. S. (2008). *A Álgebra nos Livros Didáticos do Ensino Fundamental: uma análise praxeológica*. Dissertação de Mestrado em Educação. Campo Grande: Universidade Federal de Mato Grosso do Sul.

Oliveira, A. B. de. (2010). *Prática pedagógica e conhecimentos específicos: um estudo com um professor de matemática em início de docência*. Dissertação de Mestrado em Educação Matemática. Universidade Federal de Mato Grosso do Sul

Pantoja, L. F. L, (2017) Transposição didática interna: as transformações adaptativas realizadas sobre o saber matemático função afim para o ensino na Educação de Jovens e Adulto. Tese de doutorado em Educação em Ciências e Matemática. Universidade Federal do Mato Grosso.

Pjanić, K. (2017). One-step multiplication and division word problems in the 3rd grade textbooks in Bosnia and Herzegovina. In: Proceeding of the *Second International Conference on Mathematics Textbook Research and Development*, pp. 206-213

Ramalho, L. V. (2016). *Trigonometria em livros didáticos do 9o do ensino fundamental. Dissertação de Mestrado em Educação Matemática*. Campo Grande: Universidade Federal de Mato Grosso do Sul.

Shulman, L. (2001) Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*. Tradução: Alberto Ide. 57(1), 163-196.

Souza, N. F. (2014). *Contextualização no ensino da Álgebra: Análise de livros didáticos do 7o ano*. Dissertação de Mestrado em Educação Matemática. Campo Grande: Universidade Federal do Mato Grosso do Sul.

Steen, N. & Madsen, M. S.. (2017). The textbook in mathematics: findings from a systematic review. In: Proceeding of the *Second International Conference on Mathematics Textbook Research and Development*, pp. 214-225.

Thompson, D. & Senk, (2014) S. Lessons learned from three decades of textbook research. In: Proceeding of the *International Conference on Mathematics Textbook Research and Development*, University of Southampton, UK. Pp.51-56.

Usiskin, Z. (2013). Studying Textbooks in an Information Age - A United States Perspective. *ZDM Mathematics Education,* 45: 713–723

Verbisck, J. T. S. (2019). *Uma análise praxeológica da proposta de ensino de probabilidade em livros didáticos da educação básica*. Dissertação de Mestrado em Educação Matemática. Campo Grande: Universidade Federal do Mato Grosso do Sul.

Vergnaud, G. (1990). La théorie de champs conceptuels. *Recherches en Didactique de Mathématiques,* 10(2.3), 133-170.