The Mathematics Enthusiast

Manuscript 1600

(Re)Humanizing Mathematics with Story A Review of Keith Devlin's Finding Fibonacci: The Quest to Rediscover the Forgotten Mathematical Genius Who Changed the World

Cynthia Nicol

(Re)Humanizing Mathematics with Story
A Review of Keith Devlin*'s Finding Fibonacci: The Quest to Rediscover the Forgotten Mathematical Genius Who Changed the World*

Cynthia Nicol[1]
University of British Columbia

Indigenous scholar Richard van Camp of the Dogrib Nation shares how "a great story reminds us of what it means to be human" (van Camp in Galloway, 2020). Keith Devlin's book, *Finding Fibonacci: The Quest to Rediscover the Forgotten Mathematical Genius Who Changed the World*, does just that – reminding us what it means to be human, but in this case through mathematics. In *Finding Fibonacci,* Devlin shares his story and journey to learn more about mathematician Leonardo Pisano (Fibonacci) and Leonardo's manuscript *Liber Abbaci* (Book of Calculation), one of the first manuscripts written on practical arithmetic in Western Europe, and what Devlin argues "would change the world" (p. 17).

Today we know Leonardo Pisano by his nickname Fibonacci (Family of Bonacci), a name coined more than 100 years after Leonardo completed his first edition of *Liber Abbaci* in 1202. During Leonardo's time, as son of an Italian merchant, he was known throughout Italy and his hometown of Pisa as a mathematician who, with his book *Liber Abbaci*, offered new ways of writing and computing numbers – calculating methods that could impact business and trade practices. Leonardo did not invent these new ways of working with numbers; he acquired them while living in North Africa and observing Arabic merchants use ways of working with a number-system they acquired from Hindu merchants. Through *Liber Abbaci*, with over 600 pages of hand scripted procedures and applications of mathematics[2], Leonardo made this new number system accessible to European merchants, initiating new forms of international commerce. In *Finding Fibonacci*, Devlin brings his own journey of searching for Leonardo's story to readers following the mathematical thread over time through *Liber Abbaci*, its translations,

[1] cynthia.nicol@ubc.ca
[2] Leonardo actually wrote two massive volumes in Latin Liber Abbaci and De Practica Geometrie as well as a condensed liber abbaci book, a liber minoris guies (a book in a smaller manner).

The Mathematics Enthusiast, **ISSN 1551-3440**, vol. 20, nos. 1, 2 & 3, pp. 152-161.

scribed copies, interpretations by historians of medieval mathematics, and its global impact.

Finding *Liber Abbaci*

As a mathematician dedicated to making mathematics accessible to the general public, Devlin has written many books including *The Man of Numbers*, but *Finding Fibonacci* is Devlin's first historical writing. The book takes us on a 10-year journey as Devlin seeks to learn more about Leonardo and his work. The journey includes travelling to Italy, interviewing mathematics historians, searching Italian libraries and special medieval collections for original scribed copies of Leonardo's manuscripts, and reading the first and only English translation of *Liber Abbaci* completed in 2002. In this journey, Devlin finds that even though Leonardo was well-known in medieval time, today there exist few historical records of his life. Even more surprising is that it was not until the 1960s and 1980s with the discovery, cataloguing and translation of an extensive genre of hand-written abbacus manuscripts (commercial arithmetic books) ranging from the 13th century to the 16th century, that we see evidence of the importance at the time given to this new way of calculating. Although Leonardo's book was written in Latin, the authors of these found manuscripts wrote in Italian with the town's local language, and using local currencies, measures and contexts in their examples. Nonetheless, these found manuscripts, written by different authors, and often not authored at all, follow the content, organization and structure of Leonardo's work.

With this information, Devlin argues, following others, that the condensed simpler version of *Liber abbaci* is the source from which the found manuscripts and other abbacus books were written. Devlin's journey includes searching for one of these manuscripts and, with the help of Italian mathematical historian Rafaella Franci, he was able to locate one dated around 1290 (a copy of Leonardo's simpler book). But why were there so many abbacus books? Thousands were scribed and copied between the 13th and 16th centuries and, once the printing press was invented around 1450, these abbaci books were some of the first printed. Why were they so popular? What created the need for there to be likely thousands scribed and copied between the 13th and 16th centuries? Devlin argues that the

reason for the proliferation and demand of abbaci books during this time is due to the changing world of finance, commerce and global trade in the regions around Italy. These books offered techniques and strategies to merchants and traders, teaching new calculating strategies in the context of local commerce contexts and financial problems.

Making Alien Calculations Relatable

Before *Liber Abbaci* and the adoption of the Hindu-Arabic system of arithmetic, merchants in Western Europe used either ancient finger arithmetic or an abacas board. With finger arithmetic, number operations up to 10,000 could be represented.[3] An abacus board, on the other hand, consisted of sets of parallel lines upon which pebbles, representing numbers, could be moved.[4] Both systems were efficient but required skill to master, and neither provided a "memory" or written record of the pathway to the answer. If a buyer and seller disagreed on a resulting calculation or its accuracy, the operations needed to be carried out again with both parties paying attention to the performed procedures. Devlin argues that the opportunity to have a written record of commercial transactions along with the relative ease of learning these new procedures helped catapult and popularize their acceptance and demand across Western Europe at a time when Europe was ripe for increased opportunities for trade.

Although there is much we do not know about Leonardo the person, Devlin's research provides hints of Leonardo's life through his publications. Following the first edition published in 1202, Leonardo continued to edit and extend his work, publishing a second edition in 1228 that included algebra strategies. This second edition of *Liber Abbaci,* shorter but still roughly 400 pages of dense text, is the one that has survived today, and the one that formed the basis of the English translation published in 2002. Devlin studied this English translation trying to place himself in the 13th century and encountering this new Hindu-Arabic number system along with these new arithmetic methods for the first time. Devlin writes, "to most people at the time, symbolic numbers and their arithmetic

[3] Digit is an English word for finger or thumb and through finger arithmetic we have adopted the word "digit" for number symbols 0-9.
[4] In Latin, the word pebble is "calculus".

were alien" (p. 89). For Leonardo then, the problem was how to communicate and popularize a creative system and way of thinking that was, for the most part, "alien" to many. How to, in effect, sell or market these ideas? And for Devlin, how to help others, centuries later, see the significance of such offerings. To help the reader understand this challenge and appreciate the "revolutionary" impact of Leonardo 's contributions, Devlin, does what many good teachers do - he tries to make the event relatable to readers with parallels between our current experiences and those likely experienced during the 13th - 16 centuries.

Devlin compares what he calls two revolutions that occurred 800 years apart: one of Leonardo's roles in popularizing the Hindu-Arabic number system in Europe through *Liber Abbaci*; and the other of American Steve Jobs, founder of Apple computers, popularizing the computer through introducing the personal computer to the public. Both Leonardo and Jobs could see the potential of their products and with their introductions, writes Devlin, "the world would never be the same again" (p. 185). Following Leonardo, others were inspired to write and study various translations of *Liber Abbaci* and to create their own versions of the text, so many so that reference to the original authors was not included and eventually connections to the original text itself were no longer needed. Similarly, argues Devlin, Steve Jobs offered a computer operating system (Apple Macintosh) that could be used without knowing and writing strings of code, and this catalyzed the development of other computer operating systems like Microsoft Windows. "Both advances," writes Devlin "resulted in systems that were so natural and easy to use, people soon forgot what a huge step each had been, and how much each changed life on Earth" (p. 190).

Beyond Fibonacci Numbers

Today Leonardo is most known for the sequence of numbers that bear his nick name: the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34 …. As Devlin mentions multiple times, Leonardo did not invent the problem that led to this sequence. Instead, the problem dates back to early Hindu-Arabic mathematicians. Devlin speculates that the problem in the context of growing rabbit populations is offered to practice with the new number system

outlined in *Liber Abbaci*. Here is Leonardo's first writing of the problem from the translated 2002 version of *Liber Abbaci*:

> How Many Pairs of Rabbits are Created by One Pair in One Year
>
> A certain man had one pair of rabbits together in a certain enclosed place, and one wishes to know how many are created from the pair in one year when it is the nature of them in a single month to bear another pair, in the second month those born to bear also. [p. 404] (in Devlin, 2020, p. 127)

Like most of his problems, Leonardo followed this rabbit problem with a detailed solution laying out development of the numbers we now refer to as the Fibonacci sequence. Devlin reminds readers that the popularity of these numbers and their fascination comes from how we "see" these numbers frequently in nature when describing the growth of plants and flowers, for instance the swirls of pinecones and pineapples, or the arrangement of leaves on a plant's stem, or the spirals of moon shells and sheep's horns.

These few examples from *Liber Abbaci* are fascinating and to some extent, more powerful than Devlin's strategy to help readers appreciate its impact with parallels to Steve Jobs' personal computer and the chain of events that followed. In reviewing the translated version of *Liber Abbaci*, Devlin notes that Chapters 1-7 outline the writing and manipulating of the Hindu-Arabic number system, the content of which is similar to the content of typical school mathematics textbooks. Later chapters of *Liber Abbaci* provide detailed real-world examples where Leonardo provides examples of how the number can be used to solve problems encountered by merchants. Some examples include:

> If one hundredweight of linen or some other merchandise is sold near Syria or Alexandria for 4 Saracen bezants, and you will wish to know how much 37 rolls are worth the… [p. 142] (in Devlin, 2020 p. 117)

For those working in business partnerships, such as a shared company, Leonardo offered applied problems with men coming together to purchase horses or ships or forming a company such as:

> Three men made a company in which the first man put 17 pounds, the second 29 pounds, the third 42 pounds and the profit was 100 pounds. [p. 220] (Devlin, 2020 p. 117).

And many recreational problems such as purse problems that involve merchants finding a purse lying on the road and comparing how the purse money would comparatively represent their wealth.

Apparently, the purchase of horses was a popular context. Here is one of 29 horse-type problems offered in *Liber Abbaci*:

> Two men having bezants found a horse for sale; as they wished to buy him, the first said to the second, if you will give me 1/3 of your bezants, then I shall have the prices of the horse. And the other man proposed to have similarly the price of the horse if he takes ¼ of the first's bezants. The price of the horse and the horse and the bezants of each man are sought [p. 337] (Devlin, 2020, p. 122)

Leonardo's detailed step-by-step written solution follows, included here to emphasize the narrative nature of the solution:

> You put ¼ 1/3 in order, and you subtract the 1 which is over the 3 from the 3 itself; there remains 2 that you multiple by the 4; there will be 8 bezants, and the first has this many. Also the 1 which is over the 4 is subtracted from the 4; there remains 3 that you multiply by the 3; there remains 9 bezants, and the other man has this many. Again you multiply the 3 by the 4; there will be 12 from which you take the 1 that comes out of the multiplication of the 1 which is over the 3 by the 1 which is over the 4; there remain 11 bezants for the price of the horse; this method proceeds from the rule of proportion, namely from the finding of the proportion of the bezants of one man to the bezants of the other; the proportion is found thus. [p. 337] (Devlin, 2020, p. 122).

Men and Commerce

Devlin provides a couple of pages explaining that when Europeans were learning the Hindu-Arabic number-system they represented fractions from the right to the left and often wrote narratively rather than symbolically. For example,

$$\frac{2}{3}\frac{3}{4}\frac{1}{5}\ 6 \text{ meant } \ldots\ldots 6 + \frac{1}{5} + \frac{3}{4\times 5} + \frac{2}{3\times 4\times 5}$$

And written narratively or spoken during Leonardo's time a merchant would say: 6 and a fifth, and three fourths of a fifth, and two thirds of a fourths of a fifth.

For travelling merchants encountering different regional currencies with varied denominations and non-standard measurements, trading and making fair conversions was a complex problem that Hindu-Arabic mathematics made easier. One of Leonardo's problems, involving Pisan currency, provides a case in point:

> Pisan hundredweights have in themselves one hundred parts each of which is called a roll, and each roll contains 12 ounces, and each of which weighs $\frac{1}{2}$ 39 pennyweights; and each pennyweight contains 6 carobs and a carob is 4 grains of corn. (Stigler in Delvin, 2020, p. 125).

As there was little need for counting in tenths or hundredths during Leonardo's time, using decimal representations was uncommon even though decimal representations can be found in Arabic texts as early as 952, as Devlin notes. Furthermore, fractions written to the left of the whole number indicated addition of fractional pieces while those written to the right indicated multiplication. For example:

$$\frac{4}{10}\frac{3}{10}\frac{1}{10}\frac{9}{10}\ 4\ \frac{1}{5} \text{ means one fifth of } 4.9134$$

Although written in Latin, Devlin does include a few photos from the pages of *Liber Abbaci*. This provided the reader a glimpse of the laborious detail provided in the narratively written solutions that contained few symbolic representations. Furthermore, one can't help but notice, although Devlin does not, that all contexts for the problems shared have contexts involving men engaged in commerce, reinforcing the image of mathematicians as men.

Teaching Mathematics through Historical Fiction

These procedures, the images of the detailed solutions from *Liber Abbaci*, the contexts or situations in which mathematical problems arose, the pedagogical strategies for popularizing these methods, and Devlin's journey as a historical researcher provide mathematics teachers and mathematics teacher educators with a wealth of stories to share with their students. Such stories have the potential to (re)humanize mathematics,

to provide opportunities for students to see and experience mathematics as what Harold Jacobs (1982) called a "human endeavour" (p. 1)

Tzanakis and Arcavi (2000) discuss both the value and challenges of integrating the history of mathematics in mathematics education. Among the challenges of bringing history to the study of mathematics is the claim that history is not mathematics. In fact, it could be argued that Devlin's *Finding Fibonacci* includes very little mathematics. Yet, there is something in this story that brings us closer to the human and cultural appreciation of mathematics, the sharing of mathematical ideas, and necessary cultural context or the readiness for accepting and taking up ideas. Thus, for mathematics educators, *Finding Fibonacci* portrays Leonardo Pisano as offering much more than the sequence of numbers that today bear his nickname.

Although plausible, we must remember that Devlin's insight into Leonardo and his intentions for writing the versions of *Liber Abbaci* are speculative. Some of Devlin's text could be challenging to justify as primary source documents are few and it is, as Devlin admits, somewhat challenging to separate fact from Fibonacci's folklore. But it is here that I imagine even further possibilities for historical fiction. Perhaps far more powerful than Devlin's parallels between current and past revolutionary events might be an historical fiction that is set within Leonardo's time using what is known about the time period to create the conditions for the characters. Similar to the 2020 Netflix series *The Queen's Gambit* about Beth Harmon, a fictional chess player in the 1960s and her efforts to become a world-class chess champion, *Finding Fibonacci* has the potential to be a well-told fictional documentary. Although the events and characters of *The Queen's Gambit* are fictional, the chess moves are claimed to be well choreographed and realistic, and the set, scenes, and wardrobe to be accurate (McClain, 2020). Surprisingly the series has inspired a revitalization of chess with a rise in sales of chess sets leaving stores unable to fill demand.

Imagine *Finding Fibonacci* as a Netflix series, with young Leonardo travelling with his merchant father learning and seeing the potential of Hindu-Arabic mathematics for European merchants and commerce. The series, building on Devlin's historical account,

could create a plausible backstory giving us further insight into both the possible how and why Leonardo wrote *Liber Abbaci* along with how it was received, and its societal influence. The series could emphasize how religion, culture, and gender shaped the kinds of problems offered (something Devlin does not do) but also how the values of commerce and capitalism supported the acceptance of a new number system. Although Devlin's account tends to over-emphasize the importance of *Liber Abbaci* towards revitalizing Europe's domination of the world's trade and finance, a fictional documentary of Leonardo could help make the evolution and societal acceptance of our current number system an even more powerful human story.

Ojibwe Indigenous author Richard Wagamese (2019) writes that all things begin with stories, that "we all respond to that setting like children, rapt with wonder and entranced by the possibility of story. The teachers of our cultures recognized this [...] What begins in wonder is learned in earnest [...] story is our most powerful commonality" (p. 31-32). And so, it can be with mathematics. Whether it be a future Netflix series or Devlin's *Finding Fibonacci*, story can bring us closer to the human wonder and power of mathematics.

References

Devlin, K. (2017) *Finding Fibonacci: The question to rediscover the forgotten mathematical genius who changed the world*. Princeton University Press.

Jacobs, H. R. (1982). *Mathematics, a human endeavor: A book for those who think they don't like the subject* (2nd ed.). W.H. Freeman.

Tzanakis, C. & Arcavi, A. (2000). Integrating history of mathematics in the classroom: An analytic survey. In J. Fauvel & J van Maanen (Eds.), *History in mathematics education: The ICMI Study* (pp. 201-240). Kluwer.

Wagamese, R. (2019). *One drum: Stories and ceremonies for a planet*. Douglas and McIntyre.

Galloway, M. (Host) (2020, October 7). Richard van Camp on telling stories and staying connected. In *The Current*. CBC Radio. https://www.cbc.ca/listen/live-radio/1-63-the-current.

Ulaby, N. (2020, November 20). Can't find a chess set? You can thank "The Queen's Gambit" for that. *NPR*. https://www.npr.org/2020/11/20/936732591/cant-find-a-chess-set-you-can-thank-the-queens-gambit-for-that

McClain, D. (2020, November 23). I'm a chess expert: Here's what "The Queen's Gambit" gets right. *The New York Times*. https://www.nytimes.com/2020/11/03/arts/ television/chess-queens-gambit.html