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Applied Mathematics Laboratory: a Course-Based Research Internship

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ABSTRACT: The paper describes the Applied Mathematics Laboratory (AML), a course-based model of undergraduate research engagement in applied mathematics at Towson University. We provide historical background of similar programs at other institutions in the US; describe the implementation and the logic model of the AML; include an example of a recent project; and describe the place of the AML in the context of other course-based student research experiences in STEM.

Keywords: course-based undergraduate research, applied mathematics

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1 Introduction

Efforts to enrich undergraduate education with meaningful research experiences are motivated by a variety of reasons. Traditionally, it was a way to offer talented students development opportunities beyond the standard coursework. This traditional model, called *research internship* by [1], typically involves a small number of students, who either self-select or are selected by faculty. The students do not receive formal course credit for the work, but, if successful, extend their professional network and achieve a number of academically-relevant outcomes (presentations, publications, etc.). Summer research programs, known as Research Experience for Undergraduates (REUs), largely follow this research internship model.

More systematic, larger-scale efforts are often motivated by institutional need to improve recruitment and retention of students, especially those from underrepresented minority groups. An example of an institution-scale program is the Undergraduate Research Opportunity Program (UROP) founded at the University of Michigan in 1989. The program, described in [3], offers academic-year research experience to students in a wide range of disciplines. The students receive academic credit for participation. The program does have elements of self-selection (the students volunteer to participate in the program) and selection by faculty (the students and faculty are "matched"). A good example of a department-scale program is Mathematical Clinic at Harvey Mudd College. Established in early 1970s and running today, the program offers advanced mathematics students an opportunity to work on unsolved problems coming from industry or government. The department explains that the word "Clinic" was chosen because it gives students an opportunity to practice mathematics the way it is practiced by professional mathematicians, "just as a medical clinic provides interns with experience in actual medical practice" [4].

Mathematics Clinic (MC) of Harvey Mudd College was used as a model for establishing the Applied Mathematics Laboratory (AML) at Towson University (the subject of this paper) in 1980. Similar programs were introduced at other universities. "Math in the City" (MitC) at the University of Nebraska at Lincoln [2] is a more recent example; the program was started in 2006. All three programs offer course credit for students' work; with some exceptions, they all are generally available for more advanced students; and to a large extent they have elements of self-selection and selection by faculty. Another common feature of AML, MC, and MitC is emphasis on applications of mathematics rather than on fundamental research in the field.

In biology, a cross-institutional effort was launched in 2012 to address topics related to Course-Based Undergraduate Research Experiences (CURE). One of the benefits of the CURE network is access to a wide variety of programs involving students in research activities. The variety makes it possible to see more clearly the distinguishing features of different models of engagement. Similar efforts are needed in other STEM fields.

This paper aims to contribute to cross-institutional dialogue. We describe the Applied Mathematics Laboratory and provide a description of a recent AML project. We use the framework developed by [1] to place the program in the instructional continuum and develop a logic model describing short- and long-term outcomes of the AML and the activities supporting the outcomes. We conclude by briefly describing the place of the AML in the scholarship continuum identified by Boyer [5].

2 Applied Mathematics Laboratory description

The Towson University AML is one of very few experiential learning programs in applied mathematics in the country. To the best of the authors' knowledge, it is the only such program in the State of Maryland. The premise of the AML is that students receive course credit to work on a mathematical problem of current interest to a sponsoring organization. Sponsoring organizations typically come from government, the private sector or the non-profit sector. Because AML problems are sought out by mathematics faculty, personal contacts of faculty members are the primary source of AML projects.

In a typical iteration of an AML course, the problem presented to the AML by the sponsoring organization is initially ill posed and it lacks a clearly developed set of mathematical tools that can be used to address it. In the months preceding the start of the course, faculty mentors restructure the initially ill-posed problem into a well-posed problem. AML student participants are presented the well-posed problem and it is their job to develop mathematical tools with which the problem will be addressed. In most cases, the main deliverable to the sponsoring organization is a student presentation of a solution to the sponsor's problem. In some cases, students provide additional deliverables (e.g. software applications).

To construct faculty-student teams, the directors of the AML select a team of one or two interested faculty mentors to lead the project. The faculty mentors then recruit a team of 4–6 undergraduate students. To recruit these students an initial email is sent to a faculty mentor-selected group of approximately 20 students. Selection of this initial group is based both on successful completion of relevant coursework and on academic performance. Because the AML is not a required course for mathematics majors, a substantial portion of the initially contacted group of students does not express interest even in attending an information session. In this sense, the AML program has a significant element of self-selection as well. In recent history, the opportunity to participate in an AML project was extended to all students who expressed strong interest in participating.

With regard to assessment and content delivery, the AML course structure differs substantially from that of a typical mathematics course. There are no exams or other periodic assessments. Instead of traditional homework, students are assigned research tasks. A key assessment component of the course is the end-of-semester presentation given by the student team to the sponsoring organization (in a form understandable by the layman). In most cases, faculty mentors do not keep records of student performance on individual assignments. Mathematical background and theory are developed and introduced to the students whenever necessary. In particular, much of the content that is delivered is in direct response to the students' progress (or lack thereof) on their project. Compared to a traditional mathematics course, the content that is delivered is less planned and more "on the fly".

An effort is made to structure the work in a way that allows students to take ownership of the project. In particular the students are asked to come up with ideas, not just implement the ideas given by faculty. Often, smaller teams of 2-3 students are created to handle specific tasks.

2.1 Description of a recent project

The sponsoring organization for the 2019–2020 AML project was a non-profit group that advocates on behalf of those living with disabilities. The project was proposed by a former Towson University student working for the organization. Although the former student did not participate in an AML project during their time at Towson University, they were aware of the program. The AML was asked to analyze the performance of a call center that supports a disability transit service operated in the mid-Atlantic.

The primary issue communicated to the faculty mentors by the sponsoring organization was that the performance metrics used by the call center tended to overestimate the call center's performance. The call center's call log data detailed incoming call volume and a variety of hold time statistics for each 15-minute interval over which the call center was operational. The amount of data in these call logs made it difficult for those who are untrained in data analysis techniques to attempt to uncover the reasons for the perceived performance overestimates.

The students' initial investigations revealed the reason for the overestimates: it turned out that the commonly used metrics employ an averaging process that "smooths out" periods of poor performance with periods of exceptional performance. The students' investigations also revealed additional issues with performance evaluation of paratransit call centers in general (not just for the call center of interest to the sponsoring organization). For example, the metrics used for evaluating disability transit call center performance are inconsistent across regions (in fact, even if the choice of metric is fixed, reporting standards are not uniform). Another problem was that the interpretation of call center data was binary: the performance standards were either deemed to be "met" or to be "unmet". In the latter case, there was no distinction between falling just below the established threshold and a severe failure. After flushing out the major issues with regard to paratransit call center performance evaluation, the students decided to focus their efforts on developing a performance evaluation metric that is simple enough for the layman to use while also accurately reflecting call center performance.

The student team came up with the idea to use a grading rubric to evaluate the call center performance. The grading rubric assigns the usual A, B, C, D or F letter grades to the call center on each 15-minute time interval over which the call center is operational. Federally accepted guidelines were used to determine the criteria for the call center to receive grades A or B. Interviews with paratransit users allowed the students to set the standards for which service levels should be assigned grades C through F. The complete rubric is provided in Figure 1. For each day, the grades assigned on the 15-minute time intervals in the day were aggregated to create a daily grade (being careful not to obscure periods of poor performance in the aggregation process). This performance metric met the requirements that call center performance be accurately reflected. It also provided a measure of the extent to which acceptable performance was missed in the cases where performance standards were not met.

Grading Rubric
A. Meets Federal Guidelines No calls received or both of the following hold At least 99% of calls answered within 5 minutes and At least 95% of calls answered within 3 minutes
B. Nearly Meets Federal Guidelines Criteria for receiving grade A are not met and at least one of the following hold At least 98% of calls answered within 5 minutes or At least 90% of calls answered within 3 minutes
C. Does Not Meet Federal Guidelines, but Does Not Fail Neither of the criteria for receiving grades A or B are not met and at least 97% of calls answered within 5 minutes
D. Poor Service None of criteria for receiving grades A, B or C are met and either fewer than 10 calls are received or at least 99.9% of calls are answered within 8 minutes and fewer than 1/3 of calls received are abandoned
F. Unacceptable Service None criteria for grades A, B, C or D are met

Figure 1: Grading scheme used on each 15-minute time interval

To make the performance metric usable to the non data scientist, students developed a software application that provides the user with a graphical output of call center performance over user-specified time windows. The visual output allows those evaluating call center performance to quickly assess the performance on the time scale of their choosing. In the event that the visual output indicates poor performance on a particular day, the user may “zoom in” to the day under consideration to view the day’s performance on a finer time scale. Figures 2 and 3 illustrate this capability. Figure 2 shows the daily call volume and grades received in February 2020. Based on this figure, one finds that the call center’s service was rated at roughly D-level on February 4. Figure 3 shows a “zoomed in” view of call center performance on February 4 by displaying the grades received for each 15-minute time interval on which the line was operational.

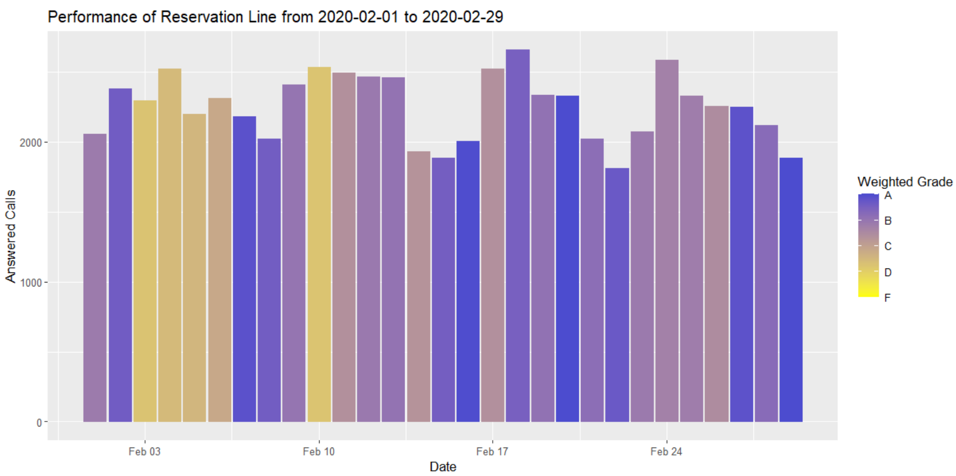

Figure 2: Daily call volume and grades for the reservation line, February 2020

With a quantitative metric in hand, students investigated two mathematically challenging optimiza-

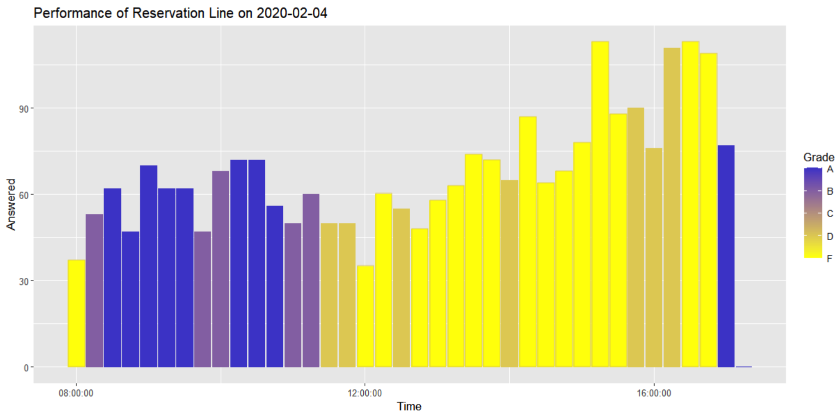

Figure 3: Reservation line call volume and grades for every operational 15-minute time interval on February 04, 2020

tion problems with regard to call center employee scheduling. The first problem sought to determine the minimal number of employees needed for a call center to achieve a prescribed level of service (as measured by the newly developed metric). The second optimization problem was to determine, given a call center's limited labor budget, the best service level attainable by the call center.

Over the course of the project students encountered a number of challenging obstacles both from a mathematical perspective and from a programming perspective. These obstacles required students to expand upon the knowledge base they had developed in their prior course work. As an example of one such obstacle, the provided call center data contained information regarding the percentages of calls having 1, 2, 3, or 5-minute hold times. However, the nuanced criterion for the grade of D required the students to estimate the percentage of calls answered within 8 minutes, see Figure 1. To obtain the estimates, students needed to implement a queueing theory model in statistical language R. (Neither queueing theory nor R are routinely covered in the standard courses for mathematics majors.) Investigation of the optimization problems required the use of tools from operations research and a fair amount of programming, beyond what is taught in standard upper-level mathematics courses. It also required students to be able to understand technical documentation of software packages.

Besides overcoming the mathematical obstacles encountered, students were also required to develop some "soft skills". For example, the students needed to interact with non-mathematical audiences, both for the purpose of gaining information that was used in the mathematical model and for the purpose of communicating the outcomes of the project. Of particular importance was the students' ability to clearly explain to the client the use and interpretation of the software application they developed.

The year-long project involved 6 students in total. Of the participating students, 3 have graduated and are employed. One of these students has explicitly credited his AML experience with the ability to secure the job. The remaining 3 of the 6 student participants are currently finishing their studies.

Overall the client was very appreciative of the students' work. In a message sent to the faculty mentors, the client said

> "The presentation is fantastic. We are so impressed. [...] Clients had the March 4, 2020 draft report from you all but your presentation was much beyond that. Please share our very high regard with [the students]. I also much appreciated the manner in which the students engaged. It was respectful and thoughtful. For example, responding with the comment that they were glad [one of the people on the call] got home safely after she shared her experience of being stranded by paratransit and afraid."

3 Is AML a CURE?

In this section, we use the taxonomy developed by [1] to identify the place of AML among different research-based or inquiry-based methods of instruction. Our methodology is to examine the AML activities along the dimensions described in Tables 1 and 2 of [1]. It is worth noting that [1] was developed with the field of biology in mind. To apply the classification in the context of applied mathematics projects, the interpretation of some of the items had to be adjusted.

3.1 Features

Table 1 of [1] compares the following five features of CUREs with those of Research internships: scale, mentorship structure, enrollment, time commitment and setting. We find that the AML is firmly in the internship category on one of the five items; it is between a CURE and an internship model on three items; and it is a CURE on the remaining item. Here are the details.

Scale and mentorship structure. While some courses elsewhere in the country that are comparable to the AML (for example, Harvey Mudd Mathematics Clinic and MitC program) enroll many students, the AML is a relatively small-scale endeavor. A typical year-long project involves 4–7 students. It is not uncommon for some students to depart and other students to join an AML project at the junction between semesters. With regard to the number of participants, the AML is comparable to a laboratory apprenticeship in the sciences. During the AML meetings, there are opportunities for one-on-one mentorship. However, the most typical type of interaction involves one or two faculty members and a group of students. On both of the scale and mentorship structure items, the AML is somewhere between a CURE and an internship, leaning towards the research internship model.

Enrollment. As discussed in Section 2, enrollment in the AML is determined by a multiphase selection process that involves both selection by faculty mentors and student self-selection. In the first phase faculty mentors select the initial group of candidates based on student performance in relevant courses. These candidates are invited to attend an informational meeting hosted by the faculty mentors where an overview of the project and expectations are given. Because the AML is not a required course for graduation, a significant number of students initially selected by the faculty mentors do not show interest in attending the informational meeting. A recent addition of the departmental honors program specified that participation in the AML can lead to an honors thesis, and graduation with mathematics honors. However, this change is very recent and was not a factor in the decision-making of the students in the project described in Subsection 2.1. In the second selection phase, student candidates self-select by expressing interest in participating in the project. In principle, if after the self-selection phase there are too many students who want to participate, faculty mentors would select further to obtain the final class roster. In recent years, after the self-selection phase few enough students have expressed interest in participating that no further selection of students by the faculty was required. Students may enroll in the AML course only with permission from the instructor. Taken together, these factors put AML in the research internship category.

Time commitment. Any college-level course requires a fair amount of out-of-class work per week. The time spent by the students on homework, projects and preparation for exams frequently exceeds the number of in-class contact hours. It would be fair to say that AML involvement requires more out-of-class work than a typical 3-credit course. So on the dimension of time commitment, the AML falls somewhere between the CURE and research internship categories.

Setting. The AML students have their own work space equipped with a few computers, a printer, and a small library. The space is not a classroom (but it turns into one, if the project requires it). It is also not a research space for the faculty member, neither in the physical sense (this is not where faculty do their primary research) nor in the intellectual sense (for the most part, the AML project topic is different from faculty primary research interests). This makes AML closer to a CURE category than an internship category.

3.2 Learning contexts

We now turn our attention to the place of AML within several teaching contexts (Table 2 of [1]). We use a similar methodology to place the AML in the continuum of teaching contexts, from traditional classroom instruction, through inquiry-based learning and CUREs, to research internships. Here, the picture is much clearer: the AML goes well beyond the traditional instruction or inquiry-based learning.

Science practices. As the project described in Section 2.1 illustrates, building and evaluating mathematical models, dealing with the real-world data (sometimes inaccurate or incomplete), interpreting the results, and communicating the findings are the key components of students' work. The instructors assist students in structuring the problem or finding a solution, but this work is student-driven.

Discovery. The purpose of an AML project is defined by the sponsoring organization with input from faculty leading the project. As we mentioned above, it is frequently the case that the tools for analysis have to be developed before analysis can begin. While the faculty leading the project may have an idea about what type of outcome is expected, the outcome itself is not known in advance (and of course, there are occasional surprises along the way). In this sense, findings from the projects are frequently novel.

Broader relevance. In every case, the relevance of AML students' work extends beyond the course in which they are enrolled. At a minimum, the students' work is relevant to the sponsoring organization. This is due, in part, to the fact that the sponsoring organization typically lacks either the capacity or the resources to analyze the problems they present to the AML. In the project described in Subsection 2.1, the students' work will continue to be relevant to the sponsoring organization well after the completion of the project. Indeed, the software application created by the students can continue to be used by the sponsoring organization to monitor the performance of the call center. In many instances, the analytical tools developed by the students are of interest outside of the sponsoring organization and to the broader applied mathematics community. In these cases, the findings are submitted for publication in peer-reviewed journals. The project described in Subsection 2.1 is one such case, the findings have been submitted for publication and are currently under review.

Collaboration is a key element of AML projects. Collaboration occurs on at least two levels: between the students, as they are working on a team assignment; and between faculty and students. These collaborative efforts focus on model design, implementation, and analysis; they produce the project deliverables: a written report to the sponsoring organization and an oral presentation. In many projects, collaboration extends to joint authorship of peer-reviewed papers. In the cases when the sponsoring organization has technical expertise, collaborative efforts include the sponsors, focusing primarily on model design activities.

Finally, **iteration** is inherent in every project. This is natural based on the fact that iteration is a common feature to virtually every applied mathematics or modeling project. In many cases, when building a mathematical model one attempts to strike a balance between accurately describing the objects or processes being modeled and the simplicity of the description. Iteration plays a major role in finding this balance. The major iterative factors in mathematical modeling, and thus in all AML projects, are illustrated in the *Work in teams* box in Figure 4. When building a mathematical model one typically designs and implements a rough-draft model. After implementation of the draft model one analyzes the results and the outcomes of these analyses inform refinements of the model's design features. This process may be repeated many times until an acceptable model is settled upon.

4 AML student outcomes

In this section we adapt the CURE logic model described in [1] to describe the relationship between key activities and outcomes of a typical AML project. The AML logic model is shown in Figure 4.

The activities collected in the group labeled "Work in teams" are iterative, each activity informs the other activities. For example, model design affects the type of analysis that can be performed; in turn, changes to model design are informed by the analysis of the model results. Communication with the sponsoring organization primarily interacts with the Model design and Model analysis activities. Work with faculty supports the student work in teams.

The expected output of an AML project includes a written report to the sponsoring organization (if appropriate, the report would contain software developed by the students) and an oral report to the organization (it is preceded by a practice presentation for a group of mathematics faculty). For several projects, the results have appeared in peer-reviewed publications (in particular, a paper with the results of the AML project described above was submitted for publication).

Short-term outcomes are supported by the repeated interaction between the relevant activities, rather than by a single activity. For example, students' technical skills are improved when model design requires the students to learn, by working with faculty, a new mathematical technique to implement it. The technical skills are also improved when communication with the sponsors requires that information be presented in an easy-to-understand way; the students then have to figure out, with the help from faculty, a data visualization technique.

Growing the skill set "organically", i.e., giving students information about relevant mathematical tools when they come up with an idea leads to the increased ownership of the project among the students.

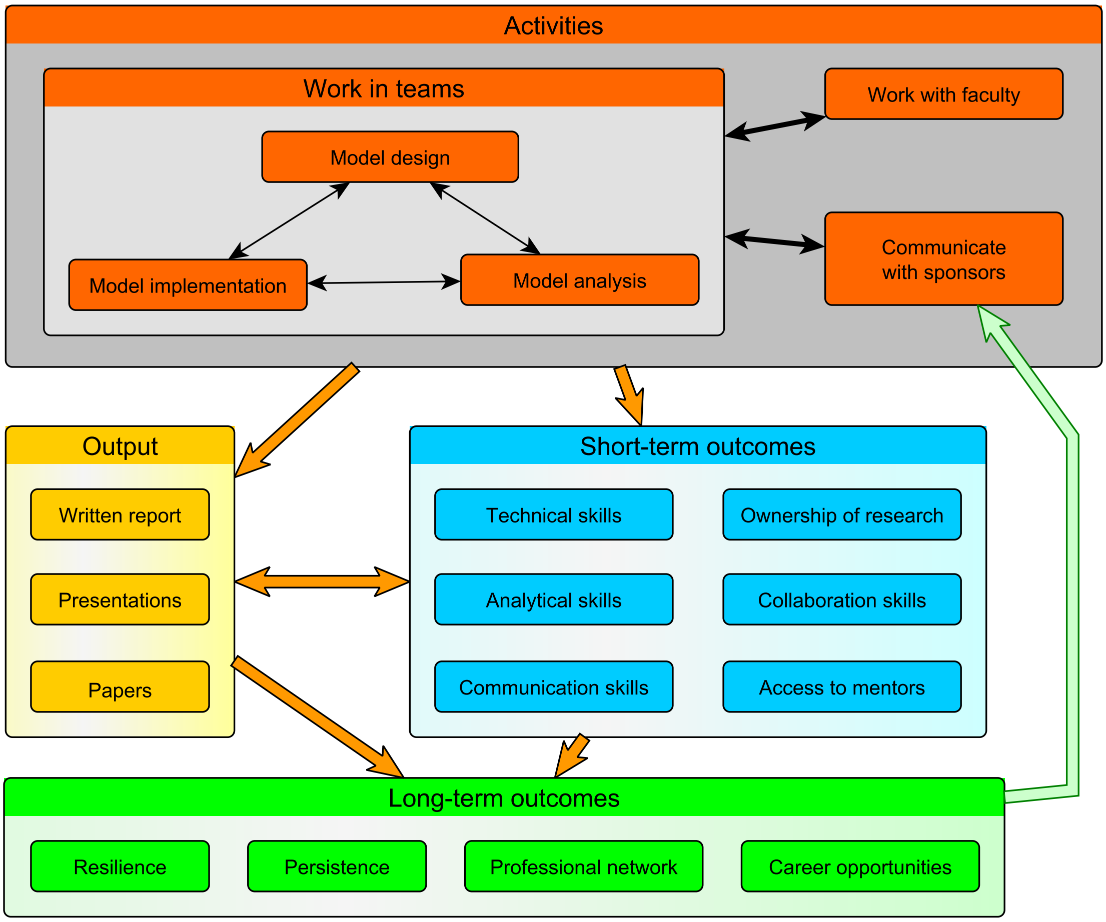

Figure 4: Logic model for the Applied Mathematics Laboratory

Close interaction with faculty also leads to the mentor relationships between the participating students and faculty.

We depart somewhat from the long-term outcomes of [1] in our description. We are taking a broader workforce development view that encompasses a range of careers beyond the science pipeline.

Former AML students reported over the years that their participation in an AML project generates substantial interest from prospective employers and provides skills that are valued by the employers. Several students credit the AML with their employment (including one of the students who was hired by the sponsoring organization), so we believe that an increase in employment opportunities should be added to the list of long-term outcomes.

Finally, we point out one unusual "arrow" in the AML logic model. One of the AML projects was brought by a former AML student. The Mathematical Clinic at Harvey Mudd College also reports that a number of their projects are brought by former students. Closing the activities/outcomes loop when former students come back as sponsors of new projects is an interesting feature of the model.

5 Conclusions and discussion

Our analysis supports the view that models of student engagement in research should be treated as a continuum, not a dichotomy. We observed that the AML exhibits elements of both a CURE and a research internship as described by [1].

We conclude by a brief discussion about needed improvements to the logic model of [1]. The framework for the discussion is the Boyer's model of scholarship [5]. Of the four main types of scholarship proposed by

Boyer, and widely adopted since, the long-term outcomes in [1] are concentrated in the area of scholarship of discovery.

Adapting assessment models for course-based student research experiences to fields outside of the natural sciences is likely to require developing a set of outcomes also for scholarship of integration, scholarship of application, as well as scholarship of teaching and learning. For each Boyer type of scholarship, there should be a set of evidence-supported short-term and long-term outcomes.

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