Baking Up Mathematics A Review of Eugenia Cheng’s How to Bake �: An Edible Exploration of the Mathematics of Mathematics

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A Review of Eugenia Cheng's *How to Bake π: An Edible Exploration of the Mathematics of Mathematics*

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Mathematics – the word alone can strike fear into the hearts of students, parents, and educators alike. This emotional response seems to be not only acceptable, but almost expected and, should you differ, then you are considered either a genius or a bit insane. "How could someone like math?", they think.

Mathematician, scientist and professor, Dr. Eugenia Cheng's book demonstrates that there is a general and fundamental misunderstanding of what mathematics truly is. It is not the monster under the bed, nor is it the most important factor in determining your intelligence: "*Whatever you think math is… let go of it now. This is going to be different*" (p. 3). This is the last sentence of the book's introduction and Cheng could not have written a more accurate set up for her very different book on mathematics.

By introducing unfamiliar concepts of mathematics into the more familiar context of a cookbook, Cheng is attempting to make mathematics more accessible to those who would not consider themselves mathematically inclined. It was my intent, as a mathematics educator and baking enthusiast, to try my hand at the content within.

Learning Through Baking

Part one of the book is very good at introducing many new topics to the reader to get them to follow aspects of mathematics. There is an over-arching theme to have the readers also understand the topics and how they might relate to their lives. Part two takes all the topics she has previously discussed and teaches the reader about category theory. She explains that category theory is "*[t]he process of working out exactly which parts of math are*

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easy, and the process of making as many parts of math [as easy] as possible" (p. 162). Using online dating, making a lasagna, money, and the human skeleton, Cheng correlates all the bits and pieces from Part one into a cohesive whole in Part two.

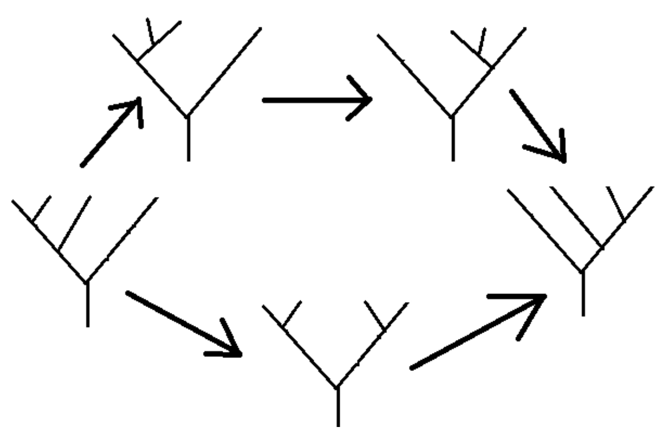
An aspect of this book I liked was the reader's ability to go at their own pace, since the chapters themselves don't need to be read in order. If a person is reading this, and they are a knowledgeable mathematician, this might seem like a pleasant read, and they might have gained some recipe knowledge. As well, if a reader is not particularly fond of mathematics, but wants to become more so, this book helps ease into different topics that might interest them while giving them interesting anecdotes and new recipes to try. Furthermore, the more mathematically intense proofs can be read over or skipped by those individuals, yet they will still comprehend Cheng's overall message. Maybe they will go back later, but the introduction has been made and, hopefully, they come away thinking that maybe math wasn't that bad, and they should see it in a more positive light.

As an educator, this book gave me new ways of approaching some of the mathematical content I teach my students, and how I can relate it to them in a different way than before. Since no two students learn exactly alike, in my experience no two students understand an anecdote the same way. While I do have a more than basic understanding of the concepts Dr. Cheng discusses in the book, I did not think to explain topology with a torus stuck on the inside of a sphere and how this is the same topographically as two interlocking circles. Cheng uses the idea of an empty blown up balloon with a ring donut on the inside. This is strikingly visual and helps explain it in a better context. Additionally, there are concepts in this book that all my students would understand. There are still others that students might have the knowledge to follow, for example, the 12-hour clock and how 11 + 3 = 2, but they lack the understanding of *why* it works, which could make for a wonderful lesson. Another example for my older students would be to understand her proof by contradiction for why $\sqrt{2}$ is, by mathematical definition, an irrational number, while my grade seven students would be able to recognize why a rational number is one that can be written as $\frac{a}{b}, b \neq 0$ but would not understand why $\sqrt{2}$ is not a rational number.

This book made me very excited for mathematics becoming more mainstream and more people, like Dr. Cheng, are making math relevant to the general population. As Cho and Kim (2010) stated: "the general public does not appreciate the assets of mathematics. [Mathematicians] have to arouse public interest towards mathematics" (p. 140). This is exactly what Dr. Cheng does with this book. While not every theorem or hypothesis was explained by the proof, I feel that in the context of what this book is planned for, that it is unnecessary and might even work against its purpose. The mathematical rigour is present when it needs to be and explained using a combination of mathematical language and layperson's terms to make the entire book more approachable to readers.

As an aside, Cheng's diagram about custard made the biology-enthusiast in me cry in confusion because that is *not* how phylogenetic trees are understood. She was explaining that category theory allows for each tree to be slightly different from each other, thus creating an interesting pentagonal shape, and this makes sense for that mathematical context. She is trying to demonstrate a category theory method, but it took me some time to come to understand what she was demonstrating, as these diagrams did not resemble what I had come to know as phylogenetic trees. So, it was an interesting moment where two subjects can use the same model and come out with different uses – sort of like how you can use a blender to make soups or purees, yet you can also use it as an emulsifier or a grinder. This is another funny example of how context matters when applying knowledge to reach understanding.

Figure 1. Photo rendition of the image on page 234 in Eugenia Cheng's book, *How to Bake π*



How to Learn: Mathematics

Cheng presents many aspects of how mathematics is learned and finishes with how category puts all this information together and simplifies them. Mathematics starts with the idea of Abstraction, which is the idea that all concepts can be thought of in an abstract way. This involves using your imagination with mathematical ideas until you run into a counter argument and let concepts go from the concrete to the abstract. There are different levels of abstraction; my favourite example being that we have a definition of a perfect square and a straight line, although these do not exist in the natural world, yet we take them as valid concepts.

Next, Cheng brings us into Principles which begins with *Conference Chocolate Cake*. This is where we get a taste for the mathematical rules that govern numbers. Some of these we learned in school: we can add, subtract, multiply and divide numbers; if you add zero to a number it remains the same, and if you multiply a number by one it also stays the same; and many others. Without an understanding of the basic guidelines, there is no way for a person to understand the ways to continue mastering this subject. This would be like not following the basic guidelines of baking: measuring your ingredients, preheating the oven, and checking the temperature of your ingredients is vital. Without these basics, it would quickly become difficult to continue baking more complex recipes.

Third, the Process of mathematics is very important; this is the ability to explain a conclusion using accurate mathematical concepts. Without this burden of proof, you would have a generalization instead of an agreement. Cheng does a very good job explaining this by giving the reader two different methods of subtraction fractions; one is the correct way and the other wrong. The answers were the same, but the methodology was not making it a "lucky guess"; this becomes important when discussing more difficult math concepts since it relies on logic rather than change. This is the same in baking. You might magically come up with an amazing dessert, but if you do not understand the reason that the recipe works, you may have a difficult time repeating the recipe or continuing your culinary adventure to more difficult techniques.

Finally, we discuss the importance of the role of Generalization in mathematics. In Chang's explanation, this is where we take a rule and start to slowly relax it a little. The example that Cheng uses is how you start at the definition of a square (a square has all four sides the same length, and all four angles the same (Cheng, 2015, p. 95)) and end with a generalized statement for all quadrilaterals (a quadrilateral is any old shape with four sides (Cheng, 2015, p. 96)) by "relaxing" the rules around the lengths of the sides and the types of angles it must have. This has also happened in baking and there are many examples of this. One of the first things I learned to make was pie. When I asked what it was, I was told a pie was a pastry that had any type of filling but that it must have a top and base of pastry. Simple enough, right? This is when I started baking cherry and apple pies a lot. As I grew up, and my tastes expanded, I realized that this definition did not seem to fit into all the pies I was making. My pumpkin and lemon meringue pie have no top of pastry. So now a pie is any dessert that has a filling with a base of pastry with or without a topping for the filling. This generalization of the rule for pie allows me to now have apple crisp, Bakewell tarts, egg pie, pot pie, and treacle tart described as pies.

As an educator, the ways that this book has laid out those four concepts has given me inspiration for new activities in my teaching that I did not have before. For example, the students that want to learn the importance of probability - it is usually explained using the weather or winning the lottery. For some students, it would be more relatable to learning about Arrow's Theorem, as Cheng discusses, and how it states that "if there are more than two people (or things) to vote for, then there is *no fair voting system*" (p. 122).

When viewing mathematical learning through the integration of abstraction, principles, process, and generalization, we can see how the mathematics curriculum tries to integrate all four of these into every learning objective for teachers to teach. These concepts are not taught apart from one another but are considered part of the whole. Here is an example: Learning about algebra. Algebra is the study of how numbers and quantities relate to one another. In the concrete world, this could be how to find the maximum number of slices that you can cut a 9" round cake into, as Cheng explains in her example of abstraction. The abstract is how to represent this relationship of number of cuts to the maximum number of slices of cake. To do this, we would use the principles of arithmetic and demonstrate our process to get to our generalized statement (the equation).

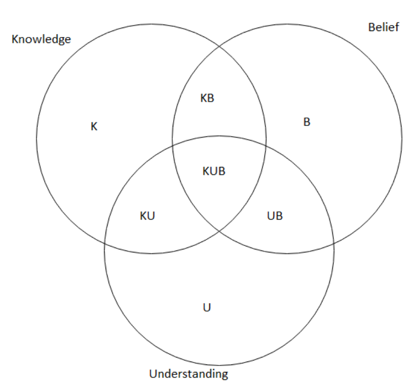
Additionally, the visuals in the book help explain many of the more abstract mathematics and is a helpful tool for educators, especially since explaining in the written word can be difficult.

Lastly, Chapter eight is explaining how students can gain an appreciation for math by understanding its applications into their daily lives. To me, this is how we can explain the exact steps in a recipe to anyone, but to truly have them appreciate what they have learned, they need to understand it on a more global scale. Cheng explains mathematics using many questions I have heard before and sometimes cannot answer. Her answer, at the end of the chapter, sums mathematics up perfectly: "So: math is easy, life is hard, therefore math isn't life. [...] The pursuit of mathematics is the process of working out exactly what is easy, and the process of making as many things easy as possible" (p. 156).

Trinity of Truth

Cheng's section on *Trinity of Truth* from pages 266 – 270 demonstrate a Venn diagram and how belief, knowledge, and understanding fit into mathematical understanding though she explains it from a personal point of view. I will try to explain it from a math educator point of view and how I believe students will interpret it.

Figure 2. Photo rendition of the image on page 267 in Eugenia Cheng's book, How to Bake π



B is where you believe what your teacher is teaching you and you cannot justify the *why* yet. For me, this is when I was first introduced to any concept in mathematics. I was told the way to do a formula and I did it that way. There was no deviating from it, and it must be done in the exact way. I could not tell you why, so I had to go on faith that I was taught it correctly.

KB is where you know and believe something, but you still don't understand it. For me, this was when I can recognize that even numbers were divisible by two with no remainders. This makes sense, but I could not understand *why* that always worked. We are still learning how the tools work, and sometimes we don't know *why* we are doing a thing, just that we must do it. This can also be when we are taught how to divide fractions. We follow the set "formula" for how to do it and we know how to apply it. We do not understand why we are doing so but we know how.

Lastly, KUB is the moment when students put all the information together. This is when students can connect what they know and believe to different problems, in different contexts, and in real-life applications. This is especially important because educators "would like to inform students that mathematics is not a foreign subject, but it is always around us to be used somewhere in our daily life" (Cho and Kim, 2010, p. 133). When students get to the point of accessing the KUB part of mathematical understanding, educators have succeeded.

Is it a Mathematical Cookbook?

I would disagree with the statement that this is a mathematical cookbook due to the ratio of baking anecdotes to other topics. Having recipes at the beginning of every chapter, and then discussing them during the introduction to the topic at hand serves to grab a reader's attention and give a quick view of what they are going to learn about in that chapter. I found that there were more analogies about other parts of her life than her baking life throughout the text. As such, I consider the baking element to be more of a framing device on Cheng's end to make the subject of mathematics more figuratively and literally palatable for the layperson. It is an extremely effective framing device, but the recipes exist to relate to the mathematics and cannot carry the book by themselves. After all, the slogan is "An Edible Exploration of the Mathematics of Mathematics" and not "A Mathematical Exploration of the Edible Matter."

Now, am I going to try these recipes? Absolutely! Part of my baking life is trying new recipes all the time. The first one? Probably the Olive Oil Plum Cake. It intrigues me because this recipe is the most successful example in the book that truly appeals to me. As a baker, I want to try this recipe that is taking it as far away from a standard cake recipe as you can while still having the final product be recognized as a cake. My existing knowledge of baking and cakes tells me this combination of flavours is going to taste delicious, and that the texture might come out like an upside-down cake meeting a spongey cookie. As a math enthusiast, I relish understanding how baking this gluten-free, dairy-free, sugar-free, and paleo-compatible recipe can be linked to ideas of generalization and geometry. As an educator, I am excited to share this example and hopefully watch a similar understanding happen for my students. As a layperson, I am disappointed that I cannot share this with my work colleagues as it contains nuts which are banned in most schools, but I can still be excited to taste it.

Conclusion

As Eugenia Cheng puts it: ""Knowledge is power," or so the adage goes. But understanding is more powerful power" (p. 278). This book is about using a different context to give the power of understanding mathematics to a wide range of readers. An example of this would be subjective probability which can be defined as "[p]robabilities quantify not events but *our information* about events" (Chernoff & Chernoff, 2015, p. 31). When we are told that there is a 30% chance of rain, this does not tell us just about the chance of rain but gives us information about the conditions leading up to and after the rain, and how humid it might be, etc. This is understanding what that probability means and not just knowing what they said or whether or not you believe it will rain.

While the slogan might lead a person to believe that this is a baking book with some mathematical content, it is really the opposite of that. *How to Bake with π* succeeds in its goal to get the reader to consider mathematics in a way that can be entirely new to them. This book managed to engage me as a mathematician, educator, and baker - thereby occupying a very particular niche. Additionally, this content coming from a female in a male-dominated field is personally significant and relatable to so many others.

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