The Mathematics Enthusiast

Volume 20
Number 1 *Number 1, 2 & 3*

Article 22

4-2023

The Birth of a Review or (How Villani Both Reinforces and Challenges Our Beliefs About Mathematics) A Review of Cédric Villani's Birth of a Theorem: A Mathematical Adventure

Manon LeBlanc

Follow this and additional works at: https://scholarworks.umt.edu/tme
Let us know how access to this document benefits you.

Recommended Citation
LeBlanc, Manon (2023) "The Birth of a Review or (How Villani Both Reinforces and Challenges Our Beliefs About Mathematics) A Review of Cédric Villani's Birth of a Theorem: A Mathematical Adventure," *The Mathematics Enthusiast*: Vol. 20 : No. 1 , Article 22.
DOI: https://doi.org/10.54870/1551-3440.1604
Available at: https://scholarworks.umt.edu/tme/vol20/iss1/22

This Article is brought to you for free and open access by ScholarWorks at University of Montana. It has been accepted for inclusion in The Mathematics Enthusiast by an authorized editor of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

The Birth of a Review or (How Villani Both Reinforces and Challenges Our Beliefs About Mathematics)

A Review of Cédric Villani's *Birth of a Theorem: A Mathematical Adventure*

Manon LeBlanc[1]
Université de Moncton

In the book *Birth of a Theorem: A Mathematical Adventure*, Cédric Villani paints a portrait of himself, from the (dare I say) obsessed mathematician who works until the wee hours of the night, to the ordinary citizen, enjoying an impressive variety of music, to the slightly odd man sporting an ornamental spider daily. From Lyon to Paris, through the despair of not being able to eat quality cheese, Villani takes us on a journey, revisiting the path that led him to win the Fields Medal. He takes us back in time and punctuates his account with anecdotes or stories of mathematicians. We recognize well-known names (Nash, Poincaré, etc.) through stories that are less well-known. He lets us in his adventure, and we jump in with both feet (e.g., see Figure 1).

Figure 1. Beginning of the proof of theorem 7.7 (Villani, 2015, p. 144)

Theorem 7.7 (Growth control via integral inequalities). *Let* $f^0 = f^0(v)$ and $W = W(x)$ *satisfy condition* **(L)** *from Subsection 2.2 with constants* C_0, λ_0, κ; *in particular* $|\tilde{f}^0(\eta)| \leq C_0 e^{-2\pi\lambda_0|\eta|}$. *Let further*

$$C_W = \max\left\{\sum_{k\in\mathbb{Z}^d_*} |\hat{W}(k)|,\ \sup_{k\in\mathbb{Z}^d_*} |k|\,|\hat{W}(k)|\right\}.$$

Let $A \geq 0,\ \mu \geq 0,\ \lambda \in (0, \lambda^*]$ *with* $0 < \lambda^* < \lambda_0$. *Let* $(\Phi(k,t))_{k\in\mathbb{Z}^d_*,\, t\geq 0}$ *be a continuous function of* $t \geq 0$, *valued in* $\mathbb{C}^{\mathbb{Z}^d_*}$, *such that*

$$\forall t \geq 0,\quad \left\|\Phi(t) - \int_0^t K^0(t-\tau)\Phi(\tau)\,d\tau\right\|_{\lambda t+\mu}$$
$$\leq A + \int_0^t \left[K_0(t,\tau) + K_1(t,\tau) + \frac{c_0}{(1+\tau)^m}\right] \|\Phi(\tau)\|_{\lambda\tau+\mu}\,d\tau, \quad (7.22)$$

where $c_0 \geq 0$, $m > 1$, *and* $K_0(t,\tau)$, $K_1(t,\tau)$ *are nonnegative kernels. Let* $\varphi(t) = \|\Phi(t)\|_{\lambda t+\mu}$. *Then*

(i) Assume $\gamma > 1$ *and* $K_1 = cK^{(\alpha),\gamma}$ *for some* $c > 0$, $\alpha \in (0, \bar{\alpha}(\gamma))$, *where* $K^{(\alpha),\gamma}$ *is defined by*

$$K^{(\alpha),\gamma}(t,\tau) = (1+\tau)d \sup_{k\neq 0,\, \ell\neq 0} \frac{e^{-\alpha|\ell|}\, e^{-\alpha\left(\frac{t-\tau}{t}\right)|k-\ell|}\, e^{-\alpha|k(t-\tau)+\ell\tau|}}{1+|k-\ell|^\gamma},$$

[1] manon.leblanc@umoncton.ca

2023 © The Authors & Department of Mathematical Sciences, University of Montana

When I started reading this book, I thought about the many times we have heard that students need to act like mathematicians. Many students are not in the same frame of mind as mathematicians when doing mathematics. Some are afraid (afraid of failing or succeeding, afraid of not finding what they are looking for or finding what they are not looking for), while others are downright terrified. Is the emotional component sometimes so cumbersome that it cannot be overcome? Are mathematicians built differently? How are they able to handle the mathematical unknown? Can we really treat students as mathematicians? How many of us truly understand what they (the mathematicians) really do? Are their mathematics the same as the students'?

I realize that this kind of questioning can be greatly influenced by our beliefs about mathematics, and it is with this perspective that I continued my reading, wondering if his account would invalidate what the didactic world has been putting forward for decades or validate what many have been believing since their (often agonizing) school days.

Before proceeding further, I feel the need to point out that this book is paradoxical. It is about mathematics, but not really. I humbly confess that when reading some of the more mathematical passages (for an example, see Figure 1, the beginning of a proof that elegantly spans 11 pages), I felt like I was hearing the parents talking in a Charlie Brown cartoon. But even if, without mathematics, there is no story to tell, strangely enough, understanding them is not a prerequisite for enjoying this book. The story goes beyond mathematics, even if it is part of the story.

Think That Mathematicians Are Solitary "Creatures" That Rarely See the Light of Day? Think Again!

There has been an interest for some time in the image that people have of people who are interested in STEM and, more specifically, with mathematicians. These special beings that many have difficulty understanding because they have consciously chosen to make a living doing mathematics, and who are often met with a half-reverent, half-perplexed silence after telling someone how they make their living. People sometimes perceive them as obsessive (Cheryan *et al*, 2011), non-attractive (Bian, Leslie, & Cimpian, 2017; Garriott *et al*, 2017; Starr & Leaper, 2019), socially awkward (Cheryan *et al*, 2011; Garriott *et al*, 2017; Starr & Leaper, 2019), alone (Bian, Leslie, & Cimpian, 2017; Garriott *et al*, 2017; Towers *et al*, 2018; Starr & Leaper, 2019), mostly silent (Towers *et al*, 2018), and with a personal hygiene that leaves something to be desired (Garriott *et al*, 2017). Apparently, people who work with mathematics are far from having an enviable image in society.

But, contrary to popular belief, mathematicians are social beings. Villani puts this forward throughout his story, sharing with us the many exchanges he had with his sidekick Clément Mouhot. We listen carefully to their phone conversations, we watch for the next email to come in to read the content, we secretly attend conversations over meals, and we understand how important collaboration and communication are in the work of mathematicians. New ideas are being generated through collaboration. Progress is made more quickly, thanks to the criticism of others. If there was ever a good example of social constructivism, it is this one!

Think That Mathematicians Are Geniuses? Think Again!

The image of mathematicians presented in the book is more nuanced than the one often found in society. Although some passages reinforce certain widespread beliefs, others reconcile us with the idea that mathematicians are just like anyone else (or almost). For example, Villani describes Paul Erdős as a restless genius wanderer. Someone who wrote fifteen hundred articles (yes, you read that right!), walking around with suitcase in hand, worn out clothes on his back, without a family and without a job. On the other hand, the image of Villani that takes shape, from page to page, leads us to meet a man who has a unique sense of style, with his scarves and ornamental spiders, who shared his life with a woman, must sometimes take care of his children, likes to interact with people, and loves manga, music, and good food. But there are cracks in this quasi-normal image of the epicurean family man, who, by chance, is also a mathematician. For instance, we feel his despair when he describes the evening of January 28, 2009, as he locks himself in a room, in the dark, pacing around while he tries to generate an idea that clings just far enough for him to grasp, or when, nine months later, he states that Landau damping occupies his every waking moment. Suddenly, the line between genius and madness is getting thinner.

This image of being a genius (whether positive or negative, depending on how we define it) or possessing a natural talent for mathematics is often associated with mathematicians (Boaler *et al*, 2018; Storage *et al*, 2016). The problem is that it often reinforces the idea that only certain people can do mathematics. In French, the expression *avoir la bosse des maths* ("to have the mathematical bump") is used to talk about people who are naturally gifted in mathematics. It refers to an imaginary bump that experts look for but cannot find. Even though neuroscience research has shown that the brain can learn and develop mathematics pathways when learning occurs (Anderson *et al*, 2018; Boaler, 2016; Dweck, 2016), people keep believing in this innate ability required for doing math (Heyder *et al*, 2019). Such beliefs are detrimental to learning as they provide an easy excuse for those who are struggling and looking for a way out (Boaler *et al*, 2018). Villani (2015) points out that "neither genetic inheritance nor environment can explain everything. We should be grateful that this is so" (p. 71) and insists that what really makes a difference is having a positive attitude towards and being curious and passionate about mathematics... and a little luck! While this is consistent with the views of several mathematics educators, and many would wish that Villani's story was that of a child who had great difficulty in mathematics and worked his whole life to end up winning the Fields Medal, this is not the case. In fact, reading Villani's book, one might consider admitting that there is such a thing as a math bump, especially when he refers to a voice in his head that he attributes to divine intervention (after checking, such a direct line does not seem to be available with the usual phone plans).

> I went into semi-automatic pilot, drawing on the whole of my accumulated experience... but in order to be able to do this, first you've got to tap into a certain line – the famous direct line, the one that connects you to God, or at least the god of mathematics. Suddenly you hear a voice echoing in your head. It's not the sort of thing that happens every day, I grant you. But it *does* happen. (Villani, 2015, p. 142)

Like a lot of mathematics educators, Villani does not believe in geniuses. In an interview on French television, when told that he is a math genius, he responds by saying that "there are no geniuses, except those who come out of lamps, Aladdin, and all that" (Larmeaux, 2011), but the fact remains that he projects the image of a genius. And if you do not believe in geniuses, and your direct line to the "god of mathematics" is always busy, you can always, like Villani, rely on magic or miracles! Apparently, to do mathematics, you must be good at it, but also believe in miracles. More than once the author mentions something, not quite known, that helps him to move forward. Apparently, he is not the only one who associates magic with mathematics. Students do it too. However, in Villani's case, it is positive (everything falls into place as if by magic), whereas for students, it is rather that success is a matter of magic, because they more or less (less than more) understand what they are doing (Lafortune et Mongeau, 2003).

Despite all this talk about geniuses, divine intervention, and magic, Villani succeeds in presenting an image of a mathematician that, while not completely dissimilar to the one present in society, is more nuanced than the latter. Reading Villani's story, one is torn between taking mathematicians down from the pedestal on which they are often placed or adding a floor to the pedestal to raise them a little higher in people's esteem.

Think That Doing Mathematics Is Always Fun for Mathematicians? Think Again!

When it comes to mathematics, people often love it or hate it! Why is this the case? How can we explain this? Representations of mathematics are negatively tinged by certain beliefs, some of which have unfortunately persisted for several decades. These beliefs are influenced by school experiences, parents, and society in general (Lampert, 1990; Towers *et al*, 2018), and greatly influence learning (De Corte & Verschaffel, 2008; Mc Leod, 1992).

Usually, when we make the connection between mathematics and emotions, we often talk about the paralyzing anxiety experienced by some students. Yet, although Villani (2015) also experiences strong emotions, sometimes even associated with suffering ("The hope of discovery and the frustration of imperfection. [...] Happiness mixed with pain", p. 162), we quickly understand that this darkness is part of the mathematical research cycle. We follow him in his intellectual adventures, and we feel the doubt he feels, we imagine the depth of the concentration he shows, we understand his passion for the subject, and we suffer a little with him from the frustration he experiences. What a delightful cocktail!

Think Mathematics Are Dry and Rational? Think Again!

While mathematics is also often associated with learning procedures and memorization (Boaler *et al*, 2018; D'Entremont, 2011; Towers *et al*, 2018) and is therefore often perceived as dry and very linear, it is refreshing to hear Villani talk about intuition and hope. He insists on the importance of giving oneself time to go round and round, to better propel oneself forward. It is clear from Villani's account that ideas take time to mature, contradicting the popular belief that people who are good at mathematics are fast (Boaler, 2016). Furthermore, although he recognizes the beauty behind a well-structured proof, Villani (2015) feels some regret at not being able to share what, for him, is the most important thing: "Probably no one who read the article that finally appeared in *Acta Mathematica* had the least inkling of the euphoria I experienced that morning. Technique is the only thing that matters in a proof. It's a pity there's no place for the most important thing of all: illumination" (p. 143). Can this illumination that Villani refers to, this moment of pure exhilaration upon discovering a longed-for answer, this mathematical intellectual orgasm that occurs at the end of a long process, take place without the preliminaries (that is all the time invested before said discovery)? Can the school really provide such a stimulating (pun intended) environment for students, given its current format? To ask the question is to answer it.

Think We Have It All Figured Out? Think Again!

Villani (2015) also emphasizes the inestimable value of the ability to make connections between different areas of mathematics, a guiding principle found in many curricula. He even compares these connections to a game of Ping-Pong, where "every discovery you make on one side helps you discover something new on the other. The connections make it possible to see more of the landscape on both sides" (p. 135). As I read through the pages of the book, I came to wonder how we have managed to distort mathematics in this way in school. Why does it seem so difficult to get back to exploring, to discovering, to finding solutions to interesting problems rather than finding ways to get to the answer at the end of a book? Villani believes that the work of the mathematician is comparable to the work of a detective, where the reasoning to find the murderer is more important (and interesting) than the identity of the murderer. Mason (1994) talks about Ah! Ha!

moments. Villani sure had a lot of those to get to the final theorem! Mason (1994) also talks about the dynamic process of problem solving and the importance of backtracking to learn from mistakes. This notion of backtracking marks Villani's narrative and takes us away from the image of the wise (wo)man who knows everything. He speaks of returning to old ideas and revisiting them in a new light, of being willing to do work for nothing, but that serves a purpose, showing that mathematics does not indeed evolve in a linear fashion. While some students are ashamed of their mistakes, we know that they are an integral part of teaching and learning mathematics (Anderson *et al.*, 2018; Astolfi, 2006).

Mathematics educators have been saying all of this for decades, mathematicians have been doing it for centuries. Where, then, does the mathematics we love so much get lost on the way to school? How do we get students to re-discover mathematics for themselves? Villani has made a book of his mistakes, his trials, his questioning, his frustrations, his questions, etc. Reading it made me wonder what we really do with all this in the classroom. I admit that there are some great things being done, but it would be an illusion to pretend that there is not still a lot of work to do.

Think I Am Done? You Are Almost Right!

Although Villani does not quite succeed in diminishing the image of the mathematical genius with an invaluable gift for understanding the language that causes headaches for many, he does succeed in making us see mathematics; real mathematics, in their natural state. From now on, every time I hear someone say, "I don't understand math and I've never liked it," I will stop and ask myself what is behind that statement. Are they talking about the same mathematics as Villani, you and I are? I would like to think not.

Knowing the impact that beliefs can have on people's motivation to do mathematics (Garriott *et al*, 2017) or to pursue a career in mathematics, but even more importantly on the relationship that people have with mathematics, one may wonder whether Villani's book nourishes existing beliefs or weakens them. It is also worth asking what is the purpose of books that aim the popularization of mathematics? What do we want to popularize to whom? Who is the target audience and who is the audience reached? Are we preaching to the choir, which would mean that we are missing a great opportunity to really popularize mathematics? I do not know. All I know is that I enjoyed reading this book.

Think you will not like this book? Think again!

References

Anderson, R. K., Boaler, J., & Dieckmann, J. A. (2018). Achieving Elusive Teacher Change through Challenging Myths about Learning: A Blended Approach. *Education Sciences, 8*(3), 98. https://doi.org/10.3390/educsci8030098

Astolfi, J.-P. (2006). *L'erreur, un outil pour enseigner*. ESF éditeur.

Bian, L., Leslie, S. J., & Cimpian, A. (2017). Gender stereotypes about intellectual ability emerge early and influence children's interests. *Science, 355*(6323), 389–391.

Boaler, J. (2016). *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching*. Jossey-Bass.

Boaler, J., Dieckmann, J.A., Pérez-Núñez, G., Liu Sun, K., & Williams, C. (2018). *Changing Students Minds and Achievement in Mathematics: The Impact of a Free Online Student Course*. Frontiers in Education, 3. https://doi.org/10.3389/feduc.2018.00026

Cheryan, S., Siy, J. O., Vichayapai, M., Drury, B. J., & Kim, S. (2011). Do female and male role models who embody STEM stereotypes hinder women's anticipated success in STEM? *Social Psychological and Personality Science, 2*, 656–664. https://doi.org/10.1177/1948550611405218

D'Entremont, Y. (2011). Croyances mathématiques chez des futurs maîtres au primaire: une activité réflexive. *The Journal of Educational Thought, 45*(3), 239-253.

de Corte E. & Verschaffel, L. (2008). Apprendre et enseigner les mathématiques: un cadre conceptuel pour concevoir des environnements d'enseignement-apprentissage stimulants. In M. Crahay, L. Verschaffel, E. de Corte et J. Grégoire (eds), *Enseignement et apprentissage des mathématiques. Que disent les recherches psychopédagogiques?* (p. 25-53). Bruxelles : Éditions De Boeck Université.

Dweck, C. S. (2016). *Mindset: The New Psychology of Success. How We Can Learn to Fulfill Our Potential*. Ballantine Books.

Garriott, P.O., Hultgren, K.M., & Frazier, J. (2017). STEM Stereotypes and High School Students' Math/Science Career Goals. *Journal of Career Assessment, 25*(4), 585-600.

Heyder, A., Weidinger, A.F., Cimpian, A., & Steinmayr, R. (2020). Teachers' belief that math requires innate ability predicts lower intrinsic motivation among low-achieving students. *Learning and Instruction, 65*. https://doi.org/10.1016/j.learninstruc.2019.101220

Lafortune, L. et Mongeau, P. (2003). Approche des mathématiques par le dessin. Une analyse qualitative et quantitative de dessins. In L. Lafortune, C. Deaudelin, P.-A. Doudin, & D. Martin (Eds), *Conceptions, croyances et représentations en maths, sciences et technos* (pp. 91-120). Presses de l'Université du Québec.

Lampert, M. (1990). *When the Problem is Not the Question and the Solution is Not the Answer: Mathematical Knowing and Teaching. American Educational Research Journal, 27*(1), p. 29-63.

Larmeaux, M. (2011, February 24). *Cedric Villani, Médaille Fields 2010, nie la variabilité de l'intelligence*. [vidéo]. YouTube. https://www.bing.com/videos/search?q=villani+franck+dubosc&&view=detail&mid=2E5C8B6692E03F5C5EDF2E5C8B6692E03F5C5EDF&&FORM=VRDGAR&ru=%2Fvideos%2Fsearch%3Fq%3Dvillani%2Bfranck%2Bdubosc%26FORM%3DHDRSC4

Mason, J. (1994). *L'esprit mathématiques*. Modulo Éditeur inc.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D.A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). Macmillan.

Starr, C.R. & Leaper, C. (2019). Do adolescents' self-concepts moderate the relationship between STEM stereotypes and motivation? *Social Psychology of Education, 22*, 1109-1129.

Storage, D., Horne, Z., Cimpian, A., & Leslie, S. (2016). The frequency of "brilliant" and "genius" in teaching evaluations predicts the representation of women and African Americans across fields. *PLoS ONE, 11*(3), 17.

Towers, J., Takeuchi, M.A., & Martin, L.C. (2018). Examining contextual influences on students' emotional relationships with mathematics in the early years, *Research in Mathematics Education,* 20(2), 146-165. https://doi.org/10.1080/14794802.2018.1477058

Villani, C. (2015). *The Birth of a Theorem: a Mathematical Adventure*. Farrar, Straus and Giroux.