Apr 27th, 3:00 PM - 4:00 PM

Music Through Math: Analyzing and Composing Scores Mathematically

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The first two measures of this song form the set: \{0,7,9\}.

Retrograde: \{0,4,7,0\}

Anton Webern

The modulo twelve system as an entry point to number theory. Let

\[ \text{Red}=\text{C}=0 \]

Cool Set

\{0,1,3,4,6,7,9,e\}

Music composition accessible to non-musicians.

The End Set: “Piano Black”

\[ \{6,8,1,1,3\} \]

You can write music by starting with numbers!

Major Scale

- \{0,2,4,5,7,9,e\}
- Diatonic scale

Minor Scale

- \{0,2,3,5,7,8,t\}
- Three forms

Octatonic Scale

- \{0,1,3,4,6,7,9,t\}
- Jazz chords

Chromatic Scale

- \{0,1,2,3,5,7,8,t\}
- Math accessible to non-mathematicians

The many would have made music with no tonal center.

Twelve tone row: All twelve pitch classes sound before any are repeated.

Katerina Set

\{e,1,7,5,6,1,2,1\}

A variety of names was tried first

It appears thus on a single staff.

Ten bars generated from this set appear below.

Chromatically Colorful Set

- Secondary Set
- Cool: a transposition of Warm.
- Secondary is a transposition of Primary.

Key Definitions

- **Pitch**: a note which we hear
- **Pitch Class**: the name for a type of pitch regardless of octave displacement
  - The pitch class of C4 and C7 is simply C
  - There are 15 pitch classes in a standard octave, with 12 chromatic notes.
  - Two pitches which sound the same on a standard octave will have the same pitch class (B♯=C).
- **Modular Congruence**: if two numbers \(b\) and \(c\) are such that \((b-c)\) is divisible by \(m\), then, \(b\) and \(c\) are said to be “congruent modulo \(m\)” \((b \equiv c \mod m)\).
- **Pitch Class Representative**: the integers modulo 12 each represent a pitch class
  - This begins with pitch class C=0. All other pitch classes are numbered by counting up half-steps from pitch class C.
- **Serialism**: Composition technique using numerical values to manipulate musical elements
- **Pitch Class Set**: a list of pitch class representatives.

Approaching Both Fields from a New Angle

Start with the Music, look at the Math

- From simple nursery rhymes to complicated symphonies, all music can be analyzed with set theory.
- Each note corresponds to a number.
- We group these numbers based on the way the notes are grouped.
  - The first two measures of this song form the set: \{0,7,9\}.

Start with the Math, look at the Music

- Composers block stymies a great many would-be music writers.
- Overcome such blocks by picking a set and and using the numbers to guide the notes.
- Say, we like the numbers 4, 7, 0, and 5.
  - Rearranging these we can produce this set: \{0,4,5,7\}. A pattern best known as “Oh when the saints…”

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- First invert, then retrograde

### Famous Sets

- **Major Scale**
  - \{0,2,4,5,7,9,e\}
- **Diatonic scale**
- **Minor Scale**
  - \{0,2,3,5,7,8,t\}
- **Three forms**
- **Octatonic Scale**
  - \{0,1,3,4,6,7,9,t\}
- **Jazz chords**

### The End Set: “Piano Black”

- **Pentatonic Scale on the black keys**
  - \{6,8,1,1,3\}

### Acknowledgments

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Terry Jimmerson – for never letting me forget the beauty within both of these fields.

### Second Viennese School

- **Chromatic - Expressionism**
- **Atonality**
- **Music with no tonal center**

Alban Berg (1885–1935)

Arnold Shoenberg (1874–1951)

Composer: Claude Debussy

Béla Bartók

Leoš Janáček

Twelve tone row: All twelve pitch classes sound before any are repeated.

### Other Post-Tonal Composers:

- Claude Debussy
- Béla Bartók
- Leoš Janáček

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University of Montana Conference on Undergraduate Research 2018

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Music Through Math: A Set Definition of Notes