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# Music Through Math: Analyzing and Composing Scores Mathematically

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# Music Through Math: A Set Definition of Notes

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## Approaching Both Fields from a New Angle

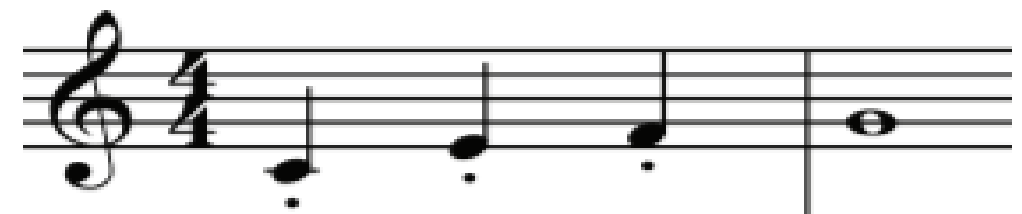
### Start with the Music, look at the Math

- From simple nursery rhymes to complicated symphonies, all music can be analyzed with set theory.
- Each note corresponds to a number.
- We group these numbers based on the way the notes are grouped.
  - The first two measures of this song form the set: {0,7,9}.



### Start with the Math, look at the Music

- Composers block stymies a great many would-be music writers.
- Overcome such blocks by picking a set and using the numbers to guide the notes.
- Say, we like the numbers 4, 7, 0, and 5.
  - Rearranging these we can produce this set: {0,4,5,7}. A pattern best known as "Oh when the saints..."



## Key Definitions

- Pitch:** a note which we hear
- Pitch Class:** the name for a type of pitch regardless of octave displacement
  - The pitch class of C4 and C7 is simply C
  - There are 12 pitch classes in a standard octave, with 12 chromatic notes.
  - Two pitches which sound the same on a standard octave will have the same pitch class (B#=C).
- Modular Congruence:** if two numbers  $b$  and  $c$  are such that  $(b-c)$  is divisible by  $m$ , then,  $b$  and  $c$  are said to be "congruent modulo  $m$ " ( $b=c \pmod m$ ).
- Pitch Class Representative:** the integers modulo 12 each represent a pitch class
  - This begins with pitch class C=0. All other pitch classes are numbered by counting up half-steps from pitch class C.
- Serialism:** Composition technique using numerical values to manipulate musical elements
- Pitch Class Set:** a list of pitch class representatives.

## Second Viennese School

Chromatic - Expressionism  
Atonality  
Music with no tonal center



Arnold Schoenberg (1874–1951)

Other Post-Tonal Composers:  
Claude Debussy  
Béla Bartók  
Leoš Janáček



Anton Webern (1883–1945)

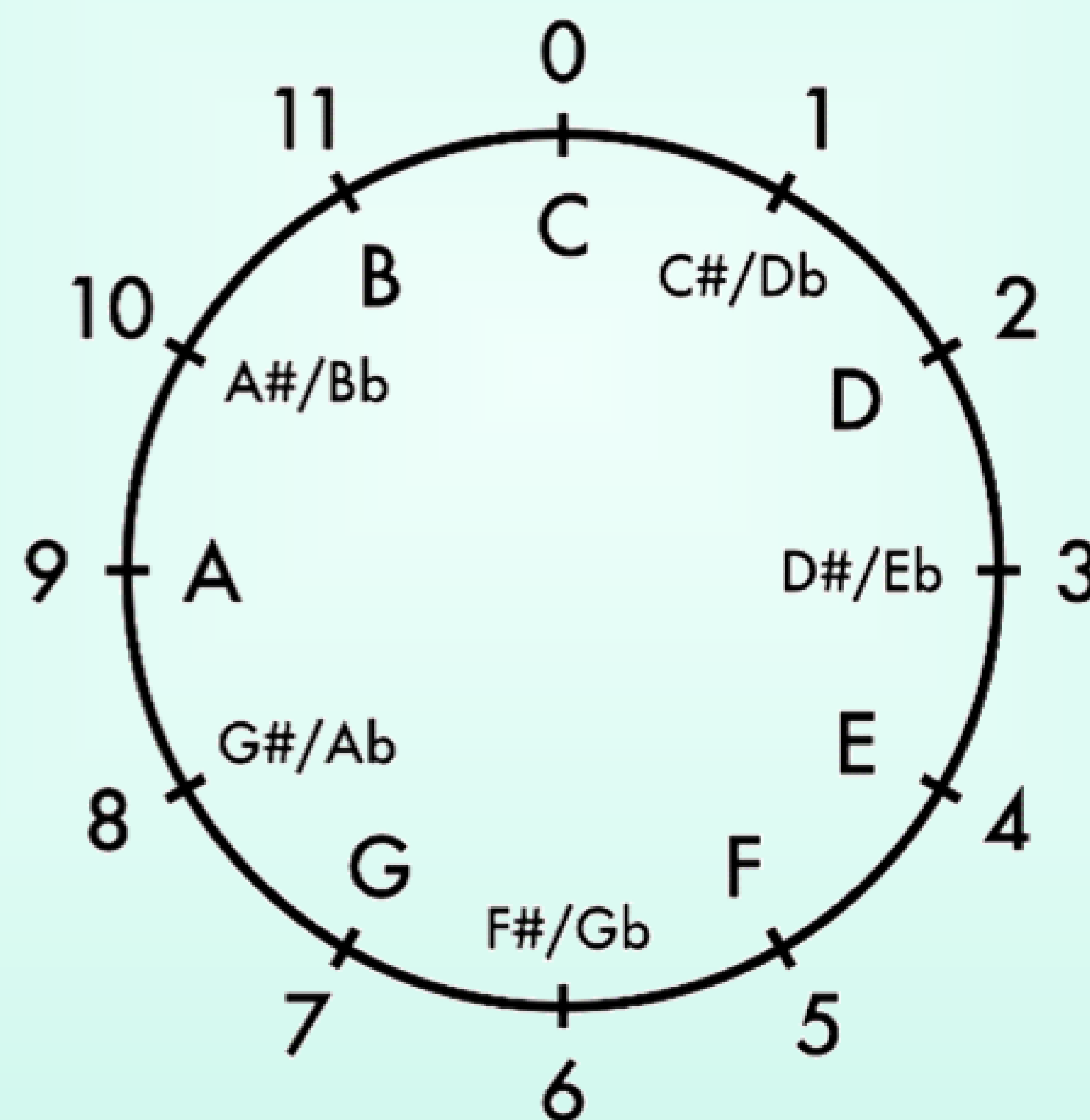


Alban Berg (1885–1935)

Twelve tone row:  
All twelve pitch classes sound before any are repeated.

## Composition Techniques With {0,4,7}

- Transposition: {4,e,8,4}**
  - Move all pitches in the set up or down by a specific interval
- Retrograde: {0,4,7,0}**
  - Reversing the order of the elements in a set
- Inversion: {0,5,8,0}**
  - A note  $n$  half-steps above C is flipped to become  $n$  half-steps below C
- Retrograde Inversion: {0,8,5,0}**
  - First invert, then retrograde

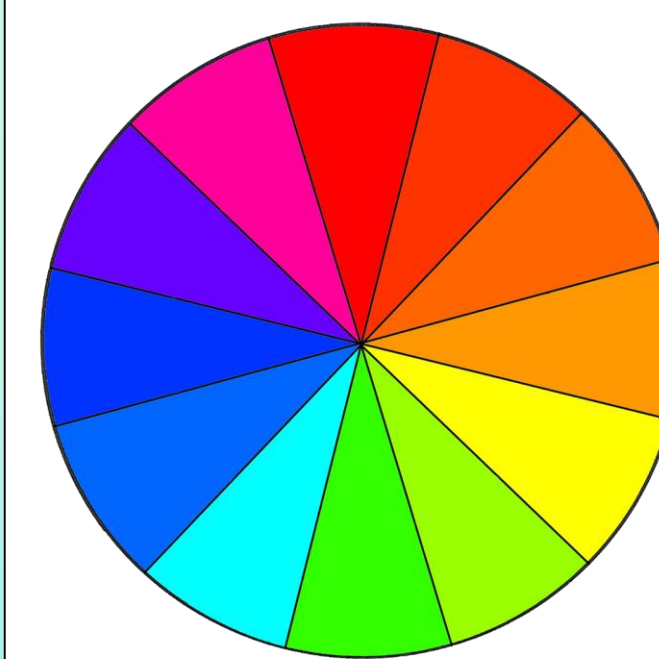


## Katerina Set

- |   |   |
|---|---|
| K | e |
| A | 1 |
| T | 7 |
| E | 5 |
| R | 6 |
| I | t |
| N | 2 |
| A | 1 |
- {e,1,7,5,6,t,2,1}
  - A variety of names was tried first
  - It appears thus on a single staff.
  - Ten bars generated from this set appear below.



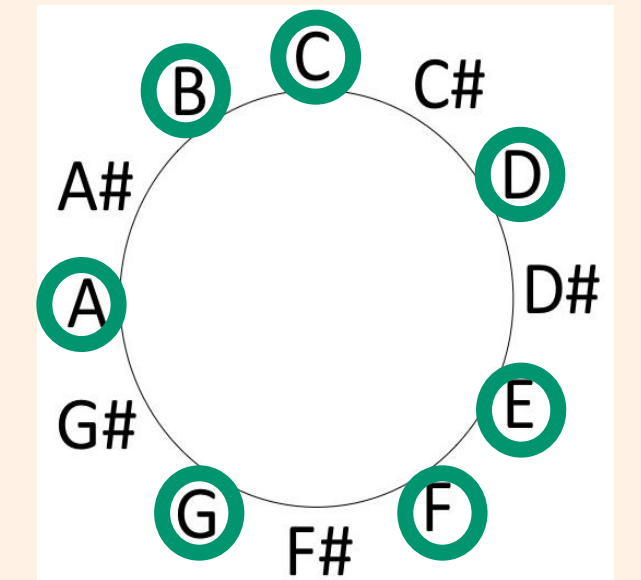
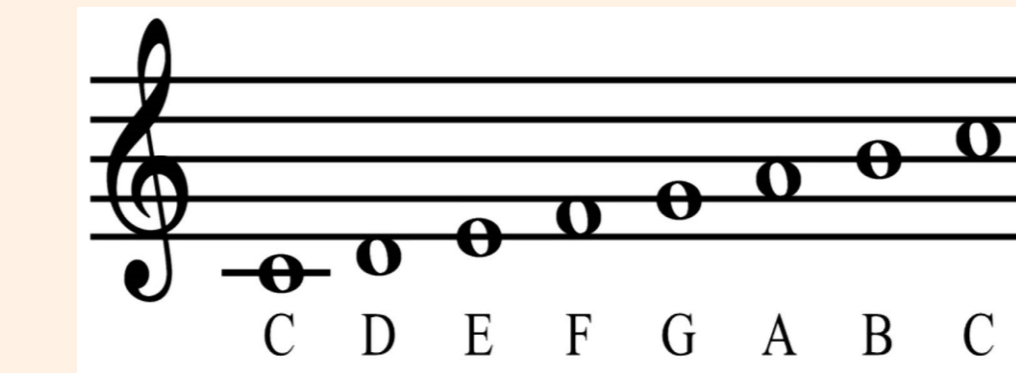
## Chromatically Colorful Set



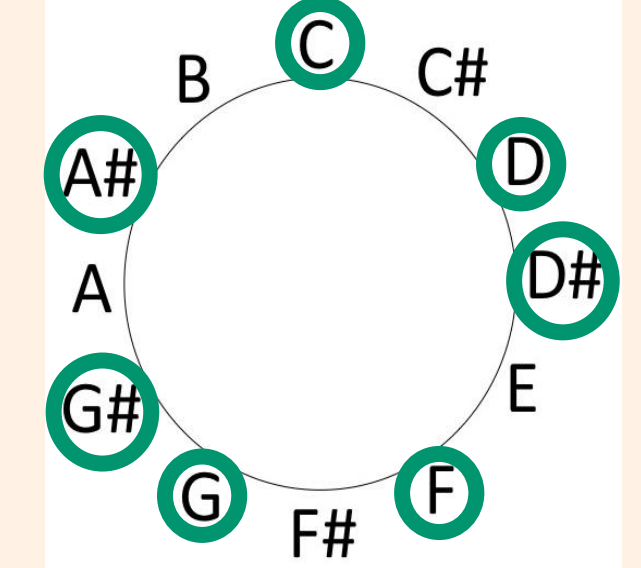
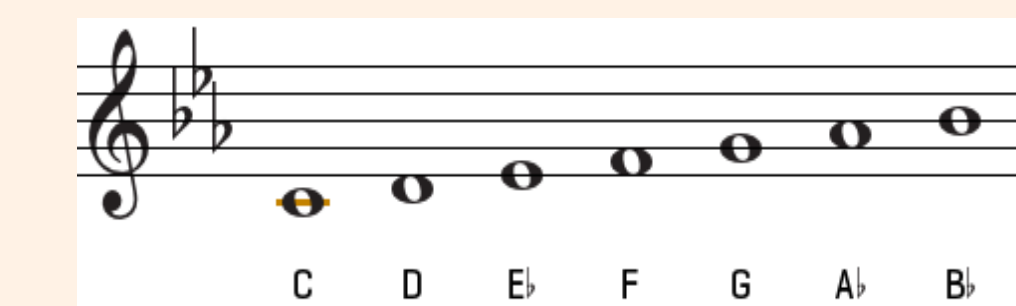
- Let Red=C=0
- Warm Set {0,1,2,3,4,5}
- Cool Set {6,7,8,9,t,e}
- Primary Set {0,4,9}
- Secondary Set {2,6,e}
- Cool is a transposition of Warm,
- Secondary is a transposition of Primary.

## Famous Sets

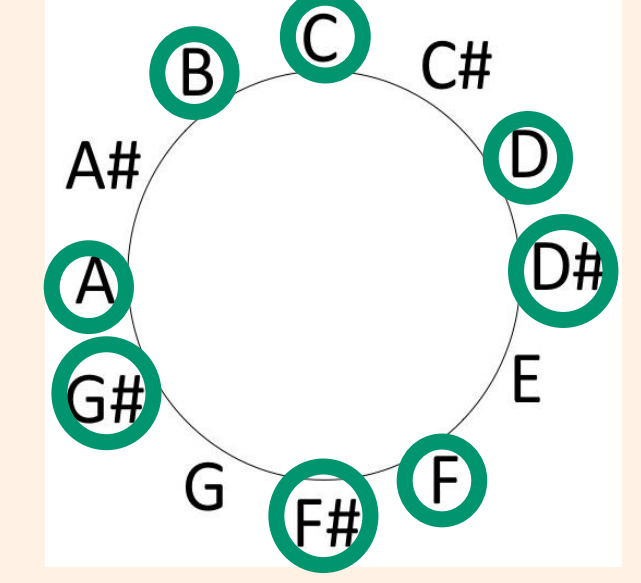
- Major Scale**
  - {0,2,4,5,7,9,e}
  - Diatonic scale



- Minor Scale**
  - {0,2,3,5,7,8,t}
  - Three forms

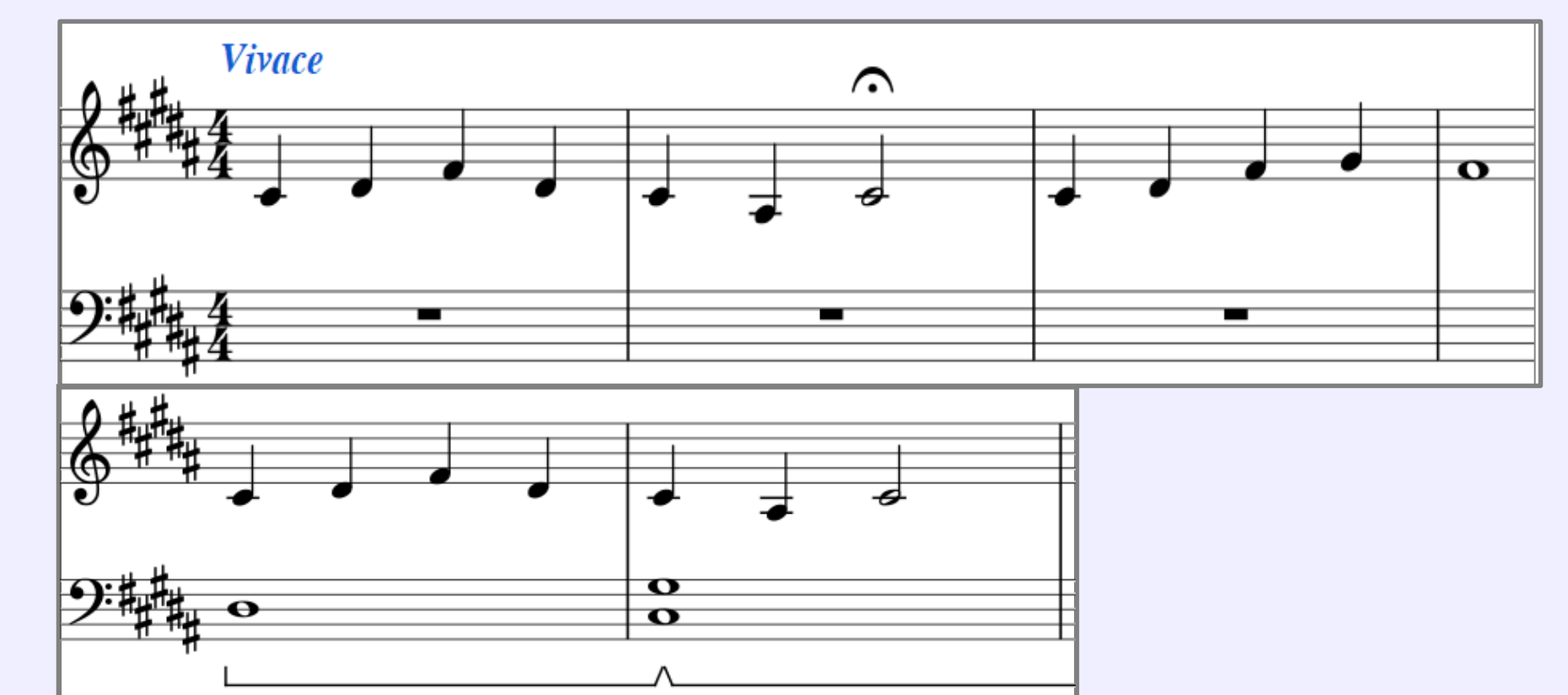


- Octatonic Scale**
  - {0,1,3,4,6,7,9,t}
  - Jazz chords



## The End Set: "Piano Black"

- Pentatonic Scale on the black keys
  - {6,8,t,1,3}



- You can write music by starting with numbers!
  - Music composition accessible to non-musicians
  - Math accessible to non-mathematicians
    - The modulo twelve system as an entry point to number theory.

## Acknowledgments

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