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RANGE OF ELECTRONS IN ALUMINUM (0.75-5.5 Mev)

By

Cheong-Mo (Edward) Chang

B.A. University of Montana, 1966

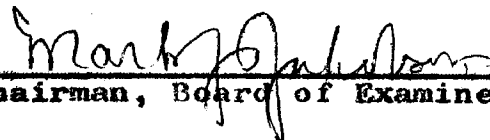
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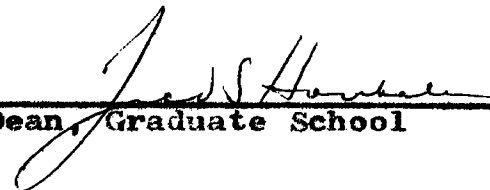
Master of Arts

UNIVERSITY OF MONTANA

1967

Approved by:

  
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## ACKNOWLEDGMENTS

The writer wishes to express his sincere gratitude to Dr. Mark J. Jakobson for suggesting the problem and for supervising the work.

He wishes also to thank Mr. Randolph H. Jeppesen for his assistance in processing the pictures.

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## INTRODUCTION

When heavy charged particles emitted by a monoenergetic source impinge on some absorption material, they will stop at various distances from the source. However, most of them will stop in a particular interval  $\Delta R$ . The distance from the source to the center of  $\Delta R$  is called the most probable range  $R$ . It is dependent upon the initial energy of the charged particles. Variation from the most probable range is called "straggle."

As soon as the charged particle enters the absorption material, it loses its energy by collisions with the charged particles in the material. These collisions reduce the energy of the incident particle. When the charged particle completely loses its energy, it stops.

The ranges of particles heavier than electrons are quite different from the ranges of electrons of the same energy because the energy losses per collision and also the angles of scattering are different. Straggling, like scattering, is much more pronounced in the case of electrons than for heavy particles. This is because heavy particles lose most of their energy in ionizing collisions with the atomic electrons, where conservation of momentum and energy permits fractional energy transfers of the order of the ratio of the masses ( $m_0/M$ ). Therefore, each collision results in the transfer of only a fraction of the energy of a heavy particle. On the other hand, an electron can lose all of its energy in an ionizing collision with atomic electrons. The incident electron would come to rest after such

a collision and the secondary electron would carry away the energy of the incident electron.

For example, a heavy particle with a kinetic energy of 3 Mev has a range in standard air of about 2.8 cm and produces about 4,000 ion pairs /mm of path. A 3 Mev electron has a range in air of over 1,000 cm and produces only about 4 ion pairs /mm of path.<sup>1</sup> This is because for a given energy the velocity of electrons is much greater than the velocity of the heavy particles, therefore the impulses due to the electrons are small as compared to heavy particles. The angle of scattering in ionizing collisions for a heavy particle is quite small compared with an electron, because the former's mass is much greater. Therefore the trajectories of heavy particles are almost straight lines and those of electrons are not, especially at low energies. Hence, the range of electrons is much more difficult to calculate than the range of heavy ions. In addition to the ionization loss there is an energy loss due to electromagnetic radiation emitted in the violent accelerations that occur during collisions.

Various methods of determining the range of electrons in different metallic foils have been used by different workers. Horszowski (1963) measured the range of electrons in a stack of aluminum foils with various energies from 0.5 to 2.5 Mev.<sup>2</sup> Leiss, Penner, and Robinson made calculations on range straggling of high-energy electrons in carbon.<sup>3</sup>

This thesis is concerned with the penetration of electrons in aluminum.

In this work stacks of aluminum foils have been used to measure the penetration of electrons of .75-0.5 Mev energies. The electrons were provided by a linear accelerator. Different energy electron beams are selected by varying an analyzing magnetic field. This analyzer magnet has been accurately calibrated by the floating wire technique.<sup>1</sup>

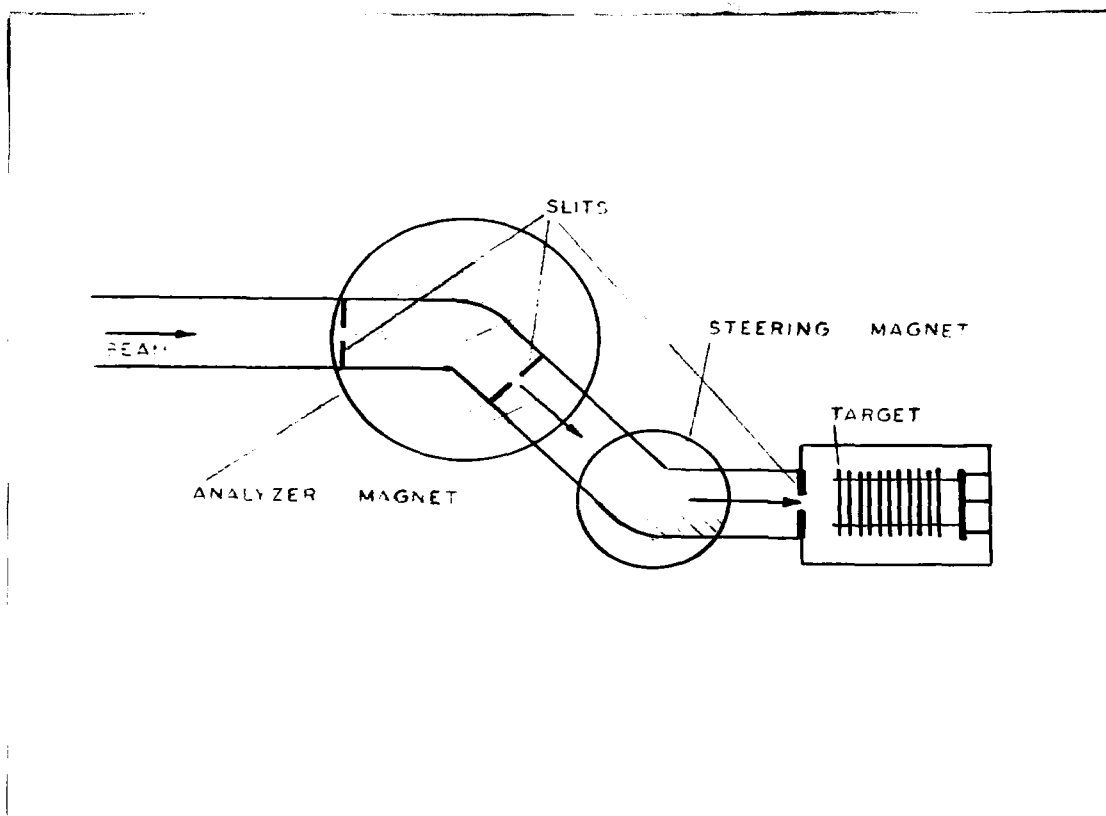


FIGURE 1

A SIMPLE DIAGRAM OF THE EXPERIMENT

The intensity of electrons stopped in the various aluminum foils were measured by a sensitive I & N galvanometer. The recorded results are reproducible to less than 5% difference.



## EXPERIMENTAL PROCEDURE

Three different sets of 20 aluminum foils of equal thickness have been used as a target (see Figure 2).

The first set was  $0.06968 \text{ g/cm}^2$  per foil (0.26 mm). It was used from 0.75 to 1.5 Mev. The second set was  $0.12864 \text{ g/cm}^2$  per foil (0.48 mm) and was used from 2 Mev to 3.5 Mev. The third set was  $0.21440 \text{ g/cm}^2$  per foil (0.8 mm) and was used from 3.5 Mev to 5.5 Mev. Only three foils of aluminum plate were drawn in Figure 2 to make the diagram simpler.

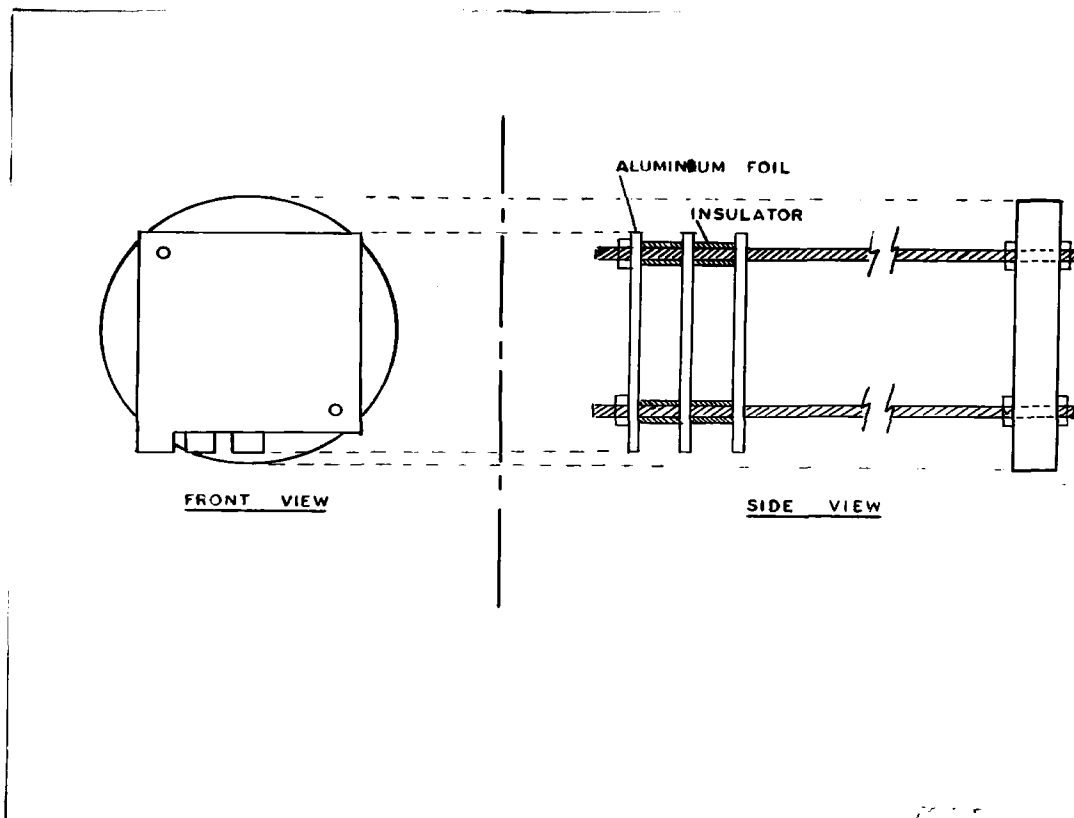


FIGURE 2

DIAGRAM OF THE TARGET

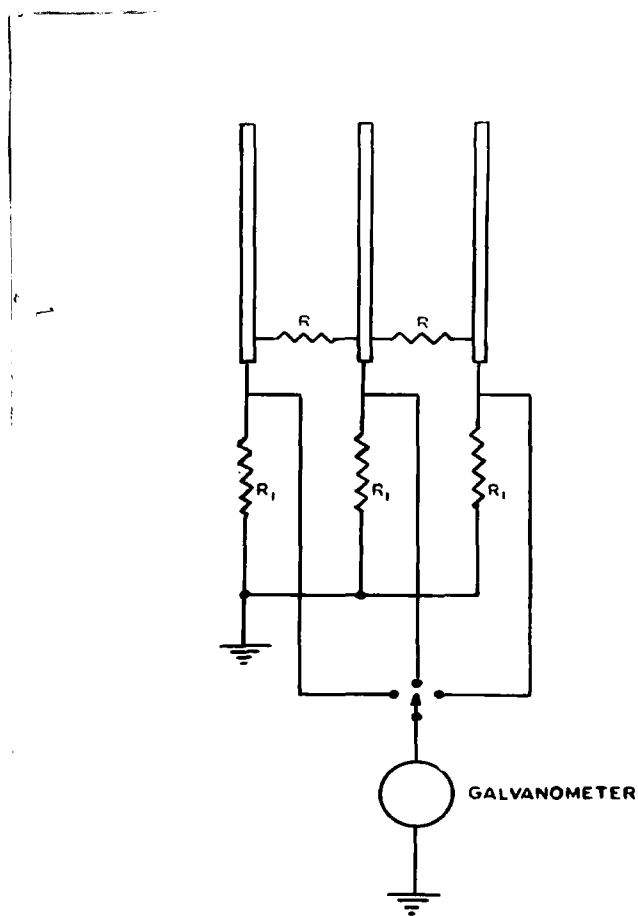


FIGURE 3

THE EQUIVALENT CIRCUIT OF THE TARGET

R represents the resistance of the insulation material. The  $R_1$ 's are  $100\text{ K}\Omega$  resistors that provide an alternate current path while the galvanometer is switched from plate to plate. Each foil was connected to the rotate switch. The center arm of rotate switch was connected to a sensitive L & N linear galvanometer. The other side of galvanometer was connected to ground. The rotate switch was controlled by relay so that it could be switched to only one

foil at a time. The deflection of the galvanometer indicated the intensity of the beam stopped at the foil.

The D-C Galvanometer (LEEDS NORTHRUP 2430) combines a high sensitivity galvanometer and a self-contained lamp and scale reading device. Its sensitivity is 0.001  $\mu$  amp. and the internal resistance is 5,000 to 8,000 ohms.

$$R \gg 10^3 R_1$$

$$R_1 = 20 R_{gal}$$

The results of the measurements are treated as if all the electrons that stop in each foil on the average travel half the thickness of the foil.

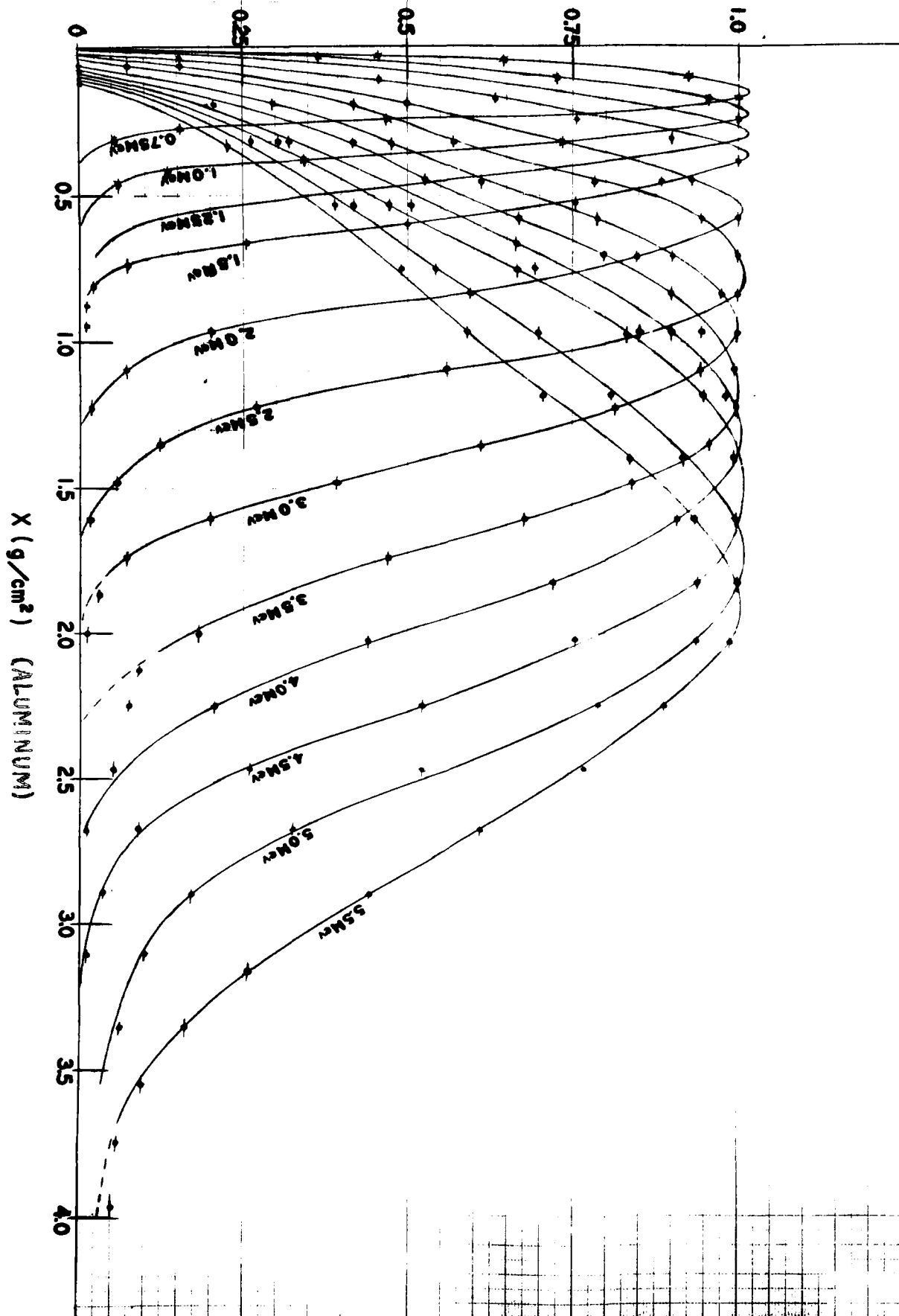
Let  $I_i$  be the average intensity of electron stopped at the center of  $i$ th foil. The differential curve, Figure 4, is plotted as relative intensity  $I_i/I_{max}$  vs. thickness  $g/cm^2$ . Figure 5 shows the integral (survival) curve. The ordinate is the normalized intensity and the abscissa is the thickness in  $g/cm^2$  of aluminum.

Figure 4 is normalized so that the maximum number stopping is the same.

Figure 5 is normalized so that the initial intensity is the same.

For each energy level, at least three sets of data have been taken with constant intensity of the incident beam. As many as nine sets of data were taken for one energy and six for some energies. Among these data three sets of 3 Mev and three sets of 3.5 Mev data

RELATIVE NUMBER OF ELECTRONS  
STOPPING AS A FUNCTION OF DEPTH



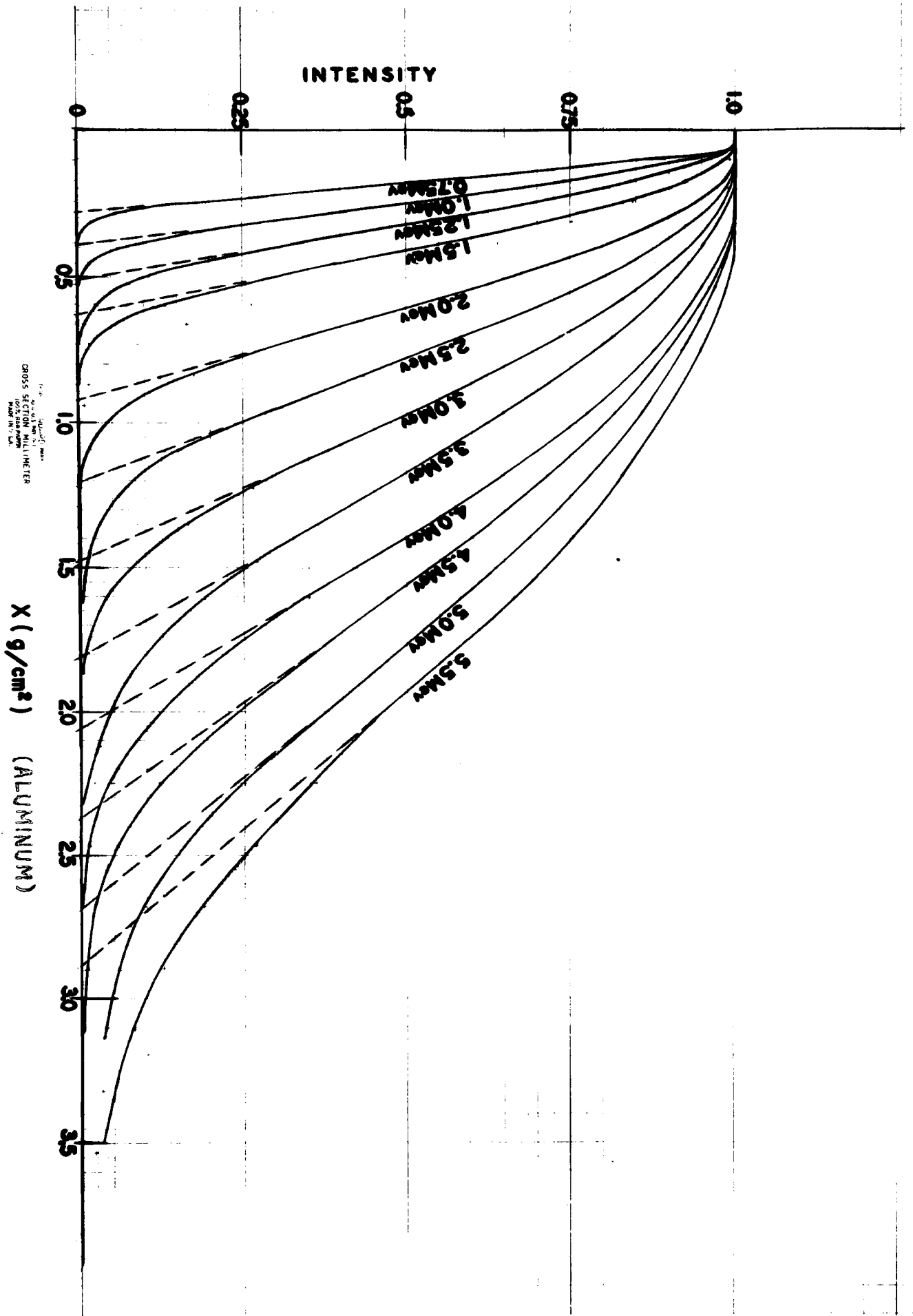


FIGURE 5 INTEGRAL (SURVIVAL) CURVE

have been eliminated because of an error in setting up the analyzer magnet. The differential and integral curves are plotted with average data for each energy level.

### CALCULATIONS

Ionization loss. To calculate the rate of energy loss by a particle of charge  $ze$  as it progresses through a medium containing  $N$  electrons  $\text{cm}^{-3}$ , the interaction volume of the charged particle is considered of the shape of a cylinder with inner and outer radii of  $b_{\text{min}}$  and  $b_{\text{max}}$ . The particle is located at the center of the cylinder. The force between the charged particle and the electron is  $e\mathcal{E}$ . Only the component of the field perpendicular to the path  $\mathcal{E}_{\perp}$  will provide a net impulse. In the collision the electron acquires an impulse such that

$$\Delta p = \int_{-\infty}^{\infty} F dt$$

$$\text{Since } F = e\mathcal{E}_{\perp}$$

$$dt = dx/v$$

$$\Delta p = \int_{-\infty}^{\infty} e\mathcal{E}_{\perp} \frac{dx}{v}$$

From Gauss's Theorem

$$\int \mathcal{E}_{\perp} dA = \int \mathcal{E}_{\perp} 2\pi b dx = 4\pi ze$$

$$\int \mathcal{E}_{\perp} dx = \frac{2ze}{b}$$

$$\Delta p = \frac{e2ze}{bv} = \frac{2ze^2}{bv}$$

the energy transfer to the electron is

$$E = \frac{(\Delta p)^2}{2m} = \frac{2z^2 e^4}{b^2 v^2 m}$$

Since the total number of electrons in the cylindrical shell of radius  $b$  is

$$N dv = N 2b \pi db dx$$

the total energy loss will be

$$\begin{aligned} -E_T &= EN2\pi b db dx \\ &= \iint 2\pi N b \frac{2z^2 e^4}{b^2 v^2 m} db dx \\ -\frac{dE_T}{dx} &= 4\pi N \frac{z^2 e^4}{v^2 m} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} \end{aligned}$$

The above equation <sup>5</sup> gives the total energy loss for heavy particles. Unfortunately the  $b_{\min}$  and  $b_{\max}$  are hard to determine. All formulas for calculating the energy loss are semi-empirical. The average energy loss can be approximated from the total energy loss equation. For high relativistic energies the total energy loss formula is

$$-\frac{dE}{dx} = \frac{4\pi e^2}{mc^2} N \left( \log \frac{2mc^2}{I} - 3/2 \log (1 - \beta^2)^{\frac{1}{2}} - \frac{1}{2} \log 8 + \frac{1}{16} \right)$$

For a given initial energy different particles may completely lose their energy in the different path lengths they have travelled. This is called straggling. A semi-empirical formula for the most probable energy loss in condensed materials for extreme relativistic

particles has been found by Goldwasser, Mills and Hanson.<sup>6</sup>

$$E_p = 2\sqrt{N}e^4 \frac{Zx}{mc^2} \left( \log \frac{x}{a_0} - 0.37 \right)$$

$$\text{or } E_p = 0.1537 \left( \sum Z / \sum A \right) D \sqrt{19.43 + \ln (D/\rho)} \text{ Mev}$$

where  $N$  = number of electrons per unit volume

$a_0$  = Bohr radius of the hydrogen atom

$x$  = path segment

$D$  = surface density of the sample

$\rho$  = volume density

Radiation loss. The calculation of the radiation loss is quite complicated. An approximate formula is

$$- \left( \frac{dE}{dx} \right)_{\text{rad}} = 4 z^2 \frac{N}{137} r_0^2 h \sqrt{E_{\text{max}}} \log \frac{183}{z^{1/3}} \quad (5)$$

However, there is another approximation for the ratio of ionization and radiation loss.<sup>1</sup>

$$\frac{(dE/dx)_{\text{rad}}}{(dE/dx)_{\text{ion}}} = \frac{EZ}{800}$$

In our case  $E_{\text{max}}$  is 5.5 Mev and  $Z$  is 13 for aluminum. The radiation loss is approximately

$$\frac{(dE/dx)_{\text{rad}}}{(dE/dx)_{\text{ion}}} = \frac{5.5 \times 13}{800} = \frac{71.5}{800} \approx 10\%$$

However, scattering is neglected in all the previous equations. To calculate the range, scattering as well as straggling has to be taken into account.



Multiple scattering. The net angle of multiple deflection is given by Fermi<sup>7</sup>

$$\overline{\theta^2} = \frac{8 \pi N z^2 Z^2 e^4}{v^2 p^2} \ln \frac{b_{\max}}{b_{\min}}$$

We can use the Monte Carlo calculation<sup>8</sup> and deal with the individual angle of deflection. The individual angle of deflection is given by

$$\cos \theta_{i+1} = \cos \theta_i \cos \theta_{i+1} - \sin \theta_i \sin \theta_{i+1} \cos \phi_{i+1}$$

where  $\theta_0 = 0^\circ$  for normal incidence.

#### COMPUTATION

A. Straggling. Although computation could not be made because the IBM 1620 computer is too small, the complete method is outlined. The Monte Carlo method makes use of statistical sampling. For example, suppose we are interested in the roll of a pair of dice. There are 36 possible outcomes. The probability of rolling a total of 12 is 1/36, 11 is 1/18, 6 is 1/6 and so on. When using Monte Carlo method, 36 numbers are set up in a box, one 12, two 11's, six 6's and so on. Then shake the box and pick a number. Record the number picked. Put it back into the box and shake again. After doing this many, many times, divide the number of times the outcome 12 is obtained by the total number of recordings. The quotient gives the probability of rolling a total of 12.

The Monte Carlo method is employed in the computations based

on Landau's distribution function for energy loss<sup>9</sup> as shown in Figure 6.

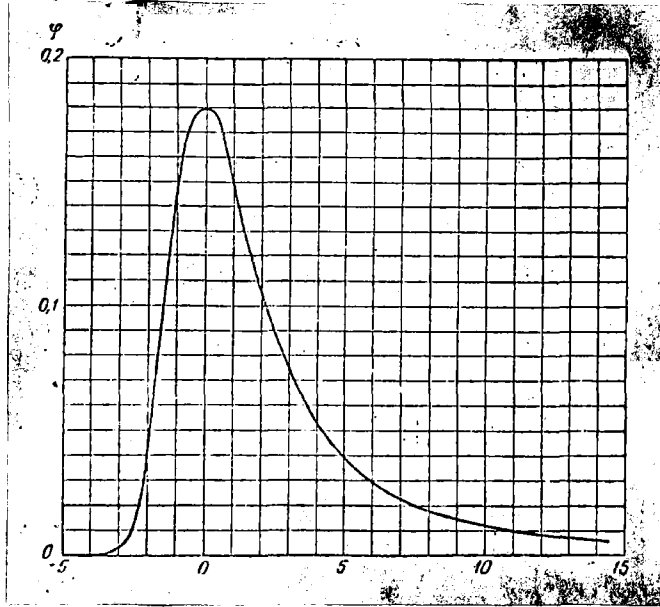


FIGURE 6

LANDAU'S DISTRIBUTION FUNCTION

For calculation purposes the range of  $\lambda$  is assumed to be from -3 to 12 and the values outside this range are neglected.

$\lambda$	$\phi$
-3	0.004
-2	0.04
-1	0.145
0	0.18
1	0.15
2	0.105

$\lambda$	$\phi$
3	0.075
4	0.052
5	0.040
6	0.03
7	0.022
8	0.018
9	0.015
10	0.012
11	0.01
12	0.008
$\phi$ total 0.906	

Suppose the random number system has 1,000 random numbers, then there will be

$$\frac{0.004}{0.906} \times 1,000 = 4 \text{ numbers for } \lambda = -3$$

$$\frac{0.04}{0.906} \times 1,000 = 44 \quad " \quad \text{for } \lambda = -2$$

$$\frac{0.145}{0.906} \times 1,000 = 160 \quad " \quad \text{for } \lambda = -1$$

$$\frac{0.18}{0.906} \times 1,000 = 199 \quad " \quad \text{for } \lambda = 0$$

and so on.

We assume that  $x$  can be from 0 to  $(R/20)$  cm which is generated from the random number in the computer. Thus  $\Delta E_{p_i}$  can be calculated

and the calculation is carried on until  $E_0 - \sum_i E_{p_i} \sim 0$ .  $E_0$  is the energy of the incident particles.

The range of electrons then can be found by

$$R = \sum_{i=0}^i X_i \cos \theta_{i+1} .$$

For a given incident energy  $R$  should be calculated many ( $\sim 1,000$ ) times.

It is assumed the electron will travel segments of  $0.015 \text{ g/cm}^2$  thickness for 0.75 and 1 Mev,  $0.03 \text{ g/cm}^2$  for 1.25 to 2 Mev,  $0.05 \text{ g/cm}^2$  for 2.5 to 3.5 Mev and  $0.1 \text{ g/cm}^2$  for 4 to 5.5 Mev, and lose a small portion of its energy. These assumptions are based on sufficient accuracy.

The most probable value of the energy loss is given by the expression

$$\Delta_0 = \xi \left( \ln \frac{\xi}{\xi'} + 0.37 \right)$$

$$\xi = x_1 \frac{2 \pi N e^4 \rho \sum Z}{m v^2 \sum A} = 0.01 \frac{2 \pi N e^4 \rho \sum Z}{m v^2 \sum A}$$

where  $N$  is Avogadro's number,  $\rho$  is density of the substance,  $\sum Z$  and  $\sum A$  are sum of the atomic numbers in the molecule and sum of atomic weights of the substance.

$$\ln \xi' = \ln \frac{\left(1 - \frac{v^2}{c^2}\right) I^2}{2m v^2} + \frac{v^2}{c^2}$$

$I$  is the ionization potential of the atom, the usually accepted value is  $I = I_0 Z$  where  $I_0 = 13.5 \text{ eV}$ .<sup>10</sup>

The energy loss for electrons that have travelled a thickness of  $0.01 \text{ g/cm}^2$  is given by

$$\lambda = \frac{\Delta_i - \Delta_0}{\xi}$$

where  $\lambda$  will be picked up each time from the random numbers according to the distribution mentioned previously.

The calculated energy loss  $\Delta_i$  is subtracted from the initial energy  $E_0$  each time until  $E_0 - \sum_i^N \Delta_i < 50 \text{ Kev}$ . Then the electron is assumed to stop.

The distance the electron travels is

$$X = \sum_i^N x_i$$

One calculation represents the distance travelled by one electron. In order to obtain reasonable accuracy for a certain initial energy it will be necessary to compute  $\sim 1,000$  trajectories.

B. Straggling and scattering. When scattering is involved, we cannot assume the electron will travel  $0.01 \text{ g/cm}^2$  in the forward direction. Therefore, the angle of scattering has to be computed.

From conservation of momentum

$$p_0 = p_1 \cos \theta_i + p' \cos \theta'_i \dots \dots \dots (1)$$

$$p_1 \sin \theta_i = p' \sin \theta'_i \dots \dots \dots (2)$$

where  $p_0$  = initial momentum,  $p_1$  = scattered momentum,  $p'$  = recoil momentum.

From equation (2)

$$\cos \theta'_i = \pm \frac{\sqrt{p_1^2 \sin^2 \theta_i + p'^2}}{p'^2}$$

Substitute into equation (1)

$$2p_1^2 \cos^2 \theta_i + 2p_1 p_o \cos \theta_i + p_o^2 - p_1^2 - p'^2 = 0$$

$$\cos \theta_i = \frac{-2p_1 p_o \pm \sqrt{4p_1^2 (2p_1^2 + 2p'^2 - p_o^2)}}{4p_1^2}$$

The differential cross section is given by N. F. Mott and H. S. W. Massey<sup>11</sup>

$$I(\theta) = \frac{Z^2 e^2}{2m^2 v \sin^4 \frac{\theta}{2}} \left(1 - \frac{v^2}{c^2} \sin^2 \frac{1}{2} \theta\right) \left(1 - \frac{v^2}{c^2}\right)$$

which is valid for all values of  $v$ , provided that  $Z/137$  is small compared with unity.

It shows that the cross section is largest as approaches zero. Therefore, the square root term for the angle of scattering is taken to be positive as the electron is expected to scatter in the forward direction.

$$\cos \theta_i = \frac{-2p_1 p_o + \sqrt{4p_1^2 (2p_1^2 + 2p'^2 - p_o^2)}}{4p_1^2}$$

Since  $E = \frac{p^2}{2m}$   $p^2 = 2mE$

$$p_o^2 = 2m (mc^2) = 2mE_o$$

$$p_1^2 = 2m'E'$$

Since  $mc^2 + m_0c^2 = \frac{1}{2}m_0v^2 + m'c^2 = \Delta + E'$

$$E' = E_0 + m_0c^2 - \Delta$$

$$p_1^2 = 2m' (E_0 + m_0c^2 - \Delta)$$

where  $m'$  can be computed from the energy  $E'$

$$p_1^2 = 2m_0\Delta$$

The angle of multiple scattering is given by<sup>7</sup>

$$\cos\Theta_{i+1} = \cos\Theta_i \cos\theta_{i+1} - \sin\Theta_i \sin\theta_{i+1} \cos\psi_{i+1}$$

where  $\cos\Theta_0 = 1$  and  $\psi_{i+1}$  will be generated by random numbers.

Therefore the depth of the electron is

$$R = \sum_{i=0}^N x_i \cos\Theta_{i+1}$$

where the limit of summation is from 0 to N.

## RESULTS

The experimental extrapolated ranges can be checked by the empirical formula.<sup>1</sup>

$$R = 412 E_0^{1.265 - 0.0954(\ln E_0)} \quad \text{for } E_0 < 2.5 \text{ Mev}$$

$$R = 530 E_0 - 106 \quad \text{for } E_0 > 2.5 \text{ Mev}$$

$E_0$ Mev	$R_{cal.} (g/cm^2)$	$R_{exp.} (g/cm^2)$	Difference $(R-R_{cal})/R$
0.75	0.284	0.28	-1.43%
1.0	0.412	0.395	-4.30%
1.25	0.544	0.51	-6.66%
1.5	0.676	0.625	-8.16%
2.0	0.943	0.92	-2.50%
2.5	1.211	1.2	-0.92%
3.0	1.484	1.48	-0.27%
3.5	1.749	1.81	3.37%
4.0	2.014	2.055	1.99%
4.5	2.279	2.37	3.84%
5.0	2.544	2.68	5.07%
5.5	2.809	2.88	2.46%

From the table we can see that most of the results agree quite well with the empirical formula. If the empirical formula for  $E_0 > 2.5$  Mev is changed to

$$R = 530 E_0 - 50$$

then the results will be improved, as shown in the following table.



$E_0$ MeV	$R_{cal.} (g/cm^2)$	$R_{exp.} (g/cm^2)$	Difference $(R-R_{cal})/R$
3.0	1.54	1.48	-4.05%
3.5	1.805	1.81	0.276%
4.0	2.07	2.055	-0.73%
4.5	2.335	2.37	1.48%
5.0	2.6	2.68	2.98%
5.5	2.865	2.88	0.521%

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