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Developing Proportional Reasoning Through Gears Investigation

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Developing Proportional Reasoning Through Gears Investigation

**Background:**
- A foundational understanding of ratio and proportion is essential for success in higher mathematics and science.
- To understand ratio and proportion concepts, students must be exposed to multiple contexts and interpretations.
- Research has identified gear-pair analysis as a promising manipulative that supports students' conceptual understanding of ratio and proportion in small-group (1-3 pupils) settings. (Lobato and Ellis, 2010)

**Research Questions:**
1. Does structured investigation of gear-pairs lead to students' ability to abstract ratio settings?
2. Does unstructured investigation of gear-pairing possibilities show evidence supporting students' ability to analyze ratio settings in pursuit of a "best solution?"
3. Does gear-pair analysis scale to whole classroom instruction?

**Montana Common Core Standard:**
7.RP.2 – Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.

**Setting and Method:**
- Research was conducted in 7th grade math classes at Washington Elementary School. Four classes participated – three regular classes and one honors class. A total of 93 students participated.
- Structured and unstructured activities were used in these seventh-grade classrooms, to facilitate this investigation.
- During the initial investigation of gears, students used manipulatives to complete a guided worksheet exploring the relationships of how different gear pairs interact.
- Next, students were given an open-ended gear-pairing problem designed to test their understanding of ratio in a bicycle gear-pair context.
- After the intervention, an analysis was made of the classroom artifacts created by students during both stages of the gears investigation.
- Data was analyzed using iterative qualitative analysis of patterns of student activity, leading to categories and counts of recurring patterns of response.

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**Results of Investigation:**
- During the structured stage of exploration:
  - Students demonstrated a mastery of tabular representations of data.
  - Students demonstrated a proficiency for arithmetic presentations of data.
  - Students demonstrated a low understanding of how to use graphical representations of data as a tool for analysis.

- During the unstructured stage of intervention:
  - Students demonstrated a strong preference for tabular and arithmetic representations of data.
  - Students used graphical interpretations ineffectively.

**Ability of Students to Represent And Interpret Data in Proportional Setting Using Gear-Pairs**

<table>
<thead>
<tr>
<th>Level of skill demonstrated:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>proficient</td>
<td></td>
<td>Y</td>
</tr>
<tr>
<td>emergent</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

**Examples of Student Work:**
- Example of guided exploration:
  - Student scales graph to look like y=x, rendering meaningful graphical analysis impossible.
  - Student correctly fills in missing values in the table.

- Student response to open-ended gear-pairing problem:
  - Student demonstrates preference for arithmetic analysis of data, despite difficult to interpret presentation.
  - Student correctly identifies equation of graph.

**Discussion:**
- Students were able to answer open-ended ratio problem in a gear-pair setting using familiar strategies for representing data.
- The intervention was ineffective in guiding students toward new strategies for representing and comparing data.
- After the initial guided exploration of gears, students were not able to make the abstract connection between a gear-pair ratio and a line on a graph.
- This activity reaffirmed student strengths in tabular and arithmetic interpretations of data.
- Specific modifications to the two-phase intervention could better support learning targets aligned with standard 7.RP.2 in order to better inform future classroom instruction.

**Informed Classroom Instruction:**
- To more effectively implement this intervention in a whole-class setting, teachers should use structured collaborative group work during initial investigation to guide use of graphical interpretation of proportional relationships between gear ratios.
- Teachers should scale axes to prevent different ratio settings from appearing as y=x.
- Teachers should design guided questions to encourage visual analysis of data.
- Teachers should include point (0, 0) in ratio tables to use students' strengths with tabular analysis to promote graphical connection.

**References:**

**Acknowledgments:**
- Dr. Matt Roscoe
- Emily J. Wilson