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Abstract: Professional noticing of mathematical thinking, as defined by Jacobs, Lamb, and Philipp (2010) can be broken down into three components: attending to relevant cues, interpreting the mathematical understanding, and deciding the next best instructional steps. Most research on this topic has been conducted with elementary children. However, there is a gap in the research on professional noticing at more advanced levels, particularly college students. The purpose of this study was to take the concept of professional noticing and apply it to mathematics education at the post-secondary level. Specifically, the question we sought to answer in this study was: *To what extent do mathematics and physics Teaching Assistants (TAs) attend and interpret student thinking when making decisions in their classroom?* Mathematics and Physics TAs (n = 20) participated in this study focusing on their professional noticing skills when analyzing a college student struggling with two calculus-based problems. Results show that the TAs struggle most with interpreting student understanding and that those with more experience are better at deciding the next steps. Additionally, there is some data to support that knowledge of the content can impact their decision-making skills.

Keywords: Noticing; Mathematics Education; post-secondary mathematics teaching

Introduction

Professors are constantly expected to make fast, in-the-moment decisions in their classrooms. There are often many students doing different things all at one time, but it is the instructor's job to attend to important pieces of information and make a quick decision about how to proceed. This is different from the more long-term decision-making instructors partake in, such as deciding what they are going to teach, when they are going to teach it, and how they might best deliver this information. However, instructors must also make smaller decisions quite frequently, decisions that come from being receptive to the student thinking and actions happening at any given moment, which is referred to as noticing. This concept has evolved over time and continues to change as researchers develop varying interpretations of the components of noticing and what thinking and actions constitute noticing (Mason, 1991; Goodwin, 1994; Stahnke et al., 2016; van Es et al., 2017; Jacobs et al., 2010). Simply put, noticing involves the ability to take in relevant cues from the environment, make sense of those cues, and then respond in an appropriate manner based on the information gathered (Thomas et al., 2014).

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There is evidence to suggest that content knowledge alone may be related to the ability to identify an issue in student work, but not necessarily related to the ability to solve that issue (Son & Sinclair, 2010). This could have something to do with underdeveloped noticing skills, specifically the interpreting and responding aspects. Understanding noticing gives crucial insight to the ways teachers make important decisions quickly in their classrooms. Mathematics content knowledge itself is important to teachers of mathematics, but so is the concept of noticing and the way teachers make sense of and respond to student thinking. This study aims to investigate the content knowledge and noticing skills of university teaching assistants.

Noticing has been defined in many different ways. From professional vision (Goodwin, 1994) to other variations such as teacher noticing (Sherin et al., 2011), or simply noticing (van Es and Sherin, 2002), the contributions to the concept of "noticing" from different researchers over time can be best described as a pool of ideas that all inform our current definition. Van Es et al., (2017) used event noticing, interpretation, teaching moves, and general noticing to watch and score the four teachers they chose to study in noticing for equity. van Es et al. (2017) explain that noticing for teachers is made of three parts: attending to the classroom during the lesson, interpreting the observations, and using this information to decide where to go next. This explanation of noticing is very similar to framework we used in this current research study.

Professional Noticing of Children's Mathematical Thinking

Although there are many ways for those in different professions to define noticing, we will be focusing specifically on *professional noticing of children's mathematical thinking* (Jacobs et al., 2010). For simplicity, professional noticing of children's mathematical thinking is referred to as "professional noticing" in our work and we also are not focusing specifically on children, rather college students. Jacobs et al. (2010) describes professional noticing of children's mathematical thinking as three components: attending, interpreting, and deciding. Attending is described as selecting relevant pieces of information from students' work, interpreting is using that information to infer what the student might understand mathematically, and deciding is using that information to choose what to do with that student moving forward. Jacobs et al. (2010) looked at professional noticing across four different groups of elementary school teachers with differing experience levels and amounts of professional development. The groups varied from preservice elementary teachers who had no experience in the field to what they called Emerging Teacher Leaders, who were practicing teachers with at least four years of professional development and leadership experience within their schools. Each of the four groups received two pieces of student work related to mathematics: one recorded video vignette of a classroom and one piece of written work. After viewing these works, they were given three prompts to respond to for each one: one attending prompt, one interpreting prompt, and one deciding prompt. These responses were then coded based on the teachers' skill level for each individual component. For example, a higher score on the interpreting portion meant the teacher showed sufficient evidence that they could interpret what the child knew based on their work. Breaking the prompts up like this allowed the researchers to evaluate the teachers at each component independently. In fact, the authors said, "our specific prompts to assess participants' expertise in attending, interpreting, and deciding how to respond could be useful discussion prompts during professional development" and that they "provide the facilitator with valuable information about participants' perspectives but participants would also have targeted opportunities to explore these important instructional skills" (Jacobs et al., 2010, p. 194).

Thomas et al. (2014) draws from the work of Jacobs et al. (2010) and describes how to apply the three components of noticing in an elementary mathematics classroom. They describe attending as noticing relevant cues as a student works through a problem. This includes both verbal and nonverbal cues such as counting on fingers, mouthing words and numbers, or even talking out loud. Interpreting is defined as relating those cues back to the student's expected math development, namely what skills they should be acquiring based on their age and grade level. From the teacher's point of view, it is understanding why the student did what they did in the attending phase. Finally, deciding is defined as making an appropriate decision about how to move forward with the student based on the information gathered. Teachers could ask another question, decide to move on, or even spend time catching the student back up to grade level standards if it appears they have fallen behind (Thomas et al., 2014). The precise definition of these components makes the process clearer, which is why we have chosen to adopt this terminology to describe the process of professional noticing. As a justification for this noticing method, it is stated in the article that "deliberately connecting instructional decisions to interpretations based on attending evidence increases the likelihood of better understanding our students and giving them thoughtful, individualized, and effective mathematical experiences" (Thomas, et al., 2014, p. 302). Noticing is thought of as a valuable process, and one that is very feasible in the classroom.

Fisher et al. (2019) focused on trying to develop professional noticing skills, specifically in preservice elementary teachers (PSETs). Of the group of PSETs, about half of them completed “an instructional module focusing on professional noticing and early algebraic thinking” (Fisher et al., 2019, p. 4). A pretest and posttest design was used, with the professional noticing abilities of both groups of PSETs being assessed before and after administration of this module. This assessment was informed by the work of Jacobs et al. (2010), as they used similar attending, interpreting, and deciding prompts to assess the PSETs’ professional noticing abilities. The answer to each prompt was scored on a scale of 0-3 using a decision tree consisting of yes or no questions, with a higher score indicating a better response. Previous work by this same team used a similar scoring process to develop numerical scores for professional noticing components (Schack et al., 2013) and it’s becoming increasingly more common to see these types of rankings and scores (Stockero & Rupnow, 2017). Ultimately, the group who completed the module had statistically significant increases in attending and interpreting scores, while the group who did not had a significant decrease in interpreting scores.

Noticing in Higher Education

As evidenced by much of the work already discussed, professional noticing is utilized in elementary classrooms (Thomas et al., 2014; Jacobs et al., 2010; Son & Sinclair, 2010; Fisher et al., 2019) and secondary classrooms (van Es & Sherin, 2002; van Es et al., 2017; Stockero & Rupnow, 2017), but studies have rarely been done at the collegiate level. While not directly studying college students, Sánchez-Matamoros and Fernández (2016) studied the development of secondary pre-service teachers’ noticing of students at the secondary level. In this study, eight undergraduate pre-service teachers (PST’s) were given both a pretest and posttest to assess their interpreting abilities. The progression of the PSTs’ noticing skills was considered better if they used more specific explanations rather than general ones. All eight PST’s started in the “low” category, meaning they showed low levels of noticing skills. The PST’s had two 7-hour study sessions and after the second one, they were given the posttest. Two stayed in the “low,” four moved to “medium” and two moved to “high.” The improvements of the PSTs’ skills was directly related to the study sessions they attended (Sánchez-Matamoros & Fernández, 2016).

More closely aligned with college student thinking, Paterson et al. (2011) focused on mathematics teachers and lecturers in higher education. They broke down in-the-moment decisions by the different approaches that math educators and mathematicians took and examined the internal conflict lecturers often experience between their educational identity and mathematics identity. In one scenario, “the teacher takes control” and the lecturer, Sandy, moved on from a graph he knew was mathematically incorrect for the sake of keeping the pace of the lecture (Paterson et al., 2011, p. 988). In the second scenario, “the mathematician takes control” and the lecturer Simon focused on the correctness and minute mathematical details of a formula, despite those details not necessarily being crucial to students (Paterson et al., 2011, p. 990). Although Sandy made his decision based on his identity as an educator and Simon made his decision based on his identity as a mathematician, neither approach is incorrect (Paterson et al., 2011). This article focused heavily on the “deciding” component of professional noticing, because each lecturer had to choose to do what they thought was best for their particular group of students. When studying a group of people very experienced in mathematics content as these lecturers were, it is important to consider the internal tension between their goals as skilled mathematicians and their goals as educators.

As an educator in higher education, being able to understand professional noticing and apply it to a group of students is essential, but the teacher must have a complete understanding of the material first. Klymchuk & Thomas (2011) studied secondary educators’ and tertiary university lecturers’ understanding of calculus problems meant for Year 13 students in New Zealand. For comparison, this is similar to the calculus most college freshmen are taught in the United States. They had respondents from both groups (secondary and tertiary) give written responses to four questions to gauge whether or not they are appropriate to be taught at the Year 13 level. Two of the questions were particularly interesting, because they were specifically designed to include small yet crucial details the educators had the potential to miss. Klymchuk & Thomas (2011) recorded that in question three, which was about using the chain rule, 73% of the secondary teachers and 80% of the university lecturers failed to see that the problem had an empty set for the domain. In problem four, which involved an improper integral that was discontinuous on the given interval, 54% of the secondary teachers answered incorrectly and 72% of the university lecturers answered incorrectly. They noted that many of the teachers had room for improvement in their noticing skills. The “research suggests that explicit training in the discipline of noticing could be a useful addition to professional development of both school teachers and university lecturers, especially those in the beginning of their career” (Klymchuk & Thomas, 2011, p. 1020). This idea is vital for our research because we are studying teaching assistants and their ability to apply professional noticing at the collegiate level.

Rationale and Research Question

As discussed, most of the aforementioned research was focused on elementary school students and teachers, with few studies focusing on higher education. However, instructors at all levels are expected to make quick decisions, meaning that noticing is not strictly an elementary concept. Little research has been done on noticing--specifically attending, interpreting, and deciding--related to college students. Another under-researched group is Teaching Assistants, a crucial group, especially for introductory mathematics and sciences courses in college. Teaching Assistants often respond to student questions, help them through worksheets, and guide their thinking, making them an important member of a successful classroom. Shultz et al. (2019) examined the differences in the ways Teaching Assistants and professors talk about their professional obligations. While they were similar in how they talked about their disciplinary obligation to mathematics, the Teaching Assistants talked about more constraints when it came to their institutional obligation to the university. This shows that while both groups may see themselves as mathematicians, the professors are more likely to see themselves as part of the institution. These differences make Teaching Assistants an interesting group and one that is worth studying, particularly because most Teaching Assistants later become professors teaching undergraduate mathematics. We will be investigating the noticing abilities of Teaching Assistants (TAs) at the college level with regards to undergraduate Calculus material. More specifically, we aim to investigate the following research question: ***to what extent do math and physics Teaching Assistants attend to and interpret student thinking when making decisions in their classroom?***

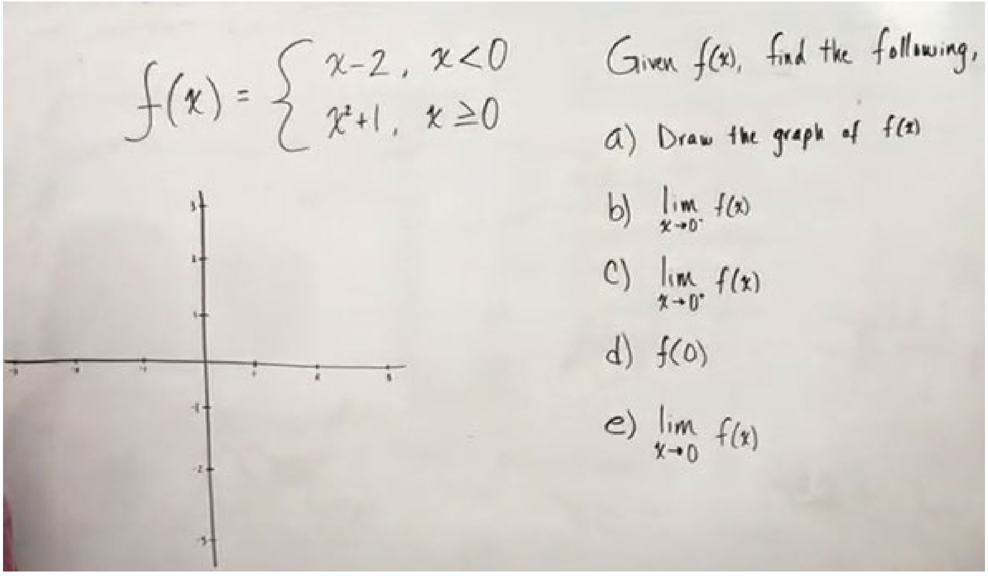
Methods

Survey Design

A survey design was used to collect data for this study. It first asked the participants to answer basic demographic questions like their age, race, gender, for what department they are a teaching assistant, what classes they have taught, how long they have been teaching, and if they have any prior teaching experience. The first question asked was a calculus question with four parts (see figure 1). Then the TAs viewed a video of a student and teacher working through the same limit question. Two of the authors of this study acted as the student and teacher in this video and scripted the video to include certain features. The problem in the video was a typical limit problem of a non-continuous piecewise function that is commonly covered in introductory calculus courses. In the video, the student correctly graphs the two equations, but failed to accurately reflect the open and closed endpoints. She then correctly identifies the limit as x approaches 0 from the left and right, and correctly found the value of $f(0)$, but then incorrectly thinks the limit as x approaches 0 is the same as the value of $f(0)$.

Figure 1

Limit Video Screenshot



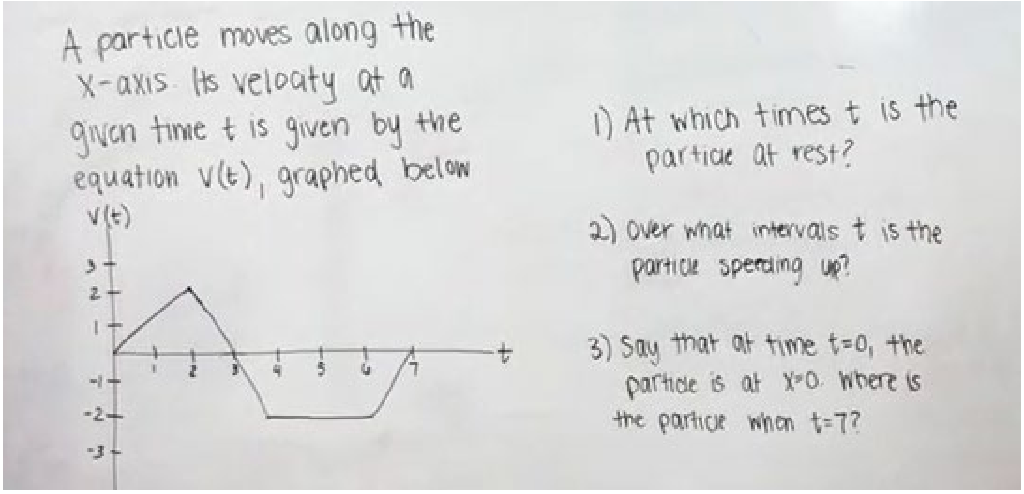
After viewing the video, we modified the questions asked by Jacobs et al. (2010) to prompt for the three professional noticing components. The prompts presented were:

1. Please describe in detail what you think the student did in response to this problem.
2. Please explain what you learned about this student's understandings.
3. Pretend that you are the teacher of this student. What problem or problems might you pose next?

The TAs were then asked to complete a physics-based question based on a graph, see Figure 2, with three parts based on the movement of a particle. They were asked to explain their answer in each part. Once again, after completing the problem themselves, the TAs viewed a video of a student and teacher working through the problem they had just answered. Then they answered the same three professional noticing prompts again. This video focused on a physics problem that is more typical of introductory level physics courses and involved a finding the location of a moving particle. The student in the video is able to identify when the particle is at rest based on the graph and circles those locations. However, she incorrectly believed the particle is speeding up only when the slope is positive, but with some questioning from the teacher regarding speed and velocity, she self-corrects her strategy to correctly identify where the particle is speeding up. Finally, to find the final location of the particle, the student incorrectly adds all of the areas instead of subtracting the areas under the x-axis.

Figure 2

Particle Video Screenshot



Participants

The participants of this survey were mathematics and physics Teaching Assistants (TA's) at a large university in the south-central area of the United States. The TAs were not contacted directly; instead, the survey was sent to the program director for each of the graduate programs who in turn forwarded it to a complete listserv of TAs. A total of 105 TAs were contacted, including 40 from the physics department and 65 from the mathematics department. Of the 20 responses received, 16 were mathematics TAs and 4 were physics TAs. These TAs had assisted in a wide range of classes, including 14 different math classes and 7 different physics classes. Years of experience varied from 0.5 to 5.5 with a median of 2 years, while age varied from 20 to 33 years old with a median of 25 years. The participants also had other types of experience, including one with experience in grades K-8, two with experience in grades 9-12, two with community college experience, three with experience at a 4-year college, four who had tutored, and one who was a TA for another university. 90% of the participants were white, 5% were Asian, and 5% were 2 or more races. 75% of the participants who provided their gender were male and 20% were female. All 20 participants were pursuing their doctorate degree in their field of study. Of these 20 responses, 7 of them were incomplete, including 6 who neglected to fill out all professional noticing questions related to the particle video. However, these participants did fill out the professional noticing questions for the limit video (besides one of these participants who did not answer the attending portion for the limit question), so they remained in the data set, as each question and components were analyzed separately.

Scoring of Survey Responses

The scoring process started with the creation of the "ideal response." First, we independently answered each of the three professional noticing prompts for both videos, then came together to compare. Through looking at similarities between our responses, we were able to work out a list of the most important elements for each prompt. We called these important elements 'salient features' (Thomas et al., 2014). For the video about limits, there were three salient features identified for the attending prompt. As shown in Table 1, attending was scored on a scale of 0-2. For the interpreting prompt, there were four salient features identified. Interpreting was also scored on a scale of 0-2. Deciding was scored slightly differently, on a scale of 0-1. There were multiple options that would earn a score of 1, but we felt only one was needed to earn a "perfect score" for deciding. For the limit question, these included asking a clarifying question related to the discontinuity in the graph or posing a new problem, such as a problem where the graph is continuous. The scoring rubric below outlines all of the salient features identified and summarizes the criteria for receiving each score.

For the video about particle movement, there were five salient features identified for the attending prompt and four for the interpreting prompt. Again, deciding was scored slightly differently, on a scale of 0-1. For the particle question, appropriate decisions included asking a clarifying question, breaking the original question down into smaller parts, or posing a new question. Specifics about each of these options are outlined in table 2, along with a description of each of the salient features identified and the criteria for receiving each score.

Table 1

Limit Question Scoring Rubric

Score	Attend	Interpret	Decide
2	All 3 of the following salient features: 1) Student used her hands/pen to trace the graph 2) Student correctly graphed the parabola and line, but failed to include the appropriate open and closed endpoints at first. 3) Student incorrectly assumed the value of the function at x=0 was also the limit as x approaches 0.	All 4 of the following salient features: 1) Student understands the graphs of quadratic and linear equations, but is less skilled with piecewise functions. She struggles with including the proper endpoints. 2) Student relies on tracing the graph to find the one-sided limits. 3) Student understands one-sided limits, but not whole limits at a given point. 4) Student does not understand graph discontinuity. Specifically, she does not understand that when the graph is discontinuous at a certain point, the limit at that point is not equal to the value of the function at that point.	
1	At least 2 of the 3 salient features	At least 2 of the 4 salient features	Asks a clarifying question to prompt the student that is specifically related to the discontinuity in the graph OR Poses a new problem with key differences from the original problem, such as a different piecewise function that is continuous at the given point (left and right limits match)
0	Less than 2 of the 3 salient features OR No Response	Less than 2 of the 4 salient features OR No Response	Does neither of the actions listed for a score of 1, or does so in a way that does not directly relate to the misconceptions in the video OR No Response

Table 2

Particle Question Scoring Rubric

Score	Attend	Interpret	Decide
2	All 5 of the following salient features: 1) Student circled where on the x-axis they thought the particle would be at rest 2) Student underlined the two sections of the graph that had a positive slope, explaining that since the slope is positive, the speed of the particle is increasing 3) When asked by the teacher the difference between speed and velocity, the student responds that there is not one 4) Student used her pen to trace the graph, and realized the particle is speeding up from 0 to 2 (graph increasing and positive) and from 3 to 4 (graph decreasing and negative). She erases the underline from 6 to 7 and changes her answer to include 0 to 2 and 3 to 4. 5) Student correctly calculates the area under the graph using the area formulas for a triangle and a trapezoid. However, she adds these areas instead of subtracting the area of the trapezoid, which lies under the x-axis.	All 4 of the following salient features: 1) Student understands that when velocity is equal to 0, the particle is at rest 2) Student knows, after prompting, that velocity includes a direction component 3) Student realizes that the particle is speeding up when velocity is both negative and decreasing or positive and increasing 4) Student knows how to find distance traveled using area under the velocity graph, but is unaware that when the graph is negative, displacement is in the negative direction (particle is moving backwards)	
1	At least 3 of the 5 salient features	At least 2 of the 4 salient features	Asks a clarifying question to prompt the student related to the sign of the graph or direction of the particle OR Breaks down the original question into smaller parts, where the graph is positive vs. where the graph is negative OR Poses a new problem with key differences from the original problem, such as providing 2 new examples with the same area on opposite sides of the x-axis to compare particle location
0	Less than 3 of the 5 salient features OR No Response	Less than 2 of the 4 salient features OR No Response	Does neither of the actions listed for a score of 1, or does so in a way that does not directly relate to the misconceptions in the video OR No Response

After establishing the scoring rubrics, the scoring process began with each researcher scoring four to ten responses individually. The research team then converged to discuss these scores, work out any discrepancies, and settle on a final score as part of the initial calibration process. Once the calibration was complete, team members scored all responses individually. After the scoring was complete, interrater reliability was calculated for each of the prompts for both videos. The percentage of instances where at least two of the three scorers agreed on a particular score as well as the percentage of instances where all three scorers agreed on a particular score are reported in the tables below. Only complete responses were factored in. Table 3 reports these values for the limit video and Table 4 reports these values for the particle video, with a separate value listed for each of the three professional noticing prompts.

Table 3

Interrater Reliability Limit Question

Limit Video	Attend	Interpret	Decide
Percent agreement between at least 2 of the 3 scorers	100%	100%	100%
Percent agreement between all 3 scorers	74%	74%	85%

Table 4

Interrater Reliability Particle Question

Particle Video	Attend	Interpret	Decide
Percent agreement between at least 2 of the 3 scorers	100%	100%	100%
Percent agreement between all 3 scorers	71%	100%	69%

After coming together and comparing scores, the team ultimately settled on one final score for every response to each prompt. When all three team members scored a response the same, that score was considered the final score for that response. When there were discrepancies between the scorers, this final score was determined through discussion of the response and the scoring rubric. Two of the three team members agreed 100% of the time for all prompts, meaning that the score settled upon was often the score shared by these two team members. However, in two instances (once for limit attending and once for limit interpreting), the final score matched that of the single team member whose score did not match the other two.

Results and Discussion

The following table summarizes the descriptive statistics of the attending, interpreting, and deciding portions of the limit question and the particle question individually. It also reports the mean and standard deviation for attending, interpreting, and deciding where the higher of the scores (on limit or particle) was kept for each participant. The higher score was determined to be the question where the sum of attending, interpreting, and deciding was higher for that participant. As this table shows, the mean scores for all three components, attending, interpreting, and deciding, were higher for the limit question than the particle question. For both the limit question and the particle question, the interpreting component had the lowest mean.

Table 5

Descriptive Statistics

	Limit		Particle		Highest of the Two	
	m	SD	m	SD	m	SD
Attend	.632	.930	.429	.495	.650	.910
Interpret	.526	.595	.357	.479	.600	.583
Decide	.600	.490	.462	.499	.600	.490

Figure 3 shows each participant's individual score for each individual component. As a reminder that attending and interpreting were on scoring scales of 0-2 which correlates a white square to a 0, a grey square to a 1, and a black square to a 2. Deciding was on a scoring scale of 0-1 meaning that a white square for deciding is a 0 and a black square is a 1. Those questions that were left blank are indicated with an X. Figure 3 shows participant 19 (P19) is the only TA who received a perfect score for the Limit Question. Along with also underlining participant 20 (P20) who almost received a perfect score for the Limit Question yet received a 0 for all components of the Particle Question. Starting with P19, they said

> "The student first graphed the function…With prompting, the student correctly represents the graph of f(x) at x = 0 with a solid dot at (0, 1) and an open dot at (0, -2). The student correctly determines both one-sided limits at x = 0, tracing over the graph of the function with their hand toward x = 0 from the left and then from the right…Finally, the student mistakenly claims that the limit of f(x) as x approaches 0 is 1 since f(0) = 1."

for attending in the Limit Question. Each section that is underlined are the salient features that P19 was able to identify, which was all three. For interpreting P19 said

> "The student appears to have a good conceptual understanding of one-sided limits, evidenced by the way they move their hand over the graph of the function toward x = 0 from each side. The student appears to think that if f(a) = b, then the limit of f(x) as x approaches a is b, which is true for continuous functions but is not true in general. The student appears to have decent procedural knowledge of how to graph and work with piecewise functions, but their struggle to evaluate f(0) suggests an incomplete conceptual understanding of what a function is and how the piecewise function notation describes it."

There were four salient features for interpreting which again are underlined in the exact response. Finally for the deciding component, P19 said

> "I might ask them to describe how parts (b) and (c) are related to part (e), that is, how one-sided limits relate to the two-sided limit. I might ask them to repeat parts (b) and (e) for an x-value other than 0 (i.e. somewhere the function is continuous). Finally, I might pose a similar problem with a different function at a point where the one-sided limits agree but the function value at that point doesn't agree with the limit (or doesn't exist)."

For this section, we were looking for a general response where the TA asks a question to the student that was either another continuous piecewise function or asks a clarifying question that helps the student identify certain parts of the piecewise function. For example, asking what the "+" and "-" signs mean in the problem and how it affects the answer. In the response that P19 gave, we underlined the exact wording that earned them the score of 1.

Figure 3

Results by Participant

Particle Deciding	X		X		X	X	X								X					X
Particle Interpreting			X		X	X	X								X					X
Particle Attending			X		X	X	X								X					X
Limit Deciding																				
Limit Interpreting																				
Limit Attending							X													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

On the table, it can be seen that P19 was very close to receiving a perfect score for all three components of Professional Noticing. For Particle Attending they were able to identify four of the five salient features but missed the fourth salient feature identifying that the student used her pen to trace the graph. For the interpreting prompt, P19 also received a 1 as shown in Figure 3 above. They were able to identify three of the four prompts with the only one they were missing being "Understands when velocity is 0, particle is at rest". We believe that the reason for them missing this simpler feature is because they had already explained that the student understood when the particle is at rest the velocity is zero in their attending answer. P19 was one of many of our participants who did this. It was interesting to see that P19 was only two sentences away from being the only person to receive a perfect score for both questions, but it is still remarkable that they were the only TA who came even close to receiving two perfect scores.

For P20, the only salient feature they missed to receive a 2 for interpreting was "still needs to trace the graph to find a limit". We believe they missed this because it is less about math concepts and more about what actions the student uses. In contrast, they did not even answer the professional noticing prompts for the particle question. P20 skipped over them and in turn received a 0 for all parts. We had a few other participants do this which accounts for many of the white squares in Figure 3.

Attending

Attending had the highest mean score for our "highest of the two" column on Table 5. We believe this is because when analyzing the highest of the two, the TAs either received a 0 or a 2 for their attending score; only one person scored a 1 and that was for the particle question. Three other TAs did have the particle question as their higher score, but they received 0's for attending; all scores of 2 came from the limit question. There was not any significant reason for the scores coming out this way and the TA who received a 1 for their attending picked out three of the five salient features. We believe that the features they selected were the most visual of the salient features and that is why they were the only TA to identify enough features to receive a score of 1 as their "higher of the two" scores. The Particle Question was difficult to understand conceptually and when we had the TAs answer the questions before watching the videos, many of the TA's answered the question incorrectly. Being able to understand the content that an educator is teaching is the first step so if our TAs did not understand the question themselves, it would be difficult to attend, interpret, and then decide the next step with the student.

Interpreting

As shown by Table 5 which summarizes our results, the TA's struggled the most with the interpreting component of professional noticing. On the interpreting portion of the limit question, only one participant was able to score a perfect score of 2. Eight participants were able to score a 1. Interestingly, seven of those eight missed the same salient feature, which was "relies on tracing graph to find one sided limits." For five of those participants, that was actually the only salient feature they missed that prevented them from scoring a perfect score of 2. That specific salient feature was clearly challenging to interpret for our participants. We think that may be due to the fact that instead of being about math content directly, that salient feature relates more to the student in the video and the actions she was taking. Again, the majority of our TA's were from the mathematics department. They may have been approaching the video more mathematically, causing them to miss this less mathematical cue to the student's understanding.

For the particle interpreting section, no TAs received a perfect score of 2. Five were able to score a 1. Again, all of these TAs missed the same salient feature, which was "student realizes that the particle is speeding up when velocity is both negative and decreasing or positive and increasing." Furthermore, this was the most missed salient feature overall for the interpreting portion of the particle question. This is likely due to the challenging nature of this concept. Thinking of velocity as a vector value that includes direction and conceptualizing the relationship between velocity and speed is something very difficult, even for advanced students. For a concept that it is hard to understand in the first place, it would be even more difficult to assess another person's understanding of that concept, which is essentially what the interpreting prompt asked our participants to do.

Deciding

When we looked at our results from deciding, we broke it down into the Limit Question and the Particle Question and then compared high and low experience TAs. We used the median experience and defined low experience as one semester to two years and high experience as two and a half to five and a half years. We had twelve low experience and eight high experience TAs. As a reminder, deciding was on a scoring scale of zero to one. For the Limit Question, the scores were eight 0's and twelve 1's. The mean of our low experience TAs was 0.500 and the high experience TAs mean score was 0.750. For the Particle Question, the scores were seven 0's and six 1's. The mean of our low experience TAs was 0.167 and the high experience TAs mean score was 0.500. We would like to note that TAs who did not answer the professional noticing questions for the Particle Question had a median experience of one and a half years. This did affect the scores of the Particle Question when calculating means, but it did not change our conclusions. When comparing experience and both questions to each other, the higher experience TAs had higher mean scores for deciding. The fact that more experienced educators are better at professional noticing is also observed by Jacobs et al. (2010). Overall, the more experienced TAs had better scoring compared to the less experienced TAs.

Implications and Significance

These results seem to indicate that professional noticing abilities are contingent on math content knowledge. Nineteen of our 20 participants were able to get the limit question correct, likely because it was an easier question. Looking specifically at interpreting, the most difficult component, one person scored a 2, seven scored a 1, and 11 scored a 0. More interesting, however, is the particle question. Only 6 of the TAs of 20 were able to correctly answer the particle question. Of these, 4 scored a 1 on interpreting, and 2 scored a 0. Of the 13 who got the particle question incorrect, only one was able to get a 1 on interpreting and the other 12 scored a 0. Of those who got the question correct, almost all of them scored a 0. Although there were many who scored a 0 when they got the question right as well, the percentage is higher when the question was answered incorrectly. This makes sense, because as discussed previously, it would be difficult for someone to notice the ways a student understands a problem that they do not first understand themselves.

The second outcome these results show is that TA experience can be linked to professional noticing abilities, specifically the interpreting and deciding components. The less experienced TAs (.5-2 years of experience; n = 12) had a mean of .50 for interpreting and a mean of .50 for deciding when looking at each individual participant's higher score out of limit or particle. The more experienced TA's (2.5-5.5 years of experience; n = 8) had higher means for both, scoring a .75 mean for interpreting and a .75 mean for deciding. Jacobs et al. (2010) found that more experienced teachers scored better on professional noticing questions. Based on our results, it appears that the same is true for TAs. The more experienced TAs performed better on the interpreting and deciding components of professional noticing.

Linking the TA experience to their attending skills proved to be more difficult. The more experienced TAs had a mean attending score of .38 while the less experiences TAs had a mean of .83. One reason for this discrepancy between the two groups could be the expectation placed on the TAs through the rubric that was developed. In order to receive a perfect score in attending, the TAs must have correctly identified every salient feature that we had identified as relevant. It may be ambitious to assume that they will note each item and perhaps a less aggressive rubric for scoring the attending response could have been helpful in further aggregating these scores.

Conclusion

Although this study did provide some interesting results, there were some limitations that we hope to remedy in future studies by expanding to include additional TA's. The first is that only four TAs were from the physics department out of the 20 surveyed. The particle question leans more toward the physics-based side of the Calculus curriculum. We wondered if the physics TAs might perform better on this question for that reason, but there were too few physics TAs to gain meaningful results. Another issue we encountered was the failure of some TAs to answer the entire survey. As previously mentioned, six TAs neglected to complete the professional noticing questions for the particle video, leaving us with only 14 complete responses for that video.

In the future, we hope to expand our study to include additional TAs. We would like a more diverse group of participants, and specifically more TAs from the physics department. Splitting the survey into smaller, less lengthy segments might encourage more TAs to respond to all professional noticing questions. We offered an incentive for completing our survey (entry into a drawing for a gift card) and that is something we would do again to further encourage participation. By including more participants, we hope to continue to find meaningful results about teaching assistants' ability to attend to and interpret student thinking in their classrooms. These results are optimistic, however, in that it reveals a training point for professors and instructors working with TAs. Considering most TAs graduate to become professors teaching undergraduate mathematics content, an increased focus on their professional noticing skills could impact their future success in the college classroom.

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