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A tribute to the life and work of David Tall

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Abstract: David Tall was my academic grandfather, a wonderful mentor, and a brilliant mathematics educator. He earned two PhD degrees in mathematics and mathematics education. His edited book, *Advanced Mathematical Thinking*, and many of his publications that followed carved the path to shape many advances in mathematics education. His work on the framework of the three worlds of mathematics resonated well with mathematicians, giving them a language to express their thought processes as they reflected on their teaching and research. His latest framework, *A Meaningful Long-term Framework for Mathematical Thinking*, extended his ideas further and captured more on the complexity of teaching at the university level. In this article, I will share a few of my memories of David Tall over the past 20 years to honor his tremendous contributions to the mathematics education community.

Keywords: David Tall, A meaningful and long-term framework for mathematical thinking, Three worlds of mathematics, Advanced mathematical thinking, Concept image and concept definition

Professor David Tall played a pivotal role in my academic journey as my mentor and academic grandfather. During my graduate studies at the University of Auckland in New Zealand, my PhD advisor, Mike Thomas (David Tall's first PhD student in Warwick), introduced me to David's influential work. His edited book *Advanced Mathematical Thinking* (Tall, 1991), a classic in our field, was my first introduction to David's work. This book, along with his renowned paper on Concept Image and Concept Definition, co-authored with Shlomo Vinner (Tall & Vinner, 1981), significantly shaped my understanding of mathematics education. I often refer to various chapters of these works and assign them as reading materials in my mathematics education graduate courses.

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When I began my PhD in mathematics education under the guidance of Mike Thomas, David's paper on "The three worlds of mathematics" (Tall, 2004) had just been published. Our initial findings from administering a linear algebra test to mathematics majors indicated that students had significant challenges grasping definitions and visualizing some linear algebra concepts (Stewart & Thomas, 2003). David's theoretical framework provided a valuable lens through which we could analyze our data, and we eagerly incorporated it into our research.

Many years later, in 2020, David wrote:

"Your early 2003 paper on linear algebra with Mike is interesting because it lists all the problems and, *at that time you knew nothing about the 3 worlds*. The paper makes no reference to my work in linear algebra because I hadn't done any! But it already had various analyses of vector spaces in geometric, algebraic and formal representations. Your 2008 papers with Mike focus on embodied, symbolic and formal thinking, now referring to my 2004 paper."

David often shared captivating stories and anecdotes detailing the genesis of his ideas, some stemming from his collaborative efforts with his PhD students and other co-authors. These personal insights enriched my understanding of his work and fostered a sense of connection and engagement with his ideas and those of others in the mathematics education community.

His passion and dedication always amazed me as he tackled these endeavors. He often reminded me that these are all part of *the journey*, that we must acknowledge them, and that we must look forward to the future.

Over the years, David provided me with extensive guidance and advice that will take me a lifetime to thoroughly analyze, revisit, and appreciate in its entirety. He has not only answered my questions and evaluated my papers but also generously shared his insights into his current and future projects. Documenting these valuable interactions would require a chapter or even a book.

During my visits to their family home in the UK, David and his wife, Sue, were incredibly generous with their time and hospitality. Their home was always open to many friends and colleagues visiting them. I often would return home with books and other literature I treasured. Sometimes, he would give me printed versions of papers I could not have had access to otherwise.

My last visit was in 2018, during my sabbatical. We talked about many ideas and plans for future papers. One idea was to write a paper that Mike Thomas and I were thinking of writing with David. David's idea was also to include Richard Skemp, showing the evolution of our ideas through four generations. I took 18 pages of notes during that visit.

He often told me that my job was not to justify the framework but to evolve it. On one occasion, he wrote:

“You don't have to follow my developments. You need to build your own framework based on your data and your experience. This will evolve over time as you gain more experience and realize how your views depend on your own interpretation. I don't have a name for the latest version of the theory, but I have a sense of what it is. I am thinking of something like LTEM (long-term evolution of mathematical thinking).”

A few years later, he named it *a meaningful long-term framework for mathematical thinking* and published it (Tall, 2023). The essence of formulating this framework is to offer:

... a long-term framework for the meaningful development of mathematical thinking that takes into account the increasing sophistication of mathematical ideas and the cognitive and emotional growth of the individual. It also offers a contextual overview to encourage the comparison and cooperation of different communities of practice. It does not predict the future. It offers a framework for readers to challenge their own beliefs to make informed choices (Tall, 2023, p. 251).

The framework of the three worlds of mathematics resonated well with many mathematicians and enabled them to express their thought processes (Stewart, 2023). After two decades of working with his framework, I felt that his new extensive framework was helping to explain and examine mathematicians' reflections on teaching further and captured more of the complexities of analyzing their rich data.

David and I wrote a paper for ICME 15 based on his extended framework (Stewart & Tall, 2024), and I presented it at the Topic Study Group on Mathematics Education Theories. Sadly, David passed away two days later, on July 15th, 2024, leaving a profound void in the mathematics education community.

David was a brilliant writer and had many excellent ideas. My last correspondence with him was around March 2024. A few months before, he wrote: “*For my part, I would like to produce a final paper of my own **as sole author** to summarise my framework where I acknowledge your extensive assistance to help with references.*”

His educational background was indeed exceptional. He obtained two PhD degrees under the guidance of two renowned experts in mathematics and mathematics education. I was captivated by David’s recounting of his experiences and anecdotes about his two PhD supervisors, Sir Michael Atiyah and Richard Skemp. Although I may have heard some of his stories before, it was just as enjoyable to listen to them again.

David loved his family and was proud of his grandchildren, and he often mentioned them in conversations and even in his papers. In addition to his passion for mathematics and academia, he enjoyed visiting with his friends, watching sports (such as rugby and football), and participating in community activities. He was also a talented musician.

I will deeply miss his tremendous insight and knowledge regarding mathematics education issues. David’s work has influenced many educators worldwide. His memories will stay with those of us who knew him. For those who have never met him, his tremendous publications will be an excellent resource for getting to know a scholar with a brilliant mind in both mathematics and mathematics education. He has left a great legacy behind, and I am grateful to have the privilege of meeting him.



David Tall and Sepideh Stewart, RUME Conference, San Diego, February 2007



David signed his book with Ian Stewart and gave it to me.

David Tall and Sepideh Stewart, David and Sue's home, Summer 2016.



David and Sue Tall, Summer 2013.

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