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PROBABLE SMOKE COLUMN HEIGHTS FROM SLASH FIRES

by

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B.S. University of Tennessee, 1962

Presented in partial fulfillment of the requirements for the degree of

Master of Science in Forestry

University of Montana

1970

Approved by:


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I am indebted to the Intermountain Forest and Range Experiment Station for allowing me to superimpose this study on the prescribed fire study and for the use of data which would have been otherwise unavailable.

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Chapter 1

INTRODUCTION

The backlog of logging slash from harvest cutting in the Northwest is increasing at a rate which is not conducive to fire safety or good management. Extensive fields of dry slash present an obvious fire hazard. If slash is not properly treated, it becomes a more or less permanent feature of the site, making access difficult and altering the pattern of regeneration.

Fire control is nearly impossible in heavy slash, and replanting is difficult. If planting is accomplished without slash treatment, there is the risk that a fire could wipe out the young trees if the slash were accidentally ignited. Logging debris is esthetically displeasing in an age when there is strong public emphasis upon keeping the forests beautiful as well as productive. Slash needs to be treated to make way for browse species for game animals and to remove impediments to the animals' movements. Prescribed fire is widely employed as an economical, effective means of reducing slash on clearcuts to acceptable levels. The growing accumulation of untreated slash dictates the need for increased use of fire in the future to keep fire hazards to a minimum and to return logged sites to production as quickly and cheaply as possible.

There is an increasing and justified concern over the effect of prescribed burning on air quality. Burning of logging debris has heretofore been performed during a short period each autumn after the

fire season and occasionally for a short time in spring. This pattern of burning, while safe from a control standpoint, produces large quantities of smoke during short periods of the year when the atmosphere is least capable of dispersing smoke.¹ The result has often been a locally overburdened atmosphere where visibility is impeded and the quality of the environment is degraded.

It is almost inevitable that an accidental wildfire will eventually consume slash that is not burned intentionally. Therefore, if we have as a stated goal the minimization of air pollution, it is only logical to pick the best times and conditions for burning which assures the least impact on air quality.

Given that logging debris is to be burned, it should be the concern of land managers to create as little air pollution as possible from planned fires. Herein lies the need for a method of predicting, on the basis of preburn measurements, the probable behavior of the smoke plume. Such a prediction would be a very useful tool in air pollution control.

¹Howard E. Graham and Owen P. Cramer. Meteorological management of slash burning operations (Graham); Atmosphere--slash smoke interrelations (Cramer). (Joint paper presented at the AMS-SAF Nat. Conf. on Fire and Forest Meteorol., Salt Lake City, Utah, March 14, 1968.)

THE PROBLEM

Prescribed fire is widely employed as a silvicultural tool to consume waste materials and to prepare logged sites for reforestation. The use of fire for disposal of slash has increased in Region 1 of the USDA Forest Service by nearly tenfold in the last 15 years, and there is a need for accelerated use in the future. The goal is to accomplish this slash treatment and at the same time to avoid polluting the air.

It is possible to separate the problem of characterizing smoke dispersal into two smaller categories, thanks to previous work in the field of air pollution meteorology. Because there has been much interest in the dispersion and diffusion patterns of smoke from industrial stacks, much of the present knowledge about the movement of smoke assumes the source of smoke to be above the surface of the earth as is the case with smoke stacks (Lowry and Boubel 1967). If it were possible to predict the elevation where the axis of a fire column becomes horizontal, the column could be looked upon as a continuous point source similar to a smoke stack. Much is known and can be said about the movement, advection, and diffusion of the smoke from this point in time and space.

According to Graham and Cramer² smoke is composed of particles that are in the aerosol size class (0.1 to 1.0 micron in diameter). Byram and Jemison (1948) reported smoke particles with a mean diameter of .25 microns. Graham and Cramer noted that 1.0-micron particles will fall

²ibid.

very slowly (3.5 feet per hour in still air) and so will essentially remain at whatever level is reached at the top of the column. That is, the particles will remain at the potential temperature where column gases become equal to the ambient air temperature. From that point in space, further smoke movement will be dependent on the patterns of isothermal surfaces, wind conditions, and lapse rates existing at the time. One major problem is that of predicting the height a smoke column will attain before its axis becomes horizontal. This study undertakes to solve the problem of describing the probable height of a slash fire smoke column.

OBJECTIVES

The objective of this study is to substantiate an existing theory or develop a new model for calculating the probable altitude a slash fire smoke column will attain. To be useful, the model must employ parameters or variables which are routinely measured or are readily available to forest land managers. Although not a stipulation for success of the study, it would be of additional usefulness if the model were one which would lend itself to fairly easy calculation in the field.

Chapter 2

LITERATURE REVIEW

There is no dearth of models to describe plume rise above fires. The models tend to fall into three general categories or approaches.

Physicists and engineers have added progressively to the body of knowledge called "plume rise theory" by employing the principles of conservation of mass, momentum, and energy. As many influencing variables as possible are accounted for while retaining sufficient simplicity to allow a solution of the resulting differential equations.

Such an approach is the one taken by Murgai and Emmons (1960). In this case, the authors attempted to take into account the effect of an arbitrary lapse rate by considering each piecewise-constant leg of the atmospheric profile as a separate problem. The final conditions of one leg become the initial conditions for the next and so on. This piecewise approach is perhaps the ideal way to explain the step-by-step progress of a fire column. An infinite number of piecewise-constant segments may be employed to account for alterations in the factors affecting the rise of the gases. A piecewise, continuous calculation is a sensitive approach in the sense that it can take into account a completely new set of conditions for each segment. The summation of the segments becomes an accurate assessment of the many factors (and changes in factors) which act upon the column as it progresses upward. Some

questions seem to remain unanswered in this approach, and the solution is not of the type which could be easily handled in the field by land managers. Perhaps future breakthroughs will refine the technique enough to make this approach the best one for universal application to problems involving the rise of hot gases. To meet present needs, we must seek a more immediate and expedient approach.

A second approach to the development of a plume rise model is the generation of empirical equations based on physical relationships and principles. This approach has been extensively used to describe plumes from industrial stacks. Some attempt has been made to apply the resulting formulas to the plumes from open fires. Lowry³ has presented a good case for using the Davidson-Bryant and Holland formulas to calculate the height to which a plume will rise before becoming level. The formulas used are these:

$$\text{Davidson-Bryant} \quad H = \left(2r \frac{W}{u} \right)^{1.4} (1 + B)$$

$$\text{Holland} \quad H = 1.5 \left(2r \frac{W}{u} \right) + C_1 \frac{L}{u}$$

where:

H = rise of a plume to the level where its axis is horizontal
(meters)

r = radius of a circular plume measured at a height "Z" meters
above the fire (meters)

³William P. Lowry. A tentative model to estimate requirements for rise of large smoke plumes from prescribed burning. (Paper presented at Pacific Northwest International Section, Air Pollution Control Association, Salem, Oregon, November 9, 1967.)

W = vertical speed at height "Z" (M/sec.)

u = gradient windspeed (M/sec.)

C_1 = a constant (unitless)

B = dimensionless buoyancy (unitless)

L = rate of heat generation (cal./sec.).

These formulas have proven empirically accurate in estimating the plume rise from industrial stacks (Moses and Strom 1961). However, their extrapolation to open fires may be of questionable value. The gradient wind appears as a denominator in both formulas, so it is inversely related to plume height. One immediately recognizes that there are severe limitations on the usefulness of these equations. For example, any windless situation yields an infinitely high plume, which is not realistic. Data from the Miller Creek fires were used in these formulas as a cursory check on their usefulness, and it quickly became obvious that ridiculous answers were resulting. Most of the Miller Creek fires were conducted under very slow windspeeds, and the weakness of using windspeed in the denominator of the model was apparent immediately (see example in Appendix B, page 65). It should also be obvious that conditions in the upper air must have some influence on the buoyancy of the plume gases as they gain altitude. These conditions are not accounted for by the equations as they stand, but Lowry⁴ has suggested some ways in which the lapse rate can be employed to modify the equations somewhat. However, the shortcomings of the Davidson-

⁴Ibid.

Bryant and Holland formulas are sufficient to make them of questionable value. In addition, some of the parameters (r , L , and W) are not preburn measurements, although Lowry's approach overcomes this difficulty to some extent.

The third general type of approach used for plume rise model development is a statistical analysis of the relationships between measurable fire parameters and the resulting column height. In applying this type of analysis, the lessons learned from physics and engineering approaches become very useful. Only with a knowledge of what fire factors exert physical influences and responses on other measurable factors can we hope to successfully formulate a dependable statistical relationship between preburn measurements and column heights. Without these established relationships one must grope for associations, with the attendant risk of arriving at nonsense correlations.

The approach in this study is to find, through a multiple regression analysis, a relationship between ultimate column height and various preburn measurements. There is general agreement in the literature on what entities influence plume rise. The choice of independent variables for a regression analysis must logically relate back to these known relationships in plume rise theory. This is necessary in order to relate known physical influences to the type of field measurements that can be made by those expected to use the resulting model.

Chapter 3

THE STUDY AREA AND SOURCE OF DATA

The location for the data-gathering portion of the study was the Miller Creek drainage on the Tally Lake District of the Flathead National Forest, in Flathead County of Montana. Experimental fires were conducted on forty-one 10-acre plots of slash. The slash was created when a mature to overmature larch-fir-spruce stand was clearcut in 1966 and 1967. Twenty-two of these fires were sufficiently documented to provide the basis for this study.

The quantity of fuel on the beds was quite uniform (within each fire), and each burn site was inventoried before burning. The depth of the partially decayed vegetable matter on the forest floor (duff) was measured. Samples of several sizes and types of fuels were collected prior to ignition for subsequent determination of their moisture content.⁵

In addition to routine observations of temperature, humidity, rainfall, and windspeed (5-minute average), standard radiosonde ascents were employed to record the temperature and humidity of the atmosphere above each fire. By tracking the radiosonde balloons, or releasing and tracking pilot balloons, winds-aloft data were also gathered at fire time.

The measurement of column height was accomplished by observations from an instrumented aircraft operated by researchers from Washington

⁵William R. Beaufait. Prescribed fire cooperative study Region 1 - Intermountain Forest and Range Experiment Station. Study Plan No. 2102-12 on file at the Northern Forest Fire Laboratory, USDA Forest Service, Missoula, Montana, 1967.

State University. Among other things, the aircraft provided altitude measurements of the top of the smoke at recorded times during the progress of each fire. Some of the column height measurements were supported by measurements by lookouts located near the study area.

Water-can fire analogs, as described by Beaufait (1966), were placed in the fire and the loss of water (by weight) from these analogs was recorded as soon after the fire as possible.

Chapter 4

ANALYSIS OF MILLER CREEK DATA

Study of modern plume theory reveals that the height achieved by a fire column is influenced by the intensity (unit area energy release rate) of the fire, the temperature/altitude profile of the atmosphere above the fire site, and to an unknown extent by the fire size and the surface windspeed at the time of the fire⁶ (Murgai and Emmons 1960). Other factors are almost certainly involved, but for an initial approach the aforementioned factors appear to be sufficiently influential to account for the major portion of the variation experienced.

ENERGY RELEASE RATE

The heat produced by a fire per unit of time is its energy release rate. The rate of energy release is a function of a number of variables, each of which can be measured or estimated prior to ignition. The fuel to be burned contains a given amount of potential energy that could be released if all of the fuel burned. However, the total amount of energy a slash fire releases is modified by the moisture content of the fuels in at least two ways:

⁶Lowry, op. cit.

1. The fraction of the fuel that is actually consumed is controlled to a large extent by how wet the fuel is.⁷

2. Even if the fuel does burn, the water in the fuel decreases the ultimate energy output (Davis 1959). Thus, the influence of fuel moisture on energy release may be thought of as multiplicative rather than simply additive.

Wind at the time of the fire also has a modifying influence on the energy release rate by accelerating the rate of spread and decreasing the time span over which the energy is released. Slope has an influence on the energy release rate similar to that of wind. However, the slopes at Miller Creek were fairly uniform and will be neglected in this first analysis. Similarly, plot size at Miller Creek was nearly constant.

In practice it has been found that fuels with diameters less than approximately 10 centimeters constitute the bulk of the fuels consumed in the usual slash fire. Only very intense fires consume the larger fuels. The 0- to 10-centimeter fuels can be categorized in three classes:

1. Needles (leaves)
2. Twigs with diameters less than 1 centimeter
3. Branches with diameters between 1 and 10 centimeters

In addition to the fuels above the surface, duff often constitutes a consumable fuel and should be accounted for. Earlier analyses of the

⁷Steering Committee. Cooperative studies of the use of fire in silviculture. Progress Report 4. 1970. (Unpublished report on file at Northern Forest Fire Laboratory, USDA Forest Service, Missoula, Montana.)

data from Miller Creek suggest that the moisture content of the duff is an indicator of the energy release one might expect from a fire.⁸

Recent experiments at the Northern Forest Fire Laboratory have shown that the potential energy of oven-dried needles, of small twigs, and of branchwood differ considerably (see Appendix A, table 2, page 52). The values used in this analysis employ the most recent heat content figures as determined by bomb calorimeter measurements on needles and twigs.

By using these heat content values and the moisture content of the various fuels at fire time, the available heat yield per pound of each fuel can be calculated (see Appendix A, page 49, Potential Heat Yield). Fuel inventories conducted at Miller Creek allow a calculation of the weight of the fuels in each category. Multiplying the heat yield per pound of each type and size of fuel by the corresponding weight of the fuel on each fire site gives the actual heat yield available from all of the fuels on the site. Note that this is not necessarily the energy that will be released by the fire because some of the fuel will not be consumed.

In summary, the inputs which are intended to account for the energy release rate are:

1. Fuel weight (separated into size classes)
2. Fuel moisture content
3. Windspeed (surface)
4. Temperature of the ambient air at the fire site
5. High heat yield of various fuels

⁸Ibid.

These five variables are combined in various ways to account for the available energy (Appendix A, page 49) and are also used individually in the regression analysis to account for the quantity of fuel burned and the rate at which it is consumed. The word schematic (fig. 1) showing the data flow path will help the reader to visualize the way each variable is used to formulate the variables that go into the multiple regression analysis. For example, one can see that a calculation of the potential heat yield requires these inputs:

1. Fuel energy content
2. Temperature (fuel at fire time)
3. Fuel weights
4. Fuel moisture content

Note also that some variables, such as fuel weight, are used directly in the regression analysis.

BUILDUP INDEX

The National Fire-Danger Rating Buildup Index (BUI) has proved to be strongly correlated to burn results in recent analyses.⁹ This is to be expected because the BUI is intended as a descriptor of the drying history of intermediate size (1.25 to 5.0 centimeter diameter) fuels. As such, the BUI is related to the fuel moisture content of those fuels. For a given loading, the BUI might be strongly related to the energy release rate and could serve as a convenient variable to account for the effect of fuel conditions on column heights.

⁹ibid.

DATA FLOW SCHEMATIC

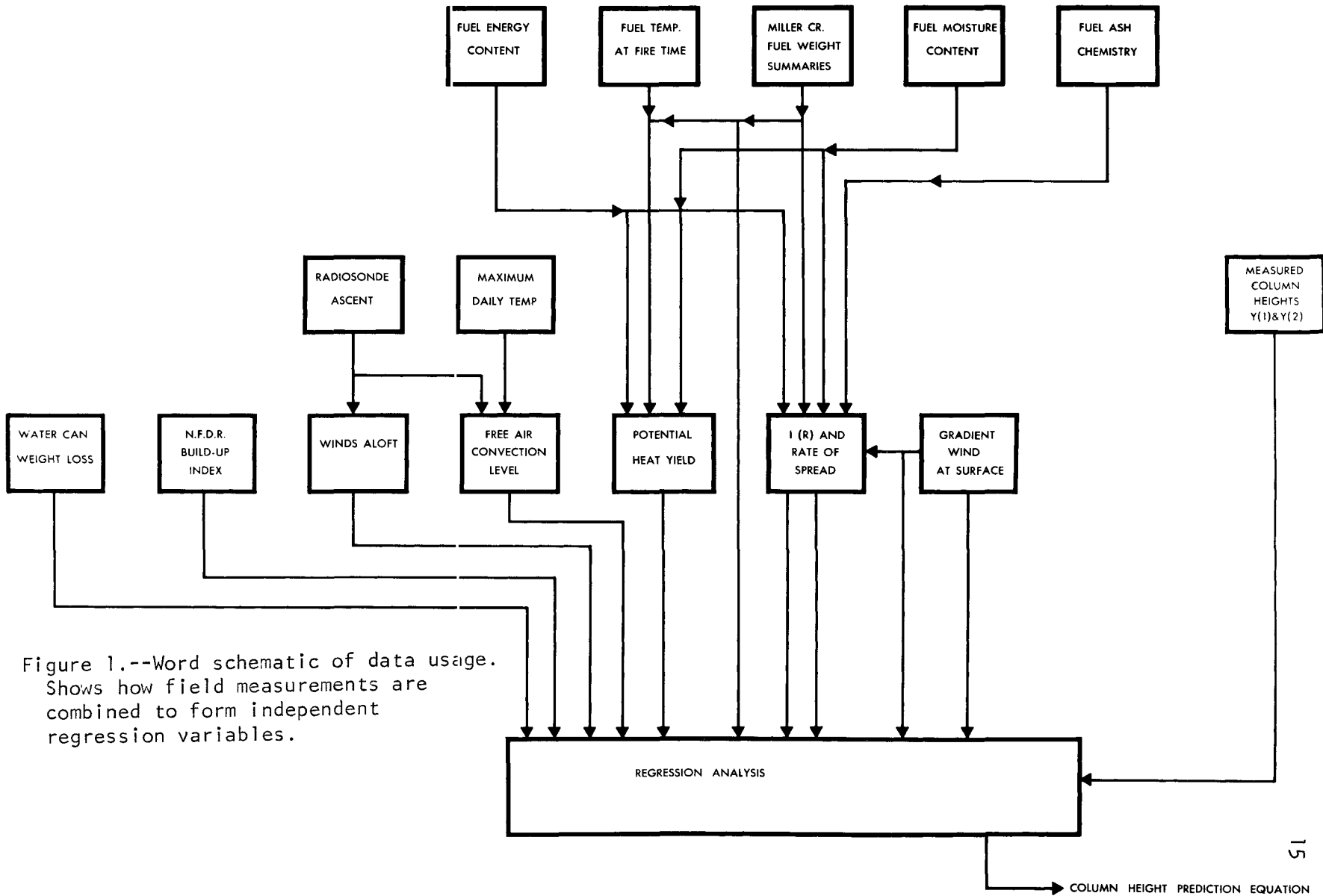


Figure 1.--Word schematic of data usage. Shows how field measurements are combined to form independent regression variables.

LAPSE RATES

The term "lapse rate" is used to describe the change in air temperature for a given altitude change. A parcel of air which is heated at the surface of the earth will tend to rise. As it rises it expands, and with no addition of energy it will become cooler. Said another way, there is no exchange of energy between the air parcel and the ambient air surrounding it. The parcel, as it rises, is simply getting larger without gaining any energy content, with the result that the temperature of the parcel decreases. This reversible process is called dry adiabatic cooling, and in our atmosphere it occurs at a rate of about 1 Celsius degree per 100 meters of altitude change ($1\text{C.}^\circ/100\text{ m.}$).

The actual existing temperature-altitude curve at any given time is composed of a continuum of many different lapse rates, stacked one upon the other. The temperature-altitude profile is constantly changing and has an infinite variety of shapes. Certain of these shapes and characteristics are of special interest. When the temperature decreases with altitude more slowly than the dry adiabatic rate, an adiabatically cooled air mass will experience a resistance to further rise because the surrounding air is increasingly warmer than the parcel. This situation is called a stable condition and is illustrated in figure 2. Conversely, when the temperature decreases more rapidly than the dry adiabatic rate an adiabatically cooled air mass experiences lifting (increased buoyancy) by virtue of growing increasingly warmer than the surrounding air. This is called an unstable condition and is illustrated in figure 3.

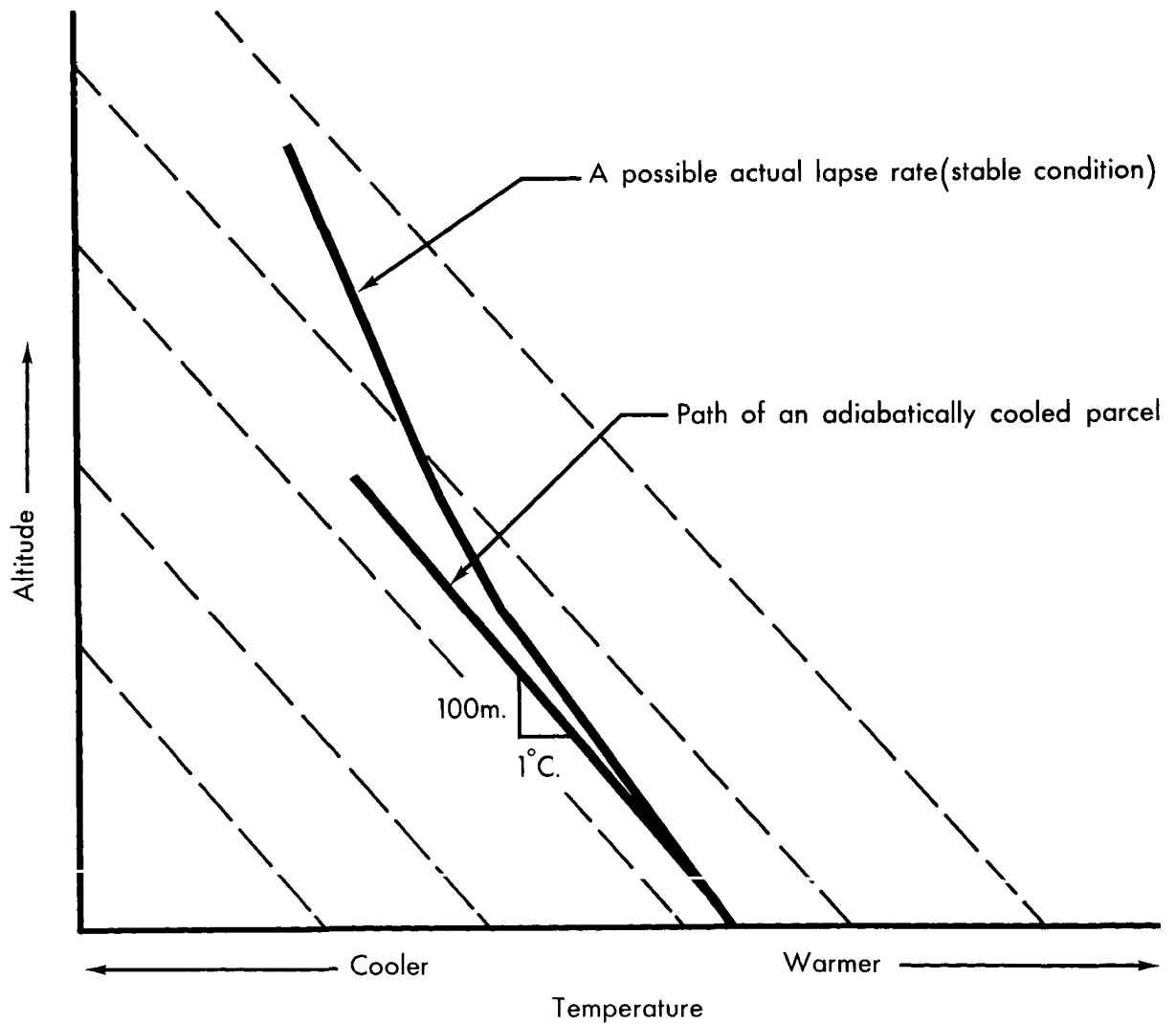


Figure 2.--Temperature relationships under a stable lapse rate condition. An adiabatically cooled airmass will experience resistance to further rise because surrounding air is increasingly warmer than the airmass.

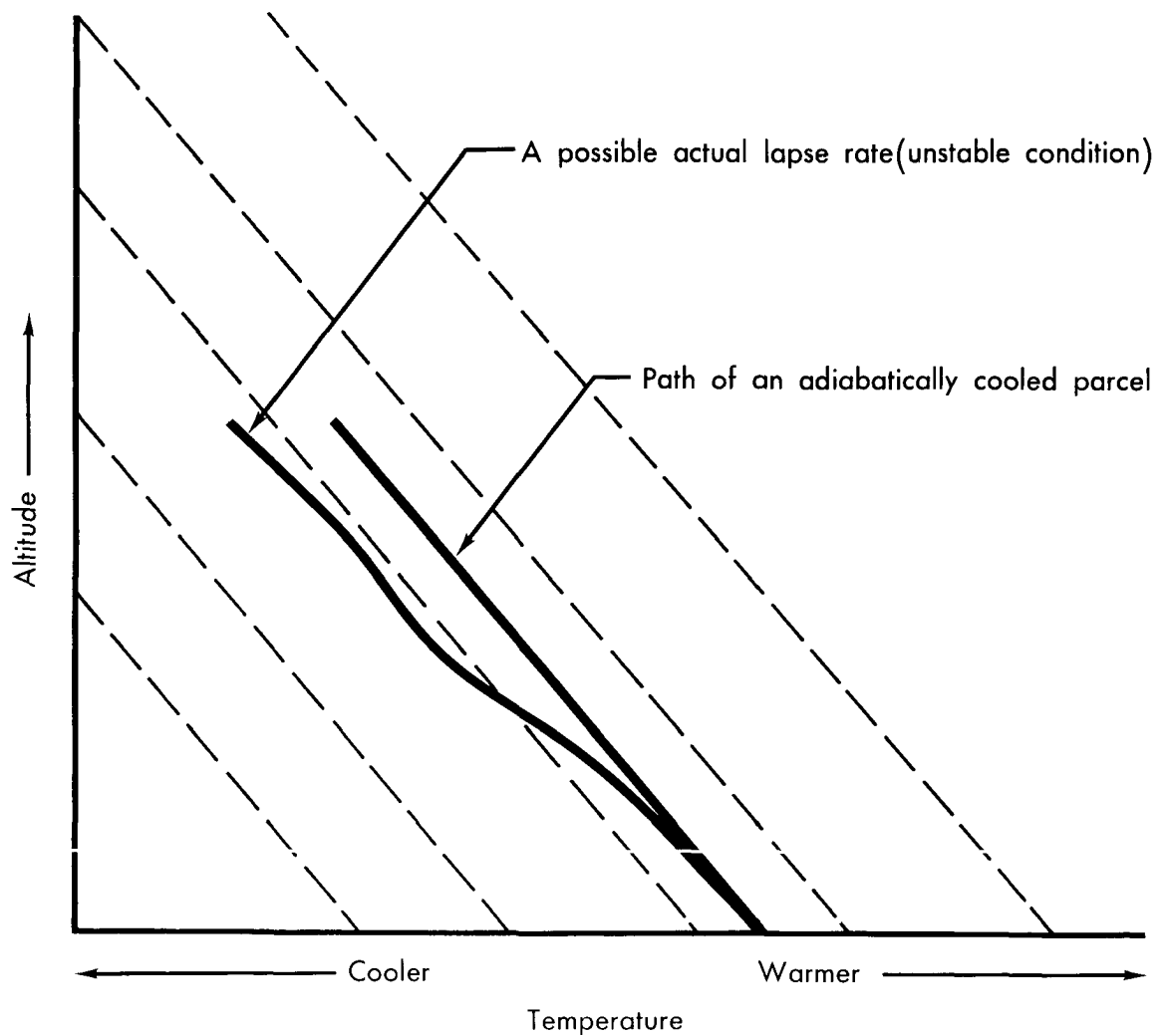


Figure 3.--Temperature relationships under an unstable lapse rate condition. An adiabatically cooled airmass will experience an increase in buoyancy by virtue of becoming increasingly warmer than the surrounding air.

COOLING OF COLUMN GASES

The gases in a fire column leave the surface at temperatures which are extremely high as compared to air temperatures around the fire. At high temperatures, the gases cool by a complex combination of mixing, expanding, and radiating which is not well known at present. In any event, the cooling curve most assuredly does not follow a dry adiabatic rate from the ground up. At some point in altitude the column gases will cease to exchange energy with the surrounding air and will begin to cool almost adiabatically. Just where this occurs is the unpredictable factor and is the result of the very complex cooling mechanisms. This complex cooling is illustrated by the three possible cooling curves in figure 4. The upper curve would describe a hot fire and the lower curves are for progressively cooler fires. Note that the curves enter into dry adiabatic cooling at different altitudes, and as a result they intersect the actual lapse rate curve at different altitudes. From the point where adiabatic cooling occurs, any further rise of the smoke in a fire column depends on the nature of the lapse rate as it exists in the atmosphere above that point. The crux of the problem of describing column behavior is to account for the shape of the cooling curve below the point where adiabatic cooling begins. If this low altitude cooling could be accurately described, it would be a simple matter to superimpose the daily lapse rate on the cooling curve and project upward to determine the correct column height. The ultimate height would be the altitude where the adiabatically cooled gases become equal to the ambient temperatures.

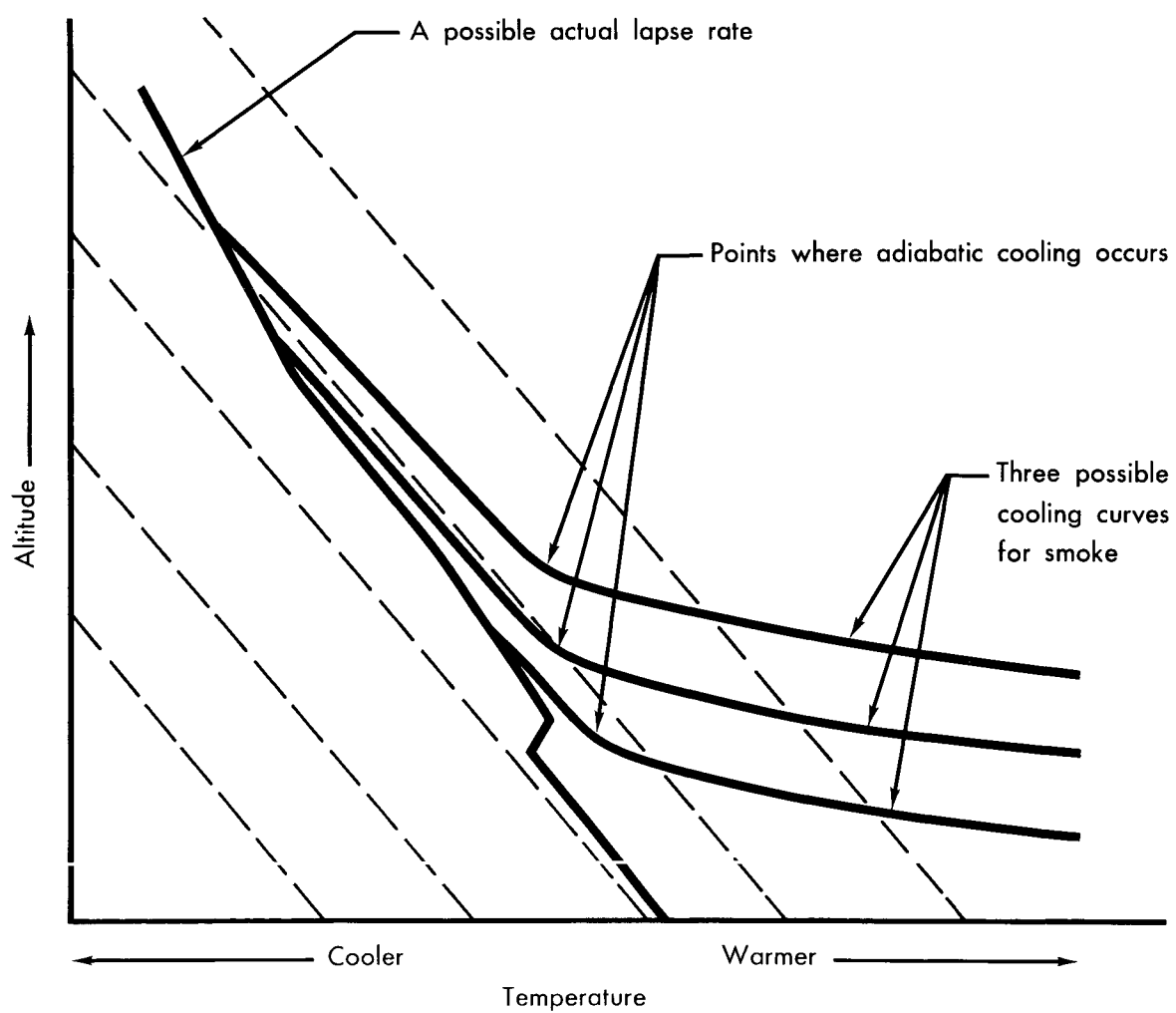


Figure 4.--Possible cooling curves for smoke. Hotter fires enter into adiabatic cooling at higher altitudes than do cooler fires.

It is possible that the windspeed near the surface influences the cooling of column gases. It is fairly well known that a high intensity fire yields a high smoke column. Therefore, one would expect that a wind-fanned fire, which tends to be intense, would tend to produce a high column. However, it should be recalled that the column gases are cooled by mixing for a portion of the distance up the column. This is especially so near the surface where the gases are very hot. Wind at the surface would most assuredly assist this form of cooling by accentuating the mixing process. By increasing the mixing, a strong surface wind could cause the gases to cool more quickly, leaving less energy in the gases to be expended by rising and expanding. The result is a shorter column.

LAPSE RATE INFLUENCES AT UPPER LEVELS

The problem of accounting for the influence of the lapse rate above the point where adiabatic cooling begins can be approached in several ways. A good physical approach would consider each leg of the temperature profile separately, calculating buoyancies from the temperature differences between the column gases and the ambient air, and employing conservation equations to determine the rate of rise to the next leg of the temperature profile, and so on.

An alternative approach stems from the fact that the pseudo-adiabatic diagram is actually an energy chart. An area on the diagram represents a given amount of energy. Lowry¹⁰ and others have suggested

¹⁰Lowry, op. cit.

that the algebraic sum of the areas above and below a dry adiabatic line and enclosed by the actual lapse rate curve could be used to modify the predicted column height. A problem arises here because one does not know the point where the smoke will begin adiabatic cooling. Therefore, the choice of which adiabat to use is simply a guess because of our presently limited knowledge of the cooling processes in the lower portion of the smoke column.

None of these approaches is realistically suited to general use by men in the field. The Free Air Convection Level (FACL), on the other hand, is a parameter which can be determined by fire-weather forecasters if a good atmospheric sounding is available. The FACL holds promise as a useful descriptor of atmospheric conditions which influence smoke rise.

FREE AIR CONVECTION LEVEL

The maximum level of free air convection (FACL) is calculated from the actual temperature lapse rate (temperature versus altitude curve) existing at the time of ignition. The predicted (or measured) maximum daily temperature is plotted as the initial surface value on a pseudo-adiabatic chart. A line is projected from the surface upward along the dry adiabat until it intersects the actual lapse rate curve. The point of intersection is defined as the Maximum Free Air Convection Level.

A preliminary regression analysis screen for best equations showed the FACL to be a moderately good predictor of ultimate column height. Therefore, the use of the FACL as an independent variable to account for the influence of the lapse rate seems justified on statistical as well as practical grounds. Because of the way the FACL is determined, it is to be expected that its accuracy as a column height predictor is better when the atmosphere is unstable so that a high FACL results. Since the FACL is determined by projecting along an adiabatic cooling line, it can only be accurate if the FACL turns out to be considerably above the point where the smoke ceases to cool by mixing and enters into adiabatic cooling. If the FACL is above this point, the method used to calculate the FACL parallels closely the actual cooling mechanism of the smoke. If the FACL prediction is below this point, its effectiveness as a predictor decreases quickly. If the lapse rate is stable from the ground up, the FACL is possibly zero (ground elevation), yet a high energy fire will drive the smoke higher than ground level.

While on the subject of lapse rates, it might be well to elaborate on a special lapse rate condition called a temperature inversion. There seems to be a growing misconception about the capacity of a temperature inversion to trap smoke or to prevent smoke from rising above a certain level. One even hears such expressions as "bursting through" an inversion, which seems to imply some rigid barrier stretched across the sky. Actually a temperature inversion is nothing more than a very stable portion of a temperature/altitude curve. Just as any stable lapse rate condition

will suppress the rise of a gas, an inversion will cause smoke (or any other gas) to stop rising only if the smoke has cooled to very nearly the ambient temperature at the altitude where the inversion exists. To illustrate this, figure 4 shows a definite and strong temperature inversion below the altitudes where any of the smoke begins to cool adiabatically. This inversion would not trap, stop, or hold the heated gases at this level. The net result of the inversion would be a slight decrease in buoyancy. It should be obvious that the gases have "burst" nothing, nor have they "overcome" an inversion. The gases are simply hotter than ambient temperatures up to and above the level of the inversion. One should also realize that, for a hot fire, the higher an inversion is, the greater is the likelihood that the inversion will suppress further rise of the smoke. However, this is also true for a lapse rate condition that is only moderately stable, so again there is nothing more special about an inversion than the fact that it is a very stable layer of air.

REACTION INTENSITY AND RATE OF SPREAD

The Fire Physics Project at the Northern Forest Fire Laboratory has recently completed the development of a mathematical fire spread model.¹¹ The model calculates two functions which may be useful in describing smoke column behavior. These two functions are the

¹¹Richard C. Rothermel. Tailoring the fire spread rate model to the field. (Paper presented at 1969 Spring Meeting of the Central States Section of the Combustion Institute, Minneapolis, Minnesota.)

Reaction Intensity (I_R) and the Rate of Spread (ROS). The I_R is one form of the unit area energy release rate experienced in moving fires. The ROS is the predicted linear speed of the movement of the fire front. The data required for the computation of I_R and ROS may be read from figure 1 by following the arrows leading into the block labeled " I_R and Rate of Spread." Work on the model is continuing to make it more versatile and more descriptive of other kinds of fires besides running fires.

MULTIPLE REGRESSION ANALYSIS

Because of the possible influence of the elevation of the fire site on column height, two measures of height were used:

Column height above mean sea level

Column height above terrain

The height above terrain is the actual length of the column from the ground to the altitude where the axis becomes horizontal. These two column height measurements constitute the dependent variables in a regression analysis where:

Y(1) = column height (ft. above m.s.l.)

Y(2) = column height (ft. above terrain)

X(1) = Maximum Free Air Convection Level (ft. above m.s.l.)

X(2) = average upper duff moisture content (percent)

X(3) = windspeed at fire time (20 feet above ground, m.p.h.)
(5-minute average)

X(4) = water-can weight loss (g.)

X(5) = National Fire-Danger Rating Buildup Index

X(8) = $1/X(2)$

X(12) = potential heat yield of fuels in X(16) (B.t.u.)

X(13) = reaction intensity (I_R)

X(14) = rate of spread (ROS)

X(15) = weight of fuels; 0-1 cm. diameter + 1-10 cm. diameter
+ needles (g./m.²)

X(16) = weight of fuels; 0-1 cm. diameter + needles (g./m.²)

X(17) = average fine fuel moisture; 0-1 cm. + needles (percent)

X(18) = $1/X(17)$

X(19) = potential heat yield of fuels in X(15) (B.t.u.)

X(20) = Maximum Free Air Convection Level (ft. above terrain)

A computer screen of all possible equations involving the above variables was performed along with machine plots of each independent variable against each dependent variable. This initial attempt revealed that a linear function of X(1), X(12), X(16), and X(17) accounts for 63.2 percent of the variation experienced in Y(1). The computer plots of the variables indicated that variables X(3), X(5), and X(18) vary as some function of the column height above mean sea level. A computer program was employed to find the single-term equations of best fit for each of these three independent variables versus column height. The resulting functions were added to the independent variable list. They are X(6), X(7), X(9), and X(11) below, and were selected as a convenience in curve fitting, not because of any physical inference. The independent variables found to be nonsignificant in the initial analysis were eliminated.

The final screen of all possible equations examined combinations of the following variables:

Y(1) = column height (ft. above m.s.l.)

Y(2) = column height (ft. above terrain)

X(1) = Free Air Convection Level (ft. above m.s.l.)

X(2) = average upper duff moisture content (percent)

X(3) = windspeed at fire time (20 feet above ground, m.p.h.)

X(4) = water-can weight loss (g.)

X(5) = National Fire-Danger Rating Buildup Index

X(6) = square root of X(5)

X(7) = $\text{Log}_{10} (X(5))$

X(8) = $1/X(2)$

X(9) = square root of X(3)

X(10) = square root of the potential heat yield of the
(0-1 cm. + needles) fuels

X(11) = square root of X(8)

Chapter 5

RESULTS OF THE ANALYSIS OF MILLER CREEK DATA

The six-variable regression equation with the smallest root mean square had these six variables:

X(1), X(2), X(3), X(4), X(9), X(11)

where:

X(1) = Free Air Convection Level (ft. above m.s.l.)

X(2) = upper duff moisture content (percent)

X(3) = windspeed (20 feet above ground, m.p.h.)

X(4) = water-can weight loss (g.)

X(9) = square root of X(3)

X(11) = square root of 1/X(2).

The R^2 (coefficient of multiple determination) for this regression is .80. That is, 80 percent of the variation in the column height as measured by variance is explained by the regression.

Source of variation	df	SS	MS	F
Due to regression	6	84,308,688	14,051,448	9.958**
Error	15	21,165,840	1,411,056	
Total	21	105,474,520		

Since tabular $F_{.01}$ with 6 and 15 degrees of freedom is 3.94, an $F_{6/15 \text{ df}} = 9.958$ is significant at the .01 level.

Standard error of the estimation = 1,188 feet.

A "t" test of the regression coefficients to determine if the coefficients are significantly different than zero yielded the following results:

<u>Coefficient</u>	<u>"t"</u>	<u>Level of significance</u>
b(1)	1.789	.10
b(2)	1.557	.20
b(3)	3.66	.01
b(4)	2.385	.02
b(9)	3.741	.01
b(11)	1.224	.30

From this test it appears that only b(3), b(4), and b(9) are statistically assured of being different than zero. Separating non-significant variables yielded the following results (see Appendix B, page 57 for computations):

Test of the gain due to X(1), X(2), and X(11) after X(3), X(4), and X(9) are in the model:

$$F_{(3, 15)} = 2.21; \text{ which is not significant at } \alpha = .05$$

Reduction due to X(3), X(4), and X(9), only:

$$F_{(3, 15)} = 17.71; \text{ which is significant at } \alpha = .01$$

The R^2 with X(3), X(4), and X(9) in the model is .711, and the standard error of the estimate is 1,302 feet (see Appendix B, page 57 for computations).

At this juncture, it is advisable to examine the validity of the regression equation on a logical as well as practical basis. It should be remembered that the ultimate intended use of this equation is the preburn calculation of probable smoke column heights. This regression equation does not fulfill the requirements because $X(4)$, which is water-can weight loss, is not known before ignition (although it can be predicted). Rather than employ a second regression equation (with its attendant error) to estimate water-can loss, it is more expedient and logical to formulate a model to describe column height directly. It is known from other analyses¹² that the Buildup Index is a good predictor of water-can weight loss. Hence, it is logical to suspect that a regression equation with some function of the Buildup Index in place of water-can weight loss would be a good descriptor of column height.

This, indeed, is the case. Recall that independent variables $X(5)$, $X(6)$, and $X(7)$ are the Buildup Index, square root of the Buildup Index, and the logarithm of the Buildup Index, respectively. Regression analyses using these variables in place of water-can weight loss yield the following results:

	<u>R²</u>
$Y(1) = f \{X(1), X(2), X(3), X(7), X(9), X(11)\}$.790
$Y(1) = f \{X(1), X(2), X(3), X(6), X(9), X(11)\}$.788
$Y(1) = f \{X(1), X(3), X(7), X(8), X(9), X(11)\}$.793

¹²Steering Committee, op. cit.

Removing the least significant variables in each of these resulted in the following:

	R^2
$Y(1) = f \{X(1), X(3), X(7), X(9), X(11)\}$.767
$Y(1) = f \{X(1), X(2), X(3), X(6), X(9)\}$.761

Also:

$Y(1) = f \{X(1), X(3), X(7), X(9)\}$.752
---------------------------------------	------

The last equation is of special interest because of its practical value. Although there are four independent variables in the equation, only three different measurements are required to calculate the independent variables. The only field measurements required are the Buildup Index, the Free Air Convection Level, and the 5-minute average windspeed at the surface (20 feet). Requiring only three parameters offers considerable advantage in practical applications. In addition to the obvious benefit of minimal data gathering, there is the added advantage of being able to present the data in simple tables. The BUI is kept current by the U.S. Forest Service as routine business, and the other two variables can be predicted by weather forecasters. The reduction in R^2 experienced in using this simpler equation in preference to one of the longer equations may well be justified because of its adaptability as a useful field technique.

The question to be answered is whether or not the simpler equation is sufficiently accurate and trustworthy for the needs at hand. The following pages offer summaries of tests of the most promising (highest

R^2 values) equations as to their significance and expected estimation accuracy. Complete calculation procedures may be followed in Appendix B (pages 57 to 59).

The simplest equation is:

$$Y = f \left(X(1), X(3), X(7), X(9) \right)$$

where:

$X(1)$ = Free Air Convection Level (ft. above m.s.l.)

$X(3)$ = 5-minute average windspeed, 20 feet (m.p.h.)

$X(7)$ = Log_{10} (BUI)

$X(9)$ = square root $\left(X(3) \right)$.

The actual regression equation is:

$$Y(1) = -5,578.3 + .0381 X(1) - 2,183.6 X(3) + 3,683.9 X(7) + 10,634.0 X(9)$$

Source of variation	df	SS	MS	F
Due to regression	4	79,334,496	19,833,624	12.899**
Error	17	26,140,032	1,537,649	
Total	21	105,474,528		

Since tabular $F_{.01}$ with 4/17 df is 4.34, the regression is significant at the .01 level.

The standard error of the estimation is 1,240.0 feet.

The coefficient of multiple determination is:

$$R^2 = \frac{\text{Regression SS}}{\text{Total SS}} = \frac{79,334,496}{105,474,540} = .752$$

A "t" test of the regression coefficients yields the following:

<u>Coefficient</u>	<u>"t"</u>	<u>Level of significance (df = 17)</u>
b(1)	1.759	.10
b(3)	4.670	.001
b(7)	2.558	.05
b(9)	5.020	.001

This test indicates that only b(1) is of questionable significance.

There exists a probability of 10 percent that assuming b(1) is not zero would lead to an erroneous association.

The confidence interval about the mean value of \hat{Y} (i.e., regression Y) for a specified set of X values can be calculated using c-multipliers.

This is done in Appendix B, page 57, and summarized below:

<u>X(1) FACL</u>	<u>95 percent confidence limits of \hat{Y}</u>
5,000 feet	10,555 to 12,617 feet
11,914 feet	11,265 to 12,435 feet
23,328 feet	11,447 to 13,122 feet

<u>X(3) windspeed (m./sec.)</u>	<u>95 percent confidence limits of \hat{Y}</u>
1.5351	11,519 to 13,019 feet
2.1330	12,019 to 13,763 feet
3.7309	11,605 to 13,553 feet

<u>X(7) Log₁₀ (BUI)</u>	<u>95 percent confidence limits of \hat{Y}</u>
1.3148	8,800 to 13,180 feet
1.5372	9,807 to 13,893 feet
1.7596	10,521 to 14,817 feet

For the sake of completeness, it is well to look at one of the regression equations that offers a somewhat higher R^2 value than the one just examined. If the addition of more independent variables offers a dependable increase in the explained variation, the more complex equation is justified as a better model.

The best six-variable regression equation (excluding those with $X(4)$) is:

$$Y(1) = f \left[X(1), X(3), X(7), X(8), X(9), X(11) \right]$$

The actual equation is:

$$Y(1) = -4,669.11 + .0369 X(1) - 1,931.92 X(3) + 3.090.4 X(7) \\ - 107,580.6 X(8) + 9,512.6 X(9) + 13,777.7 X(11)$$

<u>Source of variation</u>	<u>df</u>	<u>SS</u>	<u>MS</u>	<u>F</u>
Due to regression	6	83,617,488	13,936,248	9.564
Error	15	21,857,040	1,457,136	
Total	21	105,474,528		

Since $F_{.01}$ with 6/15 df is 4.14, the regression is deemed significant at the .01 level.

The standard error of the estimation is 1,207.

The coefficient of multiple determination is:

$$R^2 = \frac{83,617,488}{105,474,528} = .793$$

That is, 79.3 percent of the variation experienced in the column heights was explained by this regression.

A "t" test of the regression coefficients to determine if the coefficients are significantly different than zero yielded these results:

<u>Coefficient</u>	<u>"t"</u>	<u>Level of significance (df = 15)</u>
b(1)	1.562	.20
b(3)	3.774	.01
b(7)	2.119	.10
b(8)	1.366	.20
b(9)	3.966	.01
b(11)	1.409	.20

The standard error of the estimation is 1,207.

The confidence limits will not be calculated here, but the c-multipliers are supplied in Appendix B, page 62. The same procedure that was demonstrated for $Y = f \{X(1), X(3), X(7), X(9)\}$ can be employed to calculate confidence limits if desired.

The increase in R^2 of this equation over the simpler, four-variable equation is small, and of questionable validity in view of the nonsignificance of the additional coefficients.

Using the shorter equation $Y = f \{X(1), X(3), X(7), X(9)\}$, the estimated column heights were calculated for the Miller Creek fires.

These values are compared to the measured values in table 1. (Values are feet, m.s.l.).

It should be noted that the error values in table 1 are almost evenly distributed about zero. Summing the errors algebraically yields an overall error of +2 feet for all the fires in the table. This indicates that the equation $Y = f \{X(1), X(3), X(7), X(9)\}$ fits the data very well.

In word form, the equation is:

$$\begin{aligned} \text{Column height} = & -5,578 \\ & +.0381 \text{ (Free Air Convection Level)} \\ & -2,183.5 \text{ (windspeed)} \\ & +3,683.9 \text{ (Log}_{10} \text{ (BUI))} \\ & +10,634.0 \text{ (square root of windspeed)} \end{aligned}$$

Table 1.--Predicted column heights based on regression through the mean

Unit	Prediction	Observed	Error	Error percent
	<u>Feet, m.s.l.</u>	<u>Feet, m.s.l.</u>	<u>Feet</u>	
N01	12,237	10,500	1,737	16.5
N06	11,397	13,500	-2,103	15.6
N07	10,965	10,500	465	4.4
N08	13,452	13,000	452	3.5
N09	12,417	13,800	-1,383	10.0
N13	14,077	16,200	-2,123	13.1
N14	8,193	8,300	-107	1.3
N15	8,193	8,500	-307	3.6
E03	11,642	11,500	142	1.2
E04	13,337	12,200	1,137	9.3
E05	13,105	14,500	-1,395	9.6
E10	11,632	11,500	132	1.1
E14	11,632	11,500	132	1.1
S04	13,328	13,200	128	1.0
S10	13,328	14,000	-672	4.8
W04	7,742	6,700	1,042	15.6
W08	14,428	14,000	428	3.1
W10	14,544	13,500	1,044	7.7
W12	12,192	11,000	1,192	10.8
W13	12,192	10,500	1,692	16.1
W14	10,384	11,300	-916	8.1
W15	10,384	11,100	<u>-715</u>	6.4
			$\Sigma = +2$	

Chapter 6

DISCUSSION OF RESULTS

A matter of interest is the influence of windspeed on the column height, according to the model. For discussion purposes, the model is presented below in word form:

$$\begin{aligned} \text{Column height} = & -5,578.3 \\ & +.0381 \text{ (Free Air Convection Level)} \\ & -2,183.5 \text{ (windspeed)} \\ & +3,683.9 \text{ Log}_{10} \text{ (BUI)} \\ & +10,634.0 \text{ (square root of wind)} \end{aligned}$$

Notice that the coefficient of the windspeed term is negative. This says that the greater the surface wind is, the lower is the calculated column height. Contrarily, the coefficient for the square root of the wind is positive, indicating that a stronger wind yields a greater calculated smoke column height. The effect of this relationship may be seen in the column height tables (table 6, Appendix B). Note that for a given FACL and BUI, an increase of wind corresponds to a greater calculated column height up to a windspeed of about 6 miles per hour. Beyond that the calculated height decreases. This inflection point should be recognized as nothing more than a characteristic of the fitted equation. It should not be construed to be a standard or rule of thumb.

One must recognize that using the dry adiabatic lapse rate to calculate the FACL introduces some error. The dry adiabatic lapse rate

applies to dry air, and smoke is obviously not dry air. However, scant data are available to assess the moisture content of the column gases from the Miller Creek fires, so it is impossible at present to account for the effect of this moisture on the column heights. The presence of water vapor in a rising gas can result in a lesser change in temperature for a given change in altitude than is the case for dry air. The heat released by condensation is an internal source of energy and serves to warm a parcel of air (Johnson 1954). Having no other data, dry adiabatic cooling was assumed in the calculations, recognizing that some error was introduced by neglecting the effect of moisture in the smoke.

The justification for this study was founded on its contribution to the control of air pollution. The successful prediction of column heights should be an extremely valuable contribution. However, there are limitations which must be considered. One must realize that not all of the smoke is created during the active portion of the fire. A fire may require many hours or even days to burn out completely. Considerable amounts of smoke may be released during this burnout period. Since there is a relatively small rate of energy release during burnout, the smoke acts like any slightly warmed parcel of surface air and its movement is almost completely controlled by local meteorological conditions. Cramer and Graham¹³ have compiled a definitive set of guidelines which relate smoke management to meteorological conditions.

¹³Cramer and Graham, op. cit.

Their guidelines should be referred to as an aid in controlling the smoke during the burnout portion of a fire. In essence, it should be remembered that the traditional burning time in the Pacific Northwest coincides with the usually stable lapse rates of autumn. Adding to this problem is the fact that forest fuels tend to have a high moisture content at this time of year. The result is low energy fires which produce weak columns and may smolder and smoke for days. Selection of the day and the time of day for burning is of paramount importance in affecting good smoke dispersal.

It seems apparent that some changes in the annual burning schedule will be necessary in order to burn when stronger columns can be generated and conditions in the atmosphere are favorable for smoke dispersal. When fuels are dried (the BUI is high) fires burn with a high intensity and burn out in a short period of time. As a result, smoke is propelled to a high altitude where it is readily dispersed. Since the burnout period is shorter, there is less total smoke during the low intensity portion of the fire. In addition, lapse rates are conducive to high columns during the warmer months when fuels are dry, the BUI is high, and high intensity fires are possible.

At least 20 formulas have been developed which relate plume rise to various meteorological and nonmeteorological variables. This study, like most other plume rise studies, is based on fitting an equation to smoke rise measurements. The fitted equation:

$$Y(1) = -5,578 + .0381 X(1) - 2,183.6 X(3) + 3,683.9 X(7) \\ + 10,634.0 X(9)$$

is based on a limited range of the variables, so may not be valid beyond

the ranges of the samples used. Although this model is empirical in nature, it is based on physical principles and as such may prove to be more generally useful than purely empirical formulas. Although restricted in universal applicability to plume rise problems, the model holds promise of leading to a solution of a specific problem of smoke management which is narrow in scope but very wide in extent. If the model can assist in controlling the smoke from slash fires of even one forest type, it will have served a very useful purpose.

CONCLUSIONS

The model developed in this study:

$$Y(1) = -5,578 + .0381 X(1) - 2,183.6 X(3) + 3,683.9 X(7) \\ + 10,634.0 X(9)$$

employs measurements and quantities which are familiar to forest land managers. The variables required are either routinely calculated or are available as spot forecasts by meteorologists. It should not be construed that column heights in other fuels and at other locations can be calculated with certainty with this equation. The only claims made for the model are those described in the results portion of this paper. The plots were of a larch-fir forest type in western Montana and the stand was previously unmanaged. The ranges of variables examined are listed in the Appendices (page 56) along with the specific values for each plot (page 53 to 55). Although the model is not presented as a prediction equation, the rather high level of significance of the regression encourages further testing of the model at other locations to

lend extended knowledge of its validity. The simplicity of the equation would make a large scale test feasible.

The advantage of using a simple, three-variable equation is demonstrated in table 6, Appendix B, page 63. This table was generated by a short computer program and is the sort of table that can readily be employed by men in the field. All that is needed is the present Buildup Index, the Free Air Convection Level prediction, and the wind-speed (20 feet above the surface) in miles per hour. By entering the table on the correct FACL page, the estimated column height can be found at the juncture of the correct windspeed and BUI. Only two pages of the table are shown for demonstration purposes. The table ranges of the wind-speed and BUI probably cover all conditions when burning would be attempted. The length of the table (number of pages) depends on how many values of FACL are deemed necessary. In view of the small change in column height for a change in FACL, a table of values for each 1,000 feet of change in FACL would be more than adequate.

Chapter 7

FUTURE STUDIES

Any future extension of this column height study should encompass a range of slopes. As previously mentioned, slope can have an effect on the rate of spread of a fire similar to that of wind. Increasing the rate of spread could increase the rate of fuel consumption. The attendant increased energy release could influence the column height.

In view of the recognized importance of the effect of the energy release rate on column height, it is logical to suspect that the size of a fire has a considerable effect on column development. It should be recognized that the energy release rate per unit of area is what is commonly referred to as fire intensity, and intensity is the factor which is influential in creating a high energy column. The energy release rate of the whole fire tells very little about the intensity of the fire unless the size of the fire is specified. A given amount of total heat per second from a large fire will yield a lower intensity than the same total heat per second released from a smaller fire. The reason for this is that the heat release rate per unit of area is higher for this particular smaller fire. The result is that the column from the smaller fire may well go higher. It is heat yield per unit of area per unit of time (intensity) that is important. The relationship between fire size and column height for a given intensity is not well known, and a future

extension of this study should include provisions for analyzing the effects of fire size. It would seem likely that larger fires yield higher columns up to a point. The gases entrained in the column are somewhat isolated from the ambient air by the gases around the perimeter of the column and have less opportunity for cooling by mixing. Further study of column heights should include a range of fire sizes to give a basis for comparison of the effects of size.

Although the column height model developed in this study does not employ all of the variables the data offers, any future use of the model should be documented in the form of ranges of the variables not required for the model specifically. That is, values should be recorded for variables which are not needed for calculating the column height, but which may interact with model variables so as to possibly limit the range of model application. Fuel loadings and fuel moistures (including duff moisture content) particularly should be recorded. Values for some of these variables are not easily obtained at the present time. For example, the weight of the fuel to be burned is an unknown which at present requires a lengthy and costly field inventory to accurately quantify. This obstacle should be removed in the near future as analyses are completed which relate data in standard Compartment Examinations (Stage II) to fuels which result when an area is clearcut logged. Information should be available by early summer of 1970 that will provide a link between routinely collected information and the fuel parameters needed to describe the conditions under which the column height model is tested.

The analysis is being performed as part of the prescribed fire study at the Northern Forest Fire Laboratory.¹⁴ Fuel moisture measurements may also be simplified in the near future if electronic measuring techniques now under test prove to be reliable.

The effect of winds aloft is a complicating factor which more properly comes under the heading of dispersion (rather than ultimate height) of column gases. One of the initial assumptions this paper made was that the smoke column could be considered as a sort of smoke stack (a continuous point source). This assumption becomes less and less valid as the smoke nears ambient temperatures. As the cooling mechanism becomes meteorological in nature, the smoke falls under the influences of wind and lapse rate combinations which result in the various common dispersion patterns. The net result is that the phenomena of fumigating or looping may result (Slade 1968). This means that the smoke may be driven upward or downward (or both) from the altitude it would have reached under windless conditions. The column is likely to reach a different recognized height when strong winds near its expected height prematurely change the vertical motion into horizontal motion. At that point it becomes a problem of dispersion which is best handled separately from the problem of column height calculations. Studies are in progress to describe the dispersion and diffusion of smoke from slash fires. The successful completion of those studies will afford an opportunity to

¹⁴Beaufait, op. cit.

compare and consolidate information with this column height study in such a way as to describe the behavior of a fire column from its source to any point of interest down range. This ability to describe the total probable smoke column behavior is fundamental to any program of slash fire smoke control.

Chapter 8

SUMMARY

A regression equation involving the maximum Free Air Convection Level, the surface (20 feet) windspeed, and the National Fire-Danger Rating System Buildup Index can be used to calculate probable column heights. Based on 22 experimental fires, a multiple regression analysis accounts for 75 percent of the variation in ultimate column heights experienced.

The experimental fires burned slash from clearcutting a mature larch-fir forest. Burning was conducted under a wide range of fuel and weather conditions from May through October.

Although the regression equation may not be universally applicable to all slash burning, its use is encouraged in order to afford an extended field test. Column height tables for field use may be easily generated by a short computer program.

LITERATURE CITED

- Beaufait, William R.
1966. An integrating device for evaluating prescribed fires. *Forest Sci.* 12(1): 27-29.
- Byram, G. M., and Jemison, G. M.
1948. Some principles of visibility and their application to forest fire detection. U.S. Dep. Agr. Tech. Bull. No. 954, 61 p.
- Davis, Kenneth P.
1959. *Forest fire: control and use.* 584 p., illus. New York: McGraw Hill Book Co.
- Johnson, John C.
1954. *Physical meteorology.* 393 p., illus. New York: Massachusetts Institute of Technology and John Wiley and Sons, Inc.
- Lowry, William P., and Boubel, Richard W.
1967. *Meteorological concepts in air sanitation.* Oregon State Univ., Corvallis, 56 p.
- Murgai, M. P., and Emmons, H. W.
1960. Natural convection above fires. *J. Fluid Mechan.* 11(10): 455.
- Moses, H., and Strom, G. H.
1961. A comparison of observed plume rises with values obtained from well-known formulas. *J. Air Pollut. Contr. Ass.* 11(10): 455.
- Slade, David H. (Ed.)
1968. *Meteorology and atomic energy.* 445 p., illus. Oak Ridge, Tenn.: USAEC Division of Technical Information Extension.

APPENDIX A

Potential Heat Yield (Available Energy)

Davis (1959) has worked out an energy budget for the combustion of 1 pound of wood fuel which accounts for the water contained in the fuel. By employing this energy budget with the Miller Creek data for the three fuel classes, the heat energy available from all of these fuels on each study unit can be calculated.

The calculation proceeds as follows for each class of fuel:

Each pound of fuel contains a given amount of potential energy (determined by bomb calorimeter tests at the Northern Forest Fire Laboratory). This is the energy which would be released if the oven-dry wood were completely burned. Let this be H_h .

From this value we must subtract the energy losses due to water in the fuels. These are:

1. Raising the temperature of the water from ambient to boiling temperature.
(1 B.t.u./lb./°F.) x (212° - present temperature) x moisture content (fractional)
2. Separate bound water from wood (heat of desorption). Given by Davis as:

<u>Moisture content</u> <u>Percent</u>	<u>Heat of desorption</u> <u>B.t.u.</u>
0	0
10	31
25	48
50	50
100	50

3. Vaporization of the water:

$$= (972 \text{ B.t.u./lb.})(\text{fractional moisture content})$$

4. Heating of vapor from boiling temperature to flame temperature (approximately 1,600° F. - 212° F.):

$$= (700 \text{ B.t.u./lb.})(\text{fractional moisture content})$$

5. Vaporization of water of reaction (based on .559 pounds of water per pound of wood).

Given as 543 B.t.u./lb. of wood.

The total of 1 through 5 above is then subtracted from H_h to give the potential energy yield from each pound of fuel. A nominal loss of 800 B.t.u. is subtracted to account for radiation losses, and the final result is the potential heat yield per pound of fuel. Multiplying the potential heat yield by the fuel loading on the unit yields the total potential energy contained in the fuels in each of the three classes.

Finally, summing the values for the three fuel classes gives a grand total of the potential heat energy represented by the 0 to 10 centimeter twigs and the needles on the entire fire site.

These computations were accomplished by a computer program which is available for further use. The resulting potential heat yield values are listed in table 4.

Table 2.--High heat content of fuels (B.t.u./lb.)

	0-1 cm. diameter	1-5 cm. diameter	Needles
Ponderosa pine	9,798	9,409	9,300
Douglas-fir	9,422	9,215	9,016
Fir	8,618	8,521	8,477
Larch	9,667	8,605	8,020
Spruce	9,350	9,210	9,180

Table 3.--Fuel data

Unit No.	Fuel weight 0-1 cm. G./m. ² x10 ²	Fuel moisture: 0-1 cm. Percent	Fuel weight 1-10 cm. G./m. ² x10 ²	Fuel moisture: 1-10 cm. Percent	Fuel weight: needles G./m. ² x10 ²	Fuel moisture needles Percent	Duff moisture Percent
N04	2.5658	13.67	21.6251	15.67	3.1137	13.33	87.3
N06	1.8661	15.67	27.4271	19.83	2.4086	26.89	81.3
N07	3.3585	12.33	23.3860	36.66	4.0093	15.78	45.0
N08	3.0211	15.67	35.3988	19.83	3.8869	26.89	81.3
N09	2.2283	10.00	12.1622	17.50	2.9368	7.56	68.0
N13	3.1243	13.50	10.8172	17.50	3.7152	9.44	94.7
N14	2.9438	26.33	23.6847	27.67	3.7598	21.56	97.3
N15	3.0543	26.33	15.3266	27.67	3.6236	21.56	97.3
E03	2.9987	20.83	18.4248	25.67	3.3670	7.00	54.3
E04	3.7762	16.83	23.3283	38.67	4.7465	13.00	76.7
E05	2.8762	12.33	28.8668	20.00	3.5516	5.89	76.3
E10	2.9332	14.83	19.4041	17.33	3.5392	14.67	54.7
E14	2.9980	14.83	20.6089	17.33	3.6190	14.67	54.7
S04	2.3319	9.33	32.0736	18.00	2.9987	6.44	70.7
S10	3.2180	9.33	22.1543	18.00	4.1101	6.44	70.7
W04	2.9578	18.17	15.8778	29.00	2.9467	18.67	99.0
W08	4.0015	14.67	10.3348	33.67	4.7983	10.67	85.0
W10	1.9180	7.50	24.7888	9.00	2.4390	7.00	41.0
W12	2.6162	17.00	21.0665	21.83	3.0456	29.33	84.3
W13	2.8951	17.00	11.9518	21.83	2.9377	29.33	84.3
W14	2.5590	17.17	21.4690	24.00	2.9078	19.11	96.7
W15	2.2879	17.17	11.1961	24.00	2.7786	19.11	96.7

Table 4.--Water-can weight loss and potential heat yield

Unit No.	Water loss Grams	Potential heat yield: 0-1 cm. + needles	Potential heat yield: 0-10 cm. + needles
		B.t.u. x 10 ⁸	B.t.u. x 10 ⁹
N04	496.81	2.6599	3.19732
N06	302.91	1.4720	4.13912
N07	241.50	2.5658	3.13311
N08	266.33	2.0237	5.44791
N09	687.53	1.8510	2.07634
N13	609.19	1.7488	1.74876
N14	125.89	2.3614	3.53096
N15	136.97	2.3013	2.20359
E03	936.86	3.2126	2.50906
E04	479.83	4.3503	3.60255
E05	524.53	3.5026	4.26032
E10	415.14	2.6285	2.78429
E14	278.75	2.4572	2.76215
S04	407.00	1.6726	4.76706
S10	614.89	2.3913	3.30307
W04	52.53	2.3741	1.57272
W08	512.94	3.6751	1.70912
W10	519.33	1.6598	3.81018
W12	555.83	2.3149	2.73007
W13	266.36	1.8785	1.48679
W14	263.53	1.8615	2.80316
W15	228.44	1.4780	1.67549

Table 5.--Column heights and meteorological conditions

Unit No.	Column height m.s.l.	Column height a.t.	Free air convection level (m.s.l.)	Free air convection level (a.t.)	Surface windspeed		Buildup Index
	Feet	Feet	Feet	Feet	Meters/sec.	M.p.h.	
N04	10,500	5,900	6,700	2,100	1.341	3.0	35
N06	13,500	8,900	6,000	1,400	1.118	2.5	29
N07	10,500	5,900	10,800	6,200	4.470	10.0	15
N08	13,000	8,400	6,000	1,400	1.118	2.5	29
N09	13,800	8,800	11,200	6,200	.922	2.1	70
N13	16,200	11,200	15,010	10,010	3.125	7.0	49
N14	8,300	4,100	6,150	1,950	.447	1.0	24
N15	8,500	4,300	6,150	1,950	.447	1.0	24
E03	11,500	6,100	11,000	5,600	6.260	14.0	114
E04	12,200	6,800	6,500	1,100	4.447	10.0	72
E05	14,500	9,100	14,800	9,400	4.021	9.0	39
E10	11,500	6,700	3,300	0	1.341	3.0	26
E14	11,500	6,600	3,300	0	1.341	3.0	26
S04	13,200	8,600	15,950	11,350	3.125	7.0	30
S10	14,000	9,200	15,950	11,150	3.125	7.0	30
W04	6,700	2,300	6,400	2,200	.447	1.0	18
W08	14,000	9,200	13,000	8,200	3.125	7.0	64
W10	13,500	8,700	13,300	8,500	2.235	5.0	68
W12	11,000	6,200	9,300	4,500	1.341	3.0	32
W13	10,500	5,900	9,300	4,300	1.341	3.0	32
W14	11,300	6,700	9,000	4,400	.894	2.0	22
W15	11,100	6,500	9,000	4,400	.894	2.0	22

Ranges of Variables at Miller Creek

<u>Independent variable</u>	<u>Range</u>
Water-can weight loss (g.)	53 to 937
Potential heat yield (0-1 cm. + needles)(B.t.u.)	1.4720 × 10 ⁸ to 4.3503 × 10 ⁸
Potential heat yield (0-10 cm. + needles)(B.t.u.)	1.57272 × 10 ⁹ to 5.4479 × 10 ⁹
0-1 cm. fuel weight (g./m. ²)	187 to 400
1-10 cm. fuel weight (g./m. ²)	1,033 to 3,539
Needle weight (g./m. ²)	241 to 480
Moisture content of 0-1 cm. fuels (percent)	7.5 to 26.0
Moisture content of 1-10 cm. fuels (percent)	9.0 to 38.0
Moisture content of needles (percent)	6.4 to 29.3
Moisture content of upper duff (percent)	41.0 to 99.0
Wind velocity (m.p.h.)	1.0 to 14.0
Buildup Index	15 to 114
Free Air Convection Level (ft. m.s.l.)	3,300 to 15,950
Free Air Convection Level (ft. above terrain)	0 to 11,350
Air temperature at time of fire (° F.)	45 to 79
<u>Dependent variable</u>	
Column height (ft. above m.s.l.)	6,400 to 15,950
Column height (ft. above terrain)	2,300 to 11,200

APPENDIX B

Calculations Leading to F Test of Variables
with Nonsignificant Regression Coefficients

Variables in the model: X(3), X(4), X(9)

$$(\text{RMSQR})(\text{MS}\hat{y}) = \text{MS}(\text{Error})$$

$$(.3375)(5,022,597) = 1,695,126$$

$$\text{MS}(\text{Error}) \times \text{df}(\text{Error}) = \text{SS}(\text{Error})$$

$$1,695,126 \times 18 = 30,512,268$$

$$\text{Total SS} = 105,474,540$$

$$\text{SS}(\text{Error}) = \underline{30,512,268}$$

$$\text{SS}(\text{due to } 3, 4, 9) = 74,962,272$$

F Test of Variables with Nonsignificant Regression Coefficients

Source	df	SS	MS
Reduction due to 1, 2, 3, 4, 9, 11	6	84,308,688	
Reduction due to 3, 4, 9	3	74,962,272	24,987,424
Gain due to 1, 2, 11 after 3, 4, 9	3	9,346,416	3,115,472
Error	15	21,165,840	1,411,056
Total	21	105,474,540	

Test of gain due to X(1), X(2), and X(11) after X(3), X(4), and X(9) are in the model:

$$F(3, 15) = \frac{3,115,472}{1,411,056} = 2.21 \text{ (not significant at } \alpha = .05)$$

Test of the reduction due to X(3), X(4), and X(9) only:

$$F(3, 15) = \frac{24,987,424}{1,411,056} = 17.71 \text{ (significant at } \alpha = .01)$$

$$R^2_{3, 4, 9} = \frac{74,962,272}{105,474,540} = .711$$

$$\text{Standard error of estimate} = \sqrt{\text{MSQR (Error)}}$$

With X(3), X(4), and X(9) in the model

$$\text{MSQR (Error)} = \frac{30,512,256}{18} = 1,695,125$$

Standard error = 1,302 feet

Calculation of Confidence Limits for the Regression Equation

$$Y = f \left(X(1), X(3), X(7), X(9) \right)$$

The equation:

$$\begin{aligned} \hat{Y} = & -5,578.3047 + .0381 X(1)^* - 4,884.73 X(3) + 3,683.87 X(7) \\ & + 15,908.5 X(9) \end{aligned}$$

*Note, X(3) must be in meters/second for this equation.

$$\text{Standard error: } \hat{S}_Y = \sqrt{\text{Error MSQR}} = \sqrt{1.537649E06}$$

$$\hat{S}_Y = 1,240.0$$

The effect of X(1) as a predictor of \hat{Y} with the other variables held at their means is:

$$\begin{aligned} \hat{Y} = & -5,578.3 + .0381 X(1) - 4,884.7 (2.133) + 3,683.9 (1.537) \\ & + 15,908.5 (1.366) \end{aligned}$$

where:

$$X(3) = \overline{X(3)} = 2.133$$

$$X(7) = \overline{X(7)} = 1.537$$

$$X(9) = \overline{X(9)} = 1.366$$

\hat{Y} now becomes:

$$\hat{Y} = +11,395.8 + .0381 X(1)$$

Values can now be assigned to $X(1)$

Let:

$$X(1) = \overline{X(1)} - \text{S.E. of } X(1) = .5000 \times 10^4$$

$$X(1) = \overline{X(1)} = 1.1914 \times 10^4$$

$$X(1) = \overline{X(1)} + \text{S.E. of } X(1) = 2.3328 \times 10^4$$

This yields:

<u>$X(1)$</u>	<u>\hat{Y}</u>
$.5000 \times 10^4$	11,586.3
1.1914×10^4	11,849.7
2.3328×10^4	12,284.6

The confidence interval about any value of Y can be calculated from:

$$t(n, 1-1/2\alpha) S_{\hat{Y}} \sqrt{X_0' (X'X)^{-1} X_0}$$

where $t(22, 1-1/2(.05)) = 2.110 = t_{17}$, $\alpha = .05$;

and $(X'X)^{-1}$ is the vector of values sometimes referred to as the "c_{ij} multipliers" and X_0' is a vector of values for the independent variables (plus the constant).

The "c_{ij} multipliers" for this regression are:

$$c_{1,1} = .37754844 \text{ E-09}$$

$$c_{1,3} = c_{3,1} = .27146671 \text{ E-05}$$

$$c_{1,7} = c_{7,1} = -.45421223 \text{ E-06}$$

$$c_{1,9} = c_{9,1} = -.85425263 \text{ E-05}$$

$$c_{1,u} = c_{u,1} = .20812813 \text{ E-05}$$

$$c_{3,3} = .71160847$$

$$c_{3,7} = c_{7,3} = -5.2758805 \text{ E-01}$$

$$c_{3,9} = c_{9,3} = -2.1158447 \text{ E01}$$

$$c_{3,u} = c_{u,3} = .14217625\text{E01}$$

$$c_{7,7} = .134926622\text{E01}$$

$$c_{7,9} = c_{9,7} = -.14483947$$

$$c_{7,u} = c_{u,7} = -.17583046\text{E01}$$

$$c_{9,9} = .65302496\text{E01}$$

$$c_{9,u} = c_{u,9} = -.40846987\text{E01}$$

$$c_{u,u} = .52718678\text{E01}$$

The terms $X_0'(X'X)^{-1}X_0$ are:

$$\begin{aligned} & c_{1,1} X(1)^2 + c_{3,3} X(3)^2 + c_{7,7} X(7)^2 + c_{9,9} X(9)^2 + c_{u,u} (1.0)^2 \\ & + 2c_{1,3} X(1)X(3) + 2c_{1,7} X(1)X(7) + 2c_{1,9} X(1)X(9) \\ & + 2c_{1,u} X(1)(1.0) + 2c_{3,7} X(3)X(7) + 2c_{3,9} X(3)X(9) \\ & + 2c_{3,u} X(3)(1.0) + 2c_{7,9} X(7)X(9) \\ & + 2c_{7,u} X(7)(1.0) + 2c_{9,u} X(9)(1.0) \end{aligned}$$

where:

<u>X(1)</u>	<u>X(3)</u>	<u>X(7)</u>	<u>X(9)</u>	<u>Constant term</u>
.5000E04	2.133	1.537	1.366	1.0
1.1914E04	2.133	1.537	1.366	1.0
2.3328E04	2.133	1.537	1.366	1.0

so $X_0'(X'X)^{-1}X_0$ becomes:

$$\begin{aligned} &.3775E-09 X(1)^2 + .7116 (2.133)^2 + 1.349 (1.537)^2 + 6.530 (1.366)^2 \\ &+ 5.272 (1.0)^2 + 2(.2715E05) (2.133) X(1) - 2(.454E-06) (1.537) X(1) \\ &- 2(.8543E-05) (1.366) X(1) + 2(.208E-05) (1.0) X(1) - 2(.528E-01)(2.133) \\ &(1.537) - 2(2.12)(2.133)(1.366) + 2(1.422)(2.133)(1.0) - 2(.145)(1.537) \\ &(1.366) - 2(1.758)(1.537)(1.0) - 2(4.085)(1.366)(1.0) \end{aligned}$$

collecting terms, one gets:

$$X_0'(X'X)^{-1}X_0 = .106 - .897E-05 X(1) + .3775E-09 X(1)^2$$

For the three values of $X(1)$:

<u>X(1)</u>	$t_{0.05} \hat{S}_Y \sqrt{X_0'(X'X)^{-1}X_0}$
.500E04	1,030.86
1.1914E04	585.02
2.3328E04	837.25

The confidence limits are:

$$\hat{Y} \pm t \hat{S}_Y \sqrt{X_0'(X'X)^{-1}X_0}$$

<u>X(1)</u>	<u>95 percent confidence limits of \hat{Y}</u>
5,000	$10,555 \leq \hat{Y} \leq 12,617$
11,914	$11,265 \leq \hat{Y} \leq 12,435$
23,328	$11,447 \leq \hat{Y} \leq 13,122$

A similar procedure can be followed for $X(3)$, $X(7)$, and $X(9)$.

In this equation $X(9)$ is actually a function of $X(3)$, so one has to calculate them as one variable. By so doing, the vector $X_0'(X'X)^{-1}X_0$ becomes:

$$.7116 X(3)^2 + 9.2767 X(3) - 8.81896\sqrt{X(3)} - 4.232X(3)^{1.5}$$

Otherwise, the procedure is the same as for X(1).

The c_{ij} multipliers for the regression equation

$$Y(1) = f \left(X(1), X(3), X(7), X(8), X(9), X(11) \right)$$

are:

$c_{1,1} = .38327697 \text{ E-09}$	$c_{7,11} = .26909952 \text{ E-01}$
$c_{1,3} = .33547558 \text{ E-05}$	$c_{8,8} = .42584219 \text{ E-04}$
$c_{1,7} = -.69229048 \text{ E-06}$	$c_{8,9} = -83500509 \text{ E-01}$
$c_{1,8} = .12029980 \text{ E-03}$	$c_{8,11} = -6393822 \text{ E-03}$
$c_{1,9} = -.11086717 \text{ E-04}$	$c_{9,9} = .88334656 \text{ E-01}$
$c_{1,11} = .71257200 \text{ E-05}$	$c_{9,11} = -.11326192 \text{ E-02}$
$c_{3,3} = .90011770$	$c_{11,11} = .65636520 \text{ E-02}$
$c_{3,7} = -.18807280$	$c_{1,u} = .14751449 \text{ E-05}$
$c_{3,8} = -.79249513$	$c_{3,u} = .14880753 \text{ E-01}$
$c_{3,9} = -2.7706099 \text{ E-01}$	$c_{7,u} = -.8576736 \text{ E-01}$
$c_{3,11} = .33733549 \text{ E-01}$	$c_{8,u} = -.29039246 \text{ E-02}$
$c_{7,7} = .14601793 \text{ E-01}$	$c_{9,u} = -.42399845 \text{ E-01}$
$c_{7,8} = .82280798 \text{ E-01}$	$c_{11,u} = .21988754 \text{ E-01}$
$c_{7,9} = .30517250$	$c_{u,u} = .54895897 \text{ E-01}$

Table 6a. Column height tables (values are feet, mean sea level) Free Air Convection Level,
3,000 feet mean sea level

Wind M.p.h.	Buildup Index										
	10	35	60	85	110	135	160	185	210	235	260
0	3019	7634	9619	10902	11852	12607	13232	13767	14234	14649	15021
1	11471	16086	18072	19355	20305	21059	21685	22220	22687	23101	23474
2	13694	18309	20294	21577	22527	23282	23907	24442	24909	25324	25695
3	14810	19506	21491	22774	23724	24479	25105	25639	26106	26521	26893
4	15557	20172	22158	23441	24391	25145	25771	26303	26773	27187	27560
5	15884	20499	22485	23768	24718	25472	26098	26633	27100	27514	27887
6	15971	20586	22571	23855	24804	25559	26185	26720	27187	27601	27973
7	15875	20490	22475	23759	24708	25463	26089	26624	27090	27505	27877
8	15634	20249	22235	23518	24468	25222	25848	26383	26850	27264	27637
9	15276	19891	21876	23159	24109	24864	25489	26024	26491	26906	27278
10	14818	19433	21419	22702	23652	24406	25032	25567	26034	26448	26820
11	14276	18891	20877	22160	23110	23864	24490	25025	25492	25906	26279
12	13661	18276	20262	21545	22495	23249	23875	24410	24877	25291	25664
13	12982	17597	19583	20866	21816	22570	23196	23731	24198	24612	24985
14	12246	16861	18847	20130	21080	21834	22460	22995	23462	23876	24247
15	11459	16074	18060	19343	20293	21047	21673	22208	22675	23089	23462

Table 6b.--Free Air Convection Level, 16,000 feet mean sea level

Wind M.p.h.	Buildup Index										
	10	35	60	85	110	135	160	185	210	235	260
0	3514	8129	10115	11398	12347	13102	13728	14263	14730	15144	15516
1	11967	16582	18567	19850	20800	21555	22181	22715	23182	23597	23969
2	14189	18804	20790	22073	23022	23777	24403	24938	25405	25819	26191
3	15386	20001	21937	23270	24220	24974	25600	26135	26602	27016	27388
4	16052	20667	22653	23936	24886	25640	26266	26801	27268	27682	28055
5	16380	20995	22930	24263	25213	25968	26594	27128	27595	28010	28382
6	16466	21081	23067	24350	25300	26054	26680	27215	27682	28096	28469
7	16370	20985	22971	24254	25204	25958	26584	27119	27586	28000	28373
8	16129	20745	22730	24013	24963	25718	26343	26878	27345	27700	28132
9	15771	20386	22371	23655	24604	25359	25985	26520	26987	27401	27773
10	15313	19928	21914	23197	24147	24901	25527	26062	26529	26943	27316
11	14771	19386	21372	22655	23605	24359	24985	25520	25987	26401	26774
12	14156	18771	20757	22040	22990	23744	24370	24905	25372	25786	26159
13	13477	18092	20078	21361	22311	23065	23691	24226	24693	25107	25480
14	12741	17356	19342	20625	21575	22329	22955	23490	23957	24372	24744
15	11955	16570	18555	19838	20788	21543	22169	22703	23170	23585	23957

Example of the Use of the Holland Plume Rise Equation

$$H = 1.5 \left(\frac{2rW}{u} \right) + C_1(L/u)$$

as modified by Lowry¹⁵ this becomes:

$$H = 2r \frac{W}{u} (1.5 + C_2 pBr)$$

where $C_2 = 5 \times 10^{-3}$

For Miller Creek unit N-4:

$$H = 10,500 \text{ feet} = 3,200 \text{ meters}$$

$$u = \text{wind} = 2.64 \text{ meters/second}$$

$$W = 214 \text{ feet per minute} = 1.086 \text{ meters/second}$$

$$r = 201 \text{ meters}$$

Solving for B (the dimensionless buoyancy)

$$3,200 = (2)(201) \left(\frac{1.086}{2.64} \right) (1.5 + (5 \times 10^{-3})(B)(201))$$

$$B = 5.797$$

Since a value of B greater than 1.0 requires an infinitely high smoke temperature, this is an invalid result, indicating an error in the empirical equation.

¹⁵Lowry, op. cit.