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Problem Solving for Teachers – Action Research in a Cross-Listed Undergraduate and Graduate Course

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Abstract: This article describes a course-based research experience in a cross-listed mathematical problem solving course for middle school pre-service and in-service teachers. The course was conducted entirely online due to statewide and university restrictions during the coronavirus pandemic. This study examines 4 undergraduate and 5 graduate students' understanding of mathematical modeling. Undergraduate and graduate students were paired in groups to develop open-ended model-eliciting activities for their classmates to solve, and groups conducted educational action research to analyze their classmates' work on the activities. We present one group's activities and their results as an example of how action research can be conducted in a virtual teaching environment.

Keywords: Mathematical modeling, middle school pre-service teacher education

Introduction

Educational action research is viewed as a critical component to the profession of teaching and, more broadly, as the only way for reform efforts in education to have a chance to produce significant effects (National Research Council, 2002). Conducting action research can be an illuminating experience for pre-service and in-service teachers, as a way for them to bridge a gap between the theory and practice of teaching (Noffke, 2009). While enacting action research, teachers examine the process of teaching and learning in their own classrooms through four elements: descriptive reporting of an instructional practice to be examined, purposeful conversation that includes planning individual lessons and activities, collegial sharing of ideas among peer teaching partners, and critical reflection of student work and peer teaching (Miller & Pine, 1990).

The experience of conducting action research has been recommended for inclusion within teacher education courses as a way to prepare teachers to better understand student thinking (Price, 2001). Educators from the Stanford Center for Assessment, Learning & Equity (2019) developed extensive

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resources to help teacher educators in turn mentor teacher candidates to assemble Teacher Performance Assessment ("edTPA") portfolios with action research projects - including lesson plans, video recorded examples of teacher candidates' classroom instruction, their use of assessments with grade school students, and their reflections on ways to improve their instruction. In the state of Maryland in which this CURE course was offered, submission of an edTPA portfolio has been an option in lieu of taking a timed standardized content exam beginning in the summer of 2019 (Pearson Education, 2021). Beginning in the summer of 2022, the edTPA portfolio submission will become a mandatory component of teacher certification (Pearson Education, 2021).

In response to the conclusions that mathematics teachers draw when engaging in action research, teachers can create a more intentional learning environment and be better informed to make instructional decisions (Edwards & Hensien, 1999). Some examples of positive changes in classroom philosophies that teachers have made based on their action research include asking students to actively monitor their own progress rather than passively receive knowledge, presenting content with increased coherence rather than in a fragmented manner, and using assessments for students' discovery of knowledge rather than merely for fact retention (Brown, 1992).

This paper describes a three-credit mathematical problem solving content course for pre-service and in-service teachers that included educational action research. The author was the instructor of the course. The approach to problem solving in this course taken was through the lens of mathematical modeling, as suggested by (Lesh & Doerr, 2003). Modeling is one of the eight Standards for Mathematical Practice described by the Common Core State Standards Initiative (CCSSI, 2010). According to the recommendations of Jung & Brady (2016), mathematical modeling tasks should have the following characteristics: 1) be realistic for students based on extensions of their real-life experiences; 2) ask students to construct a conceptual system as they explain, extend, predict, or modify the model that they develop; 3) allow for students to self-evaluate their work; and 4) help students develop generalizable knowledge that can be used in other related situations. Integrating mathematical modeling into teacher preparation coursework is one way that Anhalt and Cortez (2015a, 2015b) believe will increase teacher knowledge of the modeling process. I also describe the action research experiences of four pre-service teachers and five in-service teachers enrolled in a mathematical problem solving class that was cross-listed for undergraduate and graduate students.

Course description

The Problem Solving for Teachers course was delivered through online instruction due to university restrictions of meeting in-person during the coronavirus pandemic. The course was cross-listed as an undergraduate and graduate mathematics course. On a weekly basis, graduate students were required to complete the undergraduate assignments, plus an additional assignment to read a journal article and write a reflection on the reading.

For the undergraduate students, this course was a required mathematical content course in their state-accredited bachelors degree programs. The pre-requisite for the undergraduate course was successful completion (grade of C or better) in Calculus 1. All of the undergraduate students had at least junior standing (completion of 60 credits or more towards the bachelors degree), and were preparing to graduate in the following spring semester. Engaging in action research in their second to last year of their bachelors degrees is especially helpful for the pre-service teachers, as the university requires all pre-service teachers to submit an edTPA portfolio (for portfolio requirements, see (Stanford Center for Assessment, Learning & Equity, 2019)) in their senior year.

For the graduate students, all of whom were in-service elementary or middle school teachers, this was a content course that would count towards their Master of Science in Mathematics Education degree programs as either an education elective or a mathematical content elective. There were no pre-requisite courses for the graduate course beyond admission into the masters degree program in mathematics education.

Course objectives listed in the course syllabus include: Students will examine the use of effective problem solving strategies, discuss and analyze strategies and solutions of their own and of their peers, reflect on the strengths and weaknesses of all work submitted and selected work of their peers; apply critical thinking skills and Usiskin's (2015) Modeling Process to solve non-routine mathematical problems; demonstrate the disposition to experiment with a problem they have not previously encountered; structure non-routine problems in a way that offers a path to the solution; develop problem posing skills; recognize links between higher level mathematics topics and middle school mathematics curricula; apply the technical skills learned in prerequisite mathematics courses to carry out a solution; organize and present coherent presentations of their approach to a solution; and appreciate and compare different approaches to solving a particular problem.

In the first half of the course, spanning the first eight weeks of the semester, students participated in Model-Eliciting Activities (MEAs) which addressed upper elementary, middle, and secondary content standards. When modeling, students are asked to develop representational descriptions for specific situations (Lesh & Lehrer, 2003). As part of the modeling process, students are expected to go through an iterative process whereby they analyze the situation or the problem, develop and formulate a model, compute a solution to the model, interpret the solution based on a real-life situation, validate their conclusions, and revise the model if needed (Anhalt & Cortez, 2015).

The MEAs selected for this course covered the four major categories (Turner, et. al., 2020): Descriptive, Predictive, Optimizing, and Rating & Ranking. For each category of MEA, students were asked to complete one MEA during their synchronous class time and complete another MEA for homework. Many of the modeling tasks were related to multiple content standards within the Common Core State Standards, as modeling tasks frequently span multiple content areas (Tam, 2011). Students were also introduced to a few web-based resources with additional examples of MEAs. A listing of the references for the activities used in the first half of the course is provided in Appendix A: Model-Eliciting Activity References used in Problem Solving for Teachers. Some of these references include libraries with multiple MEAs, so that other faculty members who wish to run a similar CURE course could have more selection of contexts.

During the semester, I invited three guest speakers who presented activities for approximately an hour each. The first speaker was another undergraduate student who conducted a geometry-based MEA on describing the shapes and patterns within snowflakes. The second speaker was a representative from the non-profit organization Population Education, who enacted several demonstration MEA lessons taken from the Classroom Resources section of the Population Education website (Population Education, 2021). The third speaker was an alumna, an in-service teacher who had previously taken this course as an undergraduate student, who implemented an MEA activity with public school students as an undergraduate student, and presented the activity at several professional development conferences prior to graduating with her bachelors degree.

In the second half of the course, students participated in an action research project, with requirements more fully described in Appendix B: Model-Eliciting Activity Presentation and Action Research Project. The instructor's own originally designed MEA (Cheng, Berezovski & Talbert, 2019), with results reported from a previous semester of undergraduate students taking the same course, was given as an example of an action research project. Students worked with one or two classmates to design an MEA and conducted it during a 75-minute session with their classmates as the participants. Within the presentation, students were asked to explain how their MEA addresses Common Core State Standards and satisfies MEA design principles (Jung, 2015). The presenting students then analyzed their classmates' work and reflected upon their implementation.

There are four main elements of action research, as identified and described by Miller and Pine (1990). In Table 1, each of these elements of action research is listed along with a description of how the action research project in this course addressed these elements.

Table 1. How Problem Solving for Teachers CURE addresses action research elements

	Action research element identified by Miller and Pine (1990)	**How this CURE course research project addressed this element**
1	Teachers describe an instructional practice to be examined.	The instructional practice is using Model-Eliciting Activities in classrooms.
2	Teachers engage in purposeful conversation that includes planning activities.	Teachers and pre-service teachers were partnered together to discuss their planning of their activity presentation.
3	Teachers share their ideas among peers, who were their classmates.	Over four consecutive weeks, one of the groups' presentations were presented each week during the virtual class sessions.
4	Teachers reflect critically on the student work and their instruction.	Presenters completed reflections on their classmates' work and their instructional activity.

A rubric published by Anhalt & Cortez (2015) was used for grading the presenters' solutions to their modeling problems. This rubric examines the fullness of explanations and justifications to demonstrate understanding of concepts present in the problem; the extent to which students show ideas that are well-connected to each other; the providing of complete work that includes assumptions and solutions that follow them; evidence of thoughtful reasoning; the appropriateness of the concepts and representations used in the solution process; and the correctness of the calculations performed.

Description of students

Nine students were enrolled in the three-credit course in spring 2021. Four were undergraduate students (pre-service teachers) and five were graduate students (in-service teachers). Of the undergraduate students, three were middle school education majors with concentrations in mathematics and science, and one was a special education major. The course is a required mathematics content course for all of these undergraduate students. All of the graduate students were certified for middle school teaching in grades 4-9, and were employed as upper elementary or middle school teachers. The five teachers worked for three different local public school districts.

A survey, initially developed by Anhalt & Cortez (2016), was administered to students at the start of the semester before they were expected to complete any background readings. The purpose of the survey was to understand students' conceptions of mathematical modeling prior to engaging in action research. Table 2 organizes the students' responses to each item by theme in a manner similar to the way Anhalt & Cortez (2016) reported their participants' responses. A post-survey was requested of the students at the end of the semester, as Jung & Brady (2016) found that even after teachers experienced as few as three mathematical modeling lessons, the teachers developed significant changes in their thinking about the way in which their students might interact with mathematical modeling tasks. But, regrettably, too few students responded to the survey, thus only pre-survey responses are reported in this article.

In Table 2, the undergraduate students are labeled as U1, U2, U3, U4 (special education major). The graduate students are labeled as G1, G2, G3, G4, G5. The responses of the two students, one undergraduate student and one graduate student, who presented the Gift Wrapping task described later in this article are denoted by U1 and G1.

Table 2: Undergraduate and graduate student responses to modeling survey responses from Spring 2021 semester students.

The themes emerged in students' responses to the Modeling Survey by Anhalt & Cortez (2016)	Under-graduate Students	Graduate Students
Question 1. One of the standards for mathematical practices is ''Model with mathematics.'' Explain what this means to you.		
Reasonably accurate understanding of mathematical modeling		
Mentioned real-world or real-life application or scenarios.	U3	G1
Mentioned or implied integrating mathematical knowledge and everyday knowledge	U2	G2, G3
Mentioned engaging in mathematics using multiple solution methods, representations or tools	U1	G1, G4, G5
Confused mathematical modeling with showing step-by-step work on a problem	U4	

Question 2. Are modeling with mathematics and solving word problems related? Explain.		
Answered yes		
Math problems can apply to real world situations through modeling	U1	G1
Modeling allows for a visualization of the real world problems	U2	G5
Models are created as possible solutions for real world problems	U3, U4	G3
Provided a way that modeling could be different from solving word problems		
Math is more open-ended and students have more autonomy in solving modeling problems than in solving traditional word problems		G1
Word problems provide an avenue to begin problem solving and critical thinking skills involved in modeling with mathematics		G2
Modeling problems include more authentic data, and word problems have more contrived data		G4
Question 3. In your own words, describe the features of Model-Eliciting Activities (MEA). Provide a specific example in your description.		
MEAs encourage different types of questioning (students have the leeway to create their own questions they wish to solve)	U1	G1, G3
MEAs are relevant to students or relate to the real world	U4	G1, G2, G4, G5
MEAs can have more than one correct solution	U3	
MEAs allow for students to test and invent their own models	U2	G1, G3, G4, G5
MEAs are interdisciplinary		G4
Question 4. How can teachers understand and prepare to teach modeling at the middle school and high school levels?		
Teacher knowledge of students' mathematical abilities	U1	
Teacher knowledge of students' interests		G3
Teacher knowledge of different modeling strategies to show students	U2	G1
Teachers' asking students to solve open-ended questions	U3	
Teachers' leading students through the thought process involved in modeling	U4	G2, G3
Teachers' engagement in MEAs themselves		G1, G4
Teachers' lesson plan alignment with state standards		G1, G5
Question 5. What role do you suppose that ''real-life'' contexts play in modeling problems?		
Real-life contexts give relevance to the mathematics	U2, U3, U4	G1, G2
Real-life contexts support students' critical thinking		G3
Real-life contexts provide motivation or interest to do math	U1	G1, G4
Real-life contexts make math content easier to understand	U3	G5

Question 6. Should modeling-eliciting activities be included in both mathematics content and mathematics methods courses in your major? Explain your response.		
Yes: Modeling applies to both the math and real worlds, so it helps to give reason and purposes to instruction.	U1, U4	G4
Yes: Pre-service teachers should have opportunities to become comfortable and familiar with MEAs, how to use them in instruction, and how they contribute to students' learning.	U2, U3	G2, G3
Yes: The way we are taught should look similar to the way we are expected to teach. Using MEAs enriches teachers' knowledge and can improve teachers' teaching skills		G1, G5

Here, some of the major take-aways of these survey results are highlighted:

- Eight of the nine students correctly articulated the meanings and features of MEAs in a way that aligned with our interpretation of mathematical modeling. One undergraduate student confounded mathematical modeling with showing work on solving problems. This highlights the importance of teaching modeling in university courses for pre-service teachers.
- Although the undergraduate students were able to articulate the more obvious features of MEAs, such as connections to real world situations and visualizations of real world problems, only one undergraduate student recognized the need for revisions within the modeling process. In contrast, four of the five graduate students mentioned the iterative nature of modeling.
- We asked the students how teachers should prepare to teach modeling in Question 4. Collectively, the responses focused on two primary types of knowledge: the teacher's mathematical knowledge for teaching including their own knowledge of mathematical modeling, and the teacher's knowledge of the students including the students' interests and mathematical preparation so that the teacher can show different strategies to the students.
- All students emphasized the role of real-life contexts as providing meaning for students.
- When asked whether MEAs should be included in college level coursework, all students answered yes, and provided a variety of explanations.

Example of an MEA created by students: the "Gift Wrapping MEA"

Model-Eliciting Activities frequently begin with a key question whose response requires significant thought. In the activity presentation provided by U1 and G1, the key question was the following: "How much wrapping paper is needed to wrap a gift?" While the context of wrapping a gift is not novel in middle school mathematics curricula (e.g., Great Minds, 2015), the presenters developed an original set of activities with modeling in mind. The implementation of the activities reported here took place during a synchronous virtual session which lasted approximately two hours. Six of the classmates attended and participated in the activity. One student was absent from the synchronous lesson.

The Common Core State Standards (CCSSI, 2010) addressed within the activity were focused on the sixth and seventh grade content standards as summarized below:

- Number Systems: Adding and subtracting decimals and rational numbers (6.NS.B.3, 7.NS.A.1.D)
- Expressions and Equations: Using variables to represent numbers and write expressions using variables to solve real-world problems (6.EE.B.6, 7.EE.B.4)
- Geometry: Representing three-dimensional figures using nets, and solving real-world problems involving surface area of three-dimensional objects (6.G.A.4, 7.G.B.6)

For an engagement activity, to help participants connect the context of gift wrapping to real life, the presenters created three slides prompting each breakout group of two students to research an international culture and how people exchange gifts in that culture. Then, they asked participants to pretend they were shopping online at Target.com, and had $20 to spend on a gift. The participants were asked to research the shipping dimensions of a box that the vendor would need to use to ship the gift to them, and then determine how much wrapping paper is needed for this gift. Participants were then asked to test their models using hands-on manipulatives. They were asked to find a rectangular prism box and wrap some blank sheets of paper, as a way to test their models. Some participants did not initially use the correct formula for surface area, (for example, they may have used the formula for volume or developed their own formula that did not produce an accurate surface area), but testing their formulas with actual wrapping paper helped participants realize whether their mathematical models were incorrect. After providing individual time for exploration, presenters asked their classmates to share and explain the formulas used. Even when participants were able to use the correct surface area formula, they only calculated the surface area of the rectangular prism shaped box, but did not take into consideration having some extra wrapping paper to overlap the paper.

An example of a participant's work of wrapping a box is shown in Figure 1. The product selected is a box of staples with dimensions 4.25 inches in length, 1 inch in height, and 1.625 inches in width.

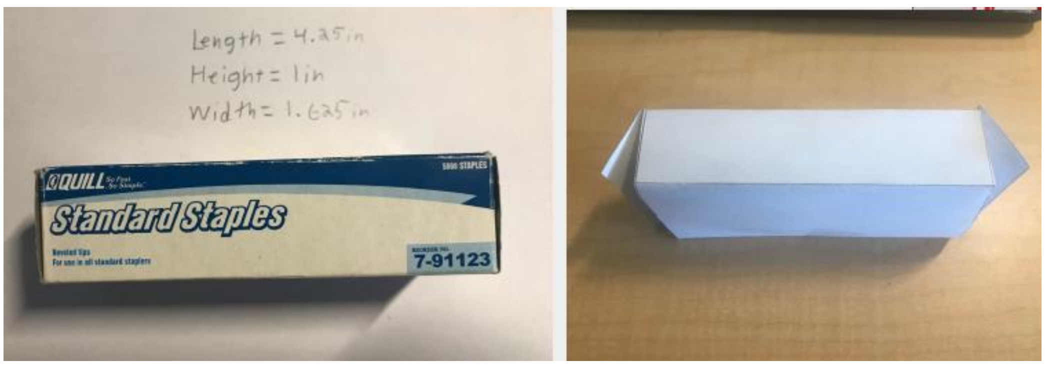

Figure 1: Participants' box and its wrapping with blank paper during the Gift Wrapping MEA

Next, as part of the modeling cycle, participants were asked how practical it would be to just use the surface area of the box to determine the amount of wrapping paper to use. The student whose work is shown above mentioned, "I would increase the measurements of all the dimensions of the wrapping paper by about 1 or 2 cm to make it a little easier to wrap around the box." Another participant wrote, "Just using the overall surface area is not very helpful. I could still use that rule, just with an additional step of figuring out the dimensions by adding ¼ of the width to the width (of the wrapping paper)." The first participant was suggesting using additive reasoning to determine the amount of an overlap, whereas the second participant was suggesting using a proportion of the box's dimensions.

Participants were then asked to answer four questions to reflect upon the activity as teachers and provide feedback to the presenters. These questions and some responses by the participants are shown in Table 3.

Table 3: Presenters' qualitative questions and sample participant feedback on Gift Wrapping MEA

	Question	**Participant Response 1**	**Participant Response 2**
1	What mathematics did you use to help you answer the key question?	"I used the concepts of area, geometric nets, and ratios / proportions to come up with a formula for the amount of wrapping paper."	"I used equations to find the areas of the sides of the box."
2	How might your rule change based on different shapes of a gift? Give an example.	"Different shapes have different surface area formulas, such as a cylinder."	"It depends on the shape, but in general I will still find the surface area and make sure to add one or two inches to wrap the gift properly."
3	Is your model practical for real-world use? Explain.	"I think it is because it is a simple formula with given measurements, but most people will probably just estimate and eyeball the amount of wrapping paper they need."	"It might be practical for real-world use, but it takes time to calculate. I think it will be practical for those who like math and have time."
4	How might you adapt or extend this lesson for your elementary or middle school students?	"I would have students try each others' models so they can get feedback on how they might change their model."	"To add cognitive complexity, you could give the class dimensions to a pre-cut piece wrapping paper, and then you could have them find gifts within a budget and use models to predict if they can wrap their gift with the wrapping paper provided. Then have the students test it out."

After each presentation, all participants were asked to complete a Likert-scale feedback form, with prompts taken from the Guidelines for Assessment & Instruction in Mathematical Modelling Education (COMAP & SIAM, 2016). This participant feedback form was set up as a GoogleForm and I monitored the number of responses that were recorded. If students did not complete their feedback forms immediately, I sent out a reminder email to the students who did not complete the forms. The responses to this participant feedback form were then given anonymously to the Gift Wrapping MEA presenters to take into consideration when writing their individual reflections.

Table 4: Participants' responses to the feedback form related to the Gift Wrapping MEA.

(Key: Each ◆ represents one student's response)

Assessment Items for Participants of MEA's to Complete for Presenters (COMAP & SIAM, 2016)	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
I understood the presenting team's interpretation of the Key question.				◆◆◆	◆◆◆
All stated assumptions were adequately justified.			◆	◆◆	◆◆◆
The model's strengths and weaknesses were addressed.			◆◆	◆◆◆	
Appropriate mathematics was used to create the model.				◆◆	◆◆◆◆
A final solution was clearly presented.				◆◆◆	◆◆◆
The mathematical model produced a plausible result.				◆◆◆	◆◆◆
Visual aids were easy to read and understand.				◆◆	◆◆◆◆
The team addressed authentic alternative scenarios and / or the need for futre work.			◆	◆	◆◆◆◆
I enjoyed the presentation; the presenters held my attention for the full extent of the talk.				◆◆◆◆	◆◆
I would like to learn more about this team's solution method.		◆	◆◆	◆◆◆	

The participants for this activity did not all finish their work within the two-hour synchronous class time, so I asked the students to finish their work as a homework assignment due the following week. After gathering their classmates' synchronous and subsequent asynchronous work, the presenters were asked to analyze their classmates' work by creating an original rubric to score the responses during the synchronous lesson (the instructions for this assignment are in Appendix B, referring to Product 4: Classmates' Work Analysis). The presenters developed the rubric in Table 5 to rate each of their classmates' work.

Table 5: Presenters' rubric used to rate classmates' work on the Gift Wrapping MEA

	Outstanding	**Good**	**Poor**
Task #1: Gift Item Selection & Details	*2 points* All information provided	*1 point* Some information provided	*0 points* No information provided
Task #2: Model Development	*3 points* Mathematical model fully developed that shows mathematical thinking and correct calculations. Amount of wrapping paper determined with provided reasoning.	*1-2 points* Some information provided	*0 points* No information provided
Task #3: Model Testing, Revisions and Application	*5 points* Images/observations documented, revisions provided with reasoning for revisions, new rule developed and application of new rule to gift item.	*1-4 points* Some information provided	*0 points* No information provided
Reflection	*4 points* All reflection questions answered fully and thoughtfully.	*1-3 points* Some information provided	*0 points* No information provided
Total Points	*14 points*	*4-10 points*	*0 points*

The presenters also wrote individual reflections on the presentation. These reflections are shown in Appendix C.

There were a total of 15 possible points available for participants to earn for attending each of their classmates' MEA presentations. Each participant received 1 point for completion of the feedback form in a timely manner, or else 0 points if the feedback form was not completed. The participant also had the opportunity to earn up to 14 points for the quality of work submitted during the Gift Wrapping MEA as rated by the presenters.

How this class addresses CURE components

Auchincloss et. al. (2014) suggested five components of Course-Based Undergraduate Research Experiences (CUREs). These components include Use of Scientific Practices, Discovery, Broadly relevant or important work, Collaboration, and Iteration. Below, I briefly summarize how the Problem

Solving for Teachers CURE course helped address these components, and describe ways in which the course could be altered to more fully address these CURE components.

1. **Use of scientific practices.** As described in the references provided in Auchincloss (2014), research within CUREs in math and science courses should include asking questions, building and evaluating models, designing studies, gathering and analyzing data, developing and critiquing interpretations and arguments, and communicating findings. Within the teacher action research experience in this CURE, the participants developed their own MEA based on a real-world key question, asked their classmates to solve the problems and provide feedback, and reflected on how to improve their lesson. In previous years' Spring semester implementations of this course, I have worked with students beyond the scope of the course and asked them to present at an annual Fall statewide teachers' professional development conference with their activities and examples of student work. This year, the statewide conference was not held due to the coronavirus pandemic. However, the MEA materials generated by these presentations could be used in a future Problem Solving for Teachers course, or as an activity with other content or methods courses for teachers.
2. **Discovery.** The pre-service and in-service teachers designed their own instructional activity and did not know in advance how the lessons would be received. In previous year's iterations of this course, I arranged for students to conduct their lessons with middle school students after they tried out the lessons with their peers. It would be natural to ask the in-service teachers enrolled in the course to allow the pre-service teachers to visit their classes to conduct the originally designed MEAs. But, during the pandemic, in Spring 2021, the in-service teachers mentioned that their middle school students were not as engaged in a virtual setting than they would have liked to see (also, G1 mentioned the activity would be better conducted in-person so that the teacher could make sure the students all had a box to wrap, and so the teacher could help the students if they had difficulties wrapping items), so we did not implement these activities with middle school students.
3. **Broadly relevant or important work.** The idea of using Model-Eliciting Activities is important because statewide mathematics standards include modeling (e.g., MSDE, 2021; CCSSI, 2011;

Tam, 2011). Using modeling in mathematics classes is broadly relevant so as to provide students with experiences of ‘thinking mathematically,’ that is, interpreting situations in a mathematical way (Lesh & Lehrer, 2003). In order for teachers to be comfortable with enacting modeling lessons with students, they should solve modeling problems themselves during their teacher preparation coursework and / or during professional development (e.g., Jung & Brady, 2016; Anhalt & Cortez, 2016).

4. **Collaboration.** Since pre-service and in-service teachers were paired together to form groups, undergraduate students were able to hear insights related to the practice of teaching that they might not otherwise have the occasion to discuss with in-service teachers. This pairing also allows for graduate students a chance to mentor undergraduate students. As seen in the reflection that U1 provided in Appendix C, U1 felt she benefitted greatly from the experiences that G1 shared during their group work time. The instructor’s role in this course is primarily to guide the groups and provide support as they develop their MEAs. The instructor also distributed a survey to the participants to provide feedback for the presenters.
5. **Iteration.** Presenters were asked to reflect upon what they would change for future implementations of the lesson, after receiving feedback from participants. However, within the time span of this course, presenters did not have a chance to reteach their lessons to a different audience.

Concluding Reflection

At Towson University, the course Problem Solving for Teachers is offered by the mathematics department once a year in the Spring semester. It began being offered as a cross-listed course for graduate students and undergraduate students in 2019. Spring 2021 was my sixth time teaching Problem Solving for Teachers since beginning to teach it in Spring 2013. In the first few years of teaching this course, I initially just had students solve interesting mathematical problems, with inspiration from National Council of Teachers of Mathematics (NCTM) teacher practitioner journals. Since the middle school pre-service teachers at my university are also seeking certification in another concentration in addition to mathematics, selected from the three choices of English / language arts, social studies, or science, I have been finding cross-disciplinary problems for the students to solve.

The students taking this course still have one more year remaining in their degree programs, thus it is still possible to work with a few students after they finish the course and before they graduate. Undergraduate middle school pre-service teachers and I co-authored one teacher practitioner article (Cheng, Moore & Wong, 2015) after they finished taking this course. Two secondary mathematics education majors (Cheng & Twillman, 2018; Cheng & Nedwick, 2020) and two graduate students (Cheng & Horner, 2014-2015; Thompson & Cheng, 2016) taught lessons to the students enrolled in the Problem Solving for Teachers course and to grade school students, with the goal of testing some originally designed instructional materials.

In the latest few implementations of the course, I have begun introducing MEAs in instruction, as the statewide standardized assessments for grade school students shifted from testing procedural knowledge to asking students to complete more modeling problems (MSDE, 2021). For the MEA project, students are welcome to adapt lessons that have already been taught but modify them in some way: for example, they could update the data that students are expected to examine, make the context more relevant to local situations, move lessons that were intended for in-person delivery to a virtual format, and / or structure the lessons into a modeling cycle format where students are asked to develop the model rather than be provided with an equation to use at the outstart. The novelties of the students' projects include asking students open-ended questions when previously published resources might have closed-ended questions, analyzing collected student work. Students have appreciated developing MEAs that they can use with their future or current students.

This CURE provides all middle school pre-service teachers enrolled in the course a chance to create MEAs alongside in-service teachers. Sometimes these projects are presented at professional development conferences and published as manuscripts in teacher practitioner journals, thus the students of this CURE course are able to collectively contribute to the knowledge base in mathematics education with their analyses of student work on the MEAs.

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References

Aguirre, J., Turner, E., Bartell, T., Kalinec-Craig, C., Foote, M., Roth McDuffie, A., & Drake, C. (2013). Making Connections in Practice: How Prospective Elementary Teachers Connect to Children's Mathematical Thinking and Community Funds of Knowledge in Mathematics Instruction. *Journal of Teacher Education*, *64*(2), 178–192. https://doi.org/10.1177/0022487112466900

Anhalt, C., & Cortez, R. (2015). Mathematical Modeling: A Structured Process, *The Mathematics Teacher*, *108*(6), 446-452. https://doi.org/10.5951/mathteacher.108.6.0446

Anhalt, C. & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation, *Journal of Mathematics Teacher Education, 19,* 523-545. DOI 10.1007/s10857-015-93098.

Auchincloss, L., Laursen, S., Branchaw, J. et al. (2014). Assessment of course-based undergraduate research experiences: A meeting report. *CBE Life Sciences Education,13*, 29-40.

Brown, A. (1992). Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings, *Journal of the Learning Sciences*, *2*(2), 141-178. DOI: 10.1207/s15327809jls0202_2

Cheng, D., Berezovski, T., & Talbert, R. (2019). Dancing on ice: Mathematics of blade tracings. *Journal of Mathematics and the Arts*, *13*(1-2), 112-130. https://doi.org/10.1080/17513472.2018.1509259

Cheng, D. & Horner, N. (2014-2015). Finding the total number of solutions to 'FOUR + ONE = FIVE': Word sum problems for elementary through graduate students. *Iowa Council of Teachers of Mathematics journal*, *41*, pp. 23-30.

Cheng, D., Moore, S., Wong, J. (2015). Project-Based Learning in a Middle School Teacher Education Problem Solving Course. *Banneker Banner*, *29*(3), pp. 12-19.

Cheng, D. & Nedwick, R. (2020). "Skating on circles: Using figure skating blades to apply circular geometry concepts." Mathematics and Sports, *1*(1), 1-12. http://libjournals.unca.edu/OJS/index.php/mas/article/view/6

Cheng, D. & Thompson, D. (2016). From Blueprints to Labyrinths. *Mathematics Teacher, 110*(4), pp. 254-257. https://doi.org/10.5951/mathteacher.110.4.0254

Cheng, D. & Twillman, M. (2018). "Double the fun: Mathematics within pairs figure skating side by side jumps." Mathematics Teacher, *111*(4), pp. 249-253. https://doi.org/10.5951/mathteacher.111.4.0249

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Available at http://www.corestandards.org

Consortium for Mathematics and Its Applications (COMAP) & Society for Industrial and Applied Mathematics (SIAM), Guidelines for Assessment & Instruction in Mathematical Modelling Education. (2016). Available at http://www.comap.com/Free/GAIMME/

Edwards, T., Hensien, S. (1999). Changing Instructional Practice Through Action Research. *Journal of Mathematics Teacher Education, 2,* 187–206. https://doi.org/10.1023/A:1009951121246

Great Minds. (2015). Lesson 19: Surface area and volume in the real world. Eureka Math. https://www.engageny.org/file/45526/download/math-g6-m5-topic-d-lesson-19-student.docx?token=2Wz6WTz9

Hirsch, C. & Roth McDuffie, A. (Eds.). (2016). Annual Perspectives in Mathematics Education: Mathematical modeling and modeling mathematics. Reston, VA: National Council of Teachers of Mathematics.

Jung, H. (2015). Strategies to support students' mathematical modeling. *Mathematics Teaching in the Middle School*, *21*(1), 42-48. https://doi.org/10.5951/mathteacmiddscho.21.1.0042

Jung, H. & Brady, C. (2016). Roles of a teacher and researcher during in situ professional development around the implementation of mathematical modeling tasks. *Journal of Mathematics Teacher Education*, *19*(2-3), 277-295.

Lesh, R. & H. Doerr, H. (Eds.) (2003). Foundations of a model and modelling perspective on mathematics teaching, learning and problem solving. In R. Lesh and H. Doerr (Eds.), *Beyond constructivism: models and modelling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum. Pp. 3-33.

Lesh, R. & Lehrer, R. (2003). Models and modeling perspectives on the development of students and teachers. (2011). *Mathematical Thinking and Learning, 5*(2-3), 109-129. https://doi.org/10.1080/10986065.2003.9679996

Maryland State Department of Education (MSDE). (2021). Maryland Comprehensive Assessment Program Resources. https://www.marylandpublicschools.org/about/Pages/DCAA/Math/MCAPResources.aspx

Miller, D. & Pine, G. (1990). Advancing professional inquiry for educational improvement through action research. *Journal of Staff Development, 11*(3), 56-61.

National Research Council. (2002). *Scientific research in education*. National Academies Press.

Noffke, S. (2009). Revisiting the Professional, Personal, and Political Dimensions of Action Research. In Noffke, S. & Somekh, B. (eds), *The SAGE Handbook of Educational Action Research*. Pp. 6-23. Los Angeles: SAGE Publications.

Pearson Education. (2021). EdTPA for Maryland. https://www.edtpa.com/PageView.aspx?f=GEN_Maryland.html

Population Education: A program of Population Connection. (2021). https://populationeducation.org/teacher-resources/

Price, J. (2001) Action research, pedagogy and change: The transformative potential of action research in pre-service teacher education, *Journal of Curriculum Studies*, *33*(1), 43-74. DOI: 10.1080/00220270118039

Stanford Center for Assessment, Learning, & Equity. (2019). EdTPA: Making Good Choices: Candidate Support Resource. http://www.edtpa.com/Content/Docs/edTPAMGC.pdf

Tam, K. (2011). Modeling in the Common Core State Standards. *Journal of Mathematics Education at Teachers College*, *2*(1). https://doi.org/10.7916/jmetc.v2i1.701

Turner, E., Aguirre, J., Foote, M., Roth McDuffie, A. (2020). Types of Mathematical Modeling Tasks. https://drive.google.com/file/d/1b2mqKdtwiiSkoSJ1Ybffa2F7plRuT2_m/view

Usiskin, Z. (2015). Mathematical modelling and pure mathematics, *Mathematics Teaching in the Middle School*, *20*(8). 476-482.

Appendix A: Model-Eliciting Activity References used in Problem Solving for Teachers

Within each of the four MEA categories described by Turner et. al. (2020), one MEA was taken from a journal article; and a second MEA was taken from a web-based library of MEAs.

Table 6: References for MEAs used in CURE course

MEA Category	**Reference**	**Authors**	**MEA Key Question**
Descriptive	Leonard, A., & Bannister, N. (2018). Dancing Our Way to Geometric Transformations, *Mathematics Teaching in the Middle School*, *23*(5), 258-267. https://doi.org/10.5951/mathteacmiddscho.23.5.0258		How can we create a group dance and describe it using geometric transformations?
	The Robertson Program: Inquiry-Based Teaching in Mathematics and Science: https://wordpress.oise.utoronto.ca/robertson/family-math-nights/	Kabatay, T., Jones, S., Jones, J., MacKinnon, S., Seo, J., Caswell, B.	"Tangram Clan Animals": How can I describe Native American clan animals made from tangram shapes?
Predictive	Schloemer. C. (2015). My Car Is Worth What? Statistical Modeling for Predicting Value. *The Mathematics Teacher*, *108*(9), 700–703. https://doi.org/10.5951/mathteacher.108.9.0700		How can I determine the value of my used car?
	"Pedagogy in Action": https://serc.carleton.edu/sp/library/mea/what.html	The Science Education Resource Center at Carleton College	"Identifying a Suspect": How can I predict a person's height from their footprint?
Optimizing	Usiskin, Z. (2015). Mathematical Modeling and Pure Mathematics. *Mathematics Teaching in the Middle School*, *20*(8), 476–482. https://doi.org/10.5951/mathteacmiddscho.20.8.0476		When should I leave from my house for the airport?
	"Guidelines for Assessment and Instruction in Mathematical Modeling Education": PDF book available at http://www.siam.org/Portals/0/Publications/Reports/gaimme-full_color_for_online_viewing.pdf?ver=2018-03-19-115454-057	Consortium for Mathematics and Its Applications (COMAP) and Society for Industrial and Applied Mathematics (SIAM)	"Hot Dog Cart": Where should I place a hot dog cart on campus? (as described on pages 62-63)
Rating & Ranking	Colen, Y., Navaratna, C., Colen, J., & Kim, J. (2012). Power Indices and U.S. Presidential Elections, *The Mathematics Teacher*, *106*(3), 184-190. https://doi.org/10.5951/mathteacher.106.3.0184		How can we rate the power that each state has towards determining the outcome of the presidential election?
	"Model Eliciting Activities": https://modelelicitingactivities.weebly.com/	University of South Florida - Stavros Center	"Cinderella's Shoe": What is the best shoe to purchase?

Appendix B. Model-Eliciting Activity Presentation & Action Research Project Description

You will design, implement, and evaluate a model-eliciting activity (MEA) with an originally designed key question. This MEA should have solutions that are possible using mathematics from grades 4-12. As part of this presentation, you will collect your peers' work on the MEA, and analyze this work. This presentation may be done individually or in a group setting (ideally, each pre-service teacher will be paired with an in-service teacher for this presentation).

Product 1: Activity Presentation (60 points)

Each presentation will span 75 minutes, including classmates' time to work on your activity. Allocation of time and even distribution of workload among group members will be considered in the grading of the presentation. <u>You should prepare a Powerpoint / Prezi / GoogleSlides or other type of formal presentation as a visual aid.</u>

Each presentation, loosely following a "5E" lesson plan model, should include all of the following:

- A statement of original problem & background information about how this relates to real-life situations (this should include a video that can be used for Engagement, either self-produced or found online)
- Time for your classmates to solve the problems.
- A presentation of your solutions of the problems.
- A summary of potential misconceptions that could occur while solving the problem
- A summary of how this problem could be adapted to students with special needs
- A summary of potential extensions: authentic alternative solutions and / or the need for future work
- A summary of Common Core State Standards – Content Standards from the middle grades (4-9) that could be addressed during the solving of the problem
- An explanation of how MEA design principles are being used [taken from Jung's (2015) article] within your activity.
- Towards the end of your presentation, include and explain a diagram similar to Figure 1.6, page 12 of Lesh & Doerr's (2003) Beyond Constructivism book chapter showing how a variety of representational media are used in your problem.

Product 2: Participation handout to be distributed to classmates

Handout Part 1: Student handout to be assigned to classmates as participation work: An open-ended, cognitively challenging problem or set of problems (that could have multiple correct solutions depending on choices of constraints) that extends the original modeling problem, provided to students to complete individually. The problem could involve internet research or other research beyond the scope of what is being presented in class. You will be collecting your classmates' work on these problems and analyzing the work for Product 4.

Handout Part 2: Answer Key: The presenting group's sample complete solutions of the problems (at least two mathematically different approaches or two versions of constraints), along with an analysis of strengths & weaknesses of the model you developed.

Product 3: Reflection on Presentation (15 points)

Following the presentation, each presenter will write an individual reflection covering the following topics:

- Include your reflections on the parts of the lesson that went well and how to improve the parts with which you are not satisfied.
- Describe the part of your lesson that you feel was most successful and the part that needs the most improvement. You may use this Lesson Analysis Tool to assist you in your analysis: **http://www.mathconnect.hs.iastate.edu/documents/CRMTLessonAnalysisTool.pdf**
    - What would you change for future implementations of this activity?
    - What would you change if you were to implement this with middle school students?
- What did you learn in the process of preparing and giving the presentation?
- [for group presentations only] – Please rate the distribution of workload among all group members. You should provide qualitative descriptions of what each group member contributed, as well as give a percentage estimate of the workload completed by each group member so that the total workload completed adds up to 100%.

Graduate students: Additional assignment for reflection: Include an explanation of how your MEA fits the criteria listed in the NCTM Modeling Book (Hirsch & Roth McDuffie, 2016): Chapter 16, about what Modeling Tasks look like; and Chapter 8, about features of Modeling tasks that support students engaging with the real world in authentic ways.

Product 4: Classmates' work analysis (15 points)
As a group, examine your classmates' solutions to the problem set that you wrote.

1. Develop a rubric to score your classmates' solutions.
2. For any incorrect solutions, determine what the error(s) were and classify the solutions based on error type.
3. Classify all of the correct & incorrect solutions by type of solution method used.
4. Prepare 1-3 slides (add to your existing presentation slides) clearly explaining the different types of correct solutions and incorrect solutions.

Rubric for Product 1: Model-Eliciting Activity Presentation (60 points)

Your activity should include all of the required materials and directions. The directions should be grade-appropriate, meaning that the language used when writing the directions should be able to be read and interpreted by middle school-aged children. You should also include variations to meet the needs of a diverse classroom.

Table 7: Grading rubric for presenters

	Outstanding	**Good**	**Poor**
A: Engagement & Instructions	5 pts. The activity is complete and with directions for children explaining how to participate in the activity. The Engagement shows how the MEA is connected to real-life.	1-4 pts. The activity is somewhat incomplete, and / or the directions could be improved so that students can understand them better. The Engagement is somewhat related to real-life but can be improved.	0 pts. The activity is not complete and the directions cannot be easily understood by children. The activity appears disorganized.
B: Modeling Problem & Solution Evaluation	30 pts. See pg. 452 of the Anhalt and Cortez (2015) *Mathematics Teacher* article.	1-29 pts. See pg. 452 of the Anhalt and Cortez (2015) *Mathematics Teacher* article	0 pts. Scores 0 in every category of rubric on pg. 452 of the Anhalt and Cortez (2015) article
C: Potential Misconceptions	5 pts. Key mathematical misconceptions are highlighted and discussed.	1-4 pts. The activity requires classmates to find solutions to problems and requires students to develop some understanding of the mathematical concepts. Conceptual and procedural tasks are somewhat equal in number.	0 pts. The activity focuses primarily on procedural understanding.
D: Special Needs Recommendations	5 pts. The activity can be adjusted to meet the needs of a diverse classroom. Suggested adaptations add usefulness to the activity.	1-4 pts. The activity can be adjusted to meet the needs of a diverse classroom, but the ways in which it can be differentiated is not entirely clear from the presentation.	0 pts. There are no variations or adaptations explicitly mentioned in the presentation.
E: Extensions	5 pts. Suggested mathematical extensions to the activity add to the complexity of the activity.	1-4 pts. The activity can be further extended, but it is not entirely clear from the presentation how to do so, or the extensions do not add to the complexity of the activity.	0 pts. There are no extensions mentioned in the presentation.
F: Connections to Common Core State Standards	5 pts. CCSS standards selected are representative of the selected activity.	1-4 pts. CCSS standards are only loosely representative of the activity.	0 pts. The activity was not aligned with CCSS at the middle or secondary level.
G: Modeling Design Process	5 pts. MEA adheres well to principles of design as described in Jung (2015). A variety of Representational Media (Lesh & Doerr, 2013) are clearly present in the activity.	1-4 pts. Model design loosely adheres to principles of modeling design.	0 pts. Model design does not adhere to principles of modeling design.
Graduate students: Written report	1-10 pts. Provides detailed explanations of how the MEA satisfies criteria listed in NCTM Modeling Book	1-10 pts. Insufficient details provided.	No written report was submitted.

For Graduate Students, Criteria C, D, E, F will each be evaluated at a maximum of 2.5 points.

Appendix C: Presenters' reflections, Assignment Product 3

Undergraduate student's reflection

Overall I was pleased with our lesson. First, my classmates researched a country of their choice and had to find facts about gifts. This got them engaged and curious about how this was going to connect with the lesson and how they would use this information. They then watched a video about how to wrap gifts before we introduced the key question. This was to subtly foreshadow to the fact that they would need more than just the surface area to wrap the gift. Then after introducing the key question, my classmates picked a gift that they would like to wrap for their friend. This was another way to make the lesson interesting and personal. Some classmates picked gifts they wanted themselves while some picked something they already had. It was very interesting to see what each person picked!

They then had to determine how they would find the amount of wrapping paper to cover the gift they chose using the given shipping dimensions. The goal of this part of the lesson was to have my classmates figure out that they had to use surface area formula (2lw+2wh+2hl).

This is where deciding to use GoogleSlides came to be very handy. We as the teachers could see what our classmates were writing as they were working. My teaching partner G1 saw that one classmate was using the incorrect formula. She then brought it to the class' attention and asked what the formula this classmate was using (without calling her out by name) was used to find. They all participated and said that it was the formula for volume and we had a volunteer or two who justified why it did not work for this problem. This lead into the hands-on portion where my classmates would test their prediction and create a rule to model the scenario best. At this point is where I would make an improvement to my lesson. My classmates definitely did not have enough time to complete this portion to the best of their ability. I think to fix that I would give my students next time less time to complete the engagement whether that be to give them a country already (eliminating the conversation in which they must make a decision), or telling them to find 1-2 facts opposed to 3. I would've also liked to have time for my classmates to complete their reflections so that they wouldn't have to complete anything outside of class. This lesson does have the flexibility to be turned into a 2 day lesson as well.

Regarding middle schoolers specifically, I have multiple ideas of how I could change or extend this lesson. Assuming this lesson would be turned into a 2 day lesson, I would have students have more constraints then the ones we used for this presentation. An example of this would be to give students "wrapping paper" with set constraints already and to have them find another object that could be wrapped with the same paper (suggested by one classmate). Students could also be given a price as we did in this one but that price would have to be for wrapping paper, tape, scissors, etc. This would allow students to use number sense and budgeting along with surface area. A final idea brought up a few times between me and G1 was to also include conversions. For example, students would be given dimensions in inches but their answer must be in centimeters. For storyline purposes, the reason for that could be because at your home you only have a ruler that reads centimeters.

This process taught me a lot!! I had actually never planned any type of math lesson plan before this one so I never knew just how much work would have to go into it. We met for hours at a time at the beginning of this process just working on the content area, key question, and overall outline of our ideas. Then once we split the work we had to be sure that our ideas aligned which I think we worked well at. I think the most difficult process of this for me because I don't have a lot of classroom experience would have to be predicting what the students' misconceptions may be. Predicting someone else's thoughts and words is very hard!! I am excited to work on getting better at that.

In terms of group distribution, I think [G1] and I did a great job on completing the same amount of work. My rating of distribution: Me (43%) - I made the PowerPoint with the idea that [G1] would work on the student handout which she did. However, we later decided that we didn't need the handout so G1 added her handout information to the slides. I made all of the student slides. I made the PowerPoint look appealing. I created my answer key. I added to the constraints slide.

[Classmate G1] (57%) – [G1] created all of the google meet links that we used to meet. [G1] knew more about how to add links (example on answer key slide) to the slide deck so she did that. She made sure that everything we needed from the syllabus was included and answered. She sent most of the emails with our questions.

We both split our slides that we would present evenly however because she is more experienced and comfortable, she asked a lot of the extension questions while I did not which I would improve for next time. [G1] was a great partner to work with and I would love to work with her again! Her teamwork was great and she taught me so many new things through sharing her experiences.

Graduate student's reflection

- *Include your reflections on the parts of the lesson that went well and how to improve the parts with which you are not satisfied.*

I thought the activity was engaging and relevant to middle school students (aligned to 6th/7th grade content standards). The hook activity exploring other cultures and their ways of gift giving created an interest in the activity and class conversation. Students were able to select their own gift and method for answering the key question, allowing for student choice and an open-ended modeling experience. I would have liked the activity to be more collaborative. Also, we did not have as much time as I would have liked to dive deeper into some of the models students created; sharing out everyone's models and solutions as we moved through the modeling process would have been beneficial for other students, as well as, help the teacher formatively assess student progress throughout the lesson. Students seemed slightly thrown off by Task #3, when they had to then apply their model to a different gift than the online Target gift. Whether in-person or in the virtual setting, there may be a better way to transition to this part of the activity but it would have been beneficial to be in-person for this particular part, to see students in action. Overall, I was satisfied with the MEA and felt that it embraced the modeling process, was flexible and aligned to several content standards and provided students an opportunity to explore mathematical content using multiple strategies.

- *Describe the part of your lesson that you feel was most successful and the part that needs the most improvement.*

The most successful part of the activity was the opportunity for student choice and open-ended structure that allowed students to show their mathematical thinking during the modeling process. Each student illustrated different abilities and ways of thinking about a problem (whether correct or incorrect) and provided opportunity for a class discussion. If I were to have completed this MEA in my actual classroom, I would have made sure to address the student thinking throughout the modeling process and checked in with each student/group, using questioning to assist students develop their thinking and possible misconceptions. The activity would likely be spread over a 2-3 days to give time to this part of the activity.

As much as I loved the idea of testing the model, and think it is absolutely necessary in order for students to experience a fully developed MEA, it is difficult to monitor or assist students with testing their models and wrapping a gift in the virtual setting. It would be more engaging and meaningful if students were able to experience this part of the activity in-person and/or collaboratively.

-*What would you change for future implementations of this activity? What would you change if you were to implement this with middle school students?*

I would extend the MEA into a 2-3 day lesson for students to allow time to work through the modeling process. Also, I would provide more scaffolding to students, as needed, throughout the lesson. Some classmates were confused about what formula to use or misused what they thought would be the appropriate formula. If the MEA was implemented in my actual classroom, it would likely follow a lesson on surface area and be used to assess new student knowledge; so there may be less confusion about formula use, in that case. But a formula sheet could be given in case students wanted to reference them. (Although, it was enlightening to see how classmates interpreted the problem and I would be curious to observe their model testing process if they used a formula incorrectly.) If we were in-person, I would likely provide boxes/gifts for students to choose from, or have students bring in their own gifts and I could provide boxes for them. This would help with the transition from the model development part of the activity (Task #1 & 2) to the model test and revisions part (Task #3), making the activity feel more connected to exploring just one gift/rectangular prism rather than two unrelated items. Also, I would likely have students work in groups of 2-4, as collaborative group work is a valuable way to learn and solve problems.

- *What did you learn in the process of preparing and giving the presentation?*

Being in a virtual setting, we had to get creative about how students would be able to participate in an accessible hands-on activity. Given that we were not able to provide materials in-person, we were able to guide students to find common items at home to manipulate during the modeling process, as well as, provide online resources and print-out options. But in the virtual setting, timing and assessing student learning can be very difficult.

– Please rate the distribution of workload among all group members. You should provide qualitative descriptions of what each group member contributed, as well as give a percentage estimate of the workload completed by each group member so that the total workload completed adds up to 100%.

[U1] and I worked well collaboratively; we discussed all items together, both editing and contributing to most items. Some items were split up and assigned; [U1] completed the MEA Design Principles and I completed the Representational Media portion. 50/50 workload.

Graduate Course - Additional assignment for reflection:
Include an explanation of how your MEA fits the criteria listed in the NCTM Modeling Book:
a. Chapter 16, about what Modeling Tasks look like
b. Chapter 8, about features of Modeling tasks that support students engaging with the real world in authentic ways

The NCTM Modeling book (Hirsch & Roth McDuffie, 2016) outlines what a modeling task looks like and the different processes of the modeling cycle. In the Gift Wrapping MEA, students are able to make choices, assumptions and estimate throughout the modeling process. Students identify essential variables by choosing a gift and determining what information is needed to formulate a mathematical model to answer the key question: how much wrapping paper is needed to wrap a gift? Then, students have to test and analyze their model by wrapping an actual gift, seeing if their model needs revision or improvement. Then, students validate their conclusions and revisit the original problem, applying their revised rule to their original gift choice. And students report their results by documenting all steps of their modeling cycle with their classmates, providing reasoning and reflection. Gift wrapping is a real world activity that can be translated into a mathematical model; students may have knowledge and experience with wrapping gifts but are able to explore the mathematical concepts involved with wrapping a gift within this MEA.

The Gift Wrapping MEA supports students engaging with the real world. The context of the MEA begins with students missing their friends due to COVID-19. Then, the activity engages students with searching online for a gift, appealing to their experiences, relationships and interests. Students will make connections between a real-world situation (buying and wrapping a gift) and the mathematical implications of purchasing and wrapping a gift. Also, students will reflect on how realistic their model can be applied in the real world. The MEA leverages students’ real world knowledge by allowing for multiple approaches and strategies to solve the problem.