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Uncertainty in an Uncertain World
A Review of Ian Stewart's *Do Dice Play God?: The Mathematics of Uncertainty*

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General Overview of Stewart's Book

In addition to the title's cheeky play on the famous words *God does not play dice with the universe* (famously attributed to Einstein), Ian Stewart's *Do Dice Play God* is a fascinating tour de force that embeds the discipline of statistics and probability within an informative and insightful socio-historical account of its development. Stewart characterizes the science of statistics and probability as the mathematical quantification of uncertainty—a very useful and productive way to think about a discipline with a reputation for being notoriously difficult and often counter-intuitive for people to understand. But his account goes beyond merely providing such an apt characterization of the discipline; he follows through by providing penetrating and clearly presented examples and analyses of the mathematics at play throughout the book's 18 chapters. One cannot help but think that Stewart—a renowned mathematician and retired mathematics professor—is a gifted teacher! All of this is presented in an informal conversational style, infused with occasional humor, and usually in sufficient mathematical detail that invites and supports the reader to participate in the development of ideas under consideration.

In the early chapters, Stewart's book may appear largely as a historical account of the emergence and relevance of statistics and probability as a scientific discipline. However, his account is set against a broader context whose running thread is the idea of uncertainty. He discusses various shades of uncertainty and developments in ways of conceiving it and coming to terms with it within Western civilization, concomitantly with the development of mathematical models and methods that are not restricted to just statistics and probability. By the end of the book, one comes away with a clearer sense that this is as much a book about uncertainty and a variety of mathematics developed to

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advance prediction and forecasting in light of it, in which statistics and probability play a central but not exclusive role.

Although set against a broad socio-historical backdrop, Stewart's focus on quantification centralizes mathematical reasoning. The examples and mathematical explanations presented are cast largely in terms of underlying reasoning—ways of thinking and understandings that, were one to possess or engage, could be adequately expressed by such explanations. In this sense, Stewart's book has a certain kinship with von Glasersfeld's method of conceptual analysis of mathematical concepts (von Glasersfeld, 1995) which has been influential within some circles in mathematics education research (Thompson, 2008).

Stewart's organizing conceptual framework for his account is what he refers to as the *Six Ages of Uncertainty*. This is essentially a socio-historical account of the development of the quantification of uncertainty that sees its progression from early eras in which anticipation and prediction of events under conditions of uncertainty were largely rooted in beliefs, superstition and divination, to later eras marked by increasingly systematic use of observational data, the development of mathematical modeling and rational analyses that undergirded the quantification of uncertainty. Through this progression the reader comes to understand how uncertainty evolved from being rooted in an essentially deterministic worldview to a view of uncertainty as an inherent feature of the world. In earlier eras uncertainty and unpredictability were often seen as temporary states reflecting human ignorance about processes which were nevertheless believed to be deterministic but as yet unknown. This view evolved over centuries to one where ideas of determinism and uncertainty could co-exist, as reflected by the mathematics of nonlinear dynamical systems which are simultaneously deterministic and chaotic. At the most modern end of this progression, uncertainty and randomness are increasingly viewed—even embraced—as inherent features of our world, as reflected in recent interpretations of quantum mechanics. One effect of this account is to paint a portrait of the discipline of statistics and probability, as well as more recent mathematical inventions such as nonlinear dynamics and cryptography, as being thoroughly grounded in human activity borne of a drive for control and prediction in situations where conditions of uncertainty prevail. This overarching portrait provides the reader with a broad context in which to embed the discipline of statistics and probability in particular, to understand its relatively humble beginnings and its development into one of the arguably most effective and consequential scientific fields with far-reaching applicability. This feature of Stewart's book, together with its accessible writing style, makes it a potentially relevant resource for courses in the history of mathematics or for courses addressing the social relevance of science and mathematics.

Specific Notable Features of Stewart's Book

In addition to the overarching features and insights outlined above, I am especially impressed by two specific aspects of the book that speak to its relevance for mathematics education on some level. The first aspect is Stewart's use of the concept of *toy models*. The second is his overview of the disciplines of weather forecasting and climatology, in which he highlights misconceptions about the concepts of weather and climate, and in which he draws on mathematics to refute arguments aimed at discrediting or throwing skepticism on the scientific evidence for human-induced climate change.

Toy Models

Stewart introduces the concept of *Toy Models*: these are simplified mathematical models in manageable contexts, involving the tossing of coins and the rolling of dice, that are useful for illustrating and developing his key ideas around quantifying uncertainty. He introduces and makes very effective use of these toy models in chapters three and four as a basis for penetrating analyses and explanations of conditional probability provided in chapter five, and for grounding his discussion of common fallacies and paradoxes within them. One simple example of this is in the way Stewart explains the "law of averages" fallacy. The idea is that at some point in a large number of flips of a fair coin (keeping a running count of the number of each outcome), if one observes an excess in the number of heads over tails, then one believes that this excess makes getting tails *more likely* thereafter. This goes counter to the idea that coin flips are independent and that their outcomes are equally likely. Stewart presents this belief as a misinterpretation of the law of large numbers, wherein the underlying intuition is that such an excess should diminish or disappear as the number of flips continues to increase thereafter. This balancing out implies that the ratio of the number of heads to tails gets closer to 1, not that the probability of any one of the two outcomes exceeds ½ in any flip.

Much of the mathematical concepts used by Stewart in the early chapters requires understanding little more than mathematics taught in upper high school: I found myself wanting to borrow from his examples and explanations in my own teaching of prospective mathematics teachers. The toy models presented by Stewart are familiar ones often used in introductory probability and statistics courses. But such courses often advance rapidly to more complex contexts and problems in which it is not necessarily clear how simple toy models can be helpful. Indeed, "real life" problems involving probabilistic prediction are generally not restricted to contexts involving tossing coins or rolling dice, and it is not unusual for students in introductory courses to ask, "how does flipping a coin help me?" when confronted with such problems. It is thus refreshing to see that such toy models can have an enduring relevance for understanding certain probabilistic ideas that arise in different and more complicated contexts, and it is insightful to see how such models could potentially be used pedagogically.

Stewart uses toy models and explanations effectively in several of the book's later chapters as well, returning to such models periodically to help explain more complex scientific and mathematical concepts such as entropy, chaos, and their relevance for concepts in a variety of fields including finance and investment, neuroscience, and quantum mechanics.

These later chapters take the reader on a tour through the application and role of mathematics in quantifying uncertainty and making predictions in complex contexts—astronomy, social sciences, law, weather and climatology, financial markets, physics, medical research, neuroscience and cryptography. In doing so, Stewart illustrates the usefulness, challenges and limitations of mathematics in quantifying uncertainty. This tour culminates with rather deep philosophically oriented discussions about reality and truth in chapters 15 and 16 which deal with quantum mechanics. Although Stewart occasionally draws insightfully on his toy model of flipping coins to help explain some of the complicated ideas within the realm of quantum mechanics, the mathematics he outlines in those later chapters are generally quite advanced and the concepts discussed are often paradoxical and "weird", to paraphrase the author. Given the complexity and unwieldiness of that subject matter, Stewart's explanatory use of toy models in those chapters is all the more impressive.

Weather Forecasting and Climate Change

Chapter 11—entitled *The Weather Factory*—is in my view both a very timely and insightful account of the history of numerical weather forecasting that, importantly, teases apart the concepts of weather and climate. I should first note that this chapter is immediately preceded by a pre-requisite chapter that provides a general overview of the mathematical field of nonlinear dynamical systems and chaos. While the mathematics described in this pre-requisite chapter requires familiarity with some advanced mathematics, Stewart's account is sufficiently non-technical and the metaphors he presents are clear enough to support understanding the key ideas of chapter 11. One of these key ideas is that it is productive to think of weather and climate, and the relationship between them, as constituting a complex system that can be modeled, at least metaphorically, as a chaotic nonlinear dynamical system. This means that the system is sensitive to even small perturbations in the behavior of its components, which can reverberate and be amplified throughout the system so as to make the entire system susceptible to unpredictable behavior. In this metaphor, *weather* is constituted by combinations of interacting atmospheric and environmental factors occurring on local and geographic time scales, and weather patterns are modeled as trajectories (solutions that evolve over time) of nonlinear differential equations. These trajectories and the patterns they form collectively constitute what is called an *attractor* of the dynamical system—this is the global behavior of the totality of weather patterns that gives rise to what we call *climate*. As Stewart explains "in an unchanging climate, there can be many paths through the same attractor, but in the long term they all have similar statistics" (p. 157). Whereas weather is constantly changing, reflecting the idea that the dynamics give rise to a variety of paths on a same attractor, climate—the *attractor*—should not change unless something extraordinary is occurring.

Stewart goes on to articulate and refute several common misconceptions about climate change by using mathematics. I'll briefly summarize one misconception here that confounds climate and weather: the belief—espoused repeatedly by climate change denier, former U.S. President Donald Trump—that "the climate has always been changing" (Kilander & Boyle, 2022). In light of the distinction between climate and weather outlined above, Stewart explains that climatologists define climate as a "30-year moving average of weather", and that averages are subtle because they are resistant to short-term fluctuations in the quantity they average. For example, consider your local climate as a 30-year moving average of temperature in your local area today: this is calculated as the ratio of the sum of daily temperatures for each day of the past 30 years to that number of days. The resulting value is very stable and would change only if temperatures differ from it over a very long period of time, and then only if they tend to increase or decrease in the same direction (hotter on average, or colder on average). Temperatures typically fluctuate around this average temperature, which lies somewhere in between such fluctuations. This is the reason why climate cannot be "always changing". Even if the weather were to change dramatically and permanently today, it would take years for this to have a significant effect on the 30-year average—the climate! This example, created to drive the point home, is just about the local climate, but climate change is really a claim about global temporal and geographic scales: it requires the average temperature to be taken over very long periods of time and over the entire planet.

Post-script

My review of this book about uncertainty was completed on the eve of the U.S. presidential election of 2020—arguably one of the most anticipated and consequential events of an already tumultuous start to the third decade of this millennium. It is an understatement to say that uncertainty abounds in the first year of COVID-19, and there is a certain tragic timeliness in reading and commenting on a book about uncertainty at this juncture.

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