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A Role for Affect in the Future of Mathematics Education (With Thoughts on Intelligence)

A Review of:

1. Alan Schoenfeld, Heather Fink, Alyssa Sayavedra, Anna Weltman, & Sandra Zuñiga-Ruiz's (2023) *Mathematics Teaching on Target*, Routledge, 164pp. ISBN (HB): 978-1-0324-4167-2;
2. Alan Schoenfeld, Heather Fink, Sandra Zuñiga-Ruiz, Siqi Huang, Xinyu Wei, & Brantina Chirinda's (2023) *Helping Students Become Powerful Mathematics Thinkers*, Routledge, 272pp. ISBN (HB): 978-1-0324-4168-9.

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Intelligence, when I was young, bore little to do with academic achievement patterns. To my parents, these patterns were scholarly skills, not specifically reflective of any traditional (or predictable) levels of cultural intelligence. “You can get straight A’s, ten papers, and a 4.0 GPA,” they’d tell me. “But what will you do when it comes to the issues that matter??” Appalled by their thinking, I thought, “I’m studying math! I go to a nice school!” But I was left pondering.

Acknowledging the affective dimensions of mathematical learning is incredibly particular in the classroom for promoting students’ mathematical proficiencies (Seah et al., 2021). By consequence, it is also important for building a student population that is able to tackle this inherently mathematized world we live in—data-filled and all—with the ability to disentangle a clever statistic when seen ripe on TV², in which a non-zero measure of lifelong mathematical practice is not only advantageous, but increasingly necessary (Clarkson et al., 2010; see also Ernest, 2002)—this would be nice. At the very least, to not be

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² Not that this should be an expectation for all students. That to the left of the percentage sign yields context, perhaps concertedly communicated—I would be happy with this. You’d be surprised what a bit of critical mathematics practice (and probability theory) can unearth here. Surprising tidbit: an infectious disease test with a .9999 chance of testing positive when patients are positive and a .9999 chance of testing negative when patients are negative *can still have* a .5 (coin-flip’s) chance of correctly guessing whether you have the disease at all, even after reading positive on the screen (this gentle example can be found in Herzog et al., 2019, pp. 5-7).

afraid of acknowledging the optimization problem, imminent when you run to the store and notice the rain and, “Oh crap, did I leave the cat out again?” Additionally desirable exercises in the cases of social mobility, as is documented by many credible sources arguing for the parallelity of both continued mathematics education and further opportunity (Moses, 2001; Burdman, 2018; Carnevale et al., 2011) only support this cause. If there *are* ways to promote mathematics learning for students, then one effective way to do this is to understand its composition.

It is difficult to conceive of a mathematics classroom as a conglomerate of both mathematical and ‘soft’ skills. The difficulty herein lying in these lingering thoughts that academic achievement in mathematics may be almost exclusively propelled by knowledge of content-matter. Surely one’s ability to study material might also play a big role. Especially for those students who sense no attachment to learning (or as it may seem here, ‘absorption’) at first sight³. Resiliency, perseverance, grit—all often talked about in the psychology literature—relating, of course, to metrics of *discipline* in a student. And what if the student grew in an environment where mathematics was often ostracized? Perhaps labeled as a difficulty or unavoidable chasm of the primary through secondary grade affairs? The student—far too easily—may develop a poor sense of “mathematics identity,” which, as we know now, can play a significant role in determining preferences regarding the felt capacities for STEM or non-STEM career choices (Cass et al., 2011). Mathematics anxiety, another equal prevailer here, is inexorably tied to course performance (see Luttenberger et al., 2018), though this is can be a no-brainer. The question, then, at the forefront of my mind, is what a mathematics classroom would like that takes care to build these skills, or perhaps positive fundamental feedback loops for students’ sense of wellbeing as mathematics learners, and the like? I believe one answer is clear and comes as follows: there is more to effective instruction than the attainment and retainment of subject-specific knowledge. How students *experience* this instruction—now there is something ripe for attention. What would a classroom that—even in partiality—valued students’ affective dimensions as a mechanism for *greater* learning, look like?

³ You may bring to mind a similar analogy from orchestra. Some musicians feel confident at the get-go: the moment of sight-reading. Others feel comfortable after time to indulge in the finer details, such as with the dynamics, staccatos, and some fine-tuning at home. Come concert-time, should there be any idealized preference between these players? The answer, given a block of time, is no.

One big question to ask, that often guides the functionality of measurement and assessment at schools, is: "What defines an educated person in mathematics?" It is not a farfetched thing to say that 'achievement' and 'intelligence' are socially contrived objects (see Borland, 1997). And, in a perfect world, it is our very own students who set the rules. But I am somewhat of a jaded person when it comes to 'optimal' teaching practices. That is, until I read these two books.

"What do we need to know in order to help teachers create classrooms from which students emerge as knowledgeable, resourceful, flexible thinkers and problem-solvers? What are the components of such powerful classrooms? Can we distill all these things down to a small number of things that teachers can pay attention to, and call them dimensions of powerful mathematics instruction?" These are the questions at the heart of Teaching for Robust Understanding (Schoenfeld, n.d.). *Mathematics Teaching on Target: A Guide to Teaching for Robust Understanding at all Grade Levels* (MToT) and *Helping Students Become Powerful Mathematics Thinkers: Case Studies of Teaching for Robust Understanding* (HSBPMT) expand our conceptions of effective mathematics instruction—what it means, what it looks like⁴. MToT speaks about the actions individual teachers can take their classrooms. HSBPMT builds on our abilities to notice and wonder about a classroom environment.

Though names have evolved since first iteration (Schoenfeld, 2013), The Teaching for Robust Understanding (TRU) framework highlights a set of axioms (i.e., 'dimensions') for analysis of effective classroom instruction. These dimensions are: (1) The Mathematics, (2) Cognitive Demand, (3) Equitable Access, (4) Agency, Ownership, and Identity, and (5) Formative Assessment. Explained further, "The Mathematics" dimension involves portraying mathematics as an open system—heady with participation, discussion, conversation, and teamwork. It is about providing "opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers" (Schoenfeld, Fink, Sayavedra et al.,

⁴ Note: the words "effective," "powerful," and "robust" are used quite frequently to determine (or, describe) the kind of mathematics instruction TRU entails. Some may view this as problematic, but let's not forget that 'happiness' is a word that also goes un(rigorously)-defined, and we make do, most notably in family get-togethers, where the occasional dinners sometime drift into non-euphoric spaces of listening, or—really—*longing* for the door.

2023, p. 35). The “Cognitive Demand” dimension regards the constant calibration of difficulty and scaffolding for students. It provides tasks and instruction that finds a balance between ease and overt difficulty, achieving a ‘productive struggle’. The “Equitable Access” dimension structures lessons in ways meant to close differential achievement gaps. There are multiple ways to access material and “make use of tasks that enable all students to engage in challenging content” (Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, p. 10). “Agency, Ownership, and Identity” is responsible for creating positive, affective outcomes for students’ lifelong mathematical practice. This means recognizing, and having ways to capitalize on, the diversity of student strengths available in your classrooms—involving, of course, a variety of conceptions for achievement and mathematical competence. Finally, the “Formative Assessment” dimension allows student thinking to come to the forefront of the classroom. It is the “ongoing monitoring of student thinking and adjusting instruction in response to it, *during the learning process*” (Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, p. 14). While both books elaborate on the TRU framework⁵, each provides a different approach to its utility.

Mathematics Teaching on Target: A Guide to Teaching for Robust Understanding at all Grade Levels (MToT) by Alan Schoenfeld, Heather Fink, Alyssa, Sayavedra, Anna Weltman, and Sandra Zuñiga-Ruiz, illustrates ways to improve classroom activities and reflect on instructional values by “Thinking in TRU terms”⁶ (Schoenfeld, Fink, Sayavedra et al., 2023, p. 12). Each chapter devotes itself to one of the five TRU dimensions. The role of each dimension in the framework is motivated, resources and guiding questions are provided, and, more importantly: concrete examples—from each of elementary, middle, and secondary grade levels—ground the text to practical examples of common-day math problems you might see in each of these classrooms. These improvements are done via ‘targets’ (‘dart board’ targets, to be precise (for the imagination)) where the closer to the center you go, qualities of ‘more robust’ learning

⁵ “Framework”: way of thinking about something. For more details about how the TRU framework was developed (i.e., its origin) see Schoenfeld (2013).

⁶ “When you’re getting started, focusing on the math and one other dimension might be a good goal for a semester or perhaps longer. Once thinking about those dimensions has become more or less habitual, you might add another dimension, while continuing to build on the progress you’ve made. Over time, as your repertoire grows and thinking in TRU terms becomes more natural, you’ll have the ideas in your head.” (Schoenfeld, Fink, Sayavedra et al., 2023, p. 12).

become obvious. For instance, when developing a target for the Formative Assessment dimension, the guiding question, “In what ways are students’ informal understandings and language use valued and built on?” is proposed, and examples such as: “Teachers know their students as human beings and as learners as well as where they are at mathematically,” and, “Students’ outside of school knowledge is considered a valued resource for learning” emphasize the human-centered practice of mathematics teaching as-a-whole are items located close to the center of the question’s target (p. 130).

These ‘targets’ are not just a visual theme. Each chapter explains the thinking behind using one of the five dimensions of effective instruction to improve lesson plans and classroom norms. They are a mode of self-reflection. Ultimately, this is where MToT really shines. Reminders to think about whether we are meeting our own goals in our teaching practice, or if we are defining mathematics instruction to be traditional drill-and-kill—with expected classroom norms—occur in every page. Furthering this, not only does each chapter provide these blank targets, they also provide blank ones, to fill-in and use during our own reading. Epistemically, this makes MToT a workbook. Something to discuss amongst educators—or aloud—during a trail-walk next to a nearby park.

MToT emphasizes a relationship between classroom norms and the established ceiling for students’ mathematical understanding. “We need to attend to students’ affective experiences as they engage in problem-solving,” (Schoenfeld, Fink, Sayavedra et al., 2023, p. 23) the authors describe. I view this book as an essential guide for learning how to teach mathematics, in an age where caring for the student is a predominant concern (see Seah et al., 2021). Through reflection and habit-formation, the questions littered on each page mediate the writing between coffee-table and workbook, to the reader’s benefit.

Helping Students Become Powerful Mathematics Thinkers: Case Studies of Teaching for Robust Understanding (HSBPMT) by Alan Schoenfeld, Heather Fink, Sandra Zuñiga-Ruiz, Siqi Huang, Xinyu Wei, and Brantina Chirinda presents three real case studies from three real math classrooms, utilizing the TRU framework as lenses for thinking about classroom activity. First, I want to note the rather brief treatment of the framework itself—in comparison to MToT’s—despite HSBPMT just being an applied

examination of the dimensions introduced in MToT. Not to cast it aside, I say this to appreciate the venerable differences between these books. While MToT focuses more on describing aspects of the framework, and how to be “Thinking in TRU terms” (Schoenfeld, Fink, Sayavedra et al., 2023, p. 12), HSBPMT omits the grainier details and sticks to real-life examples in near-entirety.

After providing an overview of the five TRU dimensions⁷, we are presented with three case studies from three different classrooms⁸. Each frames one or two of the TRU dimensions as a lens for discussion. “Chapter 2: The Car Value Problem” is a modeling activity inside an untracked Algebra II classroom. Blanca, Devin, Caleb, and Ayra—all 10th-12th graders—our protagonists—are perpetually on the brink of discovering their needed formula for the depreciation of a \$5000 car by 15% each year. The writing describes, in four “episode” installments⁹—beginning after the group receives the problem—the group’s problem solving process, as well as the teacher’s—Ms. Sierra’s—efforts to keep Blanca, Devin, Caleb, and Ayra moderately challenged, yet still afloat. Each episode is broken up by a set of focus questions for the reader, followed by commentary from the authors on the previous episode. As you may note, the theme in this study is Cognitive Demand—namely, “To what extent are students supported in grappling with and making sense of mathematical concepts?” (Schoenfeld, Fink, Sayavedra et al., 2023, p. 55). Each chapter ends with a small analysis of all five dimensions, and how how each can be observed in these visits “Chapter 3: Where is the Ten?” does the same with the Equitable Access and Agency, Ownership, and Identity dimensions. “Chapter 4: Graphing Quadratic Functions,” focuses on Formative Assessment.

⁷ pp. 3-17; a total of 14 pages. Compare this to MToT’s deeper dives, where almost twenty-one pages are devoted to understanding the particularities of a *single* dimension, exclusively.

⁸ And none of that uppity ‘private school’ shenanigans. These are three algebra classrooms (two Algebra II, one Algebra I). Two reside in urban, public high schools with high percentages of minority students and students qualifying for free or reduced lunch. One is a “sheltered algebra” classroom—a multilingual classroom where English and content instruction are integrated, and most students identify as fluent Spanish speakers (see Schoenfeld, Fink, Zuñiga-Ruiz et al., pp. 38, 53, 153)

⁹ The activity took around twenty minutes total. Each of the four episodes logs the amount of time they take in real-time. They are, however, not defined by arbitrary chunks of equal length. Each episode instead is defined by moments of uncertainty. Originating (i.e., posed) by the students themselves to each other in the group, or by the semi-socratic nudges of the teacher.

Important to note is that despite HSBPMT's page count (almost twice that of MToT's), the content reads just as fast. This is because each chapter focuses on staging its case study like a *story*. Instead of fronting the principle of research that will be made apparent later, there is dialogue and an interactions between the students and teacher. An expansive illustration of the mathematics problem at hand¹⁰ follows the introduction to each classroom, as well as context for the classroom being visited, and the episodic play-by-play of how the students in this classroom tackle their work, reading a lot like screenplay and film, with reactions, dialogue, and body language changes¹¹. The authors conclude each chapter with a small reflection on how each of the TRU dimensions manifests itself in the recorded classroom interaction.

Both books share a collaborative dialectic with the reader. The majority of the work—analysis and connection-making—is left explicitly *to* the reader, through the use of focus questions present in both editions. One of my personal takeaways as-a-whole has to be the adoptability (i.e., accessibility) of *thinking* with the TRU framework. These five dimensions can be used as a reflective tool to understand our own teaching practices, as a development tool to procure a more refined curriculum, and as a gateway to observing students' mathematical experiences. The framework is, in it of itself, *non-prescriptive*. And prescriptiveness, really, is the problem with many professional development sessions or curricular propositions. You have to do this and not that. Make sure you're using so-and-so words when you're talking about X and Y. Thinking with TRU does not need to offer a subscription service. Teachers do not

¹⁰ The word expansive, here, should not be ignored. Each case study presents from eleven to seventeen pages of 'coffee-table mathematics,' referring to the elegant choice of triarchically presenting all of (1) the math problem the students and teacher in the case study follow along to, (2) the mathematics the problem engenders to tackle (as well as its pedagogical value), and (3) step-by-step methodologies for solving the problem in, attemptedly, every possible way a student may think of (tables, graphs and figures all generously included), where elegance, here, is to say the explicit act of *involving* the reader in the mathematics and instructional decision-making (rather than allowing us to act as passive, possibly value-laden, consumers) of a classroom experience.

¹¹ As an example, the following is an excerpt from Episode 1 in The Car Value Problem: "Devin laughs, "That ain't right." He starts to whistle as he continues to pash calculator buttons. Turning his calculator to show Caleb, he continues, "Bruh, it's worth one cent after 8 years." Devin laughs, shakes his head, and puts his calculator back down on the desk" (Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, p. 42). All of this is to say that—yes—we are meant to experience these case studies as if we were in the classrooms ourselves, observing, interested in both the actions a student takes, and how that may reveal more about the cognitive process occurring inside the collective of the group.

have to change their practices as the embodiment of the framework is a reflective one. Meaning, given time to reflect on your own values as the instructor, this is a framework that gives you language to teach in ways that better reach your goals and values of the classroom. All this from understanding instruction through five difference facets. “The idea is for you to customize that target and to focus on what matters to you” (p. 12). The two books are also strengths-based and one-million percent nonjudgmental about, even, the teacher case studies in HSBPMT. It is evident that the focus of the authors is not on just improving the current student experience, but whole-heartedly the experience of teachers too.

For instance: *why? what constitutes effective instruction, and what does it look like?* Both books centralize a theme of reflection. HSBPMT, primarily, about¹² classrooms. MToT with great pedagogical missions. To empathize with the students’ mathematics¹³ or wonder about the interaction between students in group-work¹⁴—situations arising in every math teacher’s busy lives, yet allowing us the opportunity for dialogue and practice (extra encouragement to do so with collaborators of a reading team). One of MToT’s more explicit aids is helping us construct our own idealized notion of practice. Or, providing us with space to reflect on the nature of mathematics education. What it looks like, sounds like, and—hopefully—feels like for our students.

It is insurmountably clear that neither is meant to be an encyclopedic account of research in the field pertinent to effective mathematics instruction—or, for that matter, the TRU framework, which candidly characterizes such a subject. Instead, the intended audience is revealed via the style of writing and its

¹² See Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, pp. 29, 31, 35, 43-44, 49-50, 56-57, 63, 69-70, 86, 89, 93, 101, 108, 114, 121-122, 140, 147, 148, 150, 160, 171-172, and 180-181—and those are just the “focus questions”!

¹³ “What mathematical understandings might students develop from engaging in this lesson? How are those understandings important?” (Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, p. 93)

¹⁴ “At times Bernardo and Carla were somewhat dismissive of Alicia... to the point where one might be tempted to intervene as a teacher. How did things work out? What can we say about each student’s participation, and possibly their mathematical senses of self?” (Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, p. 122)

psychic distance to the subject-matter¹⁵: “We are about to join an Algebra II class as it delves into the complexity of representing exponential decay in symbolic terms. The teacher, Ms. Sierra, has asked the students to model the decreases in a car’s value over time” (Schoenfeld, Fink, Zuñiga-Ruiz et al., 2023, p. 25). These books expand knowledge on what effective mathematics instruction means and looks like, and would be in the welcome hands of curriculum designers, teachers, teachers of teachers, and—of course—anyone interested in the praxis (and complexities) of mathematics instruction.

Supporting broad notions of mathematical competence is another recurring theme in these books. And inherent in this is the topic of what it means to define intelligence, measurement, and achievement in mathematics. To this, the authors state: “There are many ways to be smart mathematically, including: making sure you understand the problem, posing interesting equations, keeping track of useful ideas, revision ideas or approaches in the light of new information, facilitation productive conversations, making interesting connections, working systematically, clarifying ideas and expressing them logically, and being able to summarize or explain your own or your group’s ideas” (Schoenfeld, Fink, Sayavedra et al., 2023, pp. 74-75). This is, unfortunately, something not well understood even by students. But how can we blame them? When much of our assessment practices are geared towards fleshing out particular, historically observed strengths related to mathematical proficiency. As I have been reminded by the writing in these books, there is an abundance to mathematics education and the research being fleshed out that, like rotundas, are only beginning to be tapped. When compared to many of our experiences in the math classroom, these ideas feel new: “Students develop their understandings regarding the nature of mathematics and expectations for how they are to engage with it from their classroom experiences” (p. 50). These books make it exceptionally clear that it is not just about allowing students to grow in mathematical content-knowledge and soft-skill competencies (see Seah et al., 2021), such as building agency and identity, but also to ask, “*Who* is getting this treatment?”

¹⁵ Psychic distance is the “distance that the reader feels between [themselves] and the events of the story” (Gardner, 1991, p. 111).

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