A Solution To An Open Problem In Random Graph Theory

John Finlay
jf142583@umconnect.umt.edu

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John Finlay

University Of Montana

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Rainbow connectivity of randomly perturbed graphs

József Balogh* John Finlay† Cory Palmer‡

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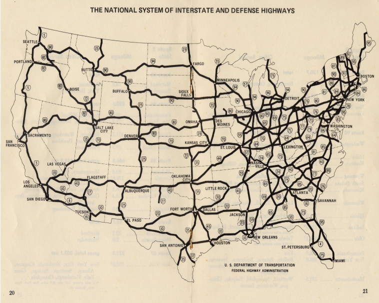
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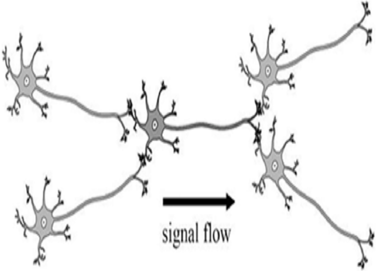
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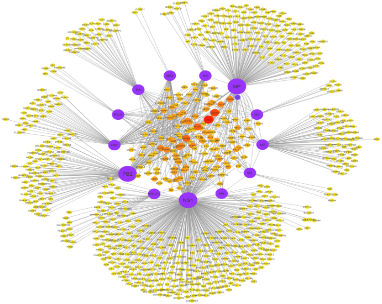
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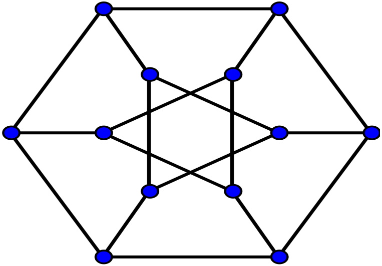

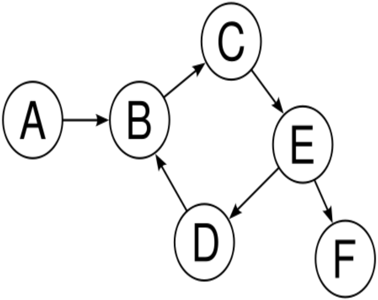
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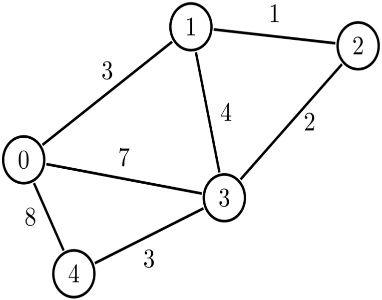
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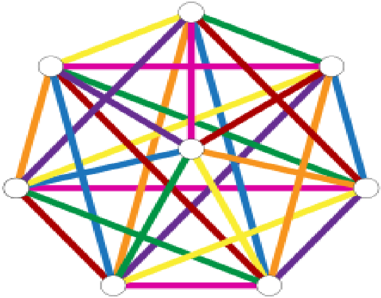

What Is A Graph

Graph

We define a graph G to be the two disjoint sets $V(G)$ and $E(G)$. We call $V(G)$ the vertex set of G or the vertices. Informally, these are the "points" in an illustration. We call $E(G)$ the edge set of G. $E(G)$ is made up of two element subsets of $V(G)$. Informally, these are the "lines" that connect the points in an illustration.

Graph Definitions

Definitions Related to Graph

- If u and v are vertices in G such that they have an edge between them, we denote that edge by uv and say that u is **adjacent** to v. Throughout this presentation we will only be using simple graph (no direction) so $vu = uv$.
- The edge uv is said to be **incident** with both u and v.
- A **path** is a sequence of vertices where each vertex is adjacent to the next one in the sequence.
- A graph G is **connected** if there exists a path between any two vertices of G.
- We denote a path whose endpoints are u and v as $u - v$.
- The number of edges incident to a vertex is the **degree** of the vertex. We denote the minimum degree of G by δ.

Rainbow Connectivity

What is rainbow connected?

- Suppose that we color every edge of a graph G using $[r] = \{1, 2, ..., r\}$ colors.
- A **rainbow path** is a path where no edge color is repeated.
- A graph G is **rainbow connected** if between any two vertices in G there exists a rainbow path.
- For a graph G the **rainbow connected number** is the smallest r such that G can be rainbow connected.

Randomly Perturbed Graphs

How to make a randomly perturbed graph

- Fix some $\delta > 0$.
- Choose some graph from the set of all graphs on n vertices with minimum degree at least δn arbitrarily, call this graph H.
- Let M be a set of edges of size m chosen at random from all of the $\binom{n}{2}$ possible edges.
- Insert the m edges into H, ignoring double edges, the resulting graph is $G_{H,m}$.
- Randomly color the edges with r colors, producing $G^r_{H,m}$

The Probabilistic Method

Recall

In a probability space, all of the event probabilities must sum to exactly 1.

The Probabilistic Method

In order to prove that something exists, rather than using a constructive argument, we define an appropriate probability space of structures and prove that a structure with the desired properties exists with positive probability. Or that the probability that it does not exist does not have probability 1.

The Union Bound

F

or any events $E_1, E_2, \ldots$ we have,

$$Pr\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} Pr(E_i)$$