Apr 17th, 3:00 PM - 4:00 PM

# Developing Proportional Reasoning Through Gears Investigation

Colt J. Davidson
*University of Montana*, colt.davidson@umontana.edu

# Developing Proportional Reasoning Through Gears Investigation

Colt Davidson
Elementary Education, University of Montana

## Background:

- A foundational understanding of ratio and proportion is essential for success in higher mathematics and science.
- To understand ratio and proportion concepts, students must be exposed to multiple contexts and interpretations.
- Research has identified gear-pair analysis as a promising manipulative that supports students' conceptual understanding of ratio and proportion in small-group (1-3 pupils) settings. (Lobato and Ellis, 2010)

## Research Questions:

1. Does *structured* investigation of gear-pairs lead to students' ability to abstract ratio settings?

2. Does *unstructured* investigation of gear-pairing possibilities show evidence supporting students' ability to analyze ratio settings in pursuit of a "best solution?"

3. Does gear-pair analysis scale to whole classroom instruction?

## Montana Common Core Standard:

**7.RP.2** – *Recognize and represent proportional relationships between quantities including those represented in Montana American Indian cultural contexts.*

1. *Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.*
2. *Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.*
3. *Represent proportional relationships by equations.*
4. *Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r), where r is the unit rate.*

## Setting and Method:

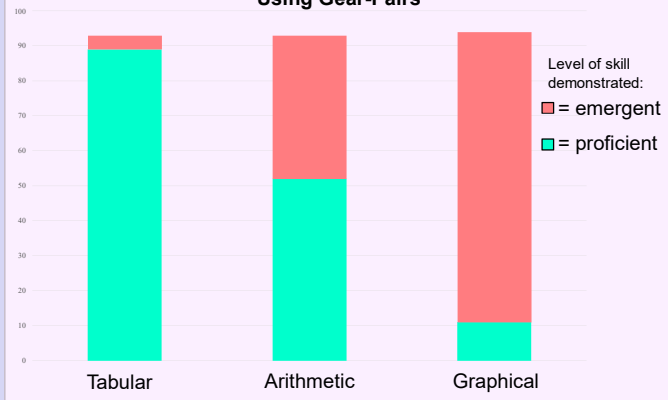
- Research was conducted in 7th grade math classes at Washington Elementary School. Four classes participated – three regular classes and one honors class. A total of 93 students participated.
- Structured and unstructured activities were used in these seventh-grade classrooms. to facilitate this investigation
- During the initial investigation of gears, students used manipulatives to complete a guided worksheet exploring the relationships of how different gear pairs interact.
- Next, students were given an open-ended problem designed to test their understanding of ratio in a bicycle gear-pair context
- After the intervention, an analysis was made of the classroom artifacts created by students during both stages of the gears investigation.
- Data was analyzed using iterative qualitative analysis of patterns of student activity, leading to categories and counts of recurring patterns of response

## Results of Investigation:

During structured stage of exploration:
- Students demonstrated a mastery of tabular representations of data
- Students demonstrated a proficiency for arithmetic presentations of data
- Students demonstrated a low understanding of how to use graphical representations of data as a tool for analysis

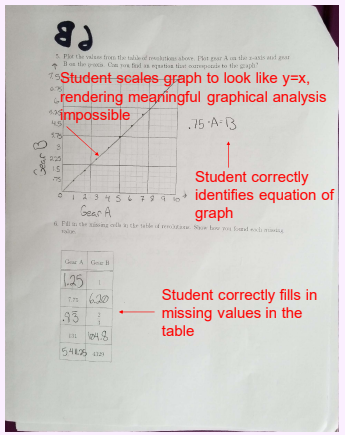
During the unstructured stage of intervention:
- Students demonstrated a strong preference for tabular and arithmetic representations of data.
- Students used graphical interpretations ineffectively.

**Ability of Students to Represent And Interpret Data in Proportional Setting Using Gear-Pairs**

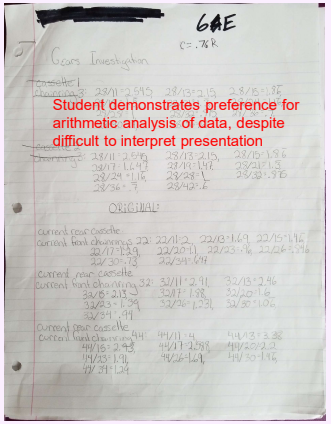



## Examples of Student Work:

Example of guided exploration:




Student response to open-ended gear-pairing problem:



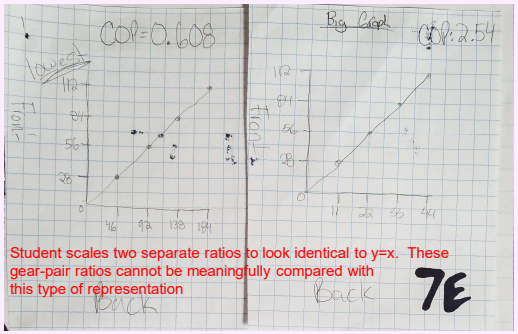

## Discussion:

- Students were able to answer open-ended ratio problem in a gear-pair setting using familiar strategies for representing data
- The intervention was ineffective in guiding students toward new strategies for representing and comparing data
- After the initial guided exploration of gears, students were not able to make the abstract connection between a gear-pair ratio and a line on a graph
- This activity reaffirmed student strengths in tabular and arithmetic interpretations of data
- Specific modifications to the two-phase intervention could better support learning targets aligned with standard **7.RP.2** in order to better inform future classroom instruction

## Informing Classroom Instruction:

- To more effectively implement this intervention in a whole-class setting, teachers should use structured collaborative group work during initial investigation to guide use of graphical interpretation of proportional relationships between gear ratios.
- Teachers should scale axes to prevent different ratio settings from appearing as y=x.
- Teachers should design guided questions to encourage visual analysis of data.
- Teachers should include point (0, 0) in ratio tables to use students' strength with tabular analysis to promote graphical connection.

Student response to open-ended gear-pairing problem:




**References:**

Lobato, J., & Ellis, A. B. (2010). Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning. National Council of Teachers of Mathematics.

**Acknowledgments:**

- Dr. Matt Roscoe
- Emily J. Wilson