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A Review of Amir D. Aczel's *Finding Zero: A Mathematician's Odyssey to Uncover the Origins of Numbers*

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In the 2003 Pixar animation *Finding Nemo,* a clownfish named Marlin is an overprotective father, and for good reason: he has recently gone through the trauma of losing both his wife and all their fertilized eggs save one. Nemo is his only remaining offspring, a son. On Nemo's first day of school, he gets caught in a fishing net and is taken far away. Marlin spends the rest of the movie looking for Nemo. He gets to know a variety characters in his search, some providing comic relief, some providing leads. But in each interaction Marlin learns a more about himself, in particular, that he is too fearful and too protective. For some, *Finding Nemo* is about Nemo; for others, it is about Marlin. That is, instead of Marlin providing a subplot, he provides the central story. *Finding Nemo* is about Marlin's personal journey of overcoming the trauma of losing his wife and almost-children. To give even more of a twist to a seemingly benign movie, there is a theory bouncing around on the Internet that Marlin imagines Nemo and everything that happens in the movie is only imagined. That is, he makes up Nemo to cope with his loss. This theory has some merit as Nemo means "nobody" in Latin.

Amir D. Aczel is a professor of statistics. He has a doctorate in statistics, but a bachelor's in mathematics and a master's in science. Although he did not go through the loss or trauma of Marlin, his framing of *Finding Zero* is similar to the framing of *Finding Nemo*. He never mentions the movie, but I am inclined to believe he was well aware of the connection, especially considering the similarity of the titles. Aczel's book is about his own search for the first occurrence of something like the modern concept of zero. In the story of his quest for this elusive number, he comes into contact with many characters who provide leads, ideas, and perspectives, each of which provide Aczel a further awareness of what the concept of zero is about. There is even comic relief at the end of *Finding Zero*

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when Aczel describes that after making a discovery of a seventh-century depiction of zero on a Cambodian (Khmer) stone, it was almost stolen by a couple of European researchers. Aczel aligns this story with Indiana Jones's losing a golden idol after expending much effort to get it. (This also shows me that Aczel is not averse to connecting with popular culture.)

The Mystery of Zero

Aczel couches the origins of zero in a mystery. As Marlin seeks Nemo in the vast, largely unknown ocean, Aczel depicts the meaning of zero as based in Eastern religion and philosophy. He presents his story as a quest for something elusive and intriguing. Midway through his journey, he writes how he "became increasingly convinced that the extreme number concepts—zero, infinity, and very large numbers—were intrinsically Eastern ideas" (p. 104). He continues his theme of mystery, suggesting that the "number system we use today developed in the East because of religious, spiritual, philosophical, and mystical reasons—not for the practical concerns of trade and industry in the West" (p. 77). He never expands on the role of religion, spirituality, or philosophy, but the reference to them promotes excitement at the prospect that something deep and mysterious is going on. He has such a longing when he thinks of numbers.

At an early age, he became infatuated with numbers. As a young boy he watched roulette in the casino of the cruise liner his father was the captain of. He was curious about the numbers spinning while people were excitedly hoping the ball would land on the one they had chosen.

> I was fascinated by these colourful numbers—ornate signs that beckoned me by their mystery, and which as I matured, I would understand to represent fundamental abstract concepts that rule our world….I fell in love with their magic, associating them in my mind with something alluring and forbidden, an unknown pleasure awaiting discovery. (p. 5)

The sense of excitement never left him. Just as he felt in those early days, Aczel's current quest takes him to cities around the world and to many different people, including scholars and random strangers, all in an attempt to find leads to where and how zero came

to be formulated. Aczel presents himself as swimming in this world of history and culture looking for the zero that represents, according to him, the key to the modern number system. But like Marlin in *Finding Nemo*, Aczel's story is less about the object sought and more about the seeker.

As with Marlin who is potentially imagining his search for his son Nemo, Aczel's approach in this book is to make zero as a singular idea; one of the most important mathematical ideas to have ever been invented. This kind of stance reveals the intrigue and excitement of a young boy in a casino. It is unfortunate, however, that his attempt to convey his youthful exuberance in his current search, he frames zero as if it were a lone fish in a world of complexity. Zero, however, is in fact complex and multifarious. The notion of zero does not have to be tied to Cambodian jungles or Indian temples or even attached to open mindedness. He describes that one of the reasons he believes that zero was an invention of the East was because "Eastern peoples of the tenth century had no hang-ups about sex and sexuality… The freedom exhibited… openness and sheer excitement about life and its pleasures that I felt certain it pointed to a fundamental difference between East and West" (p. 53).

Zero and its many natures are intriguing. Zero tends to attract the curiosity even from the most inert. Why, for example, did it take so long for zero to be accepted as a number? Is zero nothing or is it something? Is it an absence of quantity? Is it emptiness or is it more than nothing?

Zero can be conceived of or approached in a number of ways. One way is as a placeholder in a place value system. Another way is as a value. As far as historical records show, this notion of zero as a value emerged in the 8th century. Zero can be conceived of as a number in relation to the positive integers, capping one end so that it is understood as an end point or as a number separating positive and negative numbers.

Mayan and Babylonian Place Value

Both the Mayans and the Babylonians used a place value number system. That is, a system in which each digit of a number lies in a column that has a particular meaning. The value of a digit is affected by the column it is in. In a base 10 system, a 4 means 4, 40, 400, etc. depending which column it is in (or how many digits preceding reading from one end). The Mayans had a base 18 and 20 system and the Babylonians had a base 60 system. In contrast to a place value system, we can think of Roman numerals or Egyptian hieroglyphs where the symbols represented numbers and were additive. For example, LX equalled the value of 60 (in base 10) in Roman numerals. L equals 50 and X equals 10 and those two numbers were added up. The Egyptian system was similar.

The place value system is more complex. Each column or digit of a number has a specific meaning. What is challenging in a place value system is how to signify an empty column. In base 10, for the number 405, the tens (the middle column) does not have a value. In modern times we put a zero but what does a civilization do if there is no symbol for zero? The Mayans actually created a specific symbol for zero while the Babylonians had an empty space. Both of these civilizations existed around 4000 years ago, much earlier than the artifact Aczel discovered in a Cambodian conservation storage room. I would like to discuss the Babylonian's representation of number since it has important connections to Aczel's book.

The Babylonian number system was based on the number 60 (sexagesimal). There were cuneiform symbols, such as "<" and "|" which had the value of 10 and 1 respectively, that were embedded into clay tablets. So "<<<" would have the value of 30. And "<<< |||" would have a value of "33", but not necessarily. This is a potential problem with the Babylonian number system. Is the number represented in one column or two? If the "<<<" was in the 60s column (remember, it's base 60 so the second column is the number of 60s) then the number represented here is 30 x 60 + 3 = 1803. The other challenge was that not having a specific symbol for zero but only an empty space could create ambiguity. For example, the number 3601 would be represented as "| |". The "|" on the left would represent the value of 60^2 and the "|" on the right would represent the value of 1. The empty space in between indicated there were no 60s to be included in the value. This was

the conventional notation of the Babylonian number system. Leaving a space empty is what constituted the positional use of zero. In the Babylonian civilization, it was assumed that the scribe or whoever was doing the calculation knew the context well enough to line up the numbers properly. In fact, it was essential to understand the context of that number so appropriate columns would be lined up.

Aczel deals with both the Babylonian and Mayan number systems in an odd way. He dismisses the Mayan representation of zero on the grounds that it did not influence others. In the only line in the book that addresses the Mayan number system, he says, "The oldest zero was of course the Mayan zero—but it was confined to Mesoamerica and went nowhere from there" (p. 79). In terms of the Babylonian zero, he writes: "the Babylonians did not use any place-holding zero" (p. 75). I find it difficult to discount the Babylonian space as a zero since although potentially ambiguous, the space did represent something like our notion of zero. The fact that it was represented by nothing (so to speak) might seem apt. The question is what sort of concept was entertained when using that convention.

When we visit the past, it is difficult to set aside our modern perspectives and not make judgments based on contemporary practices. Studying the cultural context of mathematical practice is essential to approaching a concept of that time (Netz, 2017). Our modern expression of number is well-developed and so it is easy to look back on the Babylonians and wonder why they did not use a symbol rather simply leaving a blank space. But instead of looking at this situation as falling short, we could think of it as simply a different convention. This is a challenging task since it is easy to consider an empty space as less developed than a symbol since one is limited in what can be done with an empty space. In the context in which the Babylonian calculations were carried out, however, there was no need for a symbol. The Babylonians were not writing their mathematics for the modern world. Their practice was to give an empty space a significance akin to our zero and that worked fine for them. We should think of the practice as "a convention of shared practice, not the constraint of a given concept" (Netz, p. 45). The Babylonians did have a place-holding zero, it was a space. A space is not

something done by accident; it is intentional, and it had some sort of meaning to the Babylonians.

The last half of the book describes Aczel's journey to find the K-127, something he learned had existed from his meetings and explorations during the first half of the book. The K-127 is a stone artifact originally discovered in 1891, written in old Khmer language. Aczel describes it as the earliest zero found (other than the Mayan zero). It was a dot that represented the zero in 605. So it is a positional zero. It was the inscription of a date. Aczel provides a photograph of it where he found it in the Angkor Conservation, a place dedicated to Cambodian antiques. It currently sits, in the National Museum in Phnom Penh.

Concluding Comments

Zero is fascinating. It can be used as a metaphor, as a way of including something, and as a way of excluding something. It can give meaning to a variety of mathematical statements, expressions, equations and functions. In its modern form, it is a highly complex idea that has different meanings in different contexts. From an educational perspective, there is a problem caused by Aczel's blurring of the line between each of these meanings of zero. This happens because he frames zero as a thing, as a single idea much like the fish Marlin thinks he is seeking. But this communicates less about zero and more about how Aczel approaches the subject. For much of the narrative of his book, it is his imagination that we are reading about. But, yet, the history of zero and its uses could fill volumes. The general educational approach found within *Finding Zero*, is in its appeal to a sense of mystery. It is by blurring the meaning of zero and connecting it with the unknown world of Eastern spirituality that Aczel achieves this. Maybe the mystery will inspire others to study zero in more detail. Aczel's method is consistent with the notion that to teach, one cannot at first reveal everything there is to know about a topic. There are stages, and sometimes the beginning stages will not be fully accurate.

Another aspect of pedagogy that emerges from this book has to do with looking back at history with a modern perspective. How can we understand the past without interpreting

it based on our modern sensibilities? It takes more than finding a rock artifact such as the K-127 to get a deeper sense of zero or its history. To get a sense of the mysteries of zero, understanding cultural settings and conventions is imperative. And that takes deep study. The parallel to teaching students, of course, is to accept that student expression of a concept is also dependent on setting and convention. A question then arises as to how a teacher can determine a student's understanding without comparing it to their own conceptions.

At the end of *Finding Nemo*, Marlin is a different clownfish than when he first set out to find Nemo. At the end of *Finding Zero*, Aczel is transformed after his personal discovery of the K-127. The narratives of both of these stories align closely, both being about learning, growth and change…and that's not nothing.

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