The Mathematics Enthusiast

Volume 20
Number 1 *Number 1, 2 & 3*

Article 3

4-2023

Encounters With “Love and Math” A Belated Review of Edward Frenkel’s Love and Math: The Heart of Hidden Reality

Rina Zazkis

Encounters With "Love and Math"
A Belated Review of Edward Frenkel's *Love and Math: The Heart of Hidden Reality*

Rina Zazkis[1]
Simon Fraser University

In "Love and Math" the author intertwines his personal experiences as a student of mathematics and as a research mathematician with an exposition of modern mathematics, focusing on its elegance and beauty. Much is said in previous reviews (e.g., Grosholz, 2015) of this *New York Times* bestseller and winner of the Euler Book Prize from the Mathematical Association of America. All praise the author for his fascinating narratives in which he reveals the joy of intellectual discovery and provides a passionate account of exciting ideas of modern mathematics. There is no need in repetition, so this review is rather personal. I start with my personal encounters with "Love and Math" and then share what I learned from the book as a teacher and mathematics educator, focusing on issues that are often overlooked in acclaimed accounts of Frenkel's mathematical expositions.

First Encounter

Back in 2013 or 2014, a few pages of this book were circulated amongst my acquaintances over internet. As it often happens, I first got the text from a friend in Israel in its Hebrew translation and recovered the English original. Months later I got the same text in Russian from my father.

The text is republished in several outlets[2]. It is an account of an oral entrance exam at the MGU (Moscow State University) in which a Jewish applicant was deliberately failed by his examiners, despite having solved all the problems and addressed perfectly well all the

[1] zazkis@sfu.ca
[2] For example: http://www.ruthfullyyours.com/2012/10/04/the-fifth-problem-math-anti-semitism-in-the-soviet-union-edward-frenkel/

The Mathematics Enthusiast, **ISSN 1551-3440**, vol. 20, nos. 1, 2 & 3, pp. 13-20.

questions. I found out that this text was an excerpt from "Love and Math" by Edward Frenkel and I bought the book.

The described incident took place in 1985, but it resonated with similar experiences of Jews of the (former) Soviet Union. However, at the time when Edward Frenkel was declined admission to MGU (and later admitted into a less prestigious institution), I have already obtained a Master's degree in Mathematics and started a PhD program in Mathematics Education at the Technion - Israel Institute of Technology. The choice of my parents to immigrate to Israel from (what some believe to be) the most antisemitic city at the time helped me avoid the experience described by Frenkel. Nevertheless, an earlier experience with discernible similarity may have shaped the destiny of my family and my career.

In the early 70's in Kiev[3], I joined several hundred youth who participated in a city-wide Mathematics Olympiad for grade-6 students. In my school there was no special mathematics program, but every school had to send several students for this competition, and I was among the three "chosen" by my teacher. I recall my first time at a large University auditorium, with steep stairs and writing panels adjusted to each chair… We were given 5 problems and 90 minutes. I still recall one of the problems and my multiple attempts to exemplify its solution. The names could have been different, but here is the gist of it:

> *Peter has two friends: Alex, who lives in A and Bob who lives in B, where A and B are in opposite directions from where Peter lives. There is a train to A every hour and a train to B every hour. Every Sunday morning Peter arrived at some random time at the train station and boarded the first train that passed by to visit one of his friends. At the end of the year, Alex thought that Peter visited Bob twice as many times as he visited him. Peter claimed this would not be possible, given that he arrived at the station at some random time… Do you agree with Peter? Explain.*

I won the third prize in this competition. There were several children sharing each of the "medal-deserving" places, likely determined by the achieved score. This was a huge

[3] Currently in English language media the city is referred to as Kyiv, which is transliteration from Ukrainian spelling, while Kiev is a transliteration from Russian.

honour and also a huge surprise, as some competitors participated in preparation courses for this Olympiad, and I just learned about it a day or so before it took place. I was praised for this achievement, acknowledged in the school newsletter, and congratulated in front of the whole school assembly by the principal. This could have been a happy ending, but… A few weeks later my parents learned from another teacher in my school, who was a neighbour and friend, that a team was formed from high performers at this Olympiad to train for the international mathematics Olympiad. I was not chosen for the team, and would likely not have been made aware of its exitance if it were not for our family friend. The official answer, upon my parents' inquiry, was that there was an intention to promote a "national cadre", and Jewish youth were not considered as such. This was likely the last straw that shaped my parents' decision to immigrate to Israel, a decision that materialised several months later.

On Mathematics

It is always a pleasure to learn some mathematics, especially when it is presented in an accessible manner. From Frenkel's book I learned about braid groups and their relation to symmetry groups and permutations, as the notion of a braid group was new to me. With a smile, I revisited the parallel between a field extension by $\sqrt{2}$ and complex numbers, considering the latter as a field extension by $\sqrt{-1}$.

I learned about the Langlands program, which is a project in modern mathematics that seeks and outlines connections between different subfields of mathematics—a program, that according to Frenkel aims to describe "The Grand Unified Theory". While I did not follow all the details (encouraged by the author that it was perfectly fine to skip some parts), I could appreciate the idea of mysterious and unexpected connections between different branches of mathematics and the masterfully accomplished author's plan to give readers "a sense of modern mathematics, to prove that it is really about originality, imagination and ground-breaking insights" (p. 70).

I appreciated how personal story intertwines with mathematical expositions. In fact, I tried to do something alike in my 2011 book "*Relearning mathematics: A challenge for prospective elementary school teachers*", that is, to intertwine personal mathematical experiences with mathematics education research. Having read Frenkel's book, I believe I am now better equipped to do this.

On Love

Frenkel's love of mathematics invited me to revisit the question – Do I love mathematics? My answer is negative. "Love" for me involves close relations or intimacy. So "love" is how I can relate to my husband, my children, my parents, my friends. My relationship with mathematics is that of fascination, awe, wonder, not love. However, there are quite a few mathematical topics, problems and puzzles that I love.

On Teachers and Outliers

> It is hard work being a teacher! I guess in many ways it's like having children. You have to sacrifice a lot, not asking for anything in return. Of course, the rewards can also be tremendous. (p. 129)

That is how Frenkel summarised his perspective on being a teacher. While many reviewers consider Frenkel's book as an ode for modern mathematics, I can also see it as an ode for his teachers. Starting with his parents and Evgeny Evgenievich, a teacher who "converted" him to mathematics during his teens, Frenkel writes with great respect and appreciation of all the teachers and colleagues who contributed to his career, highlighting collaborations and collaborators. He describes carefully how a conversation with a professor during an occasional visit to his dacha shaped the next project; or how discussions with his supervisor, and then with other colleagues, inspired his work on the Langlands program; or how meeting an old mathematician friend at a conference led to exploration of a new direction in connecting modern mathematics to physics. (Social constructivists would be happy as Frenkel's experience is the existence proof for their case.)

Malcolm Gladwell published “Outliers: the story of success” in 2008. With a masterful choice of examples, he carefully convinced readers that an extraordinary gift in any field is not enough for a great achievement, and that every great achievement – be this of the Beatles, Steve Jobs, concert musicians or NHL hockey players – can be described as “a story of fortunate events”, that starts with a person’s day and place of birth, and proceeds with countless areas of devotion to the chosen craft. Had Gladwell waited a few years, Edward Frenkel could have been yet another chapter in his book, presenting his success, notwithstanding the gift, as a story encounter that shaped his diverse talents.

On Teaching

My teaching, I believe, has been enhanced by my exposure to Frenkel’s book. First, I have extended my repertoire of metaphors about mathematics and mathematical problem solving. Frenkel describes mathematics in general, and the Langlands program in particular, as working on separate areas of a huge puzzle, continuously fitting pieces together, but without knowing how the final picture will look like.

> Like archeologists faces with a fractured mosaic, we try to piece together the evidence we were able to collect. Every new piece of the puzzle gives us new insights, new tools to unravel the mystery. And each time we are dazzled by the seemingly inexhaustible richness of the emerging picture. (p. 184)

The primary metaphor that Frenkel uses to introduce the Langlands program is that of the Rosetta Stone. The Rosetta Stone (not elaborated in the book) is an ancient artifact, the actual granite stone, that has inscriptions in three different semiotic codes: Egyptian hieroglyphs, Demotic and Greek. The importance of the Rosetta Stone is that it provided a breakthrough in the quest to decipher the hieroglyphs. As such, any reference to the Rosetta Stone is used to symbolize a breakthrough, or discovery that provides crucial knowledge for the solving of a puzzle or problem. This metaphor is even stronger when a puzzle involves a three-way translation. The three areas of the Rosetta Stone that Frenkel describes are Number theory, Harmonic analysis and Riemann surfaces, that can be seen for simplicity as Numbers, Functions and Shapes, or put another way, Numbers, Algebra and Geometry.

Frenkel compares different areas of mathematics to different continents, and the Langlands Program as the one that provides an ultimate teleportation device, capable of getting us instantly from one continent to another. This is a huge improvement compared to previous ways to travel between the continents, by boats or by planes. This teleportation device also serves as a translator between different languages and different cultures, to help travelers from one continent to get accustomed to another. Then an amazing new connection appears – that to Quantum Physics. Frenkel describes this as the appearance of a fourth column on the Rosetta stone. So Mathematics and Physics are looked at as different planets, with different continents on each.

Applying this metaphor, on a different scale, to my teaching, I now think of, for example, connecting Fibonacci numbers to Pascal's triangle and to the Golden Ratio as connecting different villages, where mathematical induction is my transportation device.

I do not teach advanced mathematics, so many of the beautiful descriptions related to group theory and string theory I can only enjoy as an observer. But, for my work with prospective teachers, I have extended the applicable set of tasks. For example, the next time I teach a unit on symmetry of geometric objects, I will start by asking for an example of a symmetric figure and then asking for something "more symmetric" and "even more symmetric". And when – eventually – someone will point to a circle as the "most symmetric shape", I could say that I can think of something "even more symmetric", without initially disclosing that the focus on 2-dimentional shapes is a self-imposed constraint. This progression can lead to a categorization of different symmetries.

In my experience, students find it much easier to recognise reflectional symmetry than rotational symmetry. This observation can lead to a research question: What are students' initial ideas about symmetry? How do they identify and justify the notion of being symmetric?

On Jewish Problems

While Edward Frenkel successfully solved a mathematical problem during his oral exam, even though the solution used tools far beyond what was required by the curriculum, many have not been successful. Only marginally relevant to the book, there are compilations of mathematics problems found with a few clicks on Google called "Jewish problems"[4]. Despite the immediate association that one may have with the phrase, these files are an assembly of problems that were used at universities entrance exams to fail Jewish applicants. These problems are also referred to as "coffins" or "killer problems". If one does not solve a problem presented at the (oral) exam, then their admission is automatically denied with no recourse for their application to be reconsidered.

Putting the evil history of these collections aside – if it only were possible – I am fascinated with the ingenuity in the design of these problems. The problems appear not over complicated, but each solution involves some "trick" (for example, a particular choice of representation or a particular substitution) which is very different from the conventional approaches that come to mind when one solves problems in the same domain. Using the trick, the solution appears very short and straightforward, can be even argued to be "simple". But without the trick, a solution is beyond reach.

On Publications

Reflecting on academic publishing, Frenkel noted that "Writing papers was the punishment we had to endure for the thrill of discovering new mathematics" (p. 61). Sometimes I feel that this statement is applicable to other fields, that is, the necessity to write publishable papers is the punishment for the thrill of doing research. However, sometimes there are pieces that I enjoy writing, like this one.

References

Frenkel, E. (2014). *Love and Math: The heart of hidden reality*. Basic Books.

Gladwell, M. (2008). *Outliers: The story of success*. Penguin books, LTD.

---

[4] https://arxiv.org/pdf/1110.1556.pdf

Grosholz, E. R. (2015). Book Review: Love and Math: The Heart of Hidden Reality by Edward Frenkel. *Journal of Humanistic Mathematics, 5*(1), 286-294.

Zazkis, R. (2011). *Relearning mathematics: A challenge for prospective elementary school teachers*. Information Age Publishing: Charlotte, NC.