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M 564.01: Topics in Analysis - Graph C*-Algebras

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M 564: Graph C*-algebras
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This class

C*-algebras were first invented by physicists, as a model for quantum mechanics; they have since turned out to give useful information about groups, and other structures from topology and algebra. However, as befits an analytic object, C*-algebras are defined by taking a bunch of limits. You don't necessarily know what those limits are, just that they exist, so it can be hard to get a good handle on a random C*-algebra.

Graph C*-algebras were invented to fill in this gap. Many natural questions about C*- algebras can be answered, in the case of graph C*-algebras, just by looking at the graph you started with! So, finding graphs with certain properties can help you understand whether a C*-algebra with certain other (related) properties could exist. However, these properties of graphs that we C*-algebraists care about are not always the same ones that graph theorists care about.

My hope is that this Graph C*-algebras course will interest some graph theorists in new questions about graphs, arising from C*-algebra theory; introduce people from many backgrounds to C*-algebras; and hopefully find links between your interests in algebra, topology, and/or physics with graphs and C*-algebras.

Because of all this, I will try to assume as little background as possible for M 564. I won't assume everyone has taken functional analysis, for example. I would ask that if you're taking the course, you've taken at least one analysis course – but there won't be any formal prerequisites.

Textbook

Iain Raeburn's *Graph Algebras*, published by the AMS.

Course structure

There will be weekly written homework assignments. Homework will be due on Friday, at the beginning of class.

Each of the 13 homework assignments will be graded out of 20 points, but the total number of points in the "Homework" category of your grade will be 200, to give you a buffer for busy weeks.

I encourage you to discuss the homework problems together, and take notes on these discussions, but you **must write up your solutions on your own**, and acknowledge, in your write-ups, any sources (human or otherwise) that you consulted, other than me or the course textbook.

When writing up your homework, please consult your notes, and any other resources, as minimally as possible. Before writing the proof, think through the argument. How do all the pieces fit together? What's the best order to explain things? Then, once you start writing, do your best to write the solution without reference to your notes or the textbook. If you have to look up a reference more than 2 or 3 times during the writing, you don't fully understand the solution yet. If this happens, I suggest you stop working on the final draft; work through again (using notes) how the argument should go; and take a break, perhaps by moving on to another problem. When you come back to the tricky problem later, you should have a better, deeper understanding of how the proof works.

Failure to follow these guidelines may result in homework solutions that are uncomfortably close to plagiarism. Plagiarism (copying solutions from a source other than your own brain) constitutes academic misconduct, and will result in a loss of credit for the assignment and/or disciplinary sanctions.

I'll also ask you to read the textbook before class most days, and to let me know what's interesting or confusing about the reading. The reading assignments will be available on Moodle; we may also use Perusall.

Everyone should also plan to give a presentation (20-25 minutes) about a topic of interest to you that relates to Graph C^*-algebras. You can base this presentation on a section of the textbook that we haven't covered in class, or on material from another textbook or a paper, or even material you've learned in another course if it's relevant and will be new to most of the class.

Please let me know as soon as you have a presentation topic in mind, and we'll fine-tune the choice of topic/materials and figure out when to schedule it.

Grade breakdown

Homework: 75%
Presentation: 15%
Reading: 10%