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The Siren Call of Calculus
A Review of Steven Strogatz's *Infinite Powers: How Calculus Reveals the Secrets of the Universe*

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I hate calculus. Which is a lie: I secretly love it. I enjoy teaching it, drawing intricate diagrams of where the definition of the derivative comes from, stumping my students on convoluted trigonometric integrals with solutions dependent on partial fractions, hammering mnemonics for the derivative rules—all the joys of teaching calculus. And I enjoyed learning it, too. Well, not failing it the first time, but after that. I long for my undergraduate time spent in the reading room of Rutherford South re-arranging integrals à la Fubini's Theorem, the massive lecture halls of anxious engineering students copying every stroke of chalk from the front, and the arithmetic errors costing me scores of marks from my tests. A calculus course presents the liturgy of undergraduate mathematics: students and instructors alike gather, willfully or otherwise, to engage in the ritualistic celebration of the mysteries of the infinite.

But I do mostly hate calculus.

I hate how it has become the reigning monarch of elementary education, so much so that lesser, obsequious subjects—college algebra and trigonometry in universities in the USA, and the entirety of K-12 education the world over—are bound to it in servitude, bringing students on a perilous journey to worship at its throne. It is a rite of passage, with many not surviving — women and under-represented students foremost among them (Ellis et al., 2016). Calculus largely sets the pace for our curricula from the top down. If secondary teachers are not preparing their students for calculus at university, they're setting them up for failure; if primary teachers aren't setting their students up for secondary...

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Enter *Infinite Powers*. It's a sweeping, grandiose book, covering calculus' history and current happenings, and tells an endearing story of humankind's desire to understand the world and universe around them and their place in that mystery. Read the first paragraph with me:

> Without calculus, we wouldn't have cell phones, computers, or microwave ovens. We wouldn't have radio. Or television. Or ultrasound for expectant mothers, or GPS for lost travelers. We wouldn't have split the atom, unraveled the human genome, or put astronauts on the moon. We might not even have the Declaration of Independence.

Goodness! That's quite the list. And Strogatz isn't being hyperbolic; he draws the paths linking each item on this laundry list back to calculus.

What spoke to me most is that this book provides a sense of the struggle behind those glossy pages of Stewart's Calculus (2020)—each equation, each derivative rule, though stated simply in a quarter of a lecture hour, was the result of hundreds of years of human investigation into patterns and forms, involving politics and personal tensions, often petty, inching slowly towards those terse textbook pages. Calculus isn't just *the* gateway course for our students, it was the gateway invention for all of humankind; a gift from those toiling in darkness to bring us once again into the light. This is what attracted me most to Strogatz's writing: he foregrounds, throughout the book and repeatedly, how hard-fought the incremental development of calculus was.

Perhaps most importantly, Strogatz's book re-awakens the deep-seated love I have for calculus. He writes passionately of the subject's history, breathing new air into familiar stories, and drawing connections I've never considered—that's not easy to do since calculus, and its historical tropes and mundane applications hammered out off the yellowed lecture notes of lecturers well past their expiry date, is so completely familiar to me. The book is mathematics marriage counseling: the love is there in my heart and has only been clouded by the daily sinks of dirty dishes and first principles derivative exercises. Strogatz has made love to calculus and wrote his book in the post-coital glow. *Infinite Powers* is fresh and alive and young and flush with lust for calculus's well-worn history, like a new lover tracing and discovering its creases and wrinkles with practiced and knowing fingertips. Don't let this deceive you, however—never think of bringing calculus home to meet your parents.

There is a disconnect in the profoundness of calculus and the practice of teaching calculus, one that Strogatz (2019) cannot reconcile, but does acknowledge: "Integrals have been defanged and turned into homework exercises for teenagers...[s]tudents in calculus are swimming in the fundamental theorem all the time, so naturally they take it for granted" (p. 169) The conceptual machinery of calculus is a beautiful wonder of human ingenuity. But the subject is seldom taught with that machinery and all its manifold idea cogs and belts and parts exposed. Rather, calculus is taught with the hood down, students drive it without understanding its inner workings and occasionally get where they're required to go. But that's an old gripe voiced by many educators and I won't dwell too much on it here. The upshot of the typical approach to teaching calculus, with its over-emphasis on derivative-taking and arcane integral tricks, is that students seldom get to feel the fire that forged Strogatz's words—that passion has long faded from the subject, having been routinized into a laundry list of procedures. The great human struggle and triumph that is calculus is obfuscated with the slick veneer on the pages of Stewart. Despite our intentions and our proclamations that calculus is a highly conceptual subject, students judge it by how we assess, and that's primarily procedural (Burn & Mesa, 2015; Maciejewski & Merchant, 2016; Tallman et al., 2016). Students don't appreciate the profundity of the subject through derivative-taking and integral-finding.

I can sense you nodding at the words above, so do me a favour: teach calculus from Infinite Powers. Make that your course text. Computers are far better at procedures than your students will ever be anyways, so no need for Stewart's tome. Set your students next to Strogatz's passionate fire. Let that fire seep into their souls. To hell with the taking of derivatives and integrals of functions. That's not the point anyways. Calculus is one of humanity's first Grand Unifying Theories—it brings together accumulation and change into one body, never to be separated. Be the guide for your students encountering that divine. If, that is, you think calculus needs to be taught.

One of my big discomforts with the centrality of calculus in education stems from its affordances and constraints. A colleague once told me, “what you know determines, to a large extent, what you *can* know.” By positioning calculus as the keystone of an elementary education, we educators are predisposing our students (ok, supposing we have any appreciable influence) to viewing their worlds through a calculus lens. It is historically understandable why we might want to do that: calculus was proving extremely effective at uncovering secrets of the universe and advancing technology here on earth. Strogatz (2019) takes this further when reflecting on Fermat’s use of his proto-calculus to solve a refraction problem: “This was an important early clue that calculus was somehow built into the operating system of the universe” (p.117) Of course, it isn’t—calculus is a human invention and the universe is an, ah, something other-humanly—but it certainly feels as though calculus was used in the construction of the universe. It just works that well.

I admit, the massive value of the technological, scientific, mathematical, and societal products of calculus cannot be understated. I often tell my students that it is no wonder that the industrial revolution in Great Britain followed the development of calculus—it allows us to predict the behaviour of certain things and from that we can design more things. However, the teaching of calculus is separate from the practice of calculus. Calculus teaching is a recent phenomenon, only coming into full force mid-20th century. It’s a relic of the Cold War era; its high-water mark is seen in the trajectories of the Sputnik and Apollo missions. There was an actual, genuine need in the USA and elsewhere for human calculators to apply calculus to the very real and pressing issue of defeating the Soviets in a technological race off this planet. Engineers were needed en masse and that entailed the wide-spread teaching of calculus. However, the universe that was the backdrop of the cold war was all smooth curves—from the ellipses of Kepler to Riemann’s metrics. But with the cold war over and the space race done, our experiences here on earth are increasingly rough and jagged. Understanding machine learning and social networks, both resting on a discrete, graph-theoretical foundation, are the new Grand Challenges. Might we need a mathematics that confronts this roughness, without sanding it down, better than calculus?

We use calculus to understand what's around us. Think of the order of understanding: we use the smooth to understand the stochastic; the continuous to understand the discontinuous. Now let's try to flip that order. Suppose stochastics are understood *first* and then from that, the smooth traits of calculus are derived. That motion of a physical object, for example, is not taken to be continuous but rather as the emergent result of the random, albeit directed, action of trillions of particles.

Having trouble imagining such a system? Or what I even mean by all that? Or perhaps you do understand and are thinking, "all that's so inefficient compared to smooth (differentiable) approximations of those systems!" That's entirely my point: calculus has conditioned us into thinking a certain way. Fortunately for calculus, this way of thinking forces us educators to perpetuate the centrality of calculus: we struggle in seeing an alternative. As Strogatz (2019) writes, "...calculus is the sprawling collection of ideas and methods used to study anything—any pattern, any curve, any motion, any natural process, system, or phenomenon—that changes smoothly and continuously..." (p. 272).

Let's push this idea of changing perspective out further with a thought experiment entirely within one of the primary topics of calculus: infinite approximations. Strogatz articulates nicely the fundamental idea of integral calculus, that of chopping complicated things up into smaller, easier to understand and more familiar things. We work at that chopped up level then build back up to our original, more complex problem. For many of us, those simpler things are rectangles: we approximate areas of shapes—of circles, and of regions under curves, and so on—as rectangles, calculate the areas of these with an astoundingly simple formula, then add these up to get the area we originally sought. I claim, as I'm sure others have, that the choice of rectangles is arbitrary—other shapes can do—and we choose rectangles because their roots are in our socio/cultural/historical activity, particularly in the annual flooding and re-surveying of ancient Egyptian rectangular farms along the Nile. Western civilization could have had different mathematical roots. An ancient civilization might have farmed small circular farms subject to similar annual patterns of devastation. Those ancient peoples might then have well developed surveying methods that take the circle as the base shape; that prioritizes the curvilinear over the linear. The Ancients would then be troubled with approximating the area of a rectangle with copies of their base circular shape. This isn't outlandish—plenty of cultures, especially those of Indigenous peoples, have mathematical systems that differ than the prevailing Northern European.

Try it yourself: come up with a formula for the area of a rectangle by approximating the rectangle with circles. Not easy, is it? It's not your fault and I doubt you're bad at math. You just have the weighty hindrance of socio/cultural/historical baggage to overcome.

And maybe that's where we are with science and math and technology. We're thriving—the rate of new, daily discoveries is truly astounding. But we're thriving entirely within the universe we constructed around ourselves with calculus. We have difficulty thinking within other universes. Perhaps by dethroning calculus as the monarch of elementary mathematics we open up new perspectives on the problems we've encountered in our calculus universe. We still have no Grand Unified Theory of physics, for example, with quantum mechanics and relativity as facets. Perhaps it is because both those major sub-theories are fundamentally understood in terms of calculus, even the stochastic nature of quantum mechanics is smoothed out with curves and integrals. And what might a stochastic theory of relativity look like? I don't have answers for this or the other questions; I took all the calculus courses on offer in undergrad.

Anyhow: read *Infinite Powers*. It's a good book and Strogatz is an excellent writer. But do not let it seduce you. It's time for a new lover.

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