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Marianne Nolte

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Marianne Nolte
University of Hamburg

Abstract: This article gives an overview of questions on diagnostics and procedures of high mathematical talent. Various methods such as intelligence tests, school achievement tests and checklists are presented and discussed. The conclusions favor multidimensional and multi-step approaches with a focus on special mathematical tests. As an example the approach of identifying children with a high mathematical potential used in the PriMa project at the University of Hamburg should illustrate an implementation of the issues raised.

Keywords: Diagnostic, high mathematical potential, procedure of identification

1 Introduction

Questions about the identification of high mathematical potential open a complex and broad field of questions and research. It is an undisputed fact that the further development of societies needs highly educated people (Lubinski & Benbow, 2006). However, this is no easy task: Students of today must be educated for tasks in the future, which are not yet known today.

Fostering does not always need a diagnostic process in advance. Differentiation as one of the first possibilities to foster with e.g. open-ended problems supports all students and may lead to the recognition of unexpected high abilities (Nolte & Pamperien, 2017a). However, not all students can be recognized in classrooms and not all students can develop their high potential to an appropriate level in regular classroom.

But, how shall an identification process be constructed? Any diagnostic process is prone to errors. Today in accordance with many researchers, multidimensional approaches are regarded as common and appropriate to avoid mistakes (Heller, 2004; Heller & Perleth, 2008; Heller & Schofield, 2008; Kroesbergen et al., 2016). Teacher, parents and peer nominations often based on check lists combined with achievement tests and intelligence tests are the most common approaches. In addition, personal traits like interest, endurance and motivation are used to identify high potential. However, each of these points alone is not sufficient in a diagnostic process. Before we discuss approaches to identify mathematically promising students we should take into account what questions an identification process should answer (Heller & Schofield, 2008).

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2 What is to be identified?

Many authors underline the importance of adapting the identification instruments to the purpose of the identification (Benbow & Minor, 1990; Cai et al., 1996; Lohman, 2005). Selecting students for competitions in mental calculation need other forms than a talent search for fostering problem solving (Sak, 2008). Problem solving is the focus in this article. General observations about a high potential point to the speed of working on mathematics, on the importance of working memory, to the efficiency of working on information and to the manageable complexity (Kroesbergen & Dijk, 2015; Neubauer et al., 2002). These aspects are implicitly effective in working on mathematical problems.

Going into detail, one approach for investigating hints for a high mathematical potential lies in analyzing the character of mathematical activities while solving problems. Krutetskii (1962, 1976) was a pioneer in this research. Later, many researchers confirmed e.g. abstract and generalized thinking, a reduction of information via curtailment, flexibility in handling mathematical information or reversibility as essential hints for a high mathematical potential (Coxbill et al., 2013; Kießwetter, 1985, 1994; Kontoyianni et al., 2013; Pitta-Pantazi et al., 2011; Sak, 2008; Singer et al., 2017; Sriraman, 2003; Wagner & Zimmermann, 1986). Because of the complexity of the topic, I would like to emphasize only some aspects. First of all, the character of the problems is significant for evaluating the relevance of activities as hints for a mathematical potential (Nolte, 2012b). Furthermore, also the kind of thinking processes needs deeper discussions (see e.g. Aßmus, 2017; Aßmus & Förster, 2012; Fritzlar, 2019 who worked on the role of thinking processes like reversibility or recognition of analogies). Kießwetter (1985) designates so called patterns of action that include the above mentioned aspects. Further patterns of action are e.g. organizing material in order to recognize patterns or reduction of complexity through supersigns (chunks). If students show patterns of action in complex learning environments he regards these as hints for mathematical potential. Using these in appropriate tests, a ranking may be helpful. Vilkomir & O'Donoghue (2009) developed a ranking based on student's reversibility, flexibility and ability to generalize as important cognitive components. They describe these components of mathematical ability starting from incapable to very capable students (p. 188ff), thus referring to an important point: we can observe many of the traits shown by gifted students also in students with lower potential, even in low achieving students. Therefore, Nolte (2012b) underlines that the level of challenge is essential for evaluating whether thinking skills point to high mathematical potential. Sriraman's (2002, 2003) studies provide an existence proof that more complex combinatorial problems may be used - with a call for a need to create a diagnostic tool for problem complexity for gifted students- this suggests the need for compatibility with psychological theories on the developmental stages that may be encountered, which can be expressed in German as *eine schöne begegnung*. However, such a link is difficult to establish since the Piagetian theory of linking psychological developmental structures to mathematical structures is tenuous at best. Another possibility lies in diagnostic methods, which are constructed based on psychological theories about the structure of the mind. An example for these methods is given by Kontoyianni et al. (2013) and Pitta-Pantazi (et al., 2011) who developed mathematical tasks to measure several abilities "to focus on quantitative properties" (Kontoyianni et al., 2013 p.298), the ability to focus on "causal ability" or "spatial ability" (p. 299), "qualitative ability" and "inductive and deductive ability" (p. 299f). All these skills are fundamental in mathematical activities. Although this approach can be used with different contents, it is more content specific than the approach based on patterns of action.

3 How can gifted and talented students be identified?

3.1 Special tests on mathematics

Today it is accepted, that cognitive potentials are domain specific, thus, "Gifted students should be selected, therefore, for special programs on the basis of having qualities that match the objectives of the program" (Benbow & Minor, 1990 p. 21). Consequently, special tests should test higher order mathematical thinking skills, complex problems that include building and proving of hypotheses, generalization, offer the possibility to show flexibility. Some of the problems cover several of these aspects; failing this, different problems should cover the range of aspects. To give an example for complex problems, which challenge students at primary grade level (8 years old), "Dog walking" is presented:

> Because the neighbor has broken his leg, he asks Susi if she can walk the dog. He offers to pay her for it. On the 1st day 1ct, on the 2nd day 2 ct. and so on. Every day the sum doubles. Her brother says, take every day 1€. That is better. Is he right? Can you explain this? (Nolte 2004).

Tasks based on problems like this are complex for this age level. Different ways of working on them are possible. Thus, they are harder to assess. However, they give much information about a mathematical potential. Eight years old students who recognize that the sum of money received from the previous days is always 1ct less than the double of the payment of the current day use a structure that shortens the solving process decisively. Explaining their considerations is a challenge at this age. However, they can solve the task by adding one number after the next and come to an answer in this manner. Some students may use this way to meet the expectation of a teacher. Talking with them about their solution procedure shows repeatedly that they also know other ways to solve the problem. That point to another aspect that makes the evaluation of test results difficult. However, complex and challenging problems that can be solved on different ways are often used in fostering processes. Therefore, they are important tools in identification processes. Furthermore, tasks like this can be regarded as first steps on the way to theory-building processes that are characteristic for the research work of mathematicians (Kießwetter 2006, Fritzlar, 2008) and with this can be used in fostering processes that simulate this work (Fritzlar & Nolte 2019).

3.2 Competitions

In many countries, competitions are used to identify a high mathematical potential. With competitions, students show that they understand a mathematical question very quickly. Especially participation in Mathematical Olympiads shows interest, endurance and a high level of mathematical knowledge. However, not all students with a high potential like competitions.

3.3 Achievement Tests

All tests measure a kind of achievement, but high achievements can only be considered as an indication of high talent. Rost (2000) makes a difference between high performing and gifted students. Students who work hard or learn from elder siblings may show unexpected high performance. If they are wrongly identified as highly gifted and are challenged accordingly, this can result in a loss of interest and motivation.

One of the most famous achievement tests is the College Board's Scholastic Aptitude Test (SAT) with seventh and eighth graders. Stanley observed that gifted students “at age 12 or younger score well on the mathematical sections of the College Board’s Scholastic Aptitude Test (SAT-M)” (Stanley, 1988 p. 205). Reasons for this may be the high speed of learning mathematics as a trait of students with a high potential. The SAT was very successful.

> “It has not only been shown to be useful initially, but has also been validated over a long-term basis (Benbow, 1981). Moreover, duplication of the SMPY model has been done at Duke University for 16 states.” (Stanley & Benbow, 1982 p. 5).

Important here is the aspect of acceleration. Scholastic achievement tests without acceleration may have a ceiling effect. However, although achievement tests can identify high performing students they may not recognize all promising students. Many researchers describe difficulties in identifying students who come from educationally disadvantaged families. Furthermore, Kroesbergen et al. (2016) point out that promising students who have not developed their potential "will not be considered high achievers, leading to a lack of teacher identification and nomination" (p. 18).

Taking into account, the different qualities of teaching too, not all students have the possibility to acquire contents at a high level. Thus, performance tests also reflect the learning conditions of the students.

Therefore, scholastic achievement tests can be a supplement but are not sufficient.

3.4 Intelligence tests

One of the questions, which arise with intelligence tests, is how far they are applicable to predict mathematical abilities. Problem solving processes and in general, working on mathematics, need cognitive components that are also assessed in intelligence tests. „Nearly all researchers on intelligence define „abstract and logical reasoning", „problem solving ability" and „capacity to acquire new knowledge" as central elements of „intelligence" (Rost, 2009 p. 11). For a long time intelligence tests were the most important instrument to measure giftedness.

> "A child was labeled as gifted and talented by a cutoff score on an intelligence test, which promoted an absolutist view of giftedness. All other children who did not achieve the cutoff score were viewed as "not gifted."" (Brown et al., 2005 p. 69).

This position was one of the reasons for critical discussions about the informative value of intelligence tests. One critical aspect lies in the neglect of unexpected solutions. For example, starting parts of number sequences can be continued differently. Mathematicians know that the starting part of a sequence is not equivalent with a definition of the sequence. This is not a common knowledge of psychologists. Therefore, under a mathematical perspective, the solution expected by test designers often is not the only possible (Käpnick, 1998; Nolte, 2004b).

Furthermore, according to Holling et al. (2004) even intelligence tests may show ceiling effects. They complain that most of the norm-referenced tests contain too few or no sufficiently difficult tasks. Therefore, the measurement accuracy in the upper range is very low.

Another question arises with the specific tasks used in intelligence tests. Waldmann & Weinert (1990) underline that the way and level on which cognitive skills are used cannot be separated from the character of the tasks. Although intelligence tests claim to measure the same cognitive components, their use in complex problems is quite different. Therefore, one should ask whether tasks in intelligence tests can really represent the complexity of considerations needed to solve mathematical problems. A comparison between the results of an intelligence test (CFT 20R) and those of a special test of mathematics for highly able primary grade students has found medium correlations, which indicates that both tests measure intelligence (Nolte, 2012a). Furthermore, the results of intelligence tests differ between tests and they are not as stable as many think. Although the IQ of students becomes more and more stable (Rost & Sparfeldt, 2017), especially with young students and even with teenage students the result may change during the developmental process (Hany, 2002; Ramsden et al., 2011).

Success in mathematics depends on more than intelligence. Thus, personal traits and intrapersonal variables (Gagné, 2004), interest and motivation beside environmental factors play an essential role in the development of all students, also of students with a high mathematical potential. It is generally agreed in research that a high mathematical potential is a multidimensional construct. Therefore some of the tests measure intellectual, social, and creative abilities as well as relevant personality and social moderators such as interest, motivation, and self-concept (Herrmann & Nevo, 2011).

These are some examples of considerations that have led to a critical view of intelligence tests.

3.5 Actual positions towards Intelligence Tests

Many intelligence tests include several parts with different emphases. An overall IQ gives much less information than looking at the subtests. By using differentiated analyzes the diagnostic value of intelligence tests becomes clearer. Also because of this reason various researchers underline the importance of intelligence tests (Holling et al., 2004; Rost & Sparfeldt, 2017; Warne, 2016). Thus, Warne (2016) claims: “However, it is my position in this article that gifted education researchers and practitioners should reembrace the concept of human intelligence" (p. 4). General high ability as a hint for mathematical giftedness was also shown by Krüger et al. (2019):

> "The present results indicate that the group of the mathematically talented children have also high scores in general intellectual ability. The average of the general intelligence (FSIQ) was 132.7 and ranged from 119 to 150" (p. 364).

That intelligence is important is also underlined by Lubinski & Benbow (2006) who show that even among the highly gifted differences in performance depending on IQ can be observed. This finding is in line with our personal experiences.

3.6 Checklists and teacher nomination

Teachers usually are not educated to identify mathematically gifted students. Furthermore, if we take into account the wildly spread definition orientated on IQ-testing, only about 2% of the population gets a test result of 130 and more. This leads to a restriction of teachers experience with highly able students. Therefore, teachers can recognize a high potential mostly if the academic results of students are at a high level (Hany, 1999; Hany, 1998; Kroesbergen et al., 2016). Although teachers' judgements are partly regarded very critically, the value of daily observations in the classroom should not be underestimated. However, what we need are interesting problems, curricula, and teachers who are familiar with complex problems.

If the students work on reasonably challenging tasks teachers may assess their potential by sensitive observation of students' working behavior (Nolte & Kießwetter, 1996). Checklists can support this assessment. However, Buch et al. (2006) point out that most of the checklists are too general. Thus, it is difficult for parents and teachers to get reliable impressions. This may be different if experts use more differentiated checklists. In our talent search process[1] with third graders (see Nolte, 2004a) during trial lessons as the first step in the process trained tutors use checklists to observe task specific thinking skills as hints for a mathematical potential. They checked whether students recognize special patterns, e.g. symmetries, whether they can explain their ideas, build hypotheses and prove them. Pamperien (2021) investigated the correlation between the results of the special mathematics test we developed to identify promising students and the observations of trained tutors in trial lessons. The results show that these tutors are capable

[1] This is described further below.

of evaluating the problem solving process with checklists very successfully. Because we are convinced of the importance of observation skills, we trained prospective teachers to identify patterns of action using checklists in small groups. As results, we observe a growing consciousness for the necessity to be familiar with problems and students solution spaces. The prospective teachers describe the checklists as a helpful tool. Nevertheless, as novices they struggle with the variety of tasks they are confronted with in classroom. But, many of them underlined that the necessary subject-specific preparation enabled them to take a differentiated look at the children's thought processes and thus to ask more specific questions.

4 Taken together

A search for talents poses the risk that students may be wrongly classified as especially talented or that students talents are not recognized. Every diagnostic process can be deficient. It is therefore important to keep the error rate as low as possible.

Multidimensional and multi-step approaches can contribute to this. Teacher training can help teachers to identify higher order thinking skills in student’s work that indicate particular giftedness. Therefore, they should be familiar with appropriately interesting problems that challenge students at different levels. Hopefully that way promising students who are not identified as gifted until now may be motivated and develop their mathematical achievement (Sheffield, 2017). Including achievement tests is also proposed (Kroesbergen et al., 2016). Although scholastic achievement tests do not measure high mathematical potential, without a certain level of scholastic achievement students may not recognize patterns and structures, reduce information via curtailment or handle it flexible. Intelligence tests can show strengths and weaknesses and in this manner, give hints for support in fostering processes.

However, no identification process can do without a special mathematics test, which contains the kinds of problems that make it possible to show a high potential.

5 What do we do in our project?

Since 1999 we foster mathematically gifted students starting in grade three within the PriMa-project[2] (*Pri*mary grade students on their way towards *Ma*thematics). We have conducted a talent search process with third graders (8 or 9 years olds). We send invitation letters to all primary schools in Hamburg, recommending that schools distribute this in the classrooms without preselection. Teacher's recommendations do not necessarily lead to an appropriate identification of students with a very high potential and performance at school is not necessarily in parallel with a high mathematical potential. Scholastic achievement tests have a ceiling effect, that means they did not differentiate enough between highly able students (Nolte & Pamperien, 2014) and they usually do not test problem solving competences.

On average every year about 500 students participate at the beginning in our talent search process, about 350 students participate until the end. The procedure contains trial lessons, a mathematics test and an intelligence test.

5.1 About the trial lessons

As first step we offer trial lessons in groups of up to 25 students. The tasks are far more complex than they usually get in school (see example above "dog walking". On Friday afternoon, they work on one problem for about 90 minutes, on Saturday morning they get three problems. The aims are

1. to give an impression of the kinds of problems we use in the fostering program so that students can decide whether they want to continue (self evaluation),

   In our fostering program we use so called *progressive research problems (PRP)*, investigations which simulate the work of researching mathematicians. It is self-evident that at primary grade level these can only be steps towards an enculturation in typical mathematical activities and thinking processes.

2. to compensate the different experiences in previous math lessons,

[2] PriMa is a cooperation project of the *Hamburger Behörde für Schule und Berufsbildung* (Authority for Schools and Vocational Training), the *William-Stern Society* (Hamburg) and the *University of Hamburg*. (for further information you are invited to visit the website www.prima-mathematik.uni-hamburg.de)

This is an important aim for offering trial lessons. Although problem solving and especially argumentation are goals in curricula even at primary grade level, they are often neglected in regular lessons. That is why it is necessary to show students how to work on complex questions and so to prepare them to the mathematics test.

3. to prepare students to the mathematics test,

   Because students get short questions to answer in regular tests our tasks may provoke uncertainty. So the duration alone can unsettle the children. The experience that also others struggle with our problems may encourage them. However also an important goal is to show how students can frame an answer and how to defend their ideas.

4. to get an impression of the students way of problem solving, motivation and general behavior.

   During the trial lessons, trained tutors use observation scales that are specific for the respective task. We discuss every child on Friday evening and Saturday afternoon.

The last step is a language free intelligence test (CFT 20 R, Weiß, 2008). After these steps we compare the results of every step and invite 50 students to participate in our fostering program at the university of Hamburg. In parallel in many schools math circles offer fostering programs so that all students who take part in the talent search process until the end get a place, either at the university or at a school.

5.2 About the mathematics test

As second step students get a mathematics test which is based on arithmetic and geometric tasks. Because we want the students to show "qualities that match the objectives of the program" (Benbow & Minor, 1990 p. 21), also the test is constructed on that idea. The tasks offer the possibility to show the abovementioned patterns of action and again are complex. The students get between 15 to 30 minutes to work on one question.

5.3 About the intelligence test

We use a culture fair test, knowing that even a language free test cannot compensate language barriers entirely. Because of our observations with individual cases, which showed that students even with an average IQ can participate in our project successfully, we compared the data of the intelligence test with the results of the mathematics test over nine years. The data of 1,663 students were used because they were complete (Nolte, 2012a). The results strengthened our opinion to value the results of the mathematics test higher than the results of the IQ test, especially in cases of unclear results. Although the data of the mathematics test give us important information about the mathematical potential, the results of the intelligence tests are an essential addition to all the other collected data.

6 During the covid 19 pandemic

Due to the pandemic, we had to adapt the whole process to digital versions during the last years. We had to develop alternatives for every step. For the students the trial lessons are very important for self-evaluation as well as for a training to the unknown format of the mathematics test. We could not just send tasks from our mathematics test because we did not want them to be spread among parents and teachers. Furthermore, we prepared the tasks in order to prevent the children from asking questions and to enable them to work independently. This is a point not to be overlooked, because we do not want to make the participation dependent on the level of education of the parental home. Thus, we selected problems from our project, one with a focus on numbers and one with a focus on geometrical patterns. The students got them one by one. In the first year only about 200 students participated. So it was possible to give all children a personal feed back to their solving process and their results. We made it a point to draw their attention to completeness of their findings, possible patterns, explanations and justifications. The focus was also on encouraging the students by underlining their own positive procedures and findings. Instead of an intelligence test we used a test about motivation (NFC-KIDS, (Preckel & Strobel, 2017). Because participation in this test was voluntary, we were only able to analyze data from 82 children. As a result we observed a positive medium monotonic relationship between the scores on the progressive researcher tasks and the score on the NFC ($r" = 0.487$) (Schröder, 2021). However, it is to question whether the NFC is meaningful for doing mathematics because in general the questions are not specific for mathematics. So far we can say that an intelligence test is more helpful in a talent search process.

In 2022 about 900 students registered for the talent search process and more than 600 sent us their results. Therefore, we had to change the process. Instead of a personal feedback we invited the students to digital lessons in small group and talked with them about the results of the problem.

First problem

The first problem is based on the idea of the Fibonacci sequence. Starting with two numbers and their sum each following number is the sum of the two that precede it. The starting numbers can vary and given is the number of steps, which lead to a result:

Figure 1: Problem 1

A	*B*	1. Step	2. Step	*Result*
1	3	4	7	11

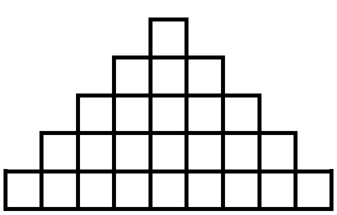
After some examples to make the structure of the problem clear, the students got tables similar to the above and questions:

1. Is it possible to get every even number? Can you see a trick? Explain it.
2. Find with this trick 1000!
3. Find starting numbers for all numbers up to 13!
4. Is it possible to find all other numbers after 13? Explain your idea!
5. Find all possible starting numbers for the result 29! How many solutions did you find? Are there more? Explain your ideas!

Second problem

The procedure for the 2nd tasks could be maintained. Following the closing date, the children received a link to a video that explained some of the many possible solutions. The aim of the second problem is to encourage the students to take different perspectives on the given pattern. Like many of the problems, we use solutions, which can be worked on at different levels. Some of the students count the squares. Because many of them are used to stop working on a question after they got the first solution it is easy to underestimate their creative potential. Due to this, we ask explicitly for different solving possibilities.

Figure 2: Problem 2



1. How many squares does this figure consist of?

   How did you find this? Explain your way.

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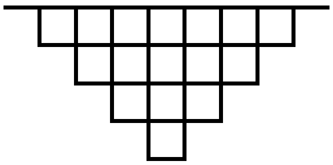
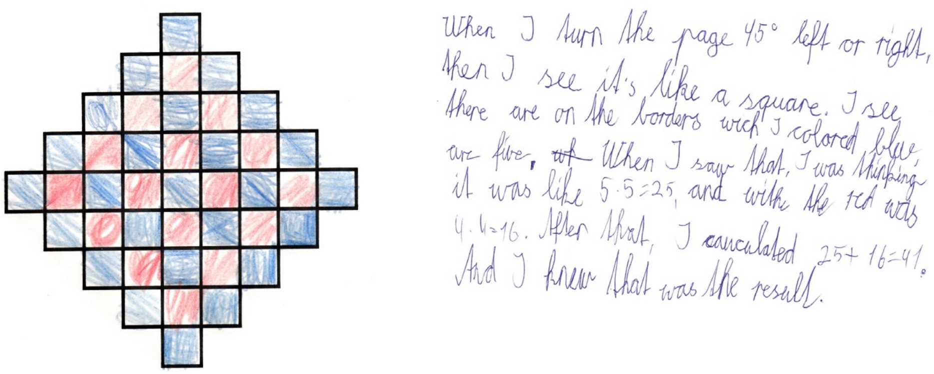


1. Describe at least 3 more ways to find the number of squares!

Besides counting the squares many students see the sequence of increasing and decreasing odd numbers. We can see both procedures often in regular lessons (Nolte & Pamperien, 2017b) whereas the following solution can be regarded as a hint for a mathematical potential (see also Nolte, 2023).

Figure 3: Eduards solution 1



7 Final remarks

Instead of adapting the talent search process to the results of different evaluations during the pandemic we had only restricted possibilities to make some pilots. Furthermore, we wanted to use another problem as last year, because the detailed feedbacks we sent may be given to students of the actual talent search process. Our first evaluation of the results shows that the first problem was easy for more children than we expected. Therefore, we are glad that we can perform a mathematics and an intelligence test this year in present.

The enforcing change of procedure during the pandemic stimulates discussions about changes of the talent search process. Trial lessons in present are very helpful. We can encourage students immediately, we can ask them to find other solving ways and not to underestimate the role of working together with other students who all are new at the university. We therefore assume that especially for students who are not placed in a privileged and supporting educational environment trial lessons in presence make sense. We also are convinced by the detailed individual feedback to the children. But whether we can do this depends on the number of children who send us their results. Although the NFC-KIDS seems to be helpful in the identification process, data of more students are necessary to come to final decisions. Although we think that most of the students worked independently, tests in presence and the combination of a mathematics test and an intelligence test give more stable information about a high potential than every attempt to test students at home.

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