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A Response and Review of Hannah Fry's *Hello World: Being Human in the Age of Algorithms*

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At the outset, Dr. Hannah Fry's book *Hello World: Being Human in the Age of Algorithms* is an engaging survey of how algorithms are being used in modern life. The book is written for a general audience with a focus on concepts and principles. As such, the narrative does not dig deeply into the details of any particular topic but instead offers a snapshot of how algorithms reside within a variety of contexts in the modern world. Through relatable examples, Dr. Fry builds a narrative that explains how algorithms are used as tools to predict, influence, augment, and mimic human behavior. Woven throughout the narrative are stories that reveal the power and pitfalls of algorithms. We learn about algorithms used by advertisers, data brokers, judges, chess masters, and healthcare workers. Along the way, Dr. Fry reveals how algorithms are being used to identify cancerous cells, build driverless vehicles, influence an online behavior, and create art. After reading the book, I was left asking practical questions such as, did that recent grocery store advertisement arrive by chance or is it based on my previous shopping behavior? How is my online browsing influencing the advertisements I see on social media? At the same time, the book inspired broader questions such as, how are algorithms being used for sentencing within the local court system? In what ways is care at the nearby hospital influenced by algorithms? In what follows, I have sought to craft a response to the book from my own lens of mathematics education rather than a detailed analysis of the particular merits. For those seeking a recommendation on whether or not to read the book, my response is: Read it. It is inextricably connected to the work that you do.

Shining through the narrative in *Hello World* is the key role that context plays in shaping how algorithms are used and misused. Likewise, context is important for understanding some of the fundamental questions that the work raises for the mathematics education

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community because mathematics educators work within different settings from schools, to businesses, to universities. They hold positions as classroom teachers, mathematics coaches, curriculum coordinators, curriculum developers, educational researchers, or higher education faculty to name a few. Many mathematics educators work across several settings on a daily basis and may hold different positions within each. Understanding how members of the field of mathematics education might view *Hello World* requires acknowledgement that individual context will play a role. A high school mathematics teacher might see very different uses or misuses of an algorithm than someone who works primarily within a School of Education. Thus, although my goal is to speak broadly from a lens of the field, my own context as a mathematics educator at a medium-sized public university shapes and influences the perspective I bring to this discussion. There is admittedly a tension in attempting to think both globally and locally. However, it seems quite akin to the tension that all educators encounter on a daily basis within the classroom where theory meets practice.

Reading Dr. Fry's book raised a range of different questions for me. Some were personal and connected to my own understanding of particular content or ideas. For example, how much do I remember about decision trees? To what extent did my undergraduate coursework focus on Bayesian statistics? Other questions were more professionally focused and oriented, such as, How might my classroom assessment benefit from an algorithm? What algorithms are behind popular programs that my students (and I) use? Still, other questions were broadly oriented toward the field of mathematics education: How are we helping PK-12 students develop the foundational knowledge needed to build the algorithms of the future? How are teacher educators and professional developers preparing mathematics teachers to work in classrooms where algorithms and the technology they control are becoming more embedded? It is this last group of questions that I have chosen to focus on and, as I will discuss, Dr. Fry's survey touches on current tensions within the field and foreshadows issues that we have yet to contend with. In doing so, *Hello World* pushes our community to ask the two foundational questions: What should we teach? How should we teach?

What Should We Teach?

> Forming an opinion on an algorithm means understanding the relationship between human and machine. Each one is inextricably connected to the people who build and use it.
>
> This means that, at its heart, this is a book about humans. It's about who we are, where we're going, what's important to us and how that is changing through technology. It's about our relationship with algorithms that are already here, the ones working alongside us, amplifying our abilities, correcting our mistakes, solving our problems and creating new ones along the way. (p. 3)

Reading *Hello World* is a tour of sorts through topics and concepts from the interdisciplinary field of data science. Among the central disciplines that data scientists draw on in their work are mathematics, statistics (including probability), and computer science. To someone outside of these fields, the distinction between them might blur. After all, each appears to use similar tools (e.g., numbers, variables, symbolic languages) and requires specialized training. To a statistician, computer scientist, or mathematician working within the field, however, there are clear distinctions between them and while it is true that the three share historical roots in mathematics, they have diverged significantly.

Within school settings, recent years have seen growth in both statistics and computer science courses. However, these topics are still far less prevalent than mathematics (Nord et al., 2011). Moreover, the number of individuals trained as teachers of statistics or teachers of computer science that work within school settings is small when compared to the number of individuals trained as teachers of mathematics. Consequently, when a course on statistics or computer science is offered within school settings, particularly at the upper levels, it is often a mathematics teacher who takes on this task due to their close connection (at least in terms of course work) to the other fields. This might suggest that the field of mathematics education would approach the teaching of statistics and the teaching of computer science as natural extensions of our current work. However, the general approach that the field has taken with regard to preparing teachers with knowledge in these two domains is different. Put simply, statistics is viewed as a natural extension of the work of a mathematics teacher, but computer science is not.

Preparing Teachers of Statistics

The need for greater focus, integration, and elevation of statistics within the school curriculum is central within discussions of the field of mathematics education. As noted by Dr. Trena Wilkerson, current President of the National Council of Teachers of Mathematics (NCTM), statistics education within school settings is the responsibility of teachers of mathematics:

> The development of statistical thinking is an imperative today. Every individual must be able to synthesize data to support decision making, make sense of our world, and prepare for the future. Given the magnitude of this challenge, it is essential that PK–12 curricula reflect an intensive, cohesive inclusion of statistics, statistical reasoning, and data analysis. We must be certain that each and every student be provided with learning opportunities to develop these essential processes and practices steeped in appropriate content in statistics. Furthermore, we must ensure that each and every teacher, preservice and in-service, has opportunities to engage in statistics education—whether in teacher education or in professional development programs—to develop a deep understanding of statistics concepts; facilitation in statistical reasoning; and effective, equitable teaching practices in statistics. (Para. 1)

Meeting the goals outlined by Dr. Wilkerson will require significant shifts in current teacher preparation and professional development. Teachers of mathematics are not trained as teachers of statistics. As noted in The Statistical Education of Teachers Report (Franklin et al., 2015), “the statistical content knowledge needed by teachers at all levels is substantial, yet quite different from that typically addressed in most college-level introductory statistics courses” (p. 5). Among other things, Franklin et al. clarify guidance for the preparation of teachers, outline how statistics is distinct from mathematics, and offer pedagogical recommendations. Moreover, the report calls for individuals from across the field of mathematics education to take on the responsibility of statistics education noting that:

> It is the responsibility of mathematicians, statisticians, mathematics educators, statistics educators, professional developers, and administrators to provide teachers with courses and professional development that cultivate their statistical understanding, as well as the pedagogical knowledge to develop statistical literacy in the next generation of learners. (p. 4)

The NCTM Board of Directors has further committed to the task of statistics education by adopting the recently released *Pre-K–12 Guidelines for Assessment and Instruction in Statistics Education II* (NCTM & ASA, 2020) as the official position of the council. Thus, the development of statistical literacy is a goal of the field of mathematics education. Within PK-12 settings, mathematics teachers are the teachers of statistics.

Stepping back into Dr. Fry's discussion, a solid understanding of mathematics and statistics is essential to navigate a world suffused with algorithms. Yet, is our goal only to help students steer the ship in this new ocean of algorithms and avoid obstacles? Or do we also seek to equip students with the tools needed to build new forms of transportation? If the answer to this question is yes, then students also need some fundamental understanding of computer science.

Preparing Teachers of Computer Science

In contrast to the embrace that the field of mathematics education has given to statistics, there have not been recent large-scale efforts to do the same with computer science. However, there is a history of collaboration between the two fields that has shaped the current reality. Work with micro worlds and LOGO programming in the 1970s and 1980s presented a new approach to teaching mathematics through computer science (Papert, 1980). Although there were some beneficial projects that came out of this work (Clements & Meredith, 1992), in the end it did not lead to an integration of computer science and mathematics. Now, over 40 years later, computer science and mathematics largely stand apart within school settings and the need for computer science has only increased. To address this issue, stakeholders wanting to increase the number of students engaged in computer science have sought different paths to address the problem.

Along one path are initiatives that seek to bring more students into computer science through student-friendly online tools such as Code.org (http://www.code.org). These tools teach foundational ideas and are designed to be stand-alone resources that can be used without involvement from a classroom teacher. A student might be introduced to these resources by chance or through initiatives like the Hour of Code (https://hourofcode.com/). Once introduced, students can continue to work independently outside of the classroom.

Another path has continued to look for ways to teach computer science and other subjects together. In 2006, the influential article by Jeannette Wing brought the concept of computational thinking (CT) to the forefront as "a fundamental skill for everyone, not just for computer scientists. To reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability" (p. 33). A community of educators, researchers, and policymakers have formed behind CT and sought to bring it into schools. Much of the work so far has focused on clarifying definitions of CT, developing tools to foster its development, and conceptualizing how it might integrate into specific domains (Grover & Pea, 2013; Pérez, 2018; Weintrop et al. 2016).

Taking a different approach along this same path is the BootStrap: Algebra Project (https://bootstrapworld.org/). The project is a curriculum module designed to teach concepts from algebra and geometry through programming. Moreover, the creators have taken an integrated approach and designed materials that reinforce mathematical ideas, have clear connections to the established mathematics curriculum, and support mathematics pedagogy. Initial research has demonstrated that BootStrap: Algebra can help students learn mathematical concepts while also learning programming principles (Schanzer, Fisler, & Krishnamurthi, 2018).

A third path taken to foster more computer science enrollment has focused on changing educational policy and allowing some computer science courses to replace mathematics courses in graduation requirements. In contrast to the other described paths, which do not pose significant challenges to the established mathematics course sequence, this third option has been met with a guarded response by the mathematics education community. As noted by NCTM past-president Dr. Matt Larson in his 2016 Presidential Message titled *Computer Science Education and Mathematics Education*:

> Without question, NCTM supports the growth of computer science education. At the same time, however, NCTM emphasizes that mathematics is an indispensable foundation to computer science. There is no substitute for the

> reasoning, sense making, and computational thinking that are learned in mathematics and later applied in computer science. If a student takes a computer science course in lieu of a required mathematics course, it is important to consider what mathematics that student will be missing. (Para. 6)

This perspective is woven into NCTM's position statement on Computer Science and Mathematics Education, which notes that a computer science course should only be substituted for a mathematics course "if the substitution does not interfere with a student's ability to complete core readiness requirements in mathematics" (NCTM, 2016, p. 1).

Regardless of the path taken, whether through enrichment materials, curriculum integration, or course replacement, it seems counterproductive for the field of mathematics education to hold a position that is strongly in favor of statistics integration efforts while at the same time being indifferent or wary of work within computer science. What is made plain by *Hello World* is the critical need to amplify data science within the school curriculum. Statistics and computer science are being used in tandem to redefine all areas of modern life and students require knowledge of both. The field of mathematics education is uniquely positioned to lead in this area, but doing so will require challenging conversations and actions to address existing mindsets and expectations.

How Should We Teach?

> At the heart of this new technology - as with almost all algorithms - are questions about power, expectation, control, and delegation of responsibility. And about whether we can expect our technology to fit in with us, rather than the other way around. (p. 153)

Power and control are central issues within mathematics education. They ride beneath the seemingly calm surface of our classrooms and can be seen in the ripples from questions such as, why are we learning this information? When are we ever going to use this? Can I use a calculator? Leaders within the field have engaged the community to reform the culture of mathematics education and lead with equity so that we might address the ways that power, whiteness, and privilege are at play within the field (Battey & Leyva, 2016; Gutiérrez, 2013; Martin, Gholson, & Leonard, 2010). Alongside these efforts for social and political change are new disruptions of longstanding practices and expectations powered by artificial intelligence. Although the field has considered what artificial intelligence might mean for mathematics education before (e.g., Thompson, 1989), the developments of neural networks and machine learning have created algorithms with capabilities that are far different from what occurred previously. To highlight how this new generation of artificial intelligence is already influencing power and control in mathematics education, consider two examples of algorithm-powered technology already being used: Photomath and the Assessment and Learning in Knowledge Spaces (ALEKS) intelligent tutoring system.

I write this review within the context of the COVID-19 pandemic when many classrooms around the world have moved into online environments. Within this context, stories about student's use of the smart camera calculator Photomath (https://photomath.app) abound. Just what is Photomath and what does it do? The company offers the following description on its website:

> Photomath is the world's most used math education app with over 200-million learners globally. Powered by advanced AI technology, the app instantly scans, accurately solves, and intuitively explains math problems to users through step-by-step explanations, ultimately promoting comprehension of fundamental concepts.

As the app developers note, the intention of the app is to help students understand mathematics, but research on its effectiveness in this objective is lacking. What is plainly obvious is how easily the app can be used to sidestep learning objectives and undermine procedural work in classrooms simply by scanning a particular problem and copying down the resulting information. For educators with a focus largely on the development procedural fluency, Photomath and its rivals are powerful disruptors that they have suddenly had to face.

In a different vein from smart camera calculators is the intelligent tutoring system ALEKS (https://www.aleks.com/). The creators of ALEKS aim to provide students with self-paced, self-guided, and self-contained instruction. As described on the website:

> ALEKS "knows," at each moment, with respect to each individual topic, whether each individual student has mastered that topic. If not, ALEKS knows whether the student is ready to learn the topic at that moment. ALEKS uses this knowledge to make learning more efficient and effective by continuously offering the student a selection of only the topics she is ready to learn right now. This builds student confidence and learning momentum.

Currently, ALEKS is being used in a variety of settings from traditional classrooms to homeschool environments. There is also a growing research base with evidence that ALEKS is as good as traditional classroom teaching (e.g., Fang, Ren, Hu, & Graesser, 2018).

Neither Photomath nor ALEKS suggests to me that mathematics teaching is obsolete, but each one highlights a different way that algorithms can interface with our classrooms. In contrast to changing the curriculum, these algorithms can change the work of mathematics teaching itself. How will mathematics education respond to these perceived threats to power? Borrowing a metaphor described by Kennedy (1995), we might regard mathematical knowledge as a kind of tree with the trunk representing the main body of general knowledge and the branches areas of specialization. Our typical curriculum sequence assumes that everyone must climb the trunk to access the branches. From this perspective, the work of mathematics teaching is to help students up the trunk along the familiar paths as they ascend to the branches. Algorithms such as Photomath and ALEKS not only challenge the value of the trunk but the need for teachers to help students in ascending it. Might we allow algorithms to create ladders that provide immediate access to the branches? Might we let algorithms lead students along the well-worn paths to mastery? How will the field respond to challenges that question the very work that we do?

The lens of history shows ongoing resistance to technology within mathematics education. Most notably, the continued resistance to the use of graphing calculators by mathematics teachers (Milou, 2010) despite the overwhelming research base showing benefits for students (Ellington, 2003; NCTM, 2011). Simply put, apart from some very specific contexts, not using calculators is wholly to the detriment of students. In my work with preservice teachers, I often pose the following thought experiment:

> Imagine you are somewhere that is inhospitable such as stranded on a mountaintop or lost in the desert. Now, are you more likely to have the things you need to survive (e.g., warmth, water, shelter) or a calculator?

The point of this exercise is to help students gain perspective on the widespread availability of calculators and to highlight that perhaps the only place one might not have easy access to a calculator is within a mathematics classroom. This is a stark disconnect to the real world yet something that many mathematics teachers have accepted as fact. As mobile technology continues to improve, how long will it be before we can say the same thing about smart camera calculators and internet access?

Concluding Thoughts: Is it Time to Trim the Tree?

For me, the central message from *Hello World* is that we are at a critical juncture where we must decide how to navigate an increasingly algorithm driven world. As Dr. Fry notes:

> This tendency of ours to view things in black and white — seeing algorithms as either omnipotent masters or a useless pile of junk — presents quite a problem in our high-tech age. If we're going to get the most out of technology, we're going to need to work out a way to be a bit more objective. (pp. 29–30)

Will we help students understand algorithms by incorporating new ideas into the curriculum? Will classroom practice change to incorporate new tools driven by algorithms? Drawing back on the metaphor of the tree of mathematics, are there parts of the tree that we cannot afford to have students bypass? What new opportunities for PK-12 mathematics are present among the branches? Although it is true that we can still separate algorithms from our physical classrooms, the pandemic has shown that even this assumption is mutable.

There has been remarkable growth in accessibility and access to mathematics over the past century. More people are able to do more mathematics. More mathematical questions are being asked and answered. Real-life applications abound. Algorithms present opportunities for us to collectively let go of long held assumptions and bring reform to our curriculum, classrooms, and practice. We can shift from a lens of power and control towards a system that embraces the capacity of mathematics to improve the world, dismantle systems of oppression, and humanize. I agree with Dr. Fry that the best possible future is one where humans and algorithms work together and complement each other. It seems appropriate to conclude with her words on the matter:

> This is the future I'm hoping for. One where the arrogant, dictatorial algorithms that fill many of these pages are a thing of the past. One where we stop seeing machines as objective masters and start treating them as we would any other source of power. By questioning their decisions; scrutinizing their motives; acknowledging our emotions; demanding to know who stands to benefit; holding them accountable for their mistakes; and refusing to become complacent. I think this is the key to a future where the net overall effect of algorithms is a positive force for society. And it's only right that it's a job that rests squarely on our shoulders. Because one thing is for sure. In the age of the algorithms, humans have never been more important. (pp. 233–234)

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