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A Review of David J. Hand's *The Improbability Principle*

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Have you ever struck up a conversation with a stranger in a coffee shop and been surprised to learn that you were born in the same week, but in different years? Or maybe you have that one annoying friend who always seems to win raffles and lotteries even though you never win anything, and you just can't figure out how they always get so lucky? Or perhaps, just for fun, you visited a psychic one day, and you were shocked by how much they were able to tell you about yourself. In *The Improbability Principle*, David J. Hand explains that, despite how unlikely these and other events may seem, they are in fact much more probable than we may think.

Hand begins by recounting the story of famed actor Anthony Hopkins traveling to London to purchase a copy of the novel *The Girl from Petrovka*, having recently secured the leading role in the upcoming film based on the book. Having failed to find a copy after hours of searching, Hopkins decided to return home, and while waiting for his train, he found a discarded copy of – you guessed it – *The Girl from Petrovka*! What's more, when Hopkins later had the opportunity to meet the author, he discovered that his found copy had in fact belonged to the author years before!

This and other stories of "simply unbelievable" coincidences are used to motivate a discussion of Borel's law, which states that "Events with a sufficiently small probability never occur." That is to say, certain events are so unlikely that it's unreasonable to expect them to occur. It would have been a little ridiculous, for instance, for Sir Hopkins to venture into London *expecting* to find the exact book he was looking for on the subway, let alone a copy that had been owned by the author himself. In the same way, we all know that traffic accidents are possible, but we don't let that knowledge stop us from going to

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the grocery store, because the probability of being involved in an accident is low enough that we don't worry much about it.

Yet, in *The Improbability Principle*, Hand describes many true events that seem wildly less likely than traffic accidents, including people winning the lottery multiple times and a stock market crash seemingly with a 1 in 10^{160} chance of occurring. And it seems like we're always hearing stories like these; events with a one-in-a-million chance of occurring seem like they happen all the time. If Borel's law is to be believed, then why do such unlikely events still happen, and why do they seem to happen so often? Far from disproving Borel's law, such events, Hand explains, are actually far more probable than they seem on the surface. By describing the components of his Improbability Principle, he gradually breaks down the misconceptions that lead us to believe such events are less likely than they really are.

Probability is Hard

Making judgments about probability is tough. That's not just true for the average person - even those who are skilled in computing probabilities often judge them incorrectly at first glance. As an example, when deciding which of two given events is more likely, many of us operate on intuition based on previous experiences or on what we might consider to be a more generic outcome (Tversky & Kahneman, 1974). Think about a multiple-choice quiz consisting of five questions with answer choices A, B, C, and D (Chernoff, 2012). Assuming the answer key is generated at random, which do you think is more likely to be the answer key: [A, C, D, B, C] or [C, C, C, C, B]? At a glance, the first key may seem more likely because there is a more even distribution among the answer choices, making it seem more representative of a "typical" answer key, or because it looks more like most answer keys we've seen in the past. But in reality, both answer keys are equally likely to occur at random – the surface features of the possible answer keys distract us from the underlying mathematical reality.

The idea that we make intuitive (and often inaccurate) judgments about probabilities even though we might have the capacity to make more rational (and more accurate) judgments is captured by the idea of dual processing. Dual process theories of cognition claim that the mind uses two different reasoning systems to make decisions – System 1, which is quick, intuitive, and effortless, and System 2, which is slow, reflective, and deliberate (Frankish, 2010; Kahneman, 2011). Judgments and decisions regulated by System 1 are automatic. If you hand me a cat and ask me if it's a table, most likely I won't even think about it before responding. I don't need to think very deeply about it because the answer seems clear, so System 1 can handle this judgment. But if you hold up a stool and ask me if it's a table, I might consider your question a little more carefully. I might think about other examples of things I'd call a table and how similar or different a stool is from those objects. I might even think about a definition for the word *table* and examine the stool to see if it meets the necessary criteria. System 2 kicks in here, because the situation requires a little more consideration.

System 1 thinking is constantly at work in our lives, and it saves us a lot of time. When I make toast for breakfast, I don't have to critically inspect every item in the kitchen to decide whether or not I've found the bread (although depending on how much coffee I've had and whether I'm wearing my glasses, it might not be a bad idea). In situations like the ones Hand describes in *The Improbability Principle*, System 1 judgments often dominate our reasoning as well. Instead of sitting down with a pencil and paper, considering all the possible factors influencing a given situation, and carefully grinding out probability calculations, we typically make snap assessments. We decide what seems right based on appearances and various heuristics. When we do this, we fail to consider all the factors that may influence the probability of a given event taking place, like how many opportunities there were for the event to occur, or how slight changes in circumstance can have substantial consequences for probability. Hand's writing calls the reader's attention to several such factors which, though easy to overlook, can significantly alter the true probability of an event, and he encourages the reader to pause for just a moment before making judgments to consider the possibilities.

The Improbability Principle

As a teacher, it's not uncommon that I'll have one or more students each semester who perform exceedingly well on the first exam or two, earning nearly perfect scores, and then suddenly an exam comes along on which their score is much worse. At first glance, this situation may seem troubling: my star pupil may be starting to slip. However, there is an aspect of probability, called *regression toward the mean*, at play here: the higher a student scores on an assessment, the more opportunities they have to score lower on the next assessment. For example, if my student earns a perfect score on their first exam, then they're almost guaranteed to earn a lower score on their second exam because any tiny error will bring down their score. Similarly, if they earn a zero percent on the first exam, then they're almost guaranteed to get a higher score on the second. Besides, it's not uncommon for exams to become more difficult as a school term progresses, so as time goes on, the probability increases that *any* student will perform a little worse than they have on previous assessments. So when a student who performs exceedingly well on an assessment doesn't do as well on the next assessment, it's not necessarily an indication that the student is slipping – it may just be a natural consequence of probability. We need more information than what we see on the surface.

The Improbability Principle calls our attention to probability concepts like regression to the mean that may be difficult to notice when they're influencing a situation. The Improbability Principle is really a combination of five "laws" author David Hand identifies as critical to correcting misconceptions in probability: The Law of Inevitability, the Law of Truly Large Numbers, the Law of Selection, the Law of the Probability Lever, and the Law of Near Enough. Alone, each of these laws has the potential to greatly alter the probability of an event, making it more likely than it would have been under slightly different circumstances. In reality though, as Hand points out, often two or more of these laws are at play in a given situation, and their effects combine to make the event even more likely. But it's very easy to let intuition take over and let System 1 dominate our thinking, and we often don't notice when these laws are at work. Hand's goal in *The Improbability Principle* is to make the reader aware of these laws, and in instances where a seemingly improbable event occurs, to consider whether the event might be more likely than it seems.

Hand begins with the Law of Inevitability, which he calls "a simple and often overlooked observation, and one that in a real sense underlies everything else: it's the simple fact that *something must happen*" (p. 75). Dice throws are a straightforward example: while the probability *against* any particular number coming up is 5/6, we nonetheless understand that some number *must* come up. So although it's unlikely that we'll be able to guess which number will come up on our next die roll, we are nonetheless unsurprised when *some* number comes up, regardless of what it is.

Despite its simplicity, Hand explains, the Law of Inevitability underlies a somewhat convincing stock-tipping scam. Choose your favorite stock from the stock market and suppose that, each week for five weeks, I've sent you a prediction (in advance) about whether the stock's value would rise or fall. And for five weeks, my predictions have been correct. Seems like a good record, right? I must have some insider information or just be a real pro at the stock market. So when I ask you on the sixth week if you'd like to give me some money to invest on your behalf, it seems crazy not to!

What you don't know, however, is that I've also been sending predictions to 31 other people. Knowing that the stock's value *must* either rise or fall, I sent 16 people a prediction stating the value would rise, and the other 16 got predictions stating the value would fall. Then, for the half I got right, I split them in half again, and sent 8 a prediction that the value would rise, and I sent the other 8 a prediction that the value would fall. By the time we get to week 5, you're the only one I've predicted correctly the whole time – but of course, you don't know that. Using only the simple fact that "something must happen," I've set up a pretty convincing scam I could use to bilk people out of some decent money. The Law of Near Enough is another of Hand's laws that seems obvious enough but can be tough to spot in practice. This one says that "*events which are sufficiently similar are regarded as identical*" (p. 164). Think about a standard deck of playing cards. If you choose one card at random, the probability of drawing the queen of hearts is 1/52. But the probability of drawing *any* queen is 4/52, and the probability of drawing any face card (jack, queen, or king) is 12/52. As your intuition might suggest, the probability gets larger as we relax our constraint.

Now think back to the question I asked in the introduction about chatting with a stranger and learning they were born in the same week you were. The probability of meeting someone who shares your *exact* birthday is 1/365 (remember that your birthday is already set, you just have to make sure the other person's birthday falls on yours). But if we relax our constraint and think about the probability of someone's birthday simply being in the same week as yours, the probability rises to just 1/52. We could take this further by thinking about being born during the same month – and you could pick any stranger off the street and be absolutely sure that you were both born sometime between January 1 and December 31. The Law of Near Enough, Hand explains, may indeed be responsible for results from experiments purporting to show evidence of clairvoyance or psychic ability as well.

The five laws that compose the Improbability Principle lie at the heart of many misunderstandings about probability. As Hand explains, when we hear about extremely improbable events happening over and over again, it is likely that one or more of these five laws is secretly at play.

Probability for the Layperson

Hand does an excellent job of distilling the complicated factors affecting probability to a short list of just five easily digestible laws, each of which he exemplifies with historical events or simple cases of dice throwing or coin flipping. Although I do not teach probability, many of the examples Hand uses to explicate probabilistic principles are examples I would present to students in an introductory course on probability. Most of the examples are understandable without much background knowledge of their contexts, and they are often presented conceptually in plain English, without the need for hefty computations. The text is light and readable, and Hand's somewhat dry sense of humor provides a refreshing break when the text does become a little dense. The text is a little repetitive in places, but this seems to be a deliberate effort on the author's part to provide numerous examples so the reader can see each law at work in various contexts. The same examples are sometimes used to illustrate the different laws that constitute the Improbability Principle, but as Hand makes clear throughout the text, sometimes multiple laws work together to substantially increase the probability of a particular event. The description of each law is also accompanied by numerous examples, which, though somewhat redundant, provides the reader with the freedom to not fully understand a particular example, knowing that there will be more examples from different contexts coming soon after.

One thing that makes this book a little bit of a challenge for the reader is that Hand often references aspects of the Improbability Principle before he has defined them, in several places referring the reader to chapters that have not yet come, leaving the reader somewhat in the dark. For example, in Chapter 5, Hand introduces the Law of Truly Large Numbers, which states that even if a certain event has a very small probability of occurring on any given trial, its probability of occurring *sometime* increases with the number of opportunities it has to occur. This law helps to explain why we see so many people who win the lottery twice, because many people play the lottery, and there are many lotteries to consider, making for many, many opportunities for someone to win twice. And, Hand explains, if we consider *second prize* lottery winners, then the effect is compounded by the Law of Near Enough – which is not introduced or defined until Chapter 8. The good thing, however, is that this is a book you can flip around in and not feel terribly lost. The reader could absolutely flip ahead to read a few pages of Chapter 8, then return to finish Chapter 5 without any difficulty.

Closing Thoughts

The Improbability Principle exposes readers to several essential probability concepts by taking an in-depth look at several seemingly unlikely scenarios. By taking this tack, author David J. Hand takes the traditional classroom probability lecture and flips it on its head. Instead of discussing equations and calculations and then showing applications, Hand presents the reader with a highly improbable situation that seems to repeat itself with alarming regularity, and he asks, "What's going on here?"

In so doing, *The Improbability Principle* accomplishes two goals. Most obviously, it teaches the reader a little something about important concepts in probability. Equally important, however, is the lesson it teaches about examining our assumptions and searching for alternative explanations. In judging probabilities and in many other facets of our lives, many of us make quick assessments that we never question. These automatic responses save us time, but what do we miss when we don't pause to think through a situation or to consider an alternative explanation for a strange situation or problem? By relying on these kinds of System 1 responses, we may leave ourselves susceptible to being misled – sometimes with disastrous consequences, as Hand recounts in some of the situations in his book. I'm reminded suddenly of the famous "proof" that 1=2. By relying on our algebraic instincts and reflexes, many of us can easily move through this proof from start to finish, not realizing that anything is wrong until it's too late, and we've suddenly proven a very troublesome result indeed. And the error is tricky to spot – it's only by switching to a more deliberate, critical form of thinking that most people realize what's happened.

So beyond teaching a few probability concepts, *The Improbability Principle* has an important lesson to teach about being on your guard and thinking critically before making decisions and before making judgments about stories we may hear on the news or from friends. Upon reading the sensational headline "Mathematics Student Proves 1=2!" we should not immediately jump to the conclusion that mathematics is unreliable. Rather, we should be critical and ask, "How did the student prove this?" In my opinion, this is the true pedagogical value of *The Improbability Principle*: to encourage us to read sensational headlines and stories with a critical mindset.

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