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Advances in Using Primary Historical Sources to Teach Mathematics: A Special Issue Introduction and Invitation

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Abstract. In this introductory article, the guest editors describe the genesis of this special issue of *The Mathematics Enthusiast*, provide an overview of its contents, and invite readers to join the authors of these articles and others in contributing to the ongoing efforts to bring primary historical sources to the mathematics classroom and research their impact on students and teachers.

Keywords: primary historical sources, TRIUMPHS, Primary Source Projects

1 Genesis

This special issue of *The Mathematics Enthusiast* is devoted to the teaching of mathematics via primary historical sources. It was envisioned as a National Science Foundation-funded project came into its eighth and final year. The project, TRansforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS), was a seven-institution, $1.5 million project that launched in 2015. Originally intended to be a five-year effort, the first no-cost extension was extended further as the COVID pandemic made it very challenging to complete some of the intended work, and the timeline was thus extended to 2023.

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At the core of the work of TRIUMPHS is the Primary Source Project (PSP). Aspects of the grant involved writing, testing, and revising these projects based on classroom practice, along with studies of the projects themselves, their implementation, their impact on students and instructors, and the theoretical frameworks through which some of their features can be viewed. As many of the papers in this special issue involve PSPs, it will be helpful to describe here what PSPs are and what they are not.

First, a Primary Source Project is designed to teach mathematics. In particular, while primary source texts are used in all PSPs, and historical context is generally given for these texts, the pedagogical purpose is not to teach history, but to use one or more historical artifacts to teach mathematics. Students using a PSP are given text excerpted from a primary source (in English translation, if appropriate), after which they are asked to complete a series of tasks that lead them to explore the text under consideration and to develop a deeper understanding of the underlying mathematical concepts. PSPs include other common elements that we determined to be best pedagogical practice, including an introduction that identifies learning goals, commentary on the content and context of the source texts by the project author, a conclusion that points to future study, and a "Notes to Instructor" section. These Notes give instructors extra information that could be useful for classroom implementation, such as the prerequisite knowledge expected, a sample implementation schedule, additional historical background, and commentary on project tasks that are likely to prove challenging.

Although this list of shared elements may seem to limit options for authors, in fact we found that they employed these in a multiplicity of ways, creating a diverse set of classroom projects. Some short projects (called "mini-PSPs") can be completed in as little as one or two class periods, while others may take weeks to complete. (Indeed, an entire Abstract Algebra class has been taught using a small number of PSPs.) While all projects teach mathematics, we conceive of this broadly, and the work includes projects in statistics as well. Any topic that one could reasonably expect to be taught at the undergraduate level was fair game, and the topics included in PSPs were commensurately broad. The set of 99 TRIUMPHS PSPs completed during the grant period includes algebra for a course for pre-service teachers, calculus, voting theory, real and complex analysis, number theory, and more.[1]

[1] Classroom-ready versions of these PSPs are freely available for download from the TRIUMPHS Digital Commons collection (https://digitalcommons.ursinus.edu/triumphs/). LaTeX source code is also available from the project authors upon request from instructors who wish to modify or adapt a PSP to their classroom context and learning goals (subject to terms of the Creative Commons Attribution-Share Alike 4.0 International License).

The TRIUMPHS grant was inspired by decades of effort by others in this field, especially the work done by Jerry Lodder, David Pengelley, Reinhard Laubenbacher, and others at New Mexico State University under the auspices of two earlier NSF-funded efforts.[2] Of course, teaching mathematics via primary historical sources is an idea much broader—and older—than Primary Source Projects, and a variety of different approaches to using primary sources with students are compared in [Jahnke et al., 2002] and [Jankvist, 2014]. As successful as TRIUMPHS was, we are thrilled that some of the papers in this special issue explore teaching with primary sources through other contexts.

At the same time, we are both pleased and pleasantly surprised by the variety of the work that has emerged from the efforts of those who have participated in the TRIUMPHS project, achievements which went well beyond the scope of what we envisioned when we launched the project. Perhaps nothing better exemplifies this than the diverse array of topics covered by the authors of this special issue of *The Mathematics Enthusiast*. In articles that span personal reflections of the classroom experience of teaching with primary source materials, considerations from philosophy, linguistics, education research, and ideas of diversity, equity, and inclusion, the authors in this issue make clear that engaging with these source materials in the university mathematics classroom has deep relevance, for instructors and students alike. In the next section of this introductory article, we consider what the authors of these articles have to say about the richness of teaching and learning with primary historical sources.

2 Overview

In the first two articles of this issue, Tricia Brown and Tara Davis each provide personal testimonials of their work teaching with PSPs. Brown was drawn to explore implementation of PSPs into her teaching from experience participating in the Inquiry-Based Learning (IBL) community of mathematics instructors. Interested in ways to catalyze active learning of mathematics by her students, she decided to employ PSPs, first in a combinatorics course and then in a history of mathematics course. In “Becoming a More Effective

[2] The two predecessor grants to TRIUMPHS were entitled “Teaching Discrete Mathematics via Primary Historical Sources” (2003–2006, DUE Grant no. 0231113, https://sites.google.com/view/davidpengelley/hist_projects) and “Learning Discrete Mathematics and Computer Science via Primary Historical Sources” (2008–2012, DUE Grants No. 0715392 and 0717752, http://www.cs.nmsu.edu/historical-projects/). Through their combined efforts, these grants produced 36 student modules based on the guided reading of primary historical sources that are freely available to instructors for use in courses on discrete mathematics, combinatorics, logic, abstract algebra, and computer science.

Instructor Using Primary Source Projects," she explains how utilizing PSPs in her classrooms allowed her to claim that they were "the central reason I was able to implement a lecture-free, fully inquiry-based classroom." These projects, and especially the tasks written by the PSP authors, afforded her the dynamic of connecting student thinking directly to the thinking of mathematicians in the action of solving problems of compelling interest to them in their original, historical contexts.

Davis, in "Team Teaching with Primary Historical Sources," relates her experience team-teaching with PSPs in two different sets of circumstances. One was an honors course that she co-taught with a psychologist colleague, and which, as she explains it, was designed for non-specialists in mathematics "to improve students' problem solving skills by investigating both traditional and non-traditional mathematics problems and by examining important cognitive and social psychological elements about how people reason, think, and analyze information." The other was a more traditional first course in abstract algebra for mathematics and mathematics education majors that she co-taught with one of us (Barnett), who also authored the PSP she employed with her students. She tells how, in both cases, she needed to help her students negotiate the unfamiliarity of how they were being asked to engage with mathematics via the primary sources and the accompanying student tasks from the PSP. She reports how her students gained a fresh perspective on mathematics as "a human endeavor," and how the classroom can become a space where cognitive struggles can be validated and discussed openly, and that even making mistakes can be normalized as a method for learning. She admits that working with another instructor to implement PSPs for the first time "allowed [her] the opportunity to show a vulnerability to the students" and offered her an opportunity for "greater self-reflection" to improve as a teacher.

It is striking how even instructors who have been teaching with primary source materials for many years are driven to investigate new ways of teaching that go beyond simply offering non-traditional course materials to their students. Jerry Lodder, who not only participated as a Principal Investigator with the TRIUMPHS project but also served on the teams that led TRIUMPHS's parent and grandparent grants (see footnote 1), reports on one such an investigation in this special issue. One of the challenges that students chronically face in working with primary historical sources is that because they were written in another time, they employ older—and thus different—patterns of speech and language no longer found in contemporary presentations of mathematics. In "The Figurate Numbers: From Verbal Expression to Algebraic Symbolism," Lodder explores how tools from transformational grammar, a field of linguistics, can be used to assist student attempts to parse language written in centuries past that is encountered in primary historical source materials (often presented to students in English translation). These linguistic tools can make it easier for students to extract mathematical meaning from older patterns of mathematical language; it is then much easier for them to build algebraic formulations that represent a standard and contemporary representation of these statements, statements which are found in the historical source in a non-symbolic rhetorical form only.

Anne Duffee's article, "Teaching Analysis with Primary Sources: From Geometric Intuition to Arithmetic Rigor with Bolzano," draws not from linguistics but from modern philosophy for new ways to understand the benefits of working with primary sources in the mathematics classroom. She reflects on her experience of doing close reading of Bernardo Bolzano's 1817 paper on the Intermediate Value Theorem with her students in an introductory analysis course. This helped her students gain an understanding of fundamental concepts in real analysis and the issues that compelled mathematicians of the past to formulate these concepts. She goes on to argue, with the help of Edmund Husserl's phenomenological concept of the "desedimentation" of knowledge, that the purpose of reading primary texts is to aid students' attempts to understand the genesis of these mathematical ideas as solutions to problems that engaged the authors of those primary texts.

The primary source texts that sit at the center of this approach to teaching mathematics are products of their own times and circumstances and often refer to or hint at the non-mathematical environment in which the source author wrote. Because of this, these texts compel some unexpected reactions from students, many of which are only tangentially related to the mathematical topics at the fore. Abe Edwards, Dominic Klyve, and Ken Monks, in their article "Inclusive Pedagogy in Mathematics via Primary Source Projects," discuss how students, in their individually unique ways, can find unexpected and exciting connections to mathematics and its practice through the words of primary source authors and their circumstances, even as they speak from long ago. Through a series of "happy accidents," the authors of this paper tell how these unexpected connections can be activated simply by the fact that students are reading the words of a particular mathematical writer from a particular time and place in history. Serendipitously, these connections can thus help instructors support students' sense of belonging in the mathematics classroom and foster an inclusive classroom experience. The authors conclude by issuing a call to action for more intentionality on the part of authors and instructors with regard to the role that PSPs can play in promoting inclusive pedagogy.

In "Situating the Statistical Legacies of Galton and Fisher: Multi-layered Discussions in the Mathematics Classroom," Deborah Kent and Jemma Lorenat push even further on the non-mathematical benefits of the use of primary sources in the classroom. Independently, the two discuss their experiences in using the writings of Sir Francis Galton and Sir Ronald Fisher—two towering figures in the history of the theory of statistics and its practice—to explore ethical (*not* mathematical) issues in the practice of data gathering and analysis. In a course for students of data science, students read from the works of Galton, and in a masters-level course in the history of science, students studied writings by Fisher. The work of both men, noted eugenicists from the late-nineteenth and early-twentieth centuries respectively, provides students much to ponder concerning the use of scientific methods and their impact on human beings in society, and "invites students to consider how subjective values inform the technical content of their data- or statistics-heavy courses." Students of Kent and Lorenat "worked with digital scans of the primary sources, [and dealt with] the added challenge of deciphering handwriting, uncovering additions and deletions, and categorizing the documents as correspondence or manuscript drafts;" they further analyzed Galton's racist conclusions of superiority of one race of people over another, or considered Fisher's connection to the academic work of the Nazi eugenicist Josef Mengele. Their students thus saw firsthand how ethical issues can impact the practice of mathematics, with the result "that acknowledging our discomfort [with these problematic issues] … is empowering for students and creates a sense of learning together."

Education theorists are also bringing the tools of their discipline to reflect on the efficacy of teaching with primary sources in mathematics. In the final two papers of this special issue, Abe Edwards makes use of Sfard's theory of commognition "to provide a justification for the trust required" by instructors who yield agency to their students through their use of PSPs in the classroom; and Mark Watford, Kathy Clark, and Cihan Can utilize the frameworks of transgressions theory and situated learning theory in order to analyze survey data from students in four classrooms in which PSPs were incorporated by instructors of abstract algebra courses.

In "Trusted Together: A Commognitive Perspective on a Primary Source Project in Multivariable Calculus," Edwards reviews some of the struggles instructors have when they choose to yield the performance stage in the classroom and "get out of the way of student learning." He turns to the commognitive theory of learning to argue that these struggles merit the effort. Within that theory, learning is signaled by students' growing ability to participate in a particular mathematical discourse. Reflecting on the experience of his implementation of a PSP focused on Green's Theorem in multivariate analysis, Edwards explains how this project "emphasizes the discursive shifts that occurred as the original source authors developed increasingly sophisticated narratives" in the development of integration theory in the nineteenth century, and does so in a way that supports students' ability to make such shifts. He argues "that allowing students to trace the historical developments of a topic can create healthy conditions for learning (as defined commognitively), and challenge instructors to place more trust in their students … when it comes to learning mathematics."

In the final article of this issue, "New Methods of Capturing Students' Experiences with Primary Source Projects: Pioneering a Transgressive Lens," Watford, Clark, and Can examine student experiences of working with PSPs in their classrooms, employing the lenses of transgressions theory and situated learning theory to see "whether transgressions could be a viable theoretical perspective to help capture students' engagement with learning undergraduate mathematics." The first of these two theories proposes that "a transgressive action" in mathematics is one in which "a boundary identifying the limit of what a person has thus far been able to achieve [in applying some method or performing a computation is crossed, resulting in] an outcome which results from crossing the boundary." The authors propose that this new outcome alters the student in a way that they describe with situated learning theory: learners gain access to the mathematical community via these actions, extend their practice within this community, and thereby authentically participate in it, forging new identities as its members. The authors' analysis of the student data in their study "indicated significant learning outcomes from a situated learning theory perspective, which, in our experience as researchers and educators, is not representative of students' experiences with standard textbooks."

3 Invitation

In broad terms, the aims of this special issue—and the TRIUMPHS project more generally—were two-fold:

- Expand the use of PSPs as a resource for teaching and learning undergraduate mathematics by supporting the development of high-quality student projects and promoting their use in a broad range of courses; and

- Expand the understanding of the STEM education community by contributing to the general knowledge base through research on the effects of teaching and learning with PSPs on students and their instructors.

While the articles in this special issue are a partial reflection of the success of TRIUMPHS with respect to these two aims, they are also a forecast of the promise that the expanded use of primary historical sources offers for enhancing both students' mathematical learning and faculty's instructional practice. Looking towards the future with the aim of fulfilling that promise, a number of those who were involved with the TRIUMPHS project have recently launched a new professional society to carry on the legacy of that grant and that of its predecessor grants. We invite readers who are inspired by the articles in this special issue to join the new TRIUMPHS Society[3]—TRansforming Instruction: Understanding Mathematics via Primary Historical Sources—in promoting the proliferation of primary source-based pedagogy in mathematics through the development of new materials for classroom use, the pursuit of both old and new directions in researching their effects on students and instructions, and continued conversation about this now established, but still fresh and exciting approach to teaching and learning mathematics.

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[3] https://triumphssociety.org/