Correlation between the frequencies of harmonic partials generated from selected fundamentals and pitches of supplied tones

Chester Orlan Strom

The University of Montana

Strom, Chester Orlan, "Correlation between the frequencies of harmonic partials generated from selected fundamentals and pitches of supplied tones" (1962). Graduate Student Theses, Dissertations, & Professional Papers. 1913.
https://scholarworks.umt.edu/etd/1913
CORRELATION BETWEEN THE FREQUENCIES OF HARMONIC PARTIALS
GENERATED FROM SELECTED FUNDAMENTALS
AND PITCHES OF SUPPLIED TONES

by

CHESTER ORLAN STROM

B.M. Montana State University, 1960

Presented in partial fulfillment of the
requirements for the degree of
Master of Music

MONTANA STATE UNIVERSITY

1962

Approved by:

Chairman, Board of Examiners

Dean, Graduate School

JUL 31 1962

Date
ACKNOWLEDGMENT

The investigator is indebted to the several students, Miss Carol Stafney, and Miss Judy Sunwall for their time and patience in helping to accumulate the data of this investigation. He wishes to thank Professor Gerald H. Doty for the many fine suggestions, as well as the giving of additional time. He owes Dr. Everett Gates a debt of gratitude for his encouragement, the list of related studies, and the permission to include his table of the four main scale systems. Lastly, the writer also appreciates the efforts of Professors Mark Jakobson and Randolph Jeppesen in supplying the mathematical formula and other information pertinent to this thesis.
# TABLE OF CONTENTS

GLOSSARY ............................................. viii

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. THE PROBLEM AND DEFINITIONS OF TERMS USED ................................</td>
<td>1</td>
</tr>
<tr>
<td>The Problem ....... .......................................................................</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the problem ......... ................................................</td>
<td>2</td>
</tr>
<tr>
<td>Importance of the study ...............................................................</td>
<td>1</td>
</tr>
<tr>
<td>Delimitations ..................................................................................</td>
<td>2</td>
</tr>
<tr>
<td>Definitions of Terms Used ............................................................</td>
<td>4</td>
</tr>
<tr>
<td>Frequency ......................................................................................</td>
<td>4</td>
</tr>
<tr>
<td>Pitch .............................................................................................</td>
<td>4</td>
</tr>
<tr>
<td>Fundamental ...................................................................................</td>
<td>4</td>
</tr>
<tr>
<td>Harmonic Partial .............................................................................</td>
<td>4</td>
</tr>
<tr>
<td>Overtone .......................................................................................</td>
<td>4</td>
</tr>
<tr>
<td>Supplier and supplied tones ................................................................</td>
<td>4</td>
</tr>
<tr>
<td>Preview ............................................................................................</td>
<td>5</td>
</tr>
<tr>
<td>II. HISTORICAL DATA AND RELATED INVESTIGATIONS ..............................</td>
<td>6</td>
</tr>
<tr>
<td>III. AN EVALUATION OF INTERVALIC COORDINATING SYSTEMS ..................</td>
<td>12</td>
</tr>
<tr>
<td>The Pythagorean System ....... .......................................................</td>
<td>12</td>
</tr>
<tr>
<td>Just Intonation ...............................................................................</td>
<td>14</td>
</tr>
<tr>
<td>The Mean-tone System .......................................................................</td>
<td>17</td>
</tr>
<tr>
<td>The Overtone Series .........................................................................</td>
<td>17</td>
</tr>
<tr>
<td>Combination Tones ............................................................................</td>
<td>19</td>
</tr>
<tr>
<td>Subjective tone ...............................................................................</td>
<td>19</td>
</tr>
</tbody>
</table>

-iii-
<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference tone</td>
<td>19</td>
</tr>
<tr>
<td>Summation tone</td>
<td>19</td>
</tr>
<tr>
<td>Aural harmonics</td>
<td>19</td>
</tr>
<tr>
<td>Combination tone</td>
<td>20</td>
</tr>
<tr>
<td>Equal-temperament</td>
<td>21</td>
</tr>
<tr>
<td>Summary</td>
<td>21</td>
</tr>
<tr>
<td>IV. THE INVESTIGATION AND RESULTS</td>
<td>23</td>
</tr>
<tr>
<td>The Equipment</td>
<td>24</td>
</tr>
<tr>
<td>Stroboconn</td>
<td>24</td>
</tr>
<tr>
<td>Organ pipes</td>
<td>24</td>
</tr>
<tr>
<td>Wind instruments</td>
<td>24</td>
</tr>
<tr>
<td>The Procedure</td>
<td>24</td>
</tr>
<tr>
<td>The Results</td>
<td>28</td>
</tr>
<tr>
<td>Flutes</td>
<td>29</td>
</tr>
<tr>
<td>The seventh partial</td>
<td>29</td>
</tr>
<tr>
<td>The eleventh partial</td>
<td>29</td>
</tr>
<tr>
<td>The thirteenth partial</td>
<td>29</td>
</tr>
<tr>
<td>The fourteenth partial</td>
<td>30</td>
</tr>
<tr>
<td>Oboe</td>
<td>30</td>
</tr>
<tr>
<td>The seventh partial</td>
<td>30</td>
</tr>
<tr>
<td>The eleventh partial</td>
<td>30</td>
</tr>
<tr>
<td>Clarinets</td>
<td>31</td>
</tr>
<tr>
<td>The fourth partial</td>
<td>31</td>
</tr>
<tr>
<td>The fifth partial</td>
<td>31</td>
</tr>
</tbody>
</table>
Appendix B .............................. 44

Letter from Dr. Everett Gates, Associate Chairman, Music Education Department, Eastman School of Music, University of Rochester, Rochester, New York

Appendix C .............................. 46

Table I. Conversion of Partials to Cents

Appendix D .............................. 47

Table II. Statistics on Dependence of Tuning on Ambient Temperatures

Appendix E .............................. 48

Table III. Mean "Warm-up" Change in Cents at Various Ambient Temperatures

Appendix F .............................. 49

Table IV. Comparison of the Pythagorean, Just, Mean-tone, and Equally Tempered Scales
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Correlation Between Supplied Tones and Harmonic Partials of Fundamental C</td>
<td>26</td>
</tr>
<tr>
<td>2. Pitch Names</td>
<td>27</td>
</tr>
</tbody>
</table>
GLOSSARY

**A cappella** - A choir performing without an accompaniment.

**Cent** - A unit of interval measure. The 100th part of an equal semitone, or 1/1200th part of an octave.

**Cochlear** - An adjective relating to the spiral-shaped part of the inner ear, or cochlea.

**Comma** - A tuning error, such as the interval B#-C in Pythagorean tuning.

**Diesis** - The interval (about 1/5 tone) between any two enharmonically equivalent notes, as Ab and G#, in just intonation.

**Diatonic** - Pertaining to or designating a standard major or minor scale of eight tones to the octave.

**Ditone** - A major third.

**Ditonic comma** - Its ratio is approximately 74/73; its value is taken conveniently at 24 cents.

**Double-stops** - A combination of simultaneously played tones on violins and related instruments.

**Fauxbourdon** - Sixth-chord harmonies, e.g., E-G-C.

**Gymel** - A late medieval term based chiefly on parallel thirds.

**Just** - pure: A term applied to intervals in their natural sound.

**Limma** - Its ratio is 16/15; also the difference between the major seventh and the octave in just intonation.

**Major tone** - Its ratio is 9/8, as C to D in the C major scale of the Pythagorean system and just intonation.

**Minor tone** - Its ratio is 10/9, as D to E in the C major scale of just intonation, also the difference between the 9th and 10th partial in the harmonic series.

**Monochord** - A contrivance, probably invented by Pythagoras, consisting of a string stretched over a resonator. A movable fret is used to vary the vibrating length of the string.

**Natural scale** - The same as the scale of just intonation.
Neapolitan sixth — A first inversion of a triad on the lowered supertonic.

Picardy third — The major third as used for the final chord of a composition in a minor key.

Quarter-tone — One-half of a semitone, or about 50 cents.

Schisma — The difference between the syntonic and the Pythagorean comma, or about two cents.

Semitone — The half of a whole tone. The size varies as to the tuning system.

Stroboconn — The Stroboconn is an electronic device for visual measurement of sound frequencies to within 1/100th part of a semitone, as found in the equally tempered scale based upon the American Standard "A" of 440 cycles per second. Its testing range is from 31.772 to 4,066.8 cycles per second. Twelve scanning windows occupy positions corresponding to the white and black keys on the piano in an octave from C to B. Its tuning unit is calibrated, by means of a pointer and dial, from 50 cents flat of a standard fundamental to 50 cents sharp—a span of one octave.

Syntonic comma — The interval between a just major third and a Pythagorean third. Its value is about twenty-two cents.

Temper — To vary the pitch slightly.

Temperament — A system in which some or all intervals cannot be expressed in rational numbers.

Tetrachord — A group of four scalewise tones.

Third, just or pure — The interval between the first and third step of a scale in just intonation; the interval between the fourth and fifth partials of the harmonic series.

Third, Picardy — The major third as used for the final chord in a minor key.

Third, Pythagorean — The interval between the first and third step of the Pythagorean scale, or about 408 cents. Three such intervals result in a discrepancy of 24 cents (3 x 8)—the Pythagorean comma.

Trumpet-reeds — Organ pipes having conical resonators of full length, i.e., 8' C has a resonator of about 8' length.
CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

Innumerable opinions have often been voiced and experiments have been performed in quest of improving intonation in the performance of music. Many have limited the study to scales alone; some have acknowledged the tendency toward Pythagorean tuning in unaccompanied playing; others have claimed an inclination to just intonation in tertian harmony; but the author has found no evidence to exist on the effect of overtones from selected fundamentals produced on instrumental generators upon the frequency of a tone sounded simultaneously by a musician utilizing a wind instrument.

I. THE PROBLEM

Statement of the problem. It was the purpose of the experiment to determine an existence, if any, of a definite relationship between the frequency of a harmonic partial generating from a selected fundamental, which, in turn, was produced on organ pipes, and the frequency of a tone of similar pitch sounded on a wind instrument. The organ pipes were chosen to eliminate the hazard arising through a variation in cycles per second of tones generated on wind instruments, thus establishing a more stable experiment.

Importance of the study. Knowledge tends to increase the efficiency, as well as proficiency, of an artificer in any field. Since intonation is definitely a deciding factor to superior performance in music, it
then follows that serious instructors must not overlook a constituent of so great importance to the development of artistic knowledge and skill in the students who are under their supervision.

**Delimitations.** The calibrating was limited to tones corresponding to the frequencies of harmonic partials generated from selected fundamentals and to tones lying within the playing ranges of the wind instruments supplying the measured frequencies.

Although several pipes were selected for use in providing fundamentals, all of them were chosen from the same classification—the trumpet reeds.

Only one wind instrument tone at a time was sounded simultaneously with one organ tone: the organ tone being the selected fundamental plus its natural overtones. To have included more would involve "subjective" tones, which are a separate study in themselves. In fact, the inclusion of more tones could involve research into a combination of subjective tones and harmonic partials.

At least three avenues of research mentioned by the investigator were considered by Dr. Gates as being suitable for comprehensive areas of study. The three areas which he selected were: (1) the effect of the "subjective tones" on intonation, (2) the calibration of the frequency of harmonic partials from fundamentals sounded on various instruments, (3) the measurement of the frequency of intervals played on instruments against a standard sounded on organ pipes.

---

¹Dr. Everett Gates, Eastman School of Music, in correspondence with the investigator, February 21, 1962. See Appendix, pages 44-45 for the complete contents of this letter.
Gates's sincerity in endorsing the project was further substantiated by the following portion of the same letter:

I am happy that you have developed your interests in the directions you have mentioned in your letter. I have encouraged many students to think along the same lines and try to develop their interests in a class which I teach here, 'Science and Music Education.' In fact, Prentice-Hall have approached me to write a text on 'Science and Music Education' which will attempt to tie down some of the knowledge gleaned from research to pedagogical procedures. The encouragement of knowing that many others realize the great advantages of exact knowledge applied to music teaching and how this can increase our teaching efficiency is a factor that will definitely affect my decision regarding the book. The more music teachers we have who are informed and interested the faster we will move forward.

I hope you will go ahead with your plans for you have considered areas that are in need of investigation. I'm sorry that I cannot be of much service to you, but I hope this information will help a little. I regret that my delay has inconvenienced you, but your questions posed many problems that required careful consideration and we have been having a very hectic year because of the absence of Dr. Hanson who has been abroad with the Philharmonia.

"The effect of subjective tones on intonation" was discarded because of the anticipated complexity of the experiment. The Master's thesis of R. H. Newton, a very interesting and enlightening study, explains some of the complications involved in subjective tones.

Even though "the calibration of the harmonic partials from fundamentals on various instruments," may reveal a variation in the frequencies of similar harmonic partials generated from like notated fundamentals sounded on different instruments, this too was discarded, since the calibration of the same would not solve the problem in the most logical sequence.

Rolfe H. Newton, The Combination Tone as the Basis for Modern Harmonic Practice: Descartes to Hindemith (unpublished Master's thesis, the University of Wisconsin, Madison, 1958).
Therefore, inasmuch as "the measurement of the frequency of intervals played on instruments against a standard sounded on organ pipes" was basically the original intent of the investigator, the research and experimentation were planned in this area. Since the Stroboconn was used in this investigation, the frequency will not be measured in cycles per second, but in cents.

II. DEFINITIONS OF TERMS USED

Frequency. The term is used to denote the number of vibrations per second generating from a sound producing medium. It may be referred to in cycles per second, which is often shortened to "CPS."

Pitch. Pitch is the subjective counterpart of frequency, and cannot be measured.

Fundamental. This term refers to the lowest harmonic partial—the tone from which all overtones in a series are generated.

Harmonic partial. Harmonic partials are members of the composite sound produced by all musical instruments. The frequencies of these partials are exact multiples of the fundamental, or lower partial.

Overtone. An overtone is one of the higher tones faintly accompanying a fundamental and created by the vibration of small segments of a string or air column; hence, the first overtone is the second partial.

Supplier and supplied tones. The term "supplier" is used when referring to the person furnishing a tone on any of the various wind instruments. "Supplied tone" is the tone furnished by the supplier.
III. PREVIEW

The problem of pitch placement has been an enigma since ancient times. Every solution possessed its own peculiarity of perplexities. In order that the reader may share the investigator's perspective, the next two chapters have been devoted to this area of study.

The experimental technique is easily understood. Here, the employment of a Stroboconn\(^3\) greatly simplified an interpretation of the results, since its mechanism is so constructed as to register automatically the slightest deviation of frequency within the magnitude of 1/100th of a semitone.

This experiment does not solve all the mysteries of intonation. Its object, however, was to investigate a musician's susceptibility to the influence of the frequencies from harmonic partials.

\(^3\)See glossary.
CHAPTER II

HISTORICAL DATA AND RELATED INVESTIGATIONS

Only the imagination can picture early man's crude attempts at tuning. It is almost a certainty that these attempts, and the resulting adjustments, were made entirely by trial and error. The placement of the perfect intervals may not have posed much of a problem, while some of the other intervals may have been only a little more difficult, but here the venture became a conundrum; this becomes apparent when studying some of the scales which have been developed in various parts of the world. Since there is no general agreement among scholars on these attempts of early man, our inquiry must be confined to that of which written records exist and to our own investigations.

Aristoxenus was one of the first to raise a question in his dispute with the disciples of Pythagoras. He asked, "Are the cogitations of theorists as important as the observation of musicians themselves?"\(^1\) Ottokar Cadek\(^2\) observed that books and articles left the ear to go its own way. He soon discovered that a keen ear was not sufficient to insure dependable intonation unless the ear sensed that for which it should listen. His experience appears to validate the question of Aristoxenus.

The ancient Greek tetrachords are the basis for our modern scales. They were used until the sixteenth century, to be superseded by the


"meantone" system, which, in turn, prevailed for several centuries.

The scale of "just" intonation appeared again early in the Christian era, when Didymus and Ptolemy presented monochords that contained perfect fifths and pure thirds, only to remain dormant during the Middle Ages. Even after attaining recognition, near the beginning of the modern era, the system drew attention only occasionally and haphazardly.

Knowledge of discrepancies in scales has existed since ancient times. Regardless of how thoroughly a system has been exploited, all methods have produced their own particular problems, from which there seems to be no escape; 117 different tones must be utilized to produce the perfect system, but the use of 53 tones would make it fairly tolerant.\(^3\)

The keyboard instruments partially solved some of these problems through equal temperament. The tolerance of the human ear, as well as other factors, contributes to the success of this system. Since the tone of the piano diminishes rapidly, the imperfections of equal temperament are less marked than on any instrument producing sustained tones.\(^4\)

The introduction of equal temperament into musical practice was not one of sudden growth. Grammateus already in 1518 suggested dividing the octave into ten semitones and two of slightly smaller size. It was clearly expounded by Mersenne in 1635. Neither was it limited to

---


Europeans alone; Tsai-yu, a Chinese prince, divided the octave with extreme precision in 1596.

As previously stated, equal temperament does not solve the problem in all music. Cadek's experience led him to propose the following studies: (1) pitch recognition, (2) threshold of pitch discrimination, (3) relative consonance or fusion of simultaneously produced tones, and (4) actual objective analyses of violin intonation as performed by artists.

Even though a keen ear is a primary requisite of musical talent, research has proven that there is no gifted group of pitch discriminators possessing perfect absolute pitch. Schoen wrote, "Experimental results show that any one of two tones on an interval can be changed up to a certain point without changing the interval." Threshold varies with pitch level as well as with the individual. A quarter-tone is the smallest unit between two tones that can regularly be perceived as an interval and not merely as the fluctuation of the same tone. Thus, the musical ear, which appears to be surprisingly tolerant, adjusts easily to these small discrepancies.

Any form of intonation involves some tempering, for the pull of a resolution or a modulation may exceed even the exaggerations of Pythagoras. Perfect intervals should remain perfect, but major thirds and sixths may be tempered without serious offense to the ear. The minor thirds, sixths, and sevenths become flatted and major intervals sharpened.

---

5 Cadek, op. cit.
Sharps become higher and flats lower by the "comma" of Pythagoras (23.5 cents).

It is generally agreed that the natural scale should be preferred to the tempered scale on instruments which do not have a fixed pitch. Certain sequences or other factors demand the augmenting or contracting of an interval. Tests of performance of eleven artists on the violin show a marked deviation from both the natural and tempered scales. When unaccompanied, the placement of the actual pitch was closer to that of Pythagorean tuning; when accompanied, it came closer to that of just intonation. The deviation from the tempered scale was slightly less than that from the natural.

Stauffer considered timbre or tone color, caused by the varying strength of certain overtones, as important factors in confusing and misleading the ear in ensemble playing. The ear often judges tones to be of a different pitch when two tones are sounded which have fundamentals of exactly the same frequency but different overtones. In general, a tone richer in overtones will give the impression of a higher pitch than a relatively pure tone.

The author has noticed a similar subjective reaction being created in rooms possessing different absorption levels of higher and lower frequencies.

A rather special effect occurs when the frequencies generated

---


from two played notes are related in such a way as to put one note exactly in unison with a note from the other sound's overtone series. In slight differences of pitch a beat frequency appears, always equal to the exact difference in the frequencies of the sound sources. Rapid beat frequencies impart a "tinny" sound to a tone because they cannot be followed individually by ear.  

Cadek\(^{10}\) suggested investigations on: (1) interval discrimination for complex tones of stringed instruments, (2) an analyses of double-stops as played by a number of concert artists, (3) the relative resonances as played in just, equal, and Pythagorean double-stops, and (4) the measurement of the duration of a note necessary to effect a pitch change.

The experiments so suggested would involve subjective tones, commonly referred to as "combination" tones. These phenomena occur when two loud tones are sounded simultaneously; the amplitude need not be so intense for the higher frequencies. The frequency of a combination tone is the difference (differential or difference tone) or the sum (summation tone) of the frequencies of the two primary tones or of their multiples. The summation and difference frequencies resulting when two or more notes are sounded simultaneously are indicated by faint grayish patterns on the windows of the Stroboconn.\(^{11}\)

---


\(^{10}\)Cadek, *op. cit.*, p. 39.

\(^{11}\)C. G. Conn Ltd., *Stroboconn Operation and Service Manual* (Elkhart: Conn Band Instrument Division of C. G. Conn Ltd., 1947), p. 3.
Whether combination tones were physiological or psychological in nature has long been a controversial topic. Experimental evidence gained through the efforts of Newman, Stevens, and Davis\textsuperscript{12} would appear to remove all doubt as to the nature of combination tones. The existence of cochlear potential was experimentally determined by using the hearing mechanism of a cat as a microphone. Electric contacts made to the animal were connected to the input of an amplifier. The amplifier's output was then connected to a loudspeaker. Sound impulses reaching the cat's ear were reproduced in a perfectly intelligible manner.

Two pure tones, one of 700 cycles per second (CPS) and one of 1200 CPS were caused to impinge simultaneously on the ears of the cat with varying degrees of intensity. An electric wave analyzer, connected to the output of the amplifier of cochlear potentials, showed the existence of four subjective tones in addition to the two pure tones. The subjective tones consisted of a difference tone of 500 CPS, a summation tone of 1900 CPS, and two other combination tones of 1700 CPS and 3100 CPS. The latter two show the existence of what is termed "aural" harmonics.

CHAPTER III

AN EVALUATION OF INTERVALLIC COORDINATING SYSTEMS

An examination of intervallic coordinating systems is a pre-requisite to an investigation into the problems of intonation. No attempt has been made to analyze all systems of the past. A few systems, however, have either left their mark in history, or are still receiving further investigation. These systems, none of them perfect, do possess justifiable reasons for existence. This, then, is the purpose of the evaluation.

I. THE PYTHAGOREAN SYSTEM

The Pythagorean system of tuning has had a profound influence upon the ancient and the modern world. It is based upon the octave and the fifth, the first two intervals of the harmonic series. Using the ratio of 2:1 for the octave and 3:2 for the fifth, it is possible to tune all the notes of the diatonic scale in a succession of fifths and octaves, or, for that matter, all the notes of the chromatic scale. Thus a simple but rigid mathematical principle underlies the Pythagorean tuning. When the formula is extended to B#, a discrepancy occurs which is known as the "comma" of Pythagoras; if it is extended to more than twelve notes in the octave, a sharpened note, as B#, is higher than the synonymous flatted note, Ab, by the same degree. In this tuning the major thirds are a syntonic comma sharper than the pure thirds of just intonation.
Tuning by fifths and octaves, Pythagorean tuning may be represented by the following ratios:

\[
\frac{4}{3} \times \frac{3}{4} \times \frac{3}{2} \times \frac{3}{4} \times \frac{3}{2} \times \frac{3}{4} \times \frac{3}{2} = \frac{243}{128}
\]

(F) (C) (G) (D) (A) (E) (B)

Scalewise, Pythagorean tuning may be represented by these ratios:

\[
1, \frac{9}{8}, \frac{81}{64}, \frac{4}{3}, \frac{3}{2}, \frac{27}{16}, \frac{243}{128}, \text{ and } 2.
\]

The result of tuning a keyboard instrument by the above system is not entirely acceptable harmonically. While the fifths are perfect, the major third becomes too harsh and strident. Historically, this necessitated a sort of unconscious temperament. More consciously, these harsh Pythagorean thirds may have been improved by a slight modification of one note or another. Undoubtedly, this was being done, for we find that Gafurius, at the end of the fifteenth century, mentioned organists who asserted that fifths undergo a small diminution called temperament.¹

Two phenomena could have contributed to this avoidance of harshness caused from disagreeable intervallic relationship—harmonic partials and combination tones.

Although not stated as such, two factors mentioned by Hindemith may have been receptive to the birth and acceptance of the Pythagorean system: (1) Primitive man, in singing and playing his primordial music, applied rhythms and melodic intervals, but had no feeling for harmony. It took thousands of years before harmony as a consciously perceived

part of musical construction could be introduced.\textsuperscript{2} (2) The degree of difficulty in singing an interval is in direct proportion to its numerical ratio, in the sense that the more easily an interval is produced, the smaller are the numbers in the ratio that measure it. Thus the octave, as the easiest interval, is represented by the two lowest numbers possible, 1 and 2. The fifth, demanding a slightly greater effort for its production is expressed by 2 and 3.\textsuperscript{3}

The introduction of gymel and fauxbourdon set the stage for the evolution of harmony. With the addition of the third in the vertical structure of music, man’s ear could no longer tolerate the sharp and harsh sound of the Pythagorean third. Necessity nourished his incentive to explore a different approach, one which was susceptible to the pure third. This by no means signed a death warrant to the Pythagorean system, for many tests still suggest that the system is, at least, partly alive.

II. JUST INTONATION

No externally enforced system of temperament can influence our understanding and production of intervals in their natural purity. If we did not base our singing on pure intervals, how could we ever believe in the accurate production of the unnatural, distorted, tempered intervals? With the use of commas and tolerances, both correctness and freedom are guaranteed. Those having experience in string quartet playing can


\textsuperscript{3}\textit{Ibid.}, p. 79.
substantiate this hypothesis, where there is, as every expert knows, no such thing as a rigid adherence to pitch.\textsuperscript{4}

The greatest musical theorist of antiquity was Aristoxenus of Tarentum (4th century B.C.), who went beyond the mathematical-numerical speculations of those of the Pythagorean adherents to investigate the problems of the perception of sound by ear.\textsuperscript{5} A modern Aristoxenus may yet leave his mark on the history of music.

The following formula designates the proper ratios for the diatonic scale in just intonation:

1, $9/8$, $5/4$, $4/3$, $3/2$, $5/3$, $15/8$, and 2

C D E F G A B C

The fifth between the second and sixth steps is contracted to the point of being unfit for harmony. Nevertheless, it does possess the smoothest thirds and sixths. Since 117 pitches, 53 being a fair approximation, are required to play in all keys of this theoretically ideal system, an adaptation of this tuning to keyboard or fretted instruments is out of the question.\textsuperscript{6}

Modulation is impossible without the addition of many extra pitches. The first three tones of the G-major scale, g-a-b, have different intervals from those of the C-major scale, c-d-e. Two different

\textsuperscript{4}Ibid., pp. 88-89.

\textsuperscript{5}M. J. Politis, "Greek Music," The Encyclopaedia Americana (1953 ed.), XIII, p. 419.

tones a would be necessary, one for the sixth of g, the other for the second of g.\(^7\)

Owing to the pure thirds, just intonation has been considered ideal for "a cappella" music in the style of Palestrina. If strictly adhered to, the principle of pure triads can be maintained only at the expense of a constant lowering in pitch. If the succession of chords notated in the following example were sung in pure thirds, the loss in frequency created by the intervals between the black notes would produce a final g a syntonic comma \((81/80)\) lower than the initial g.\(^8\)

\[
1 \times \frac{5}{3} \times \frac{4}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{80}{81}
\]

Many theorists are of the opinion that just intonation is found only in occasional chords (initial, final triads) of a cappella music. Hindemith, as previously stated, claimed that tolerances would aid in stabilizing such a situation.

The third of just intonation, amazing as it must sound, was already known to Aristoxenus (c. 354-300 B.C.).\(^9\) Gymel and fauxbourdon may have brought it again to light; but, with the rebirth of this ideal system, new problems arose which were incapable of a solution by a mere scale in just, Pythagorean, or any other known system of the time.


\(\text{\footnotesize \textsuperscript{8}}\)Ibid.

\(\text{\footnotesize \textsuperscript{9}}\)Ibid., p. 744.
III. THE MEAN-TONE SYSTEM

The mean-tone system, which was in use about 1500 A.D., was so constructed as to cope with the syntonic comma. Fifths were contracted to one-fourth of a syntonic comma (about 20 cents). Four such fifths (C−G−D−A−E) led to a pure third (C−E) when brought within the octave. This created a more satisfactory triad than that of the equal-tempered scale. However, the continuation of such fifths eventually leads to so great a discrepancy that the keys are limited from two sharps to two flats. To avoid this dilemma, some organs existed in the 16th century with divided keys.

IV. THE OVERTONE SERIES

Tones from all musical instruments are actually composite sounds. A tone consists of the fundamental or main sound, plus numerous additional pure tones, called overtones. The terms, harmonics, partials, or harmonic partials, refer to the fundamental plus its overtones; hence, the second partial is the first overtone. The frequencies of overtones are exact multiples of the fundamental.

The harmonics are the cause of at least three important musical phenomena: (1) timbre, (2) the natural tones of wind instruments, and (3) the harmonics of the violin. Timbre is a term used when referring to the "color" of a tone, i.e., the difference between tones of the same pitch when sounded on various instruments. This difference is caused by the greater or lesser prominence of some of these partials over the others.
The pure sound of the flute is due to the fact that it lacks nearly all of the overtones except the first; the mellow tone of the clarinet is caused by the prominence of the odd-numbered partials, and the penetrating sound of the oboe is the result of the prominence of a great number of the partials.

According to recent investigations, the identifying partials of an instrument's tone lie within a fixed range of rather narrow limits. This "characteristic absolute range of partials" is called formant. Some enlightening investigations may be made in this area in the future.

The harmonic series with C as a fundamental appears in the following notation:

The fourth, fifth, and sixth partial form the "pure" triad of just intonation. The seventh partial is lower than the Eb of equal-temperament by 31 cents; the eleventh partial is almost midway between F and F#; the thirteenth partial is lower than the sixth tone of the scale in just intonation, one of the problem tones of that system; the fifth and the sixth partials are identical to third and fifth interval of just intonation, being 14 cents lower and 2 cents higher, respectively, than the same notation of equal-temperament.

---

10 Ibid., p. 748.
V. COMBINATION TONES

The human ear, when activated by sounds of various pitches, will register not only the separate pitches but also new sounds often having no existence outside the ear. These phenomena are referred to as subjective tones, difference tones, summation tones, combination tones, and aural harmonics.

Subjective tone. This term is employed when referring to the nature of combination tones which have no apparent entity except the aural sensation arising in the cochlea (inner ear) as it is activated by the frequencies of two or more pitches.

Difference tone. This type of subjective tone is the most easily recognized. Its presence is noted as a buzzing sound with a definite pitch, which is the difference in the frequencies of two or more primary notes.

Summation tone. A summation tone is very similar to a difference tone. While the difference tone is the difference in frequencies, a summation tone is the sum of the frequencies of two primary notes.

Aural harmonics. Kinsler and Frye\textsuperscript{11} tell us that, when a high intensity sound of say 200 CPS is introduced into the human hearing

mechanism, not only the 200 cycle note will be heard, but also the upper harmonics will be produced within the ear.

**Combination tone.** This is the commonly used term when referring to any or all of the foregoing phenomena.

The subjective tones were first discovered during the early part of the 18th century, when they were known as "Tartini's tones." There is some conjecture as to whether Tartini was or was not the first to become aware of their presence. His claim dates the discovery as taking place in 1714 at Padua, Italy.

The combination tones fall into a series similar to that of harmonic partials, usually, however, in a reverse procedure. This is especially true of the difference tone; the presence of which can be predetermined by classifying two tones into a definite harmonic series and subtracting the placement of the lower tone from the higher tone; the difference being the placement of that subjective tone in the related harmonic series. A slight "humoring" of the notes is sometimes necessary to bring the difference tone in line with the frequencies of the sounded tones.

When two tones corresponding to the 7th and 12th partials of C (b' flat and g") are sounded simultaneously, the resultant difference tone g', is heard. These two harmonics also correspond to the 3rd and 5th partials of e' flat. A slight lowering of the g" is required to make the e' flat sound in tune.

Humoring of this type is also necessary when playing tones to match harmonic partials. This is but one example of the many problems one may expect when working with combination tones or harmonic partials.
VI. EQUAL-TEMPERAMENT

The principle of equal temperament is the division of the octave into twelve equal parts. Octaves are the only acoustically correct intervals in this system. The advantages of this system solve the discrepancies, which creep into other methods of dividing the octave, in spite of having defects of its own.

Tones out of tune with the generators are sometimes rejected by the ear, since the brain has a tendency to hear through extraneous noises and to reject unwanted sounds. We are not always conscious of harmonics, which are more often than not interpreted as tonal color.

The calibrations of the stroboscope are based upon equal temperament. The instrument is commonly known by its trade name, Stroboconn, which is manufactured by C. G. Conn, Ltd., of Elkhart, Indiana. The octave is divided into 1200 equal parts, called cents; 100 cents being allotted to each semitone. The Stroboconn was used in this investigation to determine and compare pitch.

The production of a fifth, which is two cents smaller than the perfect fifth, is caused by equal temperament. By the time the circle of fifths has been completed, the Pythagorean comma (23.5 cents) has been absorbed. Major thirds are somewhat hard and harsh, but the ear quickly adjusts to tolerate them.

Summary. Attempts to rely only on any of the aforementioned coordinating systems sooner or later leads to frustration, because of the discrepancies which eventually make themselves manifest through commas.
and dieses. Yet, their influence cannot be ignored; experience and tests prove otherwise.

Keyboard instruments may surmount these inevitable, intricate perplexities through equal temperament, an adaptable system possessing faults of its own. Other instruments, however, cannot subject themselves to this rigidity.

Mere chance did not give birth to just intonation, the Picardy third, the peculiar sound of the Neapolitan sixth, or the "muddy" color of a third voiced too low in the bass. The accomplishments of Aristoxenus in discovering the scale of just intonation is, in itself, amazing; the Pythagorean scale did not suit his fancy, so he wished to conceive what the musicians heard. The only logical components accountable for these phenomena and discoveries are harmonic partials, with some help supplied by tones now classified as being subjective.

Even though fundamental C may produce a G-major scale in just intonation on the partials designated as numbers 24, 27, 30, 32, 36, 40, 45, and 48, the most dominating partials are those lying with the formant. These and the tolerances appear to be important factors of intonation.
CHAPTER IV

THE INVESTIGATION AND RESULTS

A previous pilot study was made by the investigator in the summer of 1961 at Montana State University. The experiment, even though limited and not too precise, strongly indicated a tendency toward a relationship between harmonic partials, generated from a selected fundamental, and certain "supplied tones" sounded on a wind instrument by a person, hereafter referred to as the "supplier." The supplied tones, in this case, consisted of the notes from the Bb-major scale, plus a minor seventh interval.

Sometime prior to the experiment a series of the fundamental, Bb, was recorded on a tape-recorder. During the playback, the supplied tones were sounded against the fundamental emitting from the recorder.

The results of the investigation showed that the suppliers tended to deviate from equal temperament toward the frequencies of the harmonic partials generating from the selected fundamental.

The reliability of the former investigation was hampered by:
(1) the duration of the fundamental and (2) the instability of the fundamental's frequency.

Later, during a discourse on the project in thesis class, one of the graduates suggested the use of an organ in supplying a fundamental in order to insure stability. Professor Gerald H. Doty, graduate advisor, thought that Dr. Jody Hall, acoustical engineer at C. G. Conn, Ltd., might be helpful in supplying some knowledge in this field. Since Dr. Hall's
background was in a somewhat different area of acoustical research, he recommended the name of Dr. Everett Gates, Eastman School of Music, for further reference.¹

During the course of a telephone conversation, Dr. Gates encouraged the investigator and reported that he knew of no research relating to the comparison of the frequency of a tone sounded against overtones generating from a selected fundamental on organ pipes. His letters were instrumental in the selection and delimitations of this investigation.

I. THE EQUIPMENT

**Stroboconn.** This device, as previously described, is capable of calibrating sounds to within 1/1200th of an octave or 1/100th of a semitone or less. The deviation of an overtone's frequency from the frequency of its fundamental, as related to equal temperament, is easily measured, even though the frequency of the fundamental may not be in agreement with the accepted standard, \( A = 440 \text{ CPS} \).

**Organ pipes.** This term refers to the tonal generators in a pipe organ, used in this paper when referring to the trumpet-reed stops of the pipe organ.

**Wind instruments.** The following band instruments were used as tonal generators: oboe, bassoon, 2 clarinets, 2 flutes, 3 cornets, and 1 trombone.

---

¹See Appendix B.
II. PROCEDURE

1. A continuous selected fundamental was sounded on an organ pipe.

2. The Stroboconn was synchronized to the frequency of the selected fundamental.

3. The reading of the fundamental's frequency was recorded; e.g., o plus two cents, minus three cents, or whatever the reading designated.

4. The notation for a supplied tone was selected (refer to Figure 1 on page 26 for the selected pitches). These notations correspond to the partials of the selected fundamental in the order and numeration given from left to right. The supplied tones were furnished in that sequence, but were limited to pitches within the practical range of the supplier and instrument.

5. The Stroboconn was synchronized with the frequency of the selected partial.

6. The frequency of the selected partial was then recorded.

7. The supplier of the supplied tone was instructed to listen closely to the selected fundamental generated by the organ pipe.

8. The supplier was requested to pitch the supplied tone at the point most suitable to his ear.

9. The reading of the supplied tone's frequency was then observed and recorded.

10. The calibrations of the frequencies of the selected partial and the selected supplied tone were compared and differences, if any, recorded.
| Partial | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
|---------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Flute I | o | o | * | o | o | o | o | * | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Flute II| o | o | * | o | o | o | o | * | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Oboe    | o | o | * | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Clarinet I| o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Clarinet II| o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Bassoon | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Trumpet I| o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |
| Trombone| o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o | o |

- Frequency of partial easily matched.
- Difficult or nothing definite
- Interesting phenomena

FIGURE 1

CORRELATION BETWEEN SUPPLIED TONES
AND HARMONIC PARTIALS OF
FUNDAMENTAL C
FIGURE 2

PITCH NAMES
III. RESULTS

The tabulation of results proved to be a much simpler feat than first anticipated. One student attempted to pitch the supplied tones as he thought the investigator desired them. This supplier actually pitched the supplied tones so far from the frequency of the partial that it was necessary to call his attention to the fact that he must pitch the supplied tone at the point most suitable to his ear.

Harmonic partials 15, 17, 19, 21, 22, 23, 25, and 26 were purposely omitted as being too involved for this investigation. Close attention was paid to warming up and tuning the tonal generators as a precaution to insure stability and remove a hazard arising from the ambient temperature.\(^2\)

The supplied tones marked o gave little if any trouble. Even though the tone may have been flat or sharp on the attack, the supplier immediately channeled the pitch in line with the frequency of the partial generating from the selected fundamental.

Attention must be called to the fact that the supplied tones marked o were not necessarily in pitch with equal temperament. The 5th, 10th, and 20th partials are approximately 14 cents lower than equal temperament; the 7th and 14th partials about 28 cents lower, while the 13th partial appears to be in excess of 35 cents lower than equal temperament and the 11th partial is actually closer to the semitone below than to its designated notation in equal temperament.

\(^2\)See Appendix, pp. 47 and 48.
Therefore, tones marked o in Table I refer to an agreement with that partial's frequency and not with the frequency of equal temperament.

Since the various fundamentals and their overtones produced similar effects in relation to the supplied tones, only one table of results was included in this thesis—the table on fundamental C.

Further comment, especially on tones recorded as o or i, is herewith given under the heading of the tonal generators involved.

**Flutes**

The two flautists were experienced musicians for high school students, both having previously received honors and recognition for their musicianship. Most of the tones were brought in line with little or no effort. The 7th, 11th, 13th, and 14th partials, however, either presented difficulties or produced interesting results.

**The seventh partial.** On the first test, Flute I matched the frequency of the 7th partial, b'' flat, or about 31 cents flat. During other attempts, both flutes pitched this tone at about 14 cents flat, corresponding to the a''' of just intonation. Several returns to this partial produced the same results.

**The eleventh partial.** The eleventh partial lies close to the mid-point between the semitones on either side. Both suppliers were unable to settle for a definite pitch for the supplied tone. This partial proved to be the most troublesome of all the overtones.

**The thirteenth partial.** Both suppliers were uneasy about the effects of this partial. The partial proved to be lower than they had
anticipated. When sounding a tone at a', an octave lower than the partial, they channeled the pitch in line with the 13th partial's frequency; but when they sounded a tone at a", the same notation as the partial, they pitched it in line with the 27th partial's frequency, which is about the same as that of the Pythagorean scale. The thirteenth partial is lower by more than 50 cents from the equal temperament as measured on the Stroboconn.

The fourteenth partial. After the experience with the seventh partial, the fourteenth showed no surprising results. Both suppliers pitched the supplied tone 14 cents lower than equal temperament, which again corresponds to the a'♯ of just intonation.

Oboe

The oboist was a very inexperienced musician, having played her instrument only eight months. Nevertheless, she has proven herself to be an apt student.

The seventh partial. The supplier started sharp, but quickly lowered the pitch of the supplied tone to the frequency of the seventh partial. Further trials resulted in the choice of a pitch about five cents higher than this partial's frequency. The supplier thought this pitch was more pleasing to her ear. At the present time this result is unexplainable beyond the reason already given.

The eleventh partial. The supplier failed to locate a suitable pitch corresponding to the eleventh partial.
Clarinet

Both students were fine musicians. Clarinet I would be rated highly superior; his instrument was a very fine artist model clarinet of best quality. Clarinet II played on a well-known second line instrument recommended for school use.

The fourth partial. The failure to match the frequency of the fourth partial by Clarinet II must be blamed on the instrument which did not possess very accurate twelfths. The make of instrument owned by Clarinet I has a reputation for being accurate in this respect.

The fifth partial. Both suppliers were unwilling to sacrifice tone for pitch. The supplied tone, though uncomfortable, was closer to equal temperament than to the frequency of the fifth partial.

The seventh partial. Both suppliers on the clarinet were unable to cope with the interference created by the seventh partial. This overtone is lower than its notated counterpart in equal temperament by about 3½ cents. With such great difference in frequency, good tonal color would be lost through too little embouchre support.

The tenth partial. The tenth partial, which measures 13.7 cents flat on the Stroboconn, produced a reaction very similar to that of the fifth partial, which is one octave lower. Both suppliers refused to channel the supplied note in line with the frequency of this partial.

The eleventh partial. Even though there did not appear to be much interference from the frequency of the eleventh partial, neither supplier could locate a pitch comfortable to his ear.
The thirteenth and fourteenth partials. Both the thirteenth and fourteenth partials of fundamental C lie in that part of the clarinet's range which is more apt to be sharp than flat. Consequently, both supplied tones were measured as being sharp.

Bassoon

The bassoonist was a student of about one and one-half years experience. She does, however, possess much natural talent in vocal as well as instrumental music. In the past, she has often had a tendency to play sharp, so her ability to channel the pitch of the supplied tone in line with the frequencies of the partials was somewhat surprising even though she did possess native talent.

Trumpet I

This supplier was the director of the band in the Sidney Junior High School, Sidney, Montana. Her name is Miss Carol Stafney, a first year teacher having a very fine intellect and possessing a remarkable degree of talent. Her natural ability enabled her instinctively to match the frequencies of the partials. These intervals proved to be exceptionally surprising to her, even the pure third.

The eleventh partial. The eleventh partial appears to be extremely difficult, if not impossible, to match in frequency. This supplier continued to lower the pitch of the supplied tone until the interval reached that of a perfect fourth.

The thirteenth partial. The supplier kept lowering the pitch of the supplied tone until it matched the thirteenth partial's frequency.
A tone, one octave lower than this frequency, was next attempted; here, the pitch was again brought in line with the partial's frequency. Some difficulty was connected with this overtone.

**Trombone**

The trombonist was a junior high school student in grade nine. His abilities are considerably above that of the average student of his age. This is the student who attempted to "doctor" the supplied tones.

**The seventh partial.** The notation corresponding to the seventh partial's frequency was not tested because of possible lack of control in that range. Instead, the notation one octave lower was substituted. The regular placement of pitch caused the supplier to feel insecure. After several attempts he channeled the pitch into line with the partial's frequency.

**The thirteenth partial.** A substitution for the thirteenth partial's frequency was introduced two octaves lower than this partial's frequency. A sensation of extreme insecurity was immediately noticed. The supplier's reaction was similar to Trumpet I on the eleventh partial; after many attempts, an octave of that frequency was matched.

**Combinations**

Combinations were not originally planned for any part of this investigation. The effect of the seventh and thirteenth partial on pitch placement suggested a slight but not insignificant diversion from the project. Trumpet I, the bassoonist, and the oboist were again summoned.
for further testing, while two additional trumpeters were called in for this diversion. The trumpeters were now asked to supply three tones simultaneously with the fundamental; the three supplied tones being notes corresponding to the 5th, 6th, and 7th partial. Following this procedure, the oboist and bassoonist were requested to furnish the supplied tones corresponding to the frequencies of the 5th and 7th partial. After completing this part of the diversion, two trumpeters were asked to supply $f'$ and $a''$ against a fundamental $C$.

The seventh partial. A difference in pitch placement by Trumpet I had been anticipated. This failed to materialize; the pitch placement was the same as in the original testing. Curiosity had prompted the recall of the bassoonist and oboist; this was satisfied. The oboist now channeled the supplied note to the same degree of deviation below equal temperament as the bassoonist on the fifth partial, about 14 cents flat.

The thirteenth partial. Trumpet I had been expected to play the supplied tone, which was one octave below the thirteenth partial, higher in pitch than the previous test. The addition of $f'$ did not alter the pitch placement from that of the original testing. The initial testing of this supplied tone was a calibration of over one-third of a semitone lower than that of equal temperament.

Peculiarities

Certain peculiarities should be recorded in this report for those desiring further investigation. Three factors affecting aural perception were found to be: (1) direction faced, (2) distance and position of
supplier in relation to sound source, and (3) use of one ear.

**Direction faced.** The direction in which the supplier faced created a situation more or less susceptible to an awareness of harmonic partials.

**Distance and position.** The distance and position the supplier faced created a situation more or less susceptible to harmonic partials.

**Use of one ear.** The pitch of a tone falling on the ear that is directly in the path of a tone, sounds higher in that ear than in the other ear, which is then affected by some sort of a shadow effect. This can be proved by listening to a flute with the face pointing at a ninety degree angle away from the source of sound. Cover one ear at a time; first one and then the other. The ear pointing toward the sound source interprets the pitch as being higher than the ear which is away from the sound source.

**Summary**

The eleventh partial was the only partial of those tested which appeared to be impossible to match, while the seventh, thirteenth, and fourteenth partials produced interesting phenomena.

The calibration of the frequencies of the seventh and fourteenth partials agreed with the formula of Professor Mark Jakobson. The calibrations, approximately 969 cents above the frequency of the fourth

---

3See Appendix C, p. 46.
partial and its octave, were both about thirty-one cents lower than 
equal temperament. These calibrations do not agree with the value of 
the seventh partial as stated by Apel,\(^4\) who lists its measurement at 972 
cents above the fourth partial. The frequencies of the various partials, 
according to the formula of Jakobson, fall on a definite line of a 
plotted curve. Apel's value of 972 cents is a slight deviation from 
this curve.

A disagreement also exists concerning the value of the eleventh 
partial. Apel lists this partial as being closer to the notation of a 
semitone below the notation of the partial (equal temperament). Actually, 
using the Jakobson formula for computation, the eleventh partial is about 
one and one-third cents above the mid-point between the notation of the 
partial and the notation a semitone lower, e.g., F♯-F natural.

The extreme degree of deviation of the frequency of the thirteenth 
partial below the equivalent in equal temperament is substantiated by 
Jakobson's formula. This evidently accounts for the insecurity of the 
trombonist with the frequency of the thirteenth partial.

With the exception of the third partial and its multiples, the 
clarinet students encountered difficulty with the degree of deviation of 
a partial from equal temperament, or with the degree of inaccuracy of 
their own instruments.

The direction in which the supplier faced in relation to the 
sound source and his position in the performance chamber affected his 
awareness of the harmonic partials generating from the fundamental.

\(^4\)Willi Apel (ed.), *Harvard Dictionary of Music* (seventh edition; 
CHAPTER V

CONCLUSIONS

The foregoing investigation has proven, according to the circumstances as previously given, that a definite relationship does exist between harmonic partials emanating from a selected fundamental and corresponding notated tones supplied by various individuals using wind instruments as tonal generators. An individual, no doubt, reacts to this relationship in proportion to the existing acoustics of the performance chamber and the strength and duration of the fundamental. Undoubtedly, the formant, not included in this investigation, is an additional factor in regulating the intensity of relationship between certain harmonics and corresponding supplied tones.

Further investigation may show that discrepancies between frequencies of simultaneously sounded tones are tolerated when these tones are of such short duration as to avoid the presence of a beat. Many writers, including Hindemith, have surmised that in the performance of music every number has an individual temperament with tolerances acceptable to the demands of the progressions, which may be either chordal or intervallic in nature.

The partials encourage the matching of their frequencies through the deceleration and avoidance of the interference of beats. Musicians interpret this phenomenon as "playing in tune."
Suggestions for further investigation:

1. Calibration of the frequencies of partials generating from tones produced on the wind and string instruments of the band and orchestra.

2. Calibration of the seventh partial of a dominant seventh chord in various positions and combinations.

3. Calibration of the thirteenth partial and its lower octave as it appears in a six-four chord.

4. Calibration of pitch on various instruments in relation to angle, distance, and position as a selected fundamental is sounded.

5. Calibration and comparison of the pitches of a supplied tone to a fundamental with and without the presence of an intervening body between the sound sources.
BIBLIOGRAPHY

A. BOOKS


B. DICTIONARIES


C. ENCYCLOPEDIA

D. PERIODICALS


E. UNPUBLISHED MATERIALS


F. MANUAL

APPENDIX A

Elkhart, Indiana
July 31, 1961

Mr. C. O. Strom
436 Keith Avenue
Missoula, Montana

Dear Mr. Strom:

I do not have a bibliography of any sort pertaining to your subject. Dr. Everett Gates, Eastman School of Music, University of Rochester, Rochester, New York, probably would be your best source of information.

Related to your study, of course, is the fact that an electronic organ is tuned "dead on" the tempered scale because the harmonics "line up" in integral ratios, where a grand piano is "stretched" somewhat in its tuning, and a spinet piano is stretched even more, because the harmonics of a string tone do not "line up."

It is very possible that the reason strings sometime push the pitch upwards in an orchestra (my apologies to Mr. Doty) may be due to the fact that the harmonics of the strings do not "line up" like those of the wind instruments.

Since the modes of vibration are what we are concerned with in wind instruments, not the harmonic series, you are quite right that Stauffer's work would not be of assistance.

The research you plan sounds very interesting — I'm sorry I can be of so little help. It sounds as if the first difficulty may be in exactly defining the problem.

Let me know how it comes out, and give my best regards to Mr. Doty.

Sincerely,

Jody C. Hall, Ph.D.
Chief Acoustical Engineer
C. G. CONN, LTD.
Mr. C. Orlan Strom  
Instrumental Supervisor  
Sidney Public Schools  
Sidney, Montana  

Dear Mr. Strom:

I have given a good deal of thought to the research project which you propose to undertake as part of your graduate work at Montana State University. There are at least three areas which you mention each of which would serve as a comprehensive area for study: (1) the effect of "subjective" tones on intonation, (2) the calibration of the frequency of harmonic partials from fundamentals sounded on various instruments, (3) the measurement of the frequency (not pitch, since this is the subjective counterpart of frequency, and cannot be adequately measured on the Stroboconn) of intervals played on instruments against a standard sounded on organ pipes.

Each of these studies would have certain inherent complexities that would be difficult to surmount which is the reason for Dr. Hall's statement regarding the exact definition of the problem. In my opinion, the easiest of the three problems for you to tackle would be the measurement of the frequency of the harmonic (or inharmonic) partials generated from selected fundamentals generated by various instruments.

The principal problem in studies of this nature, as I am sure you are aware, is the careful control of all the factors involved, such as ambient temperature, intensity, acoustical coupling of the source of sound with the room in which measurements are taken, etc. When you do not take all these factors into consideration and control them carefully you greatly limit the value of the study. You must consider this point most carefully and decide on your specific area of research with the limitations of your equipment and facilities kept constantly in mind.

There have been quite a number of studies made at various schools utilizing the Stroboconn. In your bibliography you do not cite any of these so I assume that you have not yet investigated them. Most of these theses are available on microfilm or on interlibrary loan. You will find many of them listed in W. S. Larson's "Bibliography of Research Studies in Music Education, 1932-1948" and the same for the years 1949-1956, both published by the MENC. Careful study of previous research projects will help you avoid some of the pitfalls which hampered other investigators.
A Master's thesis by Rolfe Newton written about 1958 at the University of Wisconsin dealt with some of the problems of subjective tones, relating these to the harmonic series, and discussing some of the implications of the phenomena might be of some interest to you. I am sure you could obtain this easily. However, as this is such a complex subject, my advice would be to save this for your doctoral thesis, as a specific area for research.

The Journal of Research in Music Education and the Journal of the Acoustical Society of America are both fertile periodicals for investigation and you must be sure to check through them. Wm. B. White’s article in the JASA on the inharmonicity of piano strings might be a starting point for your research into investigative methods in case you decide on going into the frequency of harmonics from instrumental generators. One of my students did a project limited to the calibration of the harmonics produced from string bass strings which was quite revealing. He could easily have developed this into a full-fledged thesis had this been on (sic) of the requirements of his course. This will give you some idea of the delimitation which is desirable.

I am happy that you have developed your interests in the directions you have mentioned in your letter. I have encouraged many of my students to think along these lines and try to develop their interests in a class which I teach here, "Science and Music Education." In fact, Prentice-Hall have approached me to write a text on "Science and Music Education" which will attempt to tie down some of the knowledge gleaned from research to pedagogical procedures. The encouragement of knowing that many others realize the great advantages of exact knowledge applied to music teaching and how this can increase our teaching efficiency is a factor that will definitely affect my decision regarding the book. The more music teachers we have who are informed and interested the faster we will move forward.

I hope you will go ahead with your plans for you have considered areas that are in need of investigation. I'm sorry that I cannot be of much service to you, but I hope this information will help a little. I regret that my delay has inconvenienced you, but your questions posed many problems that required careful consideration and we have been having a very hectic year because of the absence of Dr. Hanson who has been abroad with the Philharmonia.

Cordially yours,

Everett Gates, Assoc. Chairman
Music Education Department
EASTMAN SCHOOL OF MUSIC
APPENDIX C

TABLE I
CONVERSION OF PARTIALS TO CENTS*

<table>
<thead>
<tr>
<th>Partials</th>
<th>Cents</th>
<th>Partials</th>
<th>Cents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0000.00</td>
<td>9</td>
<td>3803.91</td>
</tr>
<tr>
<td>2</td>
<td>1200.00</td>
<td>10</td>
<td>3986.31</td>
</tr>
<tr>
<td>3</td>
<td>1901.95</td>
<td>11</td>
<td>4151.32</td>
</tr>
<tr>
<td>4</td>
<td>2400.00</td>
<td>12</td>
<td>4301.95</td>
</tr>
<tr>
<td>5</td>
<td>2786.31</td>
<td>13</td>
<td>4440.53</td>
</tr>
<tr>
<td>6</td>
<td>3101.95</td>
<td>14</td>
<td>4568.83</td>
</tr>
<tr>
<td>7</td>
<td>3368.83</td>
<td>15</td>
<td>4688.16</td>
</tr>
<tr>
<td>8</td>
<td>3600.00</td>
<td>16</td>
<td>4800.00</td>
</tr>
</tbody>
</table>

\[ X = 2^{Y-1} = \frac{2^Y}{2}; \quad 2X = 2^Y \]

\[ \log 2X = Y \log 2; \quad Y = \frac{\log (2X)}{\log 2} \]

To convert to cents: subtract one and multiply by twelve hundred.

---

*This conversion was based upon the method formulated by Professor Mark Jakobson, Physics Department, Montana State University.
### APPENDIX D

#### TABLE II

**STATISTICS ON DEPENDENCE OF TUNING ON AMBIENT TEMPERATURE**

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Cents per 10 degrees</th>
<th>M*</th>
<th>N**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flute</td>
<td>7.3</td>
<td>48</td>
<td>9</td>
</tr>
<tr>
<td>Oboe</td>
<td>5.6</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>Bassoon</td>
<td>4.3</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Soprano Clarinet</td>
<td>4.4</td>
<td>153</td>
<td>23</td>
</tr>
<tr>
<td>Alto Clarinet</td>
<td>6.5</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>Bass Clarinet</td>
<td>8.6</td>
<td>24</td>
<td>4</td>
</tr>
<tr>
<td>Alto Saxophone</td>
<td>7.0</td>
<td>27</td>
<td>4</td>
</tr>
<tr>
<td>Tenor Saxophone</td>
<td>10.3</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>Baritone Saxophone</td>
<td>10.5</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Cornet</td>
<td>6.4</td>
<td>246</td>
<td>39</td>
</tr>
<tr>
<td>French Horn</td>
<td>8.8</td>
<td>105</td>
<td>16</td>
</tr>
<tr>
<td>Trombone</td>
<td>7.4</td>
<td>48</td>
<td>8</td>
</tr>
<tr>
<td>Euphonium</td>
<td>8.8</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>Eb Sousaphone</td>
<td>13.0</td>
<td>159</td>
<td>20</td>
</tr>
<tr>
<td>BBb Sousaphone</td>
<td>24.5</td>
<td>237</td>
<td>32</td>
</tr>
<tr>
<td>Vibraharp</td>
<td>-2.2</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>Xylophone</td>
<td>-5.4</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>Marimba</td>
<td>-5.2</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

*M - total number of observations.

**N** - number of different instruments tested.

---

APPENDIX E

TABLE III
MEAN "WARM-UP" CHANGE IN CENTS AT VARIOUS AMBIENT TEMPERATURES*

<table>
<thead>
<tr>
<th>Instrument</th>
<th>60°F</th>
<th>70°F</th>
<th>80°F</th>
<th>90°F</th>
<th>100°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flute</td>
<td>32</td>
<td>14</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Bassoon</td>
<td>20</td>
<td>14</td>
<td>13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Soprano Clarinet</td>
<td>14</td>
<td>10</td>
<td>6</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Alto Clarinet</td>
<td>15</td>
<td>10</td>
<td>7</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Bass Clarinet</td>
<td>20</td>
<td>12</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alto Saxophone</td>
<td>11</td>
<td>9</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baritone Saxophone</td>
<td>20</td>
<td>12</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cornet</td>
<td>20</td>
<td>16</td>
<td>8</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>French Horn</td>
<td>14</td>
<td>12</td>
<td>5</td>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>Trombone</td>
<td>21</td>
<td>6</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Euphonium</td>
<td>12</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eb Sousaphone</td>
<td>12</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>BBb Sousaphone</td>
<td>10</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>-2</td>
</tr>
</tbody>
</table>

### COMPARISON OF THE PYTHAGOREAN, JUST, MEAN-TONE, & EQUALLY TEMPERED SCALES

#### Table: Pythagorean, Just, Mean-Tone, & Equally Tempered Scales

<table>
<thead>
<tr>
<th>Scale</th>
<th>Pythagorean</th>
<th>Just (A-Tone)</th>
<th>Mean-Tone</th>
<th>Equally Tempered</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOTES</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>RATIO</td>
<td>234/229</td>
<td>235/234</td>
<td>234/233</td>
<td>C</td>
</tr>
<tr>
<td>CENT DEVIATION</td>
<td>5.6 cents</td>
<td>6.0 cents</td>
<td>5.4 cents</td>
<td>4.9 cents</td>
</tr>
<tr>
<td>CENT DEVIATION</td>
<td>24 cents</td>
<td>28 cents</td>
<td>23 cents</td>
<td>20 cents</td>
</tr>
<tr>
<td>CENT DEVIATION</td>
<td>2 cents</td>
<td>2 cents</td>
<td>1 cents</td>
<td>0 cents</td>
</tr>
<tr>
<td>SYMPHONY CENTER</td>
<td>104 cents</td>
<td>108 cents</td>
<td>105 cents</td>
<td>100 cents</td>
</tr>
</tbody>
</table>

#### Diagram: Comparison of Scales

- **Pythagorean Scale**
- **Just (A-Tone) Scale**
- **Mean-Tone Scale**
- **Equally Tempered Scale**

**Pitch Standard:** A' of Equally Tempered Scale = 440 Hz/sec; C' = 264.62 Hz/sec

*EVERETT GATES 6-10-48*