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Bringing a CURE into a Discrete Mathematics Course and Beyond

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ABSTRACT: Course-based Undergraduate Research Experiences (CUREs) have been well developed in the hard sciences, but math CUREs are all but absent from the literature. Like biology and chemistry, math programs suffer from a lack of research experiences and many students are not able to participate in programs like REUs (Research Experiences for Undergraduates). CUREs are a great alternative, but the current definition of CURE (see [1]) has potential barriers when applied to mathematics (e.g. time, novelty of project). Our solution to these barriers was to develop a math CURE pathway in which students complete "Math CUREs" in targeted courses. After finishing the pathway (or part of the pathway), students complete a research project in at least one of the following areas: Lie theory, representation theory, or combinatorics. The focus of this paper is the math CURE implemented in a discrete mathematics course for math and computer science majors. We share our experiences with the development and implementation of this CURE over several iterations as well as the impact of the CURE on students experiences through participant survey data obtained from this CURE.

Keywords: CURE, Math CURE, Undergraduate Research, Undergraduate Mathematics

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Introduction

The California State University (CSU) is the largest state school system in the country. Of the 23 colleges in the CSU, California State University, Monterey Bay (CSUMB) is one of the youngest and smallest campuses. Our student population, which is around 7,500 students, is diverse with 53% first generation, 51% from an underrepresented minority population, and 32% students from low-income families (these percentages are as of Fall 2020). Additionally, CSUMB is a Hispanic Serving Institution (HSI) with 45% of our students identifying as Latino. As such, faculty at CSUMB strive to embed innovative and research driven practices into our school so that we may be the most beneficial to our student population. When we (the authors) were first introduced to CUREs, we were excited at the opportunity to bring research to a larger group of students who may not normally have access to undergraduate research.

A CURE Framework for Mathematics

There is no doubt of the positive effects a research experience can have on a student. "Students report positive outcomes, such as learning to think like a scientist, finding research exciting, and intending to pursue graduate education or careers in science"[1]. For many students barriers exist to participating in the typical undergraduate research experience like funding, knowledge, lack of positions, and competition to name a few. To increase access to research experiences more faculty are creating what are called *Course Embedded Undergraduate Research Experiences* (CUREs).

The definition of CURE we started our work from was presented in [1]. In [1] a CURE was defined to be a research project, done in tandem with a class. In addition, a CURE must be a class project where students are: engaging with the practices of the researchers in the discipline, making discoveries that are new to the student and instructor, making additions to the current body of knowledge in the discipline, and working collaboratively to solve a problem. Since science usually has some repetitious aspect, CUREs must also contain some iterative element be it repeating an experiment, using an alternate approach to a question, or revising aspects of their work. The way CUREs have been defined seems well fitted for the hard sciences. However pure mathematicians may have a difficult time creating/embedding an authentic research experience that fits all of the criteria above into a single class project. In addition, the requirement that the project be novel to the current body of knowledge maybe be extremely difficult for certain areas of mathematics. The struggle mainly stems from a lack of time.

In pure math research, a major goal is to usually to prove some conjecture. However, as mathematicians, we know that how one arrives at a conjecture varies greatly. Sometimes years of experience and intuition can lead one to making some conjecture, but conjectures often come from doing many calculations or examples and finding a pattern. As we know, there is no "correct" number of calculations or examples that will yield a conjecture. Nor is it guaranteed that a conjecture will ever come from doing calculations. So when generating a problem for a CURE, the availability of time may become an issue.

After hypothesizing a conjecture, which may be time consuming, the goal now shifts to proving it. However, in order to prove a conjecture, one has to know what they are doing! Generally, the researcher must have intimate knowledge of the problem itself and the background material required to solve the problem. Depending on the area of the problem, this may pose issues when trying to create a CURE. For example, our (the authors) area of expertise is Lie algebras and representation theory. This is a subject that is not taught in a normal undergraduate curriculum and is a subject that takes graduate students years to master. Thus, getting students research ready in our area, inside of a semester, is a tall task. On the flip side, mathematics also has the phenomenon of having problems that are easy to understand, but are extremely difficult to prove. One famous example, of course, is the twin prime conjecture. Every undergraduate student can understand the twin prime conjecture, but this is a problem that has been unsolved for hundreds of years. Again time becomes vital.

With time being a major hurdle, we decided to reformulate the definition of CURE. We wanted to define a project that has the original CURE definition at heart, but follows a natural mathematical research process. The following is our framework for a *math CURE*. A math CURE is a project involving three components with subsequent criteria:

1. Students participate in mathematical research by
    (a) generating research questions,
    (b) developing conjectures,

   (c) proving or disproving their conjectures,
   (d) and presenting their results.

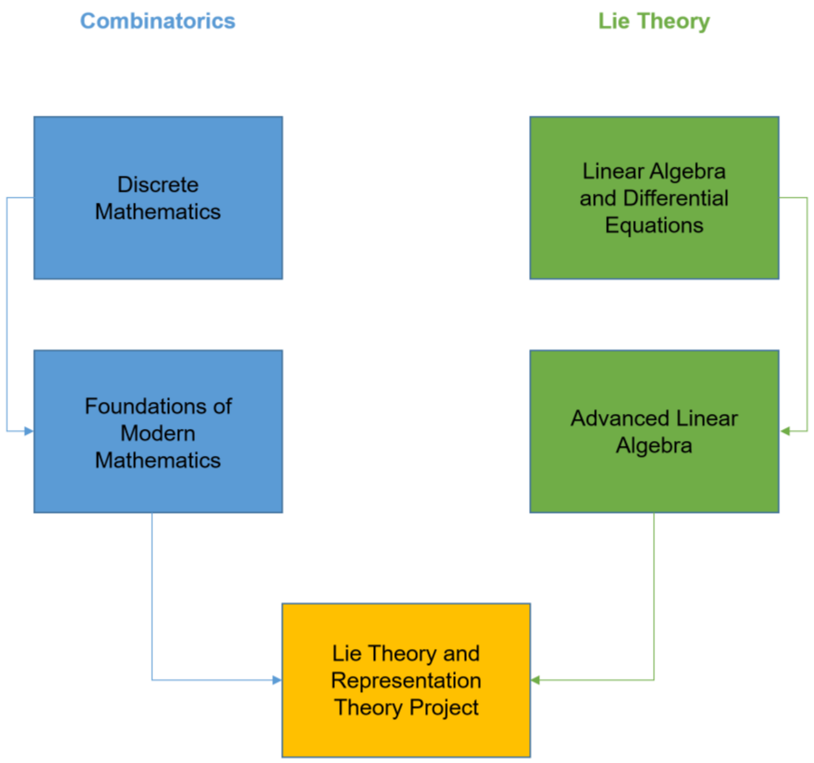
2. Students make discoveries (unique to themselves) in content that is not part of the current curriculum. Students will investigate their research by
   (a) calculating (counter)examples,
   (b) searching for patterns,
   (c) and making meaning of their examples.

3. Students develop their own identity within the broader mathematical community by
   (a) exploring topics mathematicians are currently investigating,
   (b) researching within and with a community of their peers,
   (c) presenting within the mathematics and statistics department as part of a CURE festival.

A notable change from the original CURE definition to our math CURE is that we have relaxed the novelty of the project. We only require that the problem is novel to the student (although it is still possible for the project to be novel to the math community as well). Although a math CURE may not be original, it is our ultimate goal to have students do an authentic research problem in our area. To this end, we developed a pipeline of math CUREs in targeted courses to prepare students for an eventual novel research project.

A CURE Pathway

As mentioned, one of our main goals is to have students participate in an original research project in Lie theory and representation theory. Due to the nature and difficulty of this subject area, we wanted to expose students to the necessary background over the course of many classes rather than try to make a single CURE that had all of the background and research in one semester. To do so we have developed a *math CURE pathway* in which students participate in math CUREs during the targeted courses: Discrete Mathematics, Foundations of Modern Mathematics (more commonly known as Introduction to Proofs), Differential Equations and Linear Algebra, and Advanced Linear Algebra. See picture below.



Each class was selected because of its relation to a topic required in Lie theory/representation theory research. Discrete mathematics and foundations of modern mathematics were selected as places where students can learn about combinatorics (something that is used heavily in our own research) and proof writing. Lie algebras and representation theory can be thought of as generalized linear algebra so our "differential equations and linear algebra" and "advanced linear algebra" courses were natural places to embed Lie theoretic topics. This paper is meant to focus on the implementation of a math CURE in discrete mathematics, but to give readers an idea of the other courses, we summarize them below.

In our "discrete mathematics" course, students participate in mathematical research steps investigating combinatorics questions around counting discrete structures. The CURE project is described in detail in Section 1. Students continue to explore research in combinatorics in their subsequent mathematics course "foundations of modern math." In this course, the students use the OEIS (Online Encyclopedia of integer sequence) at https://oeis.org/ to explore a counting question. They then explore different topics counted by the same sequence. Students learn to develop bijective proofs, an important proof technique in combinatorics.

In our "differential equations and linear algebra" course students spend a little over half of the semester studying linear algebra. Since there is a deep connection with Lie Theory and linear algebra, a math CURE has been created to go hand in hand with the linear algebra portion of the course. The project, which has been implemented twice, has students complete a mixture of open ended problems and conceptual problems involving Lie algebras. Additionally, the students type up a manuscript with their findings as a final project. Our last course "advanced linear algebra" has not yet had a CURE developed to fit into our pathway, but one is currently in the works.

We would like to emphasize that anyone interested in creating a CURE pathway should choose courses that best fit their plans and institutional structure. Aside from these courses being natural places to embed projects related to our research, we chose them due to their impact on math majors. Majors must have credit for these 4 courses upon graduation. Discrete mathematics and differential equations and linear algebra are lower level mathematics courses which students usually take in their first two years. However, it is possible for math majors to have transferred into CSUMB with discrete mathematics and/or differential equations and linear algebra. Since foundations of modern math and advanced linear algebra are upper division math courses, most students take these here at CSUMB. Additionally, our math CUREs are not always implemented unless one of us (the authors) is either teaching or coordinating the course (otherwise the course is taught normally without the CURE). Once we have created a CURE for advanced linear algebra, majors will have a good chance to participate in all or part of the CURE pathway and be able to partake in an authentic research project in at least one of the areas: Lie theory, representation theory, or combinatorics. Although the scope of this paper only includes the CURE in discrete mathematics, more details about the CURE pathway can be found in [4].

1 Discrete Mathematics: The Start of the Pathway

Discrete Mathematics is a freshman course required for both mathematics and computer science majors that builds the foundation for mathematical language and valid argumentation along with introducing various discrete structures. The course is divided into five modules shown below with three weeks devoted to each module. Note that in the table below Week 14 is Spring break, and so is not included.

Module #	**Topics**	**Schedule**
Module 1	The language of sets and logic	Week 1, 2, 3
Module 2	Deductive and inductive proofs	Week 4, 5, 6
Module 3	The counting methods	Week 7, 8, 9
Module 4	Graph theory and applications	Week 10, 11, 12
Module 5	Probability and relations	Week 13, 15, 16

The course is designed to be a collaborative learning experience in groups of three or four during class time. Each class consists of short lecture, followed by group work on a class activity and then a whole class discussion. There are different types of formative assessments throughout each module to be completed before class (video assignment) and after class (check your understanding quiz) followed by weekly homework assignments. The video assignments are completed in PlayPosit which allows the instructor to embed questions throughout the video. The interactive video assignments are used as a formative assessment, but are also counted as a part of their course grade. At the end of each module there is a summative assessment called a checkpoint, which is like a mini midterm exam covering that module. Students learn by working through problems and applications of each concept. The summative assessment at the end of each module assesses the knowledge of that module only. Thus, the CURE project acts as the cumulative piece by bringing core concepts together. The CURE project is designed to align with course content and it runs parallel to the course throughout the semester. For example, when students are ready to work on their project's research question they would have just completed the associated required core concepts such as proof writing and counting. By the end of the project students have the opportunity to explore and apply several core concepts.

1.1 CURE Project Description

The primary goal of the CURE project in this course is to provide students in an entry level mathematics major course with first-hand experience of conducting research in the mathematical sciences. The project runs parallel to the class with four different parts to be completed throughout the semester. Groups of three or four are assigned by the instructor in the first week of class with the goal of mixing mathematics and computer science majors evenly so that each group will have a math major and a computer science major (when possible). In order to capture the semester-long project work of each group, a Google document is created for each group that students continuously update throughout the semester. In particular, the Google document contains the guidelines for each part of the project, including prompts for what needs to be submitted at each stage. This document helps each group to navigate the semester long project in one place and helps the instructor to track their work. The project consists of four different parts and we will describe each part in detail.

CURE Part 1- Introduction to CURE and group work: The CURE project begins by introducing students to their group and getting them ready for the project by completing the following steps.

- *Getting to know your group:* Students work with a single group throughout the semester. In particular, this aligns with the element of a Math CURE identified above: *develop their own identity within the broader mathematical community by researching within and with a community of their peers.* Therefore, in order for this element to be achieved, the group itself must work well together. So, one of the early parts of the CURE experience includes each group taking part in a team building activity in class during second week so that they may get to know the group members. We use a variety of team building activities like "Tick Tock". This activity helps students negotiate and work together toward a common goal. For the activity we make a list of tasks on chart paper, assigning a point value for each job. For example: Do 25 jumping jacks (5 points); make up a nickname for each member of the class (5 points and 5 bonus points point if the nicknames are all mathematical); get every person in the class to sign a piece of paper (15 points); form a list of emotions you have about the mathematics you have learned so far (5 points); etc. We make sure to list enough tasks so that the activity is well over 10 minutes. We divide our students into their project groups and give them exactly 10 minutes to collect as many points as they can by deciding which tasks from the list to perform. We had to be creative in an online setting with virtual team building activities. Usually, by the end of the activity they feel like they know each other a bit more and seem comfortable as a group. After the team building activity, they work together on a 'setting the group norms' task to set norms and expectations for working together in this semester long group project. In this task they create a table thinking about norms for the group, what norms each of them can expect from other members and what they can do as a group to meet these norms. For example a group might put as norm "we will listen to each other" and "they will remind each other time to time to continue this norm". Each group can revisit their norms during the semester to either remind themselves or make any changes as they get to know each other better. This provides the group a framework to work together as a team in this project.

- *Understanding what a CURE is:* Students read the article "Increasing Persistence of College Students in STEM [8] that describes what a CURE is and its importance in STEM. They write a summary of their findings as a group addressing their understanding of CUREs, the benefits of participating in a CURE project, and how their experience might be beneficial for them in their growth as STEM majors. In addition, they also read the project description to understand the specific details and expectations of their discrete mathematics CURE project.

- *CURE Surveys:* We invite each student to participate in a research project (of the effectiveness of CUREs) conducted by CSUMB faculty and staff who work with the Undergraduate Research Opportunities Center, and in collaboration with discrete mathematics instructors. We explain to students how we believe our research will help us to support successful teaching and learning at CSUMB as well as at other institutions. For this research, we ask students to complete a survey at the beginning of the project which includes a few questions surrounding their attitudes towards STEM, their career plans, and their educational experiences; the survey takes roughly 5 to 10 minutes to complete. Students complete the survey individually. A post-survey that consists of the first survey with a few additional questions is also given at the end of the project. We discuss these surveys and some of the results of the surveys for this class in Section 2.

CURE Part 2 - Formulating a research question: In the next stage of their project, each group is given a mathematical word(s) to research. In particular, the group must investigate the word(s) by finding reliable resources. The objective of this part of the project has two components. First, students learn how to use the campus library and other available research tools. Second, students learn to search the existing body of research relevant to discrete structures in mathematics in the context of what word(s) they have been given. Examples of words given for students to investigate include: *standard Young tableaux, Dyck paths, monotone triangles, skew tableaux, and Young lattice*. Most of these words are completely new to students in an entry level discrete mathematics course. Lastly, for this part, students summarize their findings by describing the word mathematically. Students are required to use proper mathematical language both algebraically and visually (when possible), provide examples and non-examples, list interesting facts and construct potential research questions involving counting that arise from their investigation. Students are encouraged to develop research questions that are focused around counting as it aligns with the course content. For example a group that was studying the standard young tableaux developed the question: "How many standard young tableaux are there of a rectangular shape with two rows?" At times the research questions a group comes up with are not well formulated (which is expected) and hence cannot be used. In situations like this the instructor provides feedback to make their questions well-formulated or help them formulate a new well-formulated research question. In addition, to encourage students to look for multiple resources for their research they are required to list at least one more source in addition to Wikipedia and their textbook and they must use an appropriate citation style.

CURE Part 3 - Looking for patterns and formulating a conjecture: Equipped with a research question, the next step for each group involves investigating their research question. The learning objective of this part of the project is that students experience, first-hand, the process of inquiry of a clearly articulated research question or problem in the mathematical sciences. Each group focuses on one research question they created in the CURE Part 2 (if needed with help from the instructor). First, each group is given three examples of counting problems involving binomial coefficients to understand the process of solving a counting problem using pattern recognition. They watch videos on the problems as a group and discuss their understanding of the process. Then they are asked to work through a counting problem involving counting derangements of a positive integer using the Inclusion-Exclusion principle and summarize their understanding. Once groups have an established understanding of the process of creating and analyzing easier counting problems to observe patterns, they apply their newfound skill to help answer their research question. At this stage, groups are encouraged to investigate the research question by writing a computer program (if possible) so that they may generate many examples, create a table of their examples, and then use the OEIS (Online Encyclopedia of integer sequence) at https://oeis.org/ to look for a pattern in their example set. By the end of this portion of the CURE project, students use their findings to formulate a conjecture, which answers their original research question. For example, to answer the research question "How many standard young tableaux are there of a rectangular shape with two rows?" they provide a combinatorial counting formula as a function of n, the total number of boxes in the tableau.

CURE Part 4 - Proving or justifying the conjecture: The last part of the CURE project focuses on the objectives of helping students to identify resources, to justify and strengthen their conjecture, to identify and demonstrate appropriate proof methodologies, and to investigate when to use such methodologies. As a result students must practice the skills of proof, including to define, articulate and use terminology, concepts, and methods of proof. At the end of this part, the groups present their work to the class in both oral and written form (discussed more below).

CURE deliverable: As described above, each part has a deliverable which requires students to summarize their findings. For parts 1, 2, and 3 the groups use their structured Google document (described earlier) to capture their group work and include a summary of the respective part. The final assignment includes the completed Google document and a PowerPoint presentation summarizing their entire research on the project. The groups present their project work live and/or asynchronously to their classmates. The entire project work is supported throughout the semester by a) providing class time to bring the groups together in person, b) group activities to promote team building and group norms inside and outside of classroom, and c) meeting in person or virtually with the instructor at least once during each part of the project along with continuous office hour support.

1.2 Iterations and Online Adaptation

The CURE project for our "discrete mathematics" course has gone through 4-5 rounds of iterations since 2018 including an online adaptation in 2020-2021. The project has evolved over semesters to adapt more relevant topics and accommodate student and instructor needs. This includes adapting to provide students with adequate mentoring as well as expectations for their deliverable items. While the pandemic forced the course to develop an online iteration of the project, it also, in turn, forced us to provide appropriate tools for students to work asynchronously. Surprisingly, we found that the online CURE project experience was much better for students than their face to face experience and was well supported by the data obtained from analysing the CURE surveys described in section 2. Now that the CURE project has gone through multiple iterations, we have been able to make the project workload expectations more cohesive and transparent both for students and faculty. Over time we have been able to develop the project so that it is well-integrated into the course's common Learning Management System (Canvas) page for all sections of discrete mathematics. Having a common Canvas page has helped to make the CURE more consistent across the different sections.

One such example of how the project has improved over its iterations is the Google document that tracks students' work. In 2018, our first iteration, students were asked to create a Google document among the group members so that they may collaborate online for the project. Afterwards, at the end of each part, each group then submitted a paper summary of their part. The final cumulative work they submitted was in the form of a final paper. We realized that this approach did not provide the instructor adequate information to assess student *group* work, since all such information was developed in the Google document shared only amongst the students (and not with the instructor). Hence, in later iterations we required that students share their Google document with the instructor so that the instructor could view and assess their group work. In this iteration, we realized that every group was organizing their work in distinct ways, which provided challenges for the instructor to keep track of how students were 'logging' their work. In our latest iteration, we created a Google document which scaffolds the entire project into it. More specifically, the Google document includes each of the four parts along with its respective deliverable, making it easier to navigate. As the semester progresses, students are asked to work on a part of the project, and submit a PDF. They continue this process for all four parts. By the end of the course, the instructors have easier navigation of the semester long groupwork and are able to see their journey. In place of a final paper, students now submit a PowerPoint presentation as their final work (which summarizes the entire project) along with a recorded video presentation. All presentations are posted in a discussion board in Canvas and students must do a gallery walk of the videos commenting on at least two presentations other than their own. This format was adapted to fit the online modality of the 2020-2021 academic year, but has seemed to work well for both students and instructors. We hope to continue to use this format going forward due to the success of this format in engaging all our students at a much higher percentage.

1.3 Multi-section Challenges

The discrete mathematics course is a coordinated course, which usually runs two to three sections each semester taught by at least two instructors. The course coordinator for discrete mathematics developed the CURE project and has been working with the other instructors providing the support needed for the implementation in their sections as long as the instructors are comfortable with taking on the added workload. We have been fortunate to have dedicated instructors who, so far, have been willing to include the CURE project in their sections. However, there are challenges we have faced that we need to address and adapt as we continue. One of the main challenges is the inability to provide the professional development needed for our lecturers to learn about CURE pedagogy and how to engage students with the research work. The second challenge is the content knowledge needed to feel comfortable to mentor students through the research projects. The third challenge is the extra mentoring hours required to guide the research exploration in this Freshman course. We have tried to address the first one by having the coordinator work closely with the instructor during the first two semesters of implementing the project. The second challenge is addressed by providing extra resources for the instructors on the research topics as well as making the coordinator a mentor for them to reach out whenever needed. The third challenge is particularly hard if the class size is large and instructors have 8-10 different groups working on a CURE project. We are currently exploring options to create a CURE study hall where students can go to any instructor for mentoring in the project. However, the workload on instructors still remains a challenge. After a full year of distance learning, however, we are learning new tools that may help reduce face to face mentoring time with students, while maintaining the integrity of the project. For example, we saw that the use of both Zoom and Google documents provided more efficient, accessible and effective mentoring of students.

1.4 Lessons Learned

As instructors we have learned much from our iterations in all the courses where a CURE was implemented. First, we realized that we need to devote more course preparation time to the planning of the implementation of the CURE project. In particular, instructors needed to provide more time to groups so that they may understand the process of completing the projects and as well as the expectations. We also learned that we have to be flexible to each group's process and ability to work through a research question. For example we must accept that some group projects may result in a final deliverable with unanswered questions and in such situations we must have a way to assess their work based on the semester long process instead of the final outcome. It is critical that your CURE project is explicit in its guidelines and its expectations in terms of defining group work and final deliverable(s). Additionally, creating space for students to safely fail is paramount and should be coupled with frequent and timely feedback to iterate or revise their process (allowing for productive failure). Another important need that arose was to incorporate more frequent discussions about the project in and out of class to keep team members on track and more engaged throughout the semester. Meeting with each group was very important to the process and final outcomes. There is no doubt that it is demanding of instructors' time which is particularly challenging for the sustainability of CURE projects. It is important to mention that the Fall 2020 implementation of the CURE in discrete mathematics showed a much better student experience as you will see in the data presented in the next section. This could be a result of incorporating many lessons we learned over the last 4 iterations of this CURE project for discrete mathematics. We believe we will see similar results for the other two CUREs after more iterations.

2 Data Collection and Analysis

At the end of the semester, a student survey (see [3] and [13] from which the survey was modified) was administered to students. In particular, in our CURE survey we ask about student perception around their STEM attitudes, efficacy and beliefs. In addition the survey asks students about their perceived gains in various outcomes related to their research experience. It is the students perceived gains in several areas that we will consider in this section, specifically reflecting on why in the Fall 2020 semester students seemed to express increased positive gains. Since Fall 2018, we have been collecting the survey data for Discrete Mathematics in all semesters except for Fall 2019. We have been continuously encouraged by the results as they show that students attitudes have improved over the course of the semester as a result of their CURE experience. In addition, due to the significant changes in our modality from previous semesters to the Fall 2020 semester, we were interested in knowing how the online changes affected these positive results. What we saw surprised us. Students had shown on average even higher positive attitudes. We present here a comparison of our survey responses from Fall 2018 - Spring 2020 (with $N \approx 100$) with the survey responses from this past Fall 2020 (with $N \approx 30$). The survey is comprised of the following eight categories

- Future Research Intentions
- Analytical Skills
- Self-Efficacy
- Tolerance
- Ownership and External Validation
- Motivation
- Discovery and Relevance
- Collaboration in Class

We saw gains in all categories. In our analysis below we selected three categories in which we saw significant gains that we found interesting. In particular we present and discuss the results across the three categories 'Analytical Skills', 'Self-Efficacy' and 'Motivation' in which more students reported significant gains from the Spring to Fall semester. You can see this below in the subsequent subsections where we provide both the questions asked and a summative results table.

2.1 Analytical Skills

As part of the post survey, students were asked the following: *"How much did you GAIN in the following areas as a result of your most recent research experience? In other words, how much easier is it for you to perform the tasks described below since participating in your most recent research experience?*

1. *Analyzing data for patterns*
2. *Figuring out the next step in a research project*
3. *Problem-solving in general*
4. *Formulating a research question that could be answered with data*
5. *Identifying limitations of research methods and designs"*

In the data below, we see students reported +30% increase in their ability to "determine the next step in a research project." This may be a result of the increased meeting time with instructors and the increased feedback from instructors. In particular, in the Google documents, instructors provided students with detailed, specific feedback which allowed students to be confident in how they should be progressing in the next steps of their project.

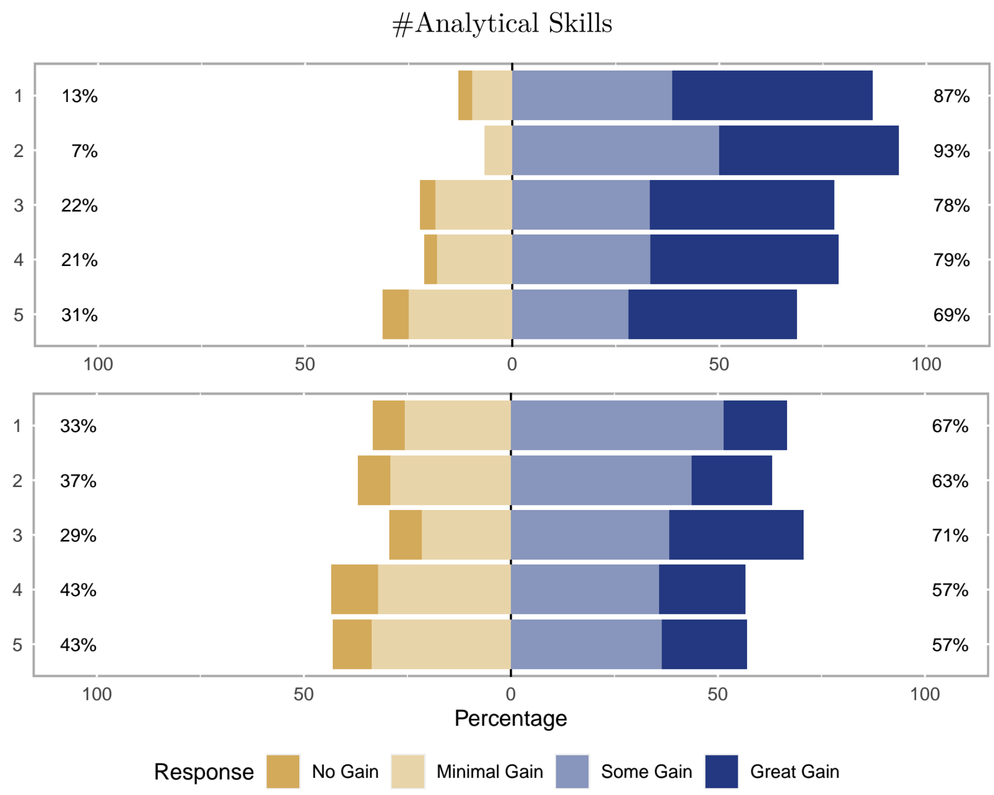


Figure 1: Comparison of Discrete Math 'Analytical Skills' Post Survey Results for Fall 2020 (top) and Fall 2018 - Spring 2020 (bottom).

2.2 Self-Efficacy

As part of the post survey, students were asked the following: *"How much did you GAIN in the following areas as a result of your most recent research experience?*

1. *Confidence in my ability to contribute to math*
2. *Comfort in discussing math concepts with others*
3. *Comfort in working collaboratively with others*
4. *Confidence in my ability to do well in future math courses*
5. *Ability to work independently"*

In the data below, we see the greatest increase in students' perception of their "ability to work independently." For this question, there was a +32% increase in students response of "some to great gain." We believe this may be attributed to the more developed scaffolding of their CURE project as well as the explicit feedback students received. Students were given enough support to feel confident proceeding on their own. We find that it is a fine balance of providing students with the support that they need, while still leaving the research to the students to discover.

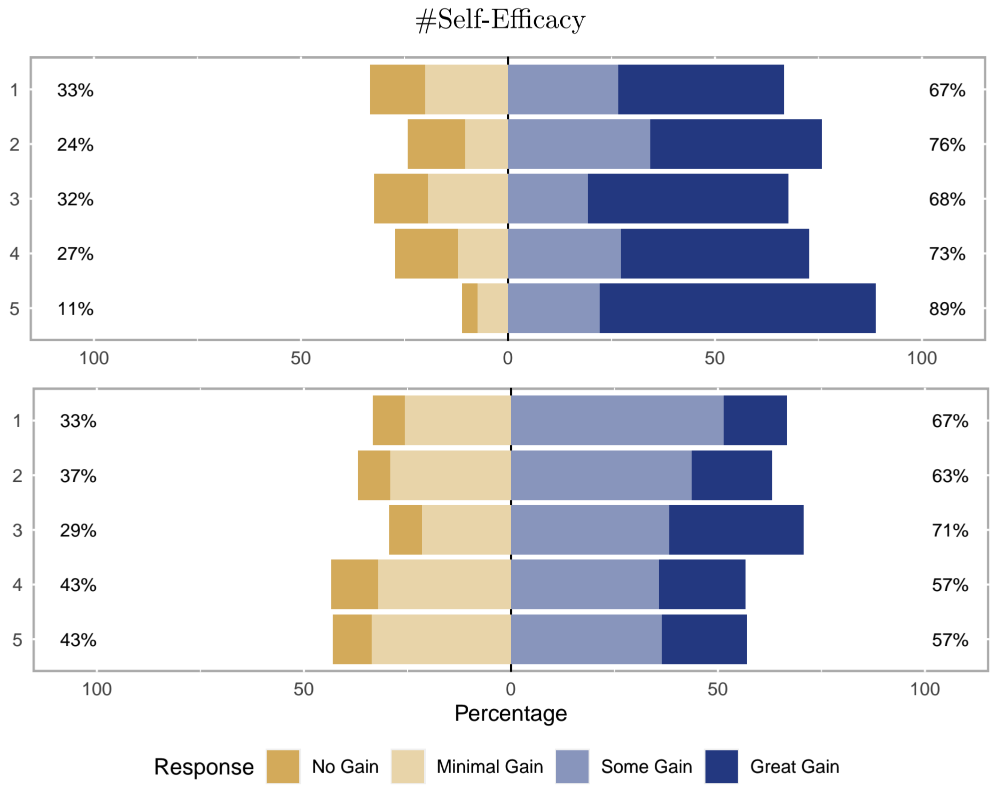


Figure 2: Comparison of Discrete Math 'Self-Efficacy' Post Survey Results for Fall 2020 (top) and Fall 2018 - Spring 2020 (bottom).

2.3 Motivation

As part of the post survey, students were asked the following: *"Compared to BEFORE doing your most recent research, HOW LIKELY ARE YOU NOW to agree with the statement:*

1. *Learning math is interesting*

2. *I am curious about discoveries in math*

3. *The math I learn is relevant to my life*

4. *Learning math makes my life more meaningful*

5. *Learning math will help me get a good job"*

Lastly, in the data below, we were thrilled to see that there was a +19% increase in students perception that "learning math is interesting." We found this particularly motivating as discrete mathematics is an introductory course to the major. We believe this is a result of all the changes we discussed above that led to better student engagement.

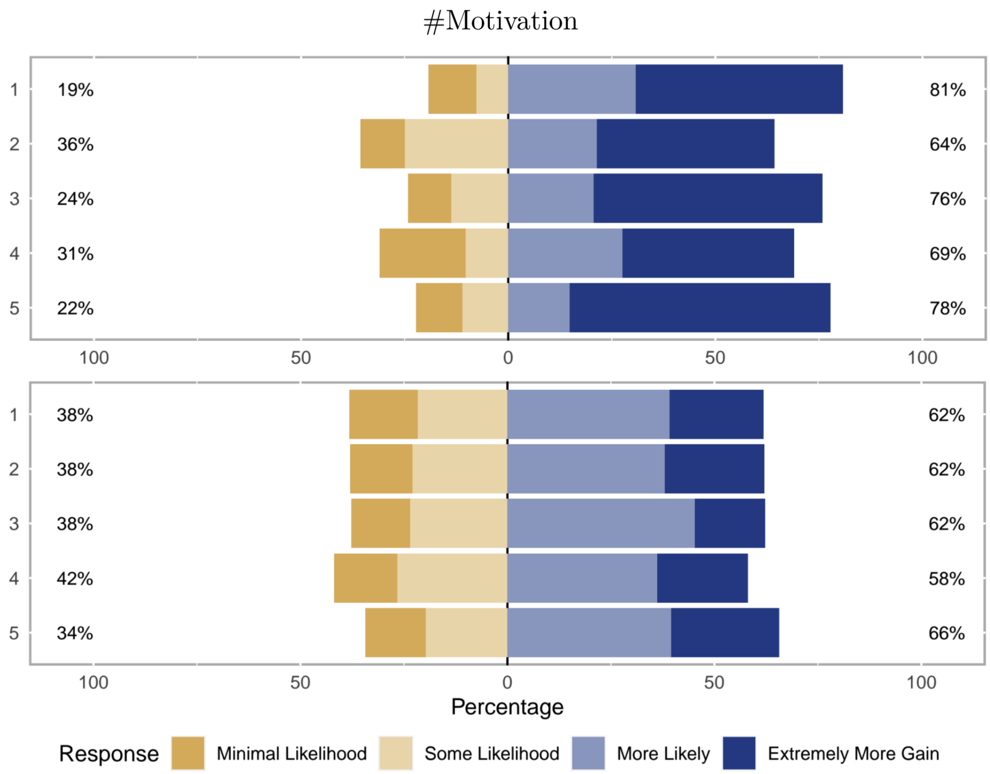


Figure 3: Comparison of Discrete Math 'Motivation' Post Survey Results for Fall 2020 (top) and Fall 2018 - Spring 2020 (bottom).

3 Conclusion

We began our work learning about and planning to implement CUREs in Summer 2018. The opportunity to provide our student population with more equitable access to undergraduate research was an enticing one and is aligned with the mission and vision of our university. Moreover, we were eager to be innovative in how to bring research experiences to undergraduates who are just beginning their mathematical program. Due to the nature of high level math research, we tweaked the definition of CURE ([1]) and created a CURE pathway. In particular, we saw our pathway as an opportunity to introduce CUREs early on, providing students continued, concentrated research allowing for more rigorous undergraduate research at the upper division level.

What we have been able to see so far is that the CURE experience in mathematics has similar benefits to those in other sciences. We feel like our work so far is promising, but we recognize that it is still a work in progress. We hope to continue to fill out our CURE pathway by developing a CURE experience in our linear algebra course. In addition, we would want to track students who have gone through the entire CURE pathway to see how the CURE experience has affected their attitudes after multiple exposures.

Lastly, discrete mathematics has had the most opportunity (in our pathway) for several iterations which included several instructors. We found that each iteration of the CURE project taught new lessons that allowed for greater improvement. In general, we observed positive gains in the Fall 2020 semester. While we do not have a large enough sample size, and data across a longer time period, we provided several hypotheses as to what might account for the positive gains overall. First, due to the online format, instructors were able to meet more with the students outside of class. In addition, it also freed up class time, allowing more times for peers to engage during class on their CURE projects. Second, the online format required us as instructors to revise our classes in general to be highly structured and explicit. In our CURE courses this also impacted the CURE project. Students were given a more scaffolded project. In particular, there was a much more explicit feedback process vis-a-vis the Google documents. Third, due to the transition to an online environment, students and instructors alike became more comfortable and more fluent in online tools supporting collaboration and online meetings such as Google documents and Zoom (both tools that we had used in previous semesters before moving to a remote learning modality). Lastly, and possibly because of these changes, we saw better student engagement in their CURE project, resulting in high quality final projects, and better gains among students. Because of the positive gains we saw, we will be implementing the revised format in our return to face to face classes. Studying the student gains upon returning face to face will help us to determine the validity of our hypotheses.

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