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The Anthropological Theory of the Didactic in Brazilian researches

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Abstract: In the present article, we undertook a systematic study of bibliographic review based on the discussions, in the academic scope, about the contribution of the Anthropological Theory of Didactic – ATD (Chevallard, 1992) linked to Mathematics teaching and learning processes. The purpose was to investigate the current appropriation of ATD in *Stricto sensu* researches in Brazil. It is worth mentioning that the present study refers to a part of a PhD thesis. Thus, we aim to present an overview of thesis and dissertations answering the following question: "What are the main programs, institutions and researchers in Brazil that have been working on ATD?" In addition, we developed a study on the investigation axes of the researches, classified as: I) Teaching: focusing on the teacher, II) Learning: the object being the student's learning and III) Analysis of documents: books, lesson plans, course projects. For this purpose, we selected as a *corpus* 69 dissertations and 37 thesis defended in the period from 2005 to 2017.

Keywords: Anthropological Theory of the Didactic (ATD), Didactics of Mathematics, Systematic Review.

Introduction

The area of Mathematics Education has developed each year with the progress of researches that seek to understand phenomena related to the teaching and/or learning processes of Mathematics, based on theoretical or experimental studies, at different school levels, in the fields of initial training and continuing education of teachers, in the development of teaching materials, in the study of daily school life, among others. In 2016, the Sociedade Brasileira de Educação Matemática (SBEM) created the working group Didática da Matemática – GT 14. The creation of this group came to definitively institutionalize discussions within the scope of Mathematics Education, the specificities of Didactics of Mathematics theoretically supported by several theories that were mostly French in origin, which justifies some labels brought to this area such as *French Didactics*. Among these theories, one can highlight the Theory of Didactical Situations, The Anthropological Theory of the Didactic, the Theory of Conceptual Fields, Semiotic Representation, Didactic Engineering and Instrumental and Documentary Approaches.

Among these theories, we highlight the Anthropological Theory of Didactic (ATD) developed by Yves Chevallard (1992) and collaborators in the early 90s. According to the founder of the theory, to describe the

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genesis and evolution of the objects of knowledge in an institution, to describe institutional and personal relationships with an object of knowledge, it is necessary to have a descriptive model of that knowledge and know-how. Yves Chevallard's Anthropological Theory of Didactic (1992, 1999) offers a conceptual and methodological framework that allows any issue related to the diffusion of knowledge in any institution to be approached as a systemic issue. It is part of the continuation of the theory of didactic transposition, placing school knowledge in its relations with other forms of knowledge and societies. The ATD considers that the role of didactics is to study the didactics that are present, each time an individual x does something to make another individual y learn a certain object of knowledge. A didactic system is established around the study of a certain question, the objective is then to provide an answer to this question, which is called work. However, as didactic systems usually age, the question is lost sight of and the answer remains its sole *raison d'être*. For this reason, ATD studies the diffusion in society of complex praxeologies or parts of these praxeologies. In fact, the Anthropological Theory of Didactics places the didactic system on a scale of didactic co-determination. This scale allows placing the didactic system and the praxeology that is transmitted in it in a set of precise places, each bringing a set of particular conditions and restrictions. To explain what people do within institutions, ATD postulates that all human activity can be modeled in terms of praxeology $[T / \tau / \theta / \Theta]$. The praxis component $[T / \tau]$ describes techniques τ to perform certain types of tasks T . While in the logos component $[\theta / \Theta]$ it describes technologies θ - which are the speeches about technology - aim to describe, explain, legitimize, produce the techniques implemented; these technologies are part of the theories that legitimize them. (Farias, 2010)

In the field of Didactics of Mathematics, ATD has been studied at an international level with regard to the advances it has shown both in research in Mathematics Education and in other areas. These works are deepened and disseminated in events such as "Congrès international sur la Théorie Anthropologique du Didactique – CITAD", a specific congress on ATD that takes place every two years in France and Spain. In addition to CITAD, we have, in Brazil, LADIMA – Congresso Latino-Americano da Didática da Matemática, which has consolidated itself as a space for the debate of knowledge related to the theories that support investigations in the fields of Didactics of Mathematics.

Given the scenario of discussions about the contribution of the Anthropological Theory of Didactic in academic investigations related to the teaching and learning process of Mathematics, one opted for conducting a bibliographic study in order to investigate the current state of ATD appropriation in *Stricto*

sensu researches in Brazil. This problem arises from the lack of studies of the type of systematic review with research in the field of Didactics of Mathematics. In Brazil, there is no mapping of these researches and, considering the recent creation of the WG and the growing number of works in the area, and still starting from an international discussion of how ATD is being developed in the investigations, this work aims to present an overview Brazilian study and ATD researchers addressing the teaching, learning and analysis of documents. As an excerpt of a PhD thesis, we seek to present, in this article, an overview of the researches answering What are the main programs, institutions and researchers in Brazil that have been working on ATD? in addition to developing a study on the research axes of the works, classified in I) Teaching: focusing on the teacher, II) Learning: the object being the student's learning and III) Analysis of documents: books, lesson plans, projects of course are the main objects of investigation.

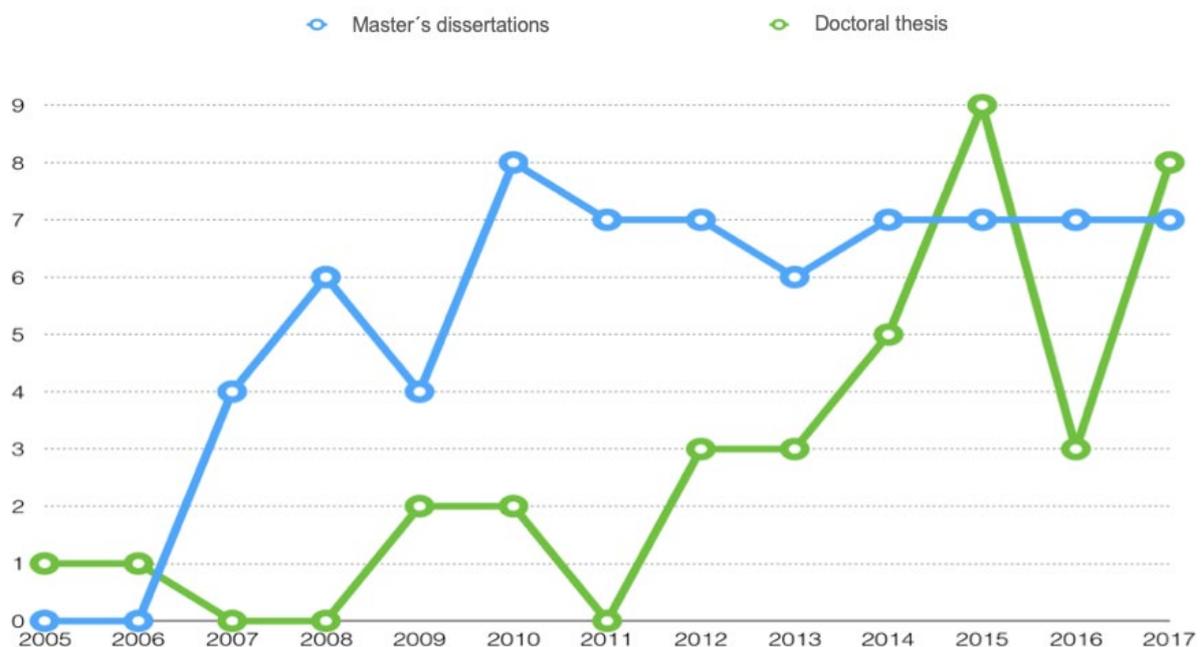
This study is inserted in the field of bibliographic research methodology of the systematic review type and is understood by us as a possibility to study a certain area of knowledge, allowing it to be evaluated through cross-views between the researches that have already been carried out, called primary studies, which are taken as a corpus of analysis. In our research, we used the Catalog of Theses and Dissertations available on the Portal da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – CAPES³, a foundation linked to the Ministry of Education of Brazil. As a descriptor, we use the expression “Anthropological Theory of Didactic”, in which 131 researches were found between theses and dissertations. As our area is Mathematics, we exclude from our repository investigations developed in other areas other than Mathematics, such as Physics, Chemistry and Biology. Thus, we had a total of 69 dissertations and 37 thesis found, defended in the period from 2005 to 2017. The year 2005 was chosen because it was the first thesis defended in Brazil with the theoretical framework of ATD.

However, it cannot be denied that, like any method, there are benefits, difficulties and gaps associated with each one of them. When it comes to the search for a *corpus* that has one element in common, this procedure can occur in such a way that the tools used cannot reach a totality. In addition, the research platforms used may consider works in categories that do not fill the whole sought, as stated by Alencar and Almouloud (2017).

³ <http://catalogodeteses.capes.gov.br/catalogo-teses/#/>

Postgraduate programmes that discuss ATD in Brazil

We took as the starting point of the period chosen to select the thesis and dissertations defended in Postgraduate programmes in Brazil, the year 2005, as this is the year in which we identified the first *Stricto sensu* work defended with the theoretical framework of ATD. It is important to highlight that it was in this year that the first edition of CITAD took place in Spain, whose objective was to "bring together researchers who currently work in the field of ATD, within the scope of Didactics of Mathematics and other fields of knowledge". To delimit the final timeframe, we established the year 2017. In order to understand the production of thesis and dissertations in the given period, we have the following graph:



Graph 1: Number of dissertations and thesis defended by year.

É possível constatar, pelos números, que a produção de dissertações é superior à de teses. A escassa produção de pesquisas no começo dos anos 2000 se justifica pelo fato de a TAD passar a ser divulgada a partir do final da década de 1990. Ademais, muitos estudantes brasileiros fizeram doutorado na França, retornando no final dos anos 1990, quando se fortaleceu a introdução da Didática da Matemática no Brasil. It is possible to verify, by the numbers, that the production of dissertations is superior to that of theses. The scarce production of research in the early 2000s is justified by the fact that ATD started to be disseminated from the end of the 1990s. In addition, many Brazilian students did doctorates in France, returning in the late 1990s, when it strengthened the introduction of Didactics of Mathematics in Brazil.

There is a time of maturity and recognition that all theory goes through; with regard to ATD, the researches, mainly Master's, have remained more constant since 2007. The amount of studies, by itself, does not reveal a fact about the state of ownership of ATD in Brazil, but it justifies, through the number of selected researches (106), the relevance of this study to the area, considering that we have just over 20 years of influence of ATD in the issues discussed within the Didactics of Mathematics.

We will contextualize these quantitative data, demonstrating the distribution of the research in question in the national territory, in order to obtain an overview of the places where the Anthropological Theory of Didactic is discussed most and in which institutions. Considering all the chosen theses and dissertations, we can verify the distribution in the country from figure 1:

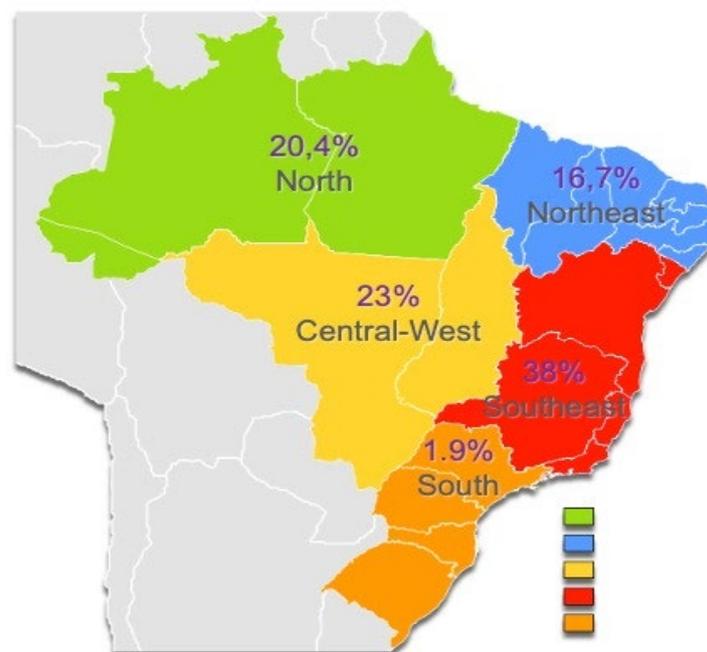
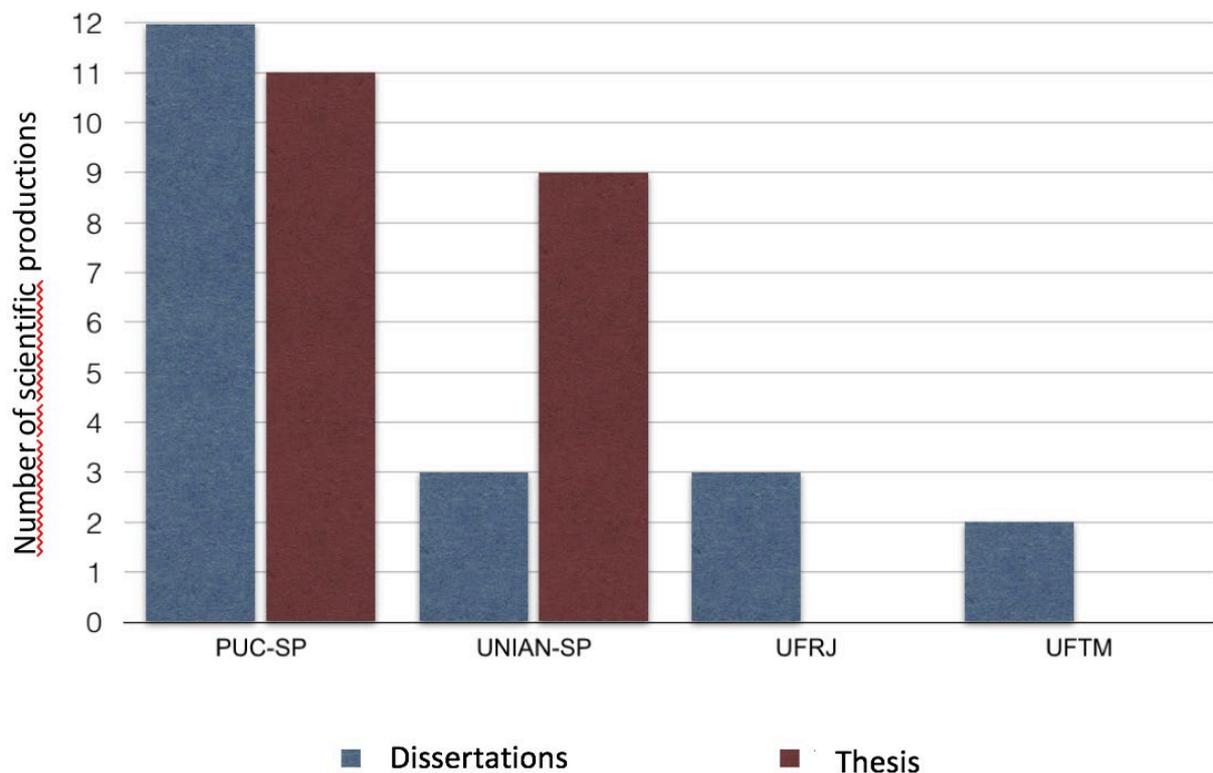


Figure 1: Percentage of thesis and dissertations published in Brazil by region.

The numbers indicate that the Southeast, Central-West and North regions have the largest number of researches on the agenda; while the South region reveals a small number of studies in relation to the whole. We will initially address the institutions belonging to these regions, highlighting those that contain more researchers in the area; for example: of the 38% belonging to the Southeast region (which corresponds to 41 works), more than 50% were defended at Pontifícia Universidade Católica de São Paulo, PUC – SP.

Starting with the largest number, which corresponds to the Southeast region, we have as main production centers PUC – SP and Universidade Anhanguera, former UNIBAN, as shown in the following graph:



Graph 2: Distribution of researches in Southeast of Brazil by university.

It is noteworthy the importance of the two institutions referred to with regard to the production of research within the field of Mathematical Education based on references from Didactics of Mathematics, especially from the Anthropological Theory of Didactics.

The Postgraduate Programme in Mathematics Education (Master's and Doctorate level) at PUC - SP was evaluated in the last quadrennium (2013-2016) with the grade 5. Since 1994, the Master's course in Mathematics at the institution started to focus on the area of Mathematics Education, which was essential for the Capes approval of the Master in Mathematics Education, in 1997. The PhD course was approved in 2002, under the justification of the need to deepen the area of Mathematics Education due to the large number of dissertations that had been defended in the Master's. The program has three areas of research: 1) Mathematics in the Curricular Structure and Teacher Formation; 2) History, Epistemology and Didactics of Mathematics; and 3) Information Technologies and Mathematical Education. The identified works (theses and dissertations) are included in line 2:

Analysis of the interrelation between the epistemology, history and didactics of mathematics with the objective to a better comprehension of the phenomena related to the teaching/learning of Mathematics, the relations between scientific and school knowledge and the historical-cultural constitution of Mathematics. (PUC, 2019)

The other institution in the Southeast region with prominence in the productions is UNIAN, former Universidade Bandeirante de São Paulo (UNIBAN). Two dissertations were defended when the university had not yet been incorporated into the Anhanguera group, in 2011. We noticed a significant number of thesis (9) defended in the period from 2012 to 2017, one being defended at UNI-BAN, in 2012, and eight doctoral thesis, from 2014 to 2017, at UNIAN.

The Master and Doctorate courses in Mathematical Education at UNIAN - SP were created in 2008. In our selection, according to graph 2, we have two dissertations and nine thesis, making up a total of approximately 10% of the total *corpus* investigated. The programme contains three areas of research that are distributed as follows: Teaching and Learning Mathematics and its Innovations; Teacher Formation, Curriculum and History; and Inclusive Mathematical Education and its Technologies. In addition to these, we had three dissertations defended at Universidade Federal do Rio de Janeiro and two at Universidade Federal do Triângulo Mineiro.

The second geographical area in Brazil with the most expressive number of publications with ATD references is the Central-Wes. Twenty-five papers were identified: 23 dissertations and 2 thesis. Of this total, 20 were defended at Universidade Federal do Mato Grosso do Sul (UFMS); the other five were at Universidade Federal do Mato Grosso.

The Master and Doctorate in Mathematics Education at UFMS started in 2007 and 2015, respectively. We highlight the Research Group DDMAT – Study Group on Didactics of Mathematics, created in 2013, coordinated by the researcher Professor Marilena Bittar and Professor José Luiz Magalhães de Freitas. According to information from the institutional website, the research group was created from the need to study the theories disseminated by the Didactics of Mathematics of the French school, such as the Theory of Didactic Situations, Theory of Anthropological Didactics, Theory of Conceptual Fields, Theory of Records of Semiotic Representation and Didactic Engineering. We noticed that the group's focus is exclusively focused on discussions in the area of interest of this research, in line with the formation of the group's leaders.

The third region discussed by us in this section is the North, which holds a percentage of 20.4% of production. It is worth noting that Universidade Federal do Pará (UFPA) was the only one within the region on the agenda to have research defended within the identified set. Thus, the North region presents a mismatch with regard to the distribution of research in the field of Didactics, especially ATD, in relation to

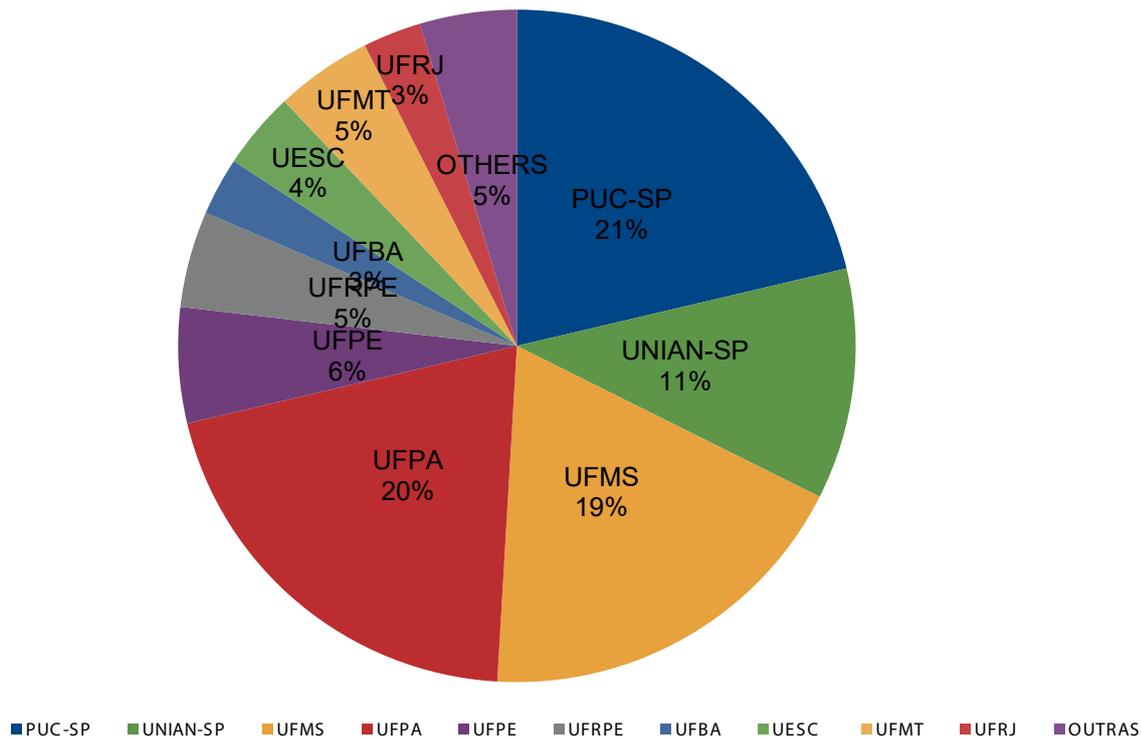
other institutions in the region. This lack of work can be justified by the lack of an academic postgraduate program in the area at Universidade Federal do Acre (UFAC), Universidade Federal Rural do Amazonas (UFRA) and other colleges and institutes of higher education. It is important to mention that, according to information from the Sucupira Platform, the North region has the lowest number of postgraduate programmes in relation to the other regions of Brazil. However, it was at UFPA that 20.4% of the searches verified on the Portal using ATD were concentrated.

In the Northeast region, we have the institutions Universidade Federal Rural de Pernambuco – UFRPE, Universidade Federal de Pernambuco – UFPE, Universidade Federal da Bahia – UFBA and Universidade Estadual de Santa Cruz – UESC, which are depositories of a total of 13 works from our *corpus* of investigation. In these institutions, we point out the following research groups: - Didactic Phenomena in the Mathematics Class, at UFRPE, coordinated by Marcelo Câmara Santos, who also has formation in France, according to information provided in the researcher's *Lattes Curriculum*; and the Interdisciplinary Center for Research, Teaching and Didactics of Science and Technology (NIPEDICMT), UFBA, being the group coordinator, professor Dr. Luiz Márcio Santos Farias, co-author of this article and who also guided the three UFBA dissertations belonging to the selected *corpus*.

In the South region, we had two dissertations defended: one at Universidade Federal de Santa Catarina (UFSC) and another at Universidade Estadual de Maringá (UEM). These figures show that, in this region of the country, studies involving the theories of Didactics of Mathematics do not yet have a strong influence on researchers in the area.

Regarding the difference in numbers related to the geographic regions of Brazil, we can infer that many of the productions are due to the training of supervisors, which, for the most part, occurred in French schools, in which theoretical studies referring to Didactics of Mathematics were pioneers. Thus, the researchers' return to Brazil, allocated in the various institutions presented, promoted the balance of the numbers, presented below.

In advance, it is possible to affirm, by analyzing graph 3, located on the next page, that four institutions (PUC – SP, UNIAN – SP, UFPA and UFMS) hold a total of 71% of the production of all research between the years of 2005 and 2017. This result is reinforced by the presence, in these institutions, of the majority of researchers who were trained in France and who developed, within the institutions to which they belong, research and study groups with references from Didactics of Mathematics.

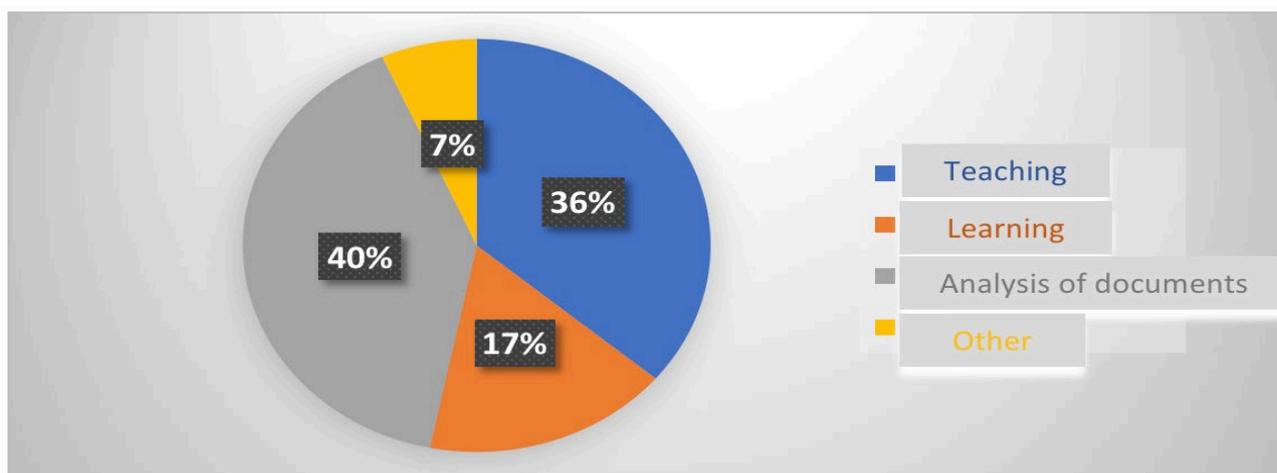


Graph 3: Distribution of researches by university.

The context of ATD in the general objectives of the researches

ATD is a theory with several strands, which allows the researcher to apply it in different approaches in the field of mathematical knowledge, such as: discussions about the epistemology of a mathematical concept, mathematical and didactic organizations developed by teachers, the praxeology of mathematical knowledge in textbooks, didactic transposition process, among others that we could verify by reading the abstracts of theses and dissertations. Thus, there are focuses on the approaches of the studies that we understand as important for the perception of the context of TAD in the general objectives proposed. Thus, we characterize the research in three axes from the identification of the problems identified in the research objectives:

- I) Teaching approach: focusing issues related to mathematics teachers;
- II) Learning approach: focusing the researches with students from basic education to undergraduate;
- III) Documental analysis: thesis and dissertations that had the documents - such as textbooks, teaching programs, evaluations, teachers and students manual, among others – are the objects of investigation and analysis supported by ATD.



Graph 4: Distribution of dissertations and thesis by axes of research.

It is important to highlight that these axes are not exclusive, having researches that carry out both studies with teachers and students and with the analysis of textbooks. We also made it clear to the reader that, by assuming a category of document analysis, research may have as its theme the teaching or learning of a particular mathematical object. However, the division was made considering the research's nuclear focus, aiming to show how ATD is being approached in the problems developed in Brazil's theses and dissertations.

As an excerpt from this research, we will discuss axis III on which most of the Brazilian theses and dissertations are concentrated, with theoretical support from ATD.

Textbook and documents analysis and ATD

Documentary research has a qualitative approach and is currently growing in the Education area.

According to Kripka, Scheller and Bonotto (2015, p. 57),

In a qualitative study, the search for data in the investigation leads the researcher to take different paths, that is, he uses a variety of procedures and instruments for the constitution and analysis of data. The instruments for constituting data generally used are: questionnaires, interviews, observation, focus groups and document analysis.

In the field of didactics, and especially in research with theoretical support from ATD, a widely used research method is the analysis of textbooks and/or institutional documents. Among these documents, we highlight course plans, lesson plans, textbooks, notebook records, teaching laws and regulations, large-scale assessments, among others that are classified as primary sources for documentary analysis studies. Some of these instruments are used for the construction of the historical and epistemological study, called by Chevallard as the Dominant Epistemological Model, constituting part of the research. According to Bosch

and Gascón apud Chaachoua and Bittar (2019), the empirical basis for the construction of this reference model is present in textbooks and institutional documents.

Regarding the methodology of analyzing books from ATD, Bittar (2017) elaborates a script on how to make this modality of appreciation, through the notions of ATD that were used in the works guided by the author. For this, it suggests that the chosen textbook should be divided into two parts, Part Course and Part Activity:

The Course Part includes an explanation of definitions, properties, results and exercises solved. In this Part, the authors of the textbook bring, even if implicitly, what they consider that students of that level of education should learn and it is in this Part that students look for clues to solve what is asked of them. The analysis of the Course Part allows to identify some types of tasks that seem important in that institution, in this case the textbook (Bittar, 2017, p. 372).

Once the analysis of the Course Part has been carried out, we move on to the Proposed Activities. At this point, we seek to analyze each activity identifying what the student's task is and what technique he or she uses to solve the task, with the support (s) praxeology (s) previously identified. A recurring, and totally pertinent, question is "how do you know that this is the technique that the author would like to be used?" To make such an inference we base ourselves on what is present in the Teacher's Manual and, mainly, on what was worked on in the Course Part. Through this analysis we look for elements that allow inferring about elm, the textbook authors would like their users to solve the activities (Bittar, 2017, p. 373).

Almouloud (2015), addresses an example of textbook analysis based on the methodology of analysis of teaching materials, according to the model developed by Chaachoua and Comiti (2010), highlighting elements that are relevant for the assessment. Among them: the moment of editing the textbook, the representativeness, the structure, the ecological and praxeological analyzes, as well as the evaluation of the tasks / techniques and technologies. For Almouloud (2015, p 12), "The analysis of textbooks remains the main entry for ecological or anthropological questioning."

In other research outlined with ATD, documentary analysis makes up the entire body of the work. Thus, considering the importance of this research method in the field of ATD, we will discuss below the objectives proposed by the dissertations and thesis of the *corpus* that made exclusive use of documentary research through the theoretical magnifying glass of ATD.

The largest number of dissertations in the selected *corpus* is located on the axis that refers to the analysis of teaching materials such as books, official documents, study notebooks, large-scale evaluations, among others, comprising the sum of 33 dissertations, representing 49% of the total . Douady's Theory of Conceptual Fields and the Change of Pictures appeared as theoretical support along with ATD in a

significant amount of work. However, most of the analyzes are based on the Mathematical and Didactic Organizations based on the Anthropological Theory of Didactics.

We started with Almeida's research (2012), whose approach concerns the content of regular polygons inscribed on the circumference contained in 9th grade mathematics textbooks evaluated and cataloged by the PNLD/2011 Plano Nacional do Livro Didático. We also have the research by Anjos (2017), whose objective was to determine the relationships established between curriculum guidance documents in Brazil, at the national, state and municipal levels, aimed at students and teachers, thinking about the teaching of scholastic content in Basic Education, specifically in elementary school. Ciabotti (2016) presented as a proposal the process of preparing a supplementary book for teaching probability, research that can be considered as an intersection with the axis of learning, according to the author's position

It is necessary to emphasize the importance of the student having contact with the reading and interpretation of texts in his/her initial education, being able to be helped with the supplementary book, thus he/she will work the probabilistic concepts in a more pleasant way. (Ciabotti, 2016, p. 10)

Other researches were classified in this axis: Costa (2013), who studied the textbooks of mathematics in the period of the Movement of Modern Mathematics, seeking to analyze the approach of Euler's Polyhedral Formula present in them; Freitas (2014), who examined numerical expressions in 6th grade books; Maia (2008), which aimed to identify the Praxeological Organization of the triangle object in the 7th grade elementary school textbooks; Martinez (2012), with the study of the division of natural numbers, and Nogueira (2008), with regard to the introduction of Algebra, investigated textbooks of Elementary Education; Souza (2017), who, in addition to ATD, used CBT to analyze convergences and divergences present in the guiding books and textbooks for the literacy cycle; and Trindade (2017), who found an absence of the Financial Education content approach in the PCN and that the textbook exercises analyzed for the high school level follow a technicist line.

Yamauti (2013), carried out a research with books of Administration graduate courses on the topic of Simple Linear Regression, in Statistics. Almeida (2015) developed a praxeological analysis, based on the ATD framework, aiming to investigate the approach to teaching polygons and spatial figures in a collection of textbooks approved by the PNLD 2013. Anjos (2014), with the theoretical framework of TCC, undertook a praxeological analysis of the addition and subtraction of natural numbers in books from the early years of

elementary school. Gonçalves (2016), studied the didactic transposition process in relation to whole numbers through the 7th grade textbook, and Souto (2010) with the concept of real number.

In the research developed by Cunha (2013), the analysis took place through statements and proof solutions by Calculus I. Freitas (2015), based on a study on the volume of geometric solids in middle-level books, concluded that there is an appreciation of the resolution techniques present in books. Luz (2007), analyzed exercises proposed on the teaching of geometric transformations in books, using TAD as an analytical reference with the notions of mathematical organizations.

Nascimento's research (2012) sought to analyze types of tasks, techniques, technology and theories, pictures and levels of knowledge of students about quadratic function in elementary, high and higher education. As a reference, the author lists:

The theoretical framework of this research is centered on the Anthropological Theory of Didactic, in particular on the notions of praxeology, ostensive and non-ostensive and, as support, the theoretical approaches are considered in terms of frame and change of frames of Douady and in the notion of levels knowledge expected from students, as defined by Robert. (Nascimento, 2012, p. 8)

Ramalho (2016) and Barbosa (2015) analyzed the proposal for teaching trigonometry; the mathematical object of Souza (2014) was the introduction of algebra in books. Santos, J. (2017) and Chaves (2016), examined in the books the approach of the quadratic function. Santana (2016) promoted a praxeological analysis of the activities of textbooks and a didactic sequence developed for teaching related functions. Silva (2017) analyzed the textbooks seeking to investigate whether the mathematical praxeologies contained in the books with regard to statistical content prepare students for large-scale assessments, such as the SA-RESP. Carvalho (2012) criticized the area object in the study guide for students of the Projovem Urbano Program - the author discussed the habitat and niche of this knowledge. Silva's work (2011, non-paged) "aimed to analyze the length, perimeter and area approaches in textbooks approved in the 2008 and 2011 Plano Nacional do Livro Didático (PNLD) in the light of the Anthropological Theory of Didactic". Reis (2010), in addition to analyzing the textbooks with regard to the study of systems of first degree equations, contemplated the PCN, the reviews of the Guia do Livro Didático and study programs of Colégio Pedro II. Abreu (2009), analyzed the use of the calculator based on activities present in textbooks.

Varela (2010) considered how authors of textbooks and bimonthly notebooks adopted by the São Paulo State education Secretariat organize the proposed tasks with regard to tests and demonstrations related to

Analytical Geometry. Almeida (2012) appears as the author of the only work, within the dissertations, which proposes to investigate how textbooks approach the study of arithmetic and geometric progressions in high school. Valenzuela (2007), in turn, investigated didactic devices, such as books, classes and productions of students, for the study of linear equations.

Due to all the dissertations of the axis referring to the analysis of documents dealing with the investigation of a mathematical object from textbooks and/or school manuals, and considering as a fundamental element of didactic research an object of knowledge, we set up, at levels of didactic co-determination, the knowledge discussed in the Master's research in Brazil, in order to understand how this research is distributed by the field of mathematical knowledge and school levels. We do not represent numbers on the co-determination scale because a diagram showing all authors is unfeasible; however, our interest is to show how it is distributed, in order to cover where the gaps of a certain knowledge may exist. We also do not indicate all levels, only those that we think are most relevant.

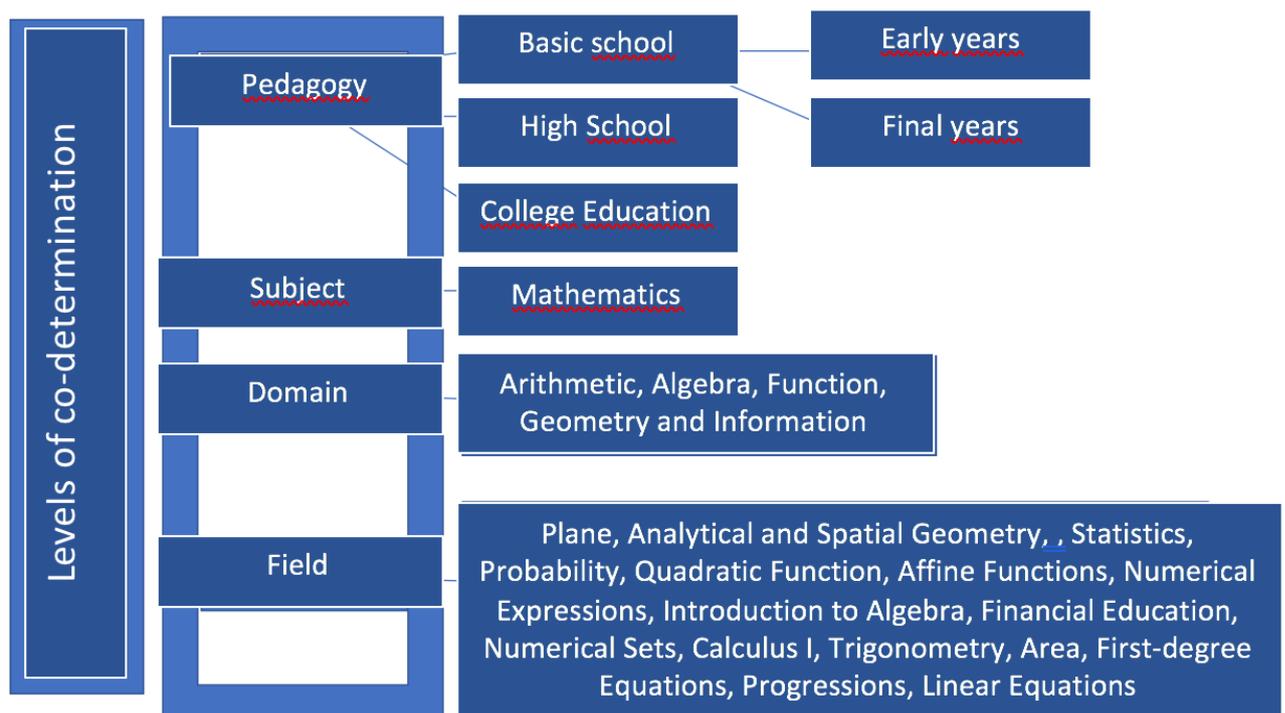


Figure 2: Levels of co-determination and research contexts.

Regarding to the thesis, we expose Andrade's research (2012, p. 45), which sought “to study the notion of exponential function in the transition between High School and Higher Education to understand the different study processes and study aid that survive and are currently being rebuilt in these school stages.”

For that purpose, the following documents were analyzed: the Lei de Diretrizes e Bases, os Parâmetros

Curriculares Nacionais para o Ensino Fundamental e Médio, a Proposta Curricular do Estado de São Paulo, four preparatory books for university examining, institutional assessments such as ENEM, UNICAMP and ENADE, in addition to the elaboration of a questionnaire. All of these documents were studied using the content analysis methodology. In this research, the author also addresses some supporting references, such as the three levels of knowledge expected from students, Robert's theory and Douady's notion of frame and frame change. the author weaves in relation to the contribution of her research to the teacher education process, explained as follows:

It is intended, with this, to enable teachers to have material for reflection so that, when they have access to their students' prior knowledge, they can make more conscious choices and, thus, conduct this process in a satisfactory manner, allowing that students are able to mobilize their previous knowledge and use it in an available way when necessary, thus being able to enrich it when they are used as explicit tools for the introduction of new concepts and new notions. (Andrade, 2012, p. 45)

Another research in this category is that of Aguiar (2014, p. 34), which aimed to answer the following question: "How do elementary school textbooks reconstruct algebraic thinking in their teaching objects?"

The work was developed with studies on Didactic Transposition and ATD, with textbooks being the objects of appreciation.

From the analysis of textbooks used in schools from 1985 to 2009, Pinheiro (2015) developed the research with the intention of "investigating the mathematical and didactic organizations of Combinatorial Analysis". The author started from a bibliographic and documentary study, based on content analysis, which was divided into two phases: the first composed the material selection; and the second, his analysis in the light of the Anthropological Theory of Didactic.

Costa (2015) also made an analysis of books through the studies of the mathematical object Analytical Geometry in Space, seeking to identify the didactic and mathematical organizations present in the textbooks for the study of Reta and Plano in higher education. The author bases her research on two aspects: praxeology, discussed through ATD by Chevallard; and didactic variables, established by Lebeau. In addition, the author's work has a particularity in relation to the book analysis process, as it addresses the material appreciation methodology developed by Chaachoua, as the author alludes: "The methodology adopted for book analysis was based on the work of Chaa-choua (2014a) who points out certain characteristics for carrying out such analyzes based on ATD" (Costa, 2015, p. 73). These characteristics are

listed by Costa (2015): time of edition, representativeness, structure, ecological analysis and praxeological analysis.

Goulart's work (2015) was also a documentary research. However, in addition to textbooks, documents such as the *Parâmetros e Diretrizes Curriculares Nacionais*, both for teaching and teacher's formation, and large-scale examinations in Brazil were part of the *corpus*. His study sought to answer:

What are the relations that we can establish between the teaching of Statistics in Basic Education and the Teaching of Statistics in Mathematics Formation courses aiming to enhance (improve) the formation of teachers for Statistical Literacy in the light of Didactic Ecology? (Goulart, 2015, p. 64)

Like Andrade (2012), Goulart (2015) reveals the intention to contribute, based on the analysis of the referred materials, to the process of formation of educators.

The thesis developed by Andrade (2013, p. 39) stated as a general objective:

To analyze the didactic transposition of measures of central tendency and dispersion for high school mathematics programs and textbooks in Brazil and France, seeking to find out if there are limitations in didactic transposition that may influence the teaching-learning process of these measures in teaching average for Brazil and France.

To this end, the author made use of two theories: ATD and the Theory of Conceptual Fields (TCF). Andrade (2013) justifies the use of the latter, developed by Vergnaud, to discuss the teaching of a concept, in particular, the concept of Central Tendency and Dispersion Measures. With regard to ATD, "The anthropological theory of the didactic adds in our research theoretical elements that allow to deepen the discussion on didactic transposition" (Andrade, 2013, p. 31), in addition to the analysis of praxeology and other elements present in programs and textbooks covered. The methodology defined by the author was bibliographic, having as a mathematical object the concepts of Measures of Central Tendency and Dispersion in the Formation of Statistics, through the comparison and analysis of programs and textbooks from Brazil and France.

Before finalizing this category, we highlight Gouveia's research (2014). The work developed by him is not restricted to the analysis of documents, as he performs part with teachers, through a questionnaire, and part with students, who answer the application of a diagnostic test. But why are we considering it on this third axis? Although the figure of the teacher and students appears, the author highlights the research component that deals with the analysis of documents and, for this reason, we understand that the focus of investigation is concentrated on the analyzed documents:

In addition to this on-the-spot research developed *in loco*, we will also do a documentary analysis of the São Paulo State Curriculum, based on the Proposta Curricular published in 2008, and the Curriculum published in 2010, comparing them to official documents at the federal level, as is the case. LDB, PCN and curricular guidelines, and also a comparison with the French Program, analyzing documents related to that program, and comparing them with the work developed in the state of São Paulo, in action in the classrooms. [...]. (Gouveia, 2014, p. 63)

Still in this axis, we framework the research of Silva (2017), a bibliographic research seeking to build a knowledge base for teaching the rate of change through the analysis of various documents. Other theoretical references were used, especially in the discussion about knowledge, in which the author made use of theorists such as Schulman (1986, 1987), Mishra and Koehler (2006), Lima and Silva (2015) and Ball, Thames and Phelps (2008). The use of ATD as a theoretical magnifying glass is justified by Silva (2017, p. 121) to support the mathematical knowledge involved in the teaching process of the rate of change:

In view of these possible sources of teaching knowledge, we are aware that, in the course of this research, other sources that may, in one way or another, assist the teacher in the construction of specific knowledge for the teaching of Mathematics, we are also convinced that the structural model of the scheme that represents a Knowledge Base for the Teaching of Mathematics built on this item is not sufficient to assist us in building a knowledge base for teaching rate of change, since this model comprises only teaching knowledge and does not include the mathematical knowledge that underlies them. For these two reasons, we use the Anthropological Theory of Didactics to assist us in identifying a set of Mathematical Organizations that will allow the development of the necessary knowledge for teaching the rate of change in Basic Education.

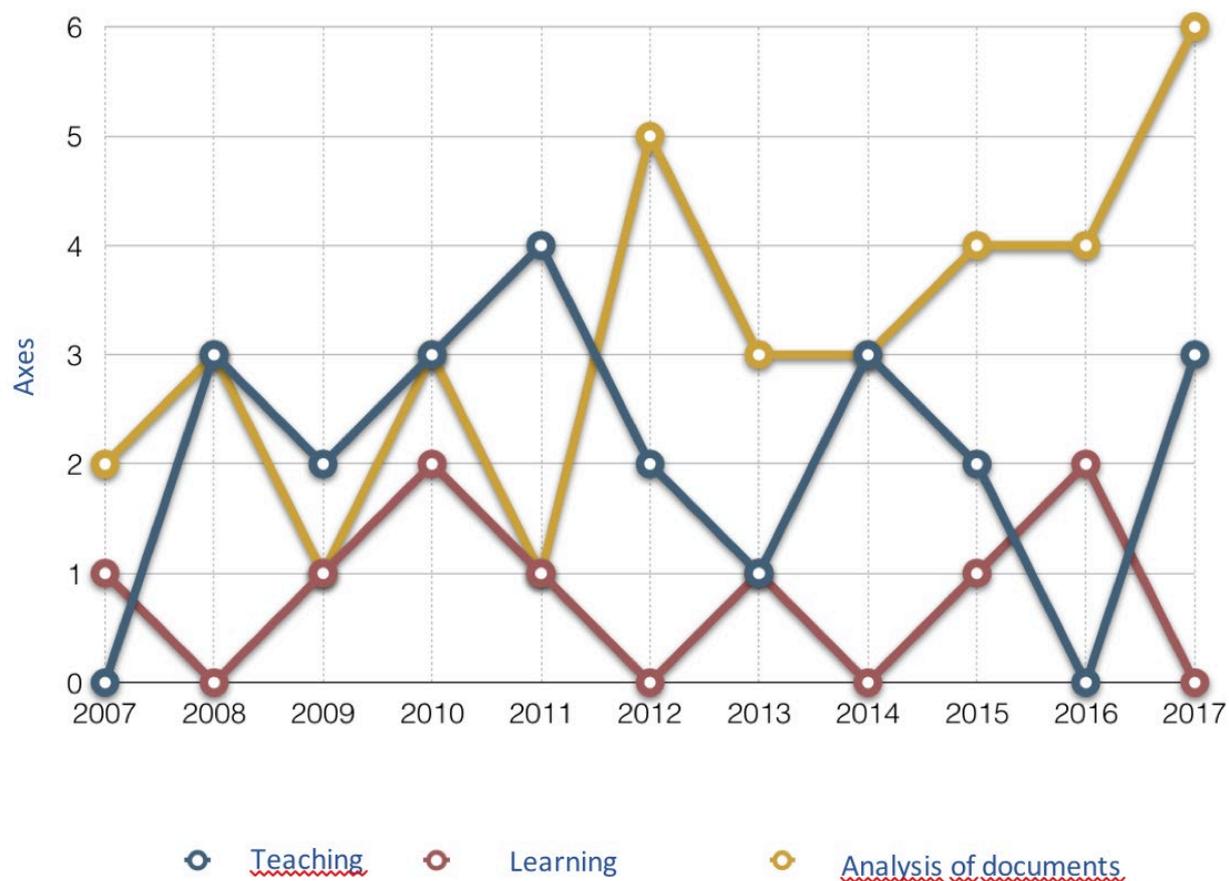
In addition to the dialogue with other scholars on pedagogical knowledge, the author makes use of several theories of Didactics of Mathematics, such as Theory of Didactic Situations, Theory of Semiotic Representation Records and Theory of Conceptual Fields to discuss didactic knowledge.

Ávila (2014, p. 31) also conducted a documentary research along with a case study, seeking

[...] to contribute to clarify the role of mediation and information exchange in learning and how work focused on this issue can help to better understand possible study processes in Higher Education -HE, which can lead to adequate and satisfactory choices of the necessary mathematical knowledge for the application of Mathematics developed in Basic Education -BE, when this knowledge is considered as explicit tools available to be applied in other sciences, in particular, in Informatics.

As a theoretical framework, the author presented the principles of Vygotsky, to discuss issues related to learning and development, and of ATD, in order to discuss the institutional expectations linked to the mathematical object Intuitive Notion of Limit.

Regarding the quantitative approach, we elaborated the following chart, which indicates the thesis and dissertations defended per year and the three axes listed by us.



Graph 5: Distribution of dissertations by axis and year.

Final considerations

We can see the predominance of ATD in dissertations that aim to analyze teaching materials (49% on the analysis axis of teaching materials, 35% on the axis with questions involving teaching and 12% related to learning). This distribution changes in relation to the theses, most of which (38%) discuss issues related to Education, followed by 27% with a focus on student learning and 24% on research aimed at document analysis. We can infer from these data that the difference in focus between theses and dissertations is due to the time required for the investigation. Considering that we have in the master's degree a time equivalent to half that dedicated to doctoral studies, and realizing that, in the analysis of documents, most of the authors focused on the study of didactic and mathematical organizations - seeking to identify types of tasks, techniques, technologies and theories in the proposed activities, both by textbooks and study materials of

students and in the documents that guide the teaching of mathematical knowledge, making use, in some cases, of ostensive and non-ostensive objects -, the longer time for the development of a thesis provides a deeper and broader theoretical analysis of the other notions of ATD. In general numbers, theses and dissertations, together, make up a percentage of 42% of the surveys that performed document analysis exclusively as a data source. It is important to emphasize that, when doing textbook analysis, we are not ignoring that this approach involves issues such as teaching and learning. However, we made this separation in order to categorize the research according to the capital analysis approach adopted by the authors. This scenario makes us reflect on questions such as: from the various studies on teaching materials, what do you have to say about improvements and changes for this universe? What contributions do these studies provide for the preparation of the materials on the agenda? Under what conditions do the research results act directly in the production and preparation of materials? These are some questions that we can leave as a possibility for new researches through the refined reality.

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