Assessing Alternative Methods For Monitoring Populus tremuloides Following Restoration Treatments

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ASSESSING ALTERNATIVE METHODS FOR MONITORING *POPULUS TREMULOIDES*

REGENERATION FOLLOWING RESTORATION TREATMENTS

By

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B.A. Environmental Science, Westminster College, Fulton, MO, 2010

Thesis

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Abstract

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Assessing Alternative Methods for Monitoring *Populus tremuloides* Regeneration Following Restoration Treatments

Dr. Elizabeth Dodson: Dr. Andrew Larson, Mr. Kevin McManigal

A variety of alternative sampling methods, commonly known as “Distance Methods”, were tested to determine if they could be a better choice for monitoring *Populus tremuloides* (quaking aspen) regeneration following aspen restoration treatments. These methods were evaluated based on their ability to accurately and efficiently estimate three common aspen stand characteristics used to gauge aspen restoration treatment effectiveness: aspen regeneration density, browse pressure, and height class distribution. Distance Method accuracy and efficiency were compared to a standard fixed-radius plot sampling method in four treated aspen stands in western Wyoming. None of the Distance Methods fulfilled all of the requirements, which were to accurately estimate all of the above stand characteristics more efficiently than fixed-radius plots. However, two Distance Methods were found to accurately estimate aspen regeneration density and browse pressure more efficiently than fixed-radius plots in all four sampled stands: one variation of Corrected Point Distance (Batcheler, 1973) and one variation of Angle Order (Morisita, 1957). The results suggest that these two Distance Methods would require less sampling time than the standard fixed-radius plots sampling method to accurately estimate many of the same stand characteristics and therefore they may be a better choice for monitoring aspen regeneration. To strengthen these conclusions, these two Distance Methods should be tested further in additional field trials.
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Introduction

Fixed-radius plots are among the most well-known and commonly used methods for sampling in the natural resource management field. While fixed-radius plots are effective at accurately estimating many common forest parameters, such as stem density, they can become tedious and time intensive. This is especially true in stands with relatively high stem densities and spatially aggregated/patchy stem patterns. Collecting fixed-radius plot data can take a lot of time because there is often many stems to count in each plot and it is necessary to collect data from more plots to achieve desired confidence levels in estimating stem density.

Other methods exist that can be used to estimate the same parameters with potentially less time spent in the field relative to the more traditional methodology using fixed-radius plots. A variety of sampling methods, commonly referred to as Distance Methods, are designed to estimate stem density using quick and simple measurements either from a selected sample point to a specific plant or from a selected plant to another plant. These Distance Methods include but are not limited to: Closest Individual (Cottam, et al., 1953; Cottam & Curtis, 1956; Pollard, 1971), T-square (Besag & Gleaves, 1973; Byth, 1982), Ordered Distance (Morisita, 1954), compound estimators (Diggle, 1975; Engeman, et al., 1994; Leavitt & St. Clair, 2011), Corrected Point-Distance (Batcheler & Bell, 1970; Batcheler, 1971; Batcheler, 1973; Warren & Batcheler, 1979), Nearest Neighbor (Cottam & Curtis, 1956; Pollard, 1971), Point-Centered Quarter (Cottam, et al., 1953; Cottam & Curtis, 1956; Pollard, 1971), Quartered Neighbor (Zhu & Zhang, 2009), and Angle Order (Morisita, 1957).

Many of these methods are inherently flawed based on the assumption that they make regarding the spatial pattern of the populations to be sampled (Persson, 1971; Pollard, 1971).
Despite this fact, their popularity persists as potential alternatives in field situations where fixed area plots are too difficult, impractical, or inefficient to implement (Engeman, et al., 1994). Moreover, there have been some methods developed that are designed to work with plant populations regardless of their spatial pattern (Morisita, 1957; Batcheler, 1971). Even though many of these methods are known to be flawed, at present it is not known if their flaws would persist if they were used in aspen regeneration. Also, potential gains in data collection efficiency using these alternative methods may be large enough to outweigh these flaws in accuracy. This investigation will help give land managers a better idea of what the tradeoff is between relative efficiency and accuracy for each of these Distance Methods.

None of these alternative Distance Methods have been extensively tested on their ability to estimate common stand parameters in stands with highly dense and patchy regeneration patterns. Regenerating aspen stands provide a good example of both of these conditions. Estimating density of forest regeneration, and particularly aspen regeneration, represents an example of a field situation in which using a fixed area plot sampling method is somewhat difficult and costly for a couple of reasons: 1) Many aspen stands have patchy regeneration patterns making variability in fixed area plot counts, and thus parameter estimates, extremely high requiring additional sample plots and more field time. 2) Stands with successful aspen regeneration tend to be extremely high in stem density (Sheperd, 1993). This requires spending a lot of time at each circular plot counting and classifying individual stems, even if plot sizes are reduced. Data collection using a Distance Method sampling strategy is likely to dramatically increase sampling efficiency relative to a traditional fixed area plot sampling strategy. Distance Methods investigated in this paper only require a maximum of four quick
distance measurements to be taken per sample point as opposed to tallying all aspen regeneration within fixed area plot boundaries.

At the time of data collection for this paper, the Wyoming Front Aspen Restoration Project (WYFARP) had implemented restoration treatments on aspen stands within four grazing allotments in the Wyoming Range of western Wyoming. A variety of monitoring objectives have been established for the WYFARP treated aspen stands, including but not limited to: 1) obtain at least 20,000 stem/acre 2 to 3 years post-treatment and 1,000 10 foot tall stems/acre 10 to 15 years post-treatment, 2) browsing limited to 30 - 40% or less of all current-year terminal leaders of aspen (used to determine if wildlife/livestock fencing is necessary), and 3) increase age/height class diversity of aspen 10 years post-treatment (Bureau of Land Management - Pinedale Field Office, 2006). The procedure used to monitor treated stands for these objectives has been adopted from the Bridger-Teton National Forest Fire Effects and Fuel Treatments Monitoring Handbook. This sampling method, hereafter referred to as Aspen Circular Plots (ACP), involves counting aspen stems within 1/500 acre fixed-radius plots and categorizing aspen stems based on height class and whether or not the terminal leader of the aspen stem has been browsed (Abendroth, et al., 2006).

Nine types of Distance Methods and 22 associated estimators, including those that are flawed, are tested in this paper to determine if one or more of them would work well enough to be considered for use as an alternative sampling strategy for monitoring aspen restoration treatments. When exploring alternative sampling methods, there are two important characteristics to consider. The first and most obvious characteristic is that the sampling method must accurately estimate the population parameters of interest. The second characteristic is the relative efficiency of the sampling method compared to other possible methods. This characteristic is just
as important as accuracy, for if there is a choice between two methods which accurately estimates the target parameters, the more efficient of the two should be chosen. Using a more efficient sampling method means that sampling time is reduced or more populations may be sampled in a single day. If any one of these Distance Methods is proven to accurately estimate these population parameters and is shown to be more efficient relative to the ACP method, they may be considered for use instead of ACP. The treated aspen stands in the WYFARP area provide a unique opportunity to test these methods because of the diversity of the stands in regards to time since treatment as well as treatment methods implemented.

Research Objectives:

The goal of this paper is to determine if any of the Distance Methods evaluated here can be used to estimate aspen population characteristics associated with a few common aspen restoration objectives. The specific parameters of interest in this paper are density, browse pressure, and height class distribution. Since these alternative methods are tested on only four aspen stands in western Wyoming, the strength of the conclusions are limited however, this investigation can at least be viewed as an exploratory case study to determine if a particular method, if any, should be further considered for potential use in aspen restoration monitoring.

In order for any of the Distance Methods to be considered further than the context of this paper, they must meet two expectations: 1) they must accurately estimate aspen sucker density, browse pressure, and relative density of height classes, and 2) they must be more efficient relative to the Aspen Circular Plots approach at estimating these parameters. In the context of this paper, relative efficiency is evaluated in terms of a combination of sample time and stem density estimator precision of each of the Distance Methods relative to ACP. Accuracy is
evaluated by comparing estimates of aspen sucker density, browse pressure, and height class distribution from each of the Distance Methods to estimates of these same parameters from the Aspen Circular Plots methodology.
Literature Review

Distance Methods have been around for over 65 years. The first mention in the scientific literature was by Cottam (1947) in which he developed a sampling strategy, ‘Random Pairs’, to estimate tree density using distance measurements between random trees and their nearest neighbor trees. Since then, a variety of other Distance Methods have been developed and improved. Generally, these methods estimate density by taking the reciprocal of mean area occupied by a stem (area per stem) to calculate mean number of trees per unit of area. Additionally, some of the Distance Methods can be used to determine the general spatial pattern or distribution of a population (Clark & Evans, 1954; Hopkins, 1954; Batcheler, 1973; Besag & Gleaves, 1973). Their popularity stems from the ease of field measurements and the speed at which they can be implemented in the field relative to traditional fixed area plots. Below is a brief summary of the history and development of each of the Distance Methods investigated in this paper.

Closest Individual and Nearest Neighbor

These methods are grouped because of the similarities in their measurements, estimating formulas, and history of development. Morisita (1954) and Clark & Evans (1954) first gave the theoretical basis for these methods proving that the mean distance between a random sample point or plant and the closest individual is equal to half of the square root of the mean area occupied by an individual under the assumption of a randomly distributed population (Poisson forest). This was shown to be true with the Closest Individual (CI) method empirically as well within a series of artificial randomly distributed populations (Cottam, et al., 1953). Cottam & Curtis (1956) came up with a modified correction factor for the Nearest Neighbor (NN) method.
to be used when sampling nearest neighbor distances at specified intervals along a transect line. They found that the mean nearest neighbor distance was equal to 66% (as opposed to 50%) of the square root of the mean area occupied by an individual plant in a population of randomly distributed individuals. So, when sampling along a transect they determined the appropriate correction factor to derive square root of mean area from NN measurements is 1.67. These methods were tested on their ability to estimate density in natural forest populations using the previously described theory, with favorable results (Cottam & Curtis, 1956).

Pollard (1971) later stated that the previously described density estimation approach for CI and NN will lead to biased results unless sample sizes are large. He went on in the same publication to theoretically develop a maximum likelihood estimator for density and variance for both of these methods. However, there still exists a problem with these methods. Sampled populations which exhibit spatial patterns other than uniformly random will cause these estimators to be biased (Pollard, 1971; Persson, 1971; Engeman, et al., 1994). Despite the potential problem with these estimators, Pollard’s maximum likelihood estimators as well as original CI and NN estimators of density are investigated in this paper.

**Compound Estimators**

This series of estimators was originally suggested by Diggle (1975) as a way to create more robust estimators that will perform reasonably well under a variety of spatial distributions. He stated that the Closest Individual and Nearest Neighbor methods tend to be biased in opposite directions depending on the type of spatial distribution exhibited by the sampled population. This is based on the observation that CI distances tend to be relatively high while NN distances tend to be relatively low when the sampled population exhibits spatially aggregated patterns while the
opposite tends to be true in spatially uniform patterns. In order to correct this bias, Diggle’s compound estimator is simply the geometric mean of CI and NN density estimates. Diggle (1975) found that this compound estimator was more robust than CI or NN over a wide range of spatial patterns. However, others have shown that this method is still affected by spatial pattern (Engeman, et al., 1994).

Others have used Diggle’s compound estimator approach and using other Distance Methods and created their own compound estimators. Leavitt & St. Clair (2011) used the geometric mean of Point-Centered Quarter and Quartered Neighbor methods, instead of CI and NN, to accurately estimate lichen density. Engeman, et al. (1994) added an additional estimator they refer to as ‘BD2N’ to one of Diggle’s compound estimators. BD2N uses distance measurements between the nearest neighbor and its nearest neighbor, also called the second nearest neighbor. These measurements are plugged into the same estimator that Cottam & Curtis (1956) use for nearest neighbor measurements. This new compound estimator they call ‘BDAV3’ is essentially the average of CI, NN, and BD2N density estimates. BDAV3 performed reasonably well except in extremely aggregated simulated point patterns (Engeman, et al., 1994). When tested on natural plant populations exhibiting a variety of spatial patterns, BDAV3 ranked highest in robustness among all Distance Methods tested (White, et al., 2008).

Diggle’s estimator (1975), Leavitt & St. Clair’s estimator (2011), and BDAV3 (Engeman, et al., 1994) are all tested in this paper.

**T-Square**

Besag & Gleaves (1973) first suggested this sampling method as an alternative way to test for spatial randomness as well as providing a simple density estimator using one of the two
measurements collected. Diggle (1975) described some compound estimators using the T-Square (TS) distances. He also tested the robustness of these estimators in extremely aggregated as well as extremely uniform simulated populations and found that the simple and compound TS estimators were more robust than CI or NN estimators. Byth (1982) conducted a simulation study testing the robustness of TS compound estimators suggested by Diggle (1975) as well as many of her own compound estimators using 4 artificial point populations; 1 spatially random and 3 spatially aggregated distributions. She found one particular compound estimator using TS measurements was the most robust except under extreme spatial aggregation. Another more recent simulation study also found that this compound estimator did not perform well even under random conditions and was outperformed by its T-Square predecessors (Engeman, et al., 1994). Others have investigated the performance of this group of methods in actual field situations concluding that they did not yield acceptable estimates of density (Kiani, et al., 2013; McGarvey, et al., 2005).

For this paper, I will be testing Byth’s compound T-Square estimator (Byth, 1982), Diggle’s reduced-bias T-Square compound estimator (Diggle, 1975), and Besag & Gleaves’ T-Square estimator (Besag & Gleaves, 1973).

**Ordered Distance**

This method was originally suggested by Morisita (1954). The field measurements are very similar to the CI method. The only difference is that the user can choose whether to measure from the sample point to the closest plant, second closest, third closest, etc. In making this decision, there is a tradeoff between accuracy and measurement difficulty. In order to describe this tradeoff, let $k$ represent to which closest plant from a given random point measurements are
made \((k = 1\) represents 1\textsuperscript{st} closest, \(k = 2\) represents 2\textsuperscript{nd} closest, and so on). Morisita (1954) and Pollard (1971) found that as \(k\) increases, variance of the associated density estimate substantially decreases. However, it becomes increasingly difficult to determine which plant is the \(k\)-th closest to the sample point as \(k\) increases (Pollard, 1971). It is generally accepted that when using the Ordered Distance method, users should measure no farther than the third closest individual \((k = 3)\) to a sample point (Krebs, 1999).

This method has been tested in a variety of situations. In simulation studies using artificial point populations, this method performs well in randomly distributed populations. However, as with many of the other methods, bias tends to increase as populations deviate from spatial randomness (Engeman, et al., 1994; Hijbeek, et al., 2013). The Ordered Distance method has also been tested in some natural populations with varying spatial patterns. Two variations of this method \((k = 2\) and \(3)\) estimated density fairly accurately in uniform and random spatial patterns, but were inaccurate in aggregated patterns (White, et al., 2008).

For this paper, I will test two variations of this method. I collected distance measurements to the second closest (OD2) and third closest (OD3) aspen stem.

**Point-Centered Quarter**

This method was suggested as a way to expand upon the CI method by dividing the area around a sample point into equiangular quadrants (four quadrants in this case) and measuring to the closest individual within each quadrant (Dice, 1952). The Point-Centered Quarter (PCQ) method was first developed empirically by Cottam, et al. (1953) and theoretically by Morisita (1954) proving that the average of the distances measured to the closest individual in each of four quadrants around a sample point is equal to the square root of the mean area occupied by a
plant in the population. This particular approach of the PCQ was tested in three forested areas and provided accurate estimations of density in each case (Cottam & Curtis, 1956). Years later, Pollard (1971) showed that the previous density estimator for PCQ was biased. He presented a very slightly altered density estimator as well as a formula to estimate variance and in turn, produce confidence intervals for the density estimate. As with many of the other Distance Methods, PCQ was developed assuming a spatially random distribution of population individuals. This estimator for PCQ was tested on artificial populations across a variety of spatial distributions. As expected, PCQ estimated density reasonably well within random spatial distributions but performed poorly in aggregated patterns (Engeman, et al., 1994). Others have tested PCQ performance on natural plant populations with mainly negative results (Laycock & Batcheler, 1975; Oldemeyer & Regelin, 1980). Both the original PCQ estimator as well as Pollard’s estimator are tested in this paper.

**Quartered Neighbor**

Quartered Neighbor (QN) is a relatively new method compared to the others investigated in this paper, although it is very similar to PCQ. This method is basically the Nearest Neighbor alternative to the Point-Centered Quarter method in that measurements are made to the nearest neighbor of the closest plant in each quadrant around a sample point (Zhu & Zhang, 2009). The density estimating formula used by this method is the same as the original PCQ formula used by Cottam, et al. (1953). Zhu & Zhang (2009) also compared QN to PCQ and fixed area plots in its ability to accurately and precisely estimate spacing between trees in two field situations. They found that QN was as accurate as PCQ and was more precise.
QN has not been further tested in its ability to estimate density relative to fixed area plots across a wider range of spatial distributions or other natural plant populations. Given its similarity to the other Distance Methods it is not expected to perform well outside of random spatial distributions. Leavitt & St. Clair (2011) tested this method along with PCQ in subalpine lichen populations and suggested that a compound PCQ/QN estimator using Diggle’s (1975) geometric mean approach is best to correct for any potential bias introduced by spatial pattern. For this project, I will use the original estimator, Pollard’s (1971) unbiased PCQ estimator substituting in QN measurements, as well as Leavitt & St. Clair’s suggested estimator using a Diggle compound estimate of PCQ and QN.

**Corrected Point Distance**

The Corrected Point Distance (CPD) method was developed to improve estimates of density in non-random spatial distributions, as well as providing a feasible method to measure density in sparsely populated areas (Batcheler & Bell, 1970; Batcheler, 1971; Batcheler, 1973). At this point, it was widely recognized that many of the traditional Distance Methods provided biased estimates of density in non-randomly distributed populations (Morisita, 1957; Lyon, 1968; Pielou, 1969). This method attempts to correct the inherent bias of the CI method caused by non-random spatial patterns using a variation of the CI method estimator, ratios of nearest neighbor and closest individual measurements, and ratios of second nearest neighbor and nearest neighbor measurements to be used in cases of extreme aggregation. These ratios either increase or decrease the biased CI density estimate depending on the type of non-random spatial distribution being sampled (Batcheler & Bell, 1970). This method was improved later by adding in the coefficient of variation of the CI measurements to the bias correction portion of the estimate (Warren & Batcheler, 1979).
The CPD method creators tested the original estimator on a variety of artificial and natural populations. Density estimates were within 30% of actual density in nearly all cases (Batcheler & Bell, 1970; Batcheler, 1973). The original CPD has been tested in other cases as well. Laycock & Batcheler (1975) found that CPD estimated two grassland species within 20% of true density and that it outperformed other popular Distance Methods in efficiency and accuracy. However, Engeman, et al. (1994) found that the original method yielded highly inaccurate density estimates in severely aggregated spatial patterns and actually performed worse in these situations than many of the earlier Distance Methods. Rempel, et al. (2012) tested the newer CPD estimator against traditional belt transects on a variety of simulated point populations as well as natural populations (snag and shrub density). They concluded that CPD performed well and was more efficient than belt transects since less time was required to sample using CPD. For this project, I will test the original and new CPD calculations.

**Angle-Order**

The Angle-Order (AO) method was created with the purpose of being utilized as a density estimator in populations regardless of their spatial pattern. Essentially, it is a combination of the Point-Centered Quarter and Ordered Distance methods in that the area around a sample point is divided into four equal quadrants, as with PCQ and QN, and the distance to the \( n \)-th nearest plant in each quadrant is measured. This method can be modified to include any number of equal-sized sectors around a point, however Morisita (1957) advises that the area around each point should be divided into four equal sectors and measurements should be made to the 3\(^{rd}\) nearest plant in each sector (hereafter referred to as AO3) for both optimum practicality and accuracy. The method presumes to overcome the common problem of other Distance Methods by assuming that an entire population which exhibits non-random spatial pattern can be broken up into units of
area small enough in which points within the smaller area are distributed randomly (Morisita, 1957). So, there still is an underlying assumption of randomness in the theoretical basis of the AO method, but this assumption applies to a much smaller scale than stand level unlike the other Distance Methods.

The performance of the AO method has been tested in both artificial and natural point populations. Morisita (1957) tested her method on spatially random, uniform, and aggregate artificial populations and found that it performed well in each of the three cases. Laycock (1965) tested the AO method against fixed-area plots and other Distance Methods in sagebrush-grass rangelands and found that it performed well, but suggested for practicality reasons that it should only be used in situations where no more than two species are being quantified. Out of all Distance Methods tested, Engeman, et al. (1994) concluded that AO3 was the best performing across all spatial patterns followed by AO2 (distance measurements made to the 2nd closest individual in each quadrant). However, they suspected spatial pattern does still appear to have an effect on estimator error with this method. As the spatial pattern became more aggregated, error increased although not nearly as dramatically as the other Distance Methods. White, et al. (2008) did a similar comparison among the better-performing Distance Methods from Engeman, et al. (1994) using a variety of natural populations. Although AO3 was bested by some of the other Distance Methods in random and uniform spatial patterns, it performed best of all in aggregated spatial patterns and its error stayed relatively stable across all spatial patterns tested.

For this project, I will test the AO2 and AO3 variations of this estimator.

Using Distance Methods to Estimate Browse Pressure and Height Class Distribution
Cottam, et al. (1953) first suggested a simple way to estimate relative density by species using two Distance Methods (PCQ and Random Pairs). Basically, they recorded the species of each plant that measurements were made to, counted the number of times each species was encountered, and divided by the total number of measurements made. For accurate results to be obtained, Cottam, et al. (1953) suggest that about 30 individuals of a species be sampled whether using a fixed area plot or Distance Method approach. Cottam & Curtis (1956) extended this approach to derive relative dominance by measuring the basal area of each individual encountered. These calculations are summarized by Mitchell (2010) as they apply to the PCQ method, though they can easily be applied to the other Distance Methods in this study (for example, Leavitt & St. Clair (2011)). Beasom & Haucke (1975) used this technique in live oak forests with PCQ, NN, and CI methods. All methods were reasonably accurate when compared to actual population parameters, especially with PCQ since this method inherently encounters more plants at each point. Rempel, et al. (2012) used a similar approach in collecting counts for each species to calculate relative density and species diversity from Corrected Point Distance data in shrubs. They found no significant differences in species diversity estimates between belt transect and CPD estimators. There were some differences in relative density estimates but the authors attribute this to the apparent sensitivity of belt transects to small scale thick patches of a species.

This approach is very similar to ACP methodology (Abendroth, et al., 2006), with the exception that under ACP each stem encountered is categorized by height class categories and categorized based on whether the terminal leader is browsed or un-browsed as opposed to classifying each stem by species. There is another more complicated approach that can be used to estimate these two parameters. When the number of species of the population is known, or if
there are a limited number of species of interest, distances from a random sample point are
measured to the closest individual of each species. These distances are then used to derive stem
density estimates for each individual species. Laycock (1965) provides an example of this
approach in grasslands using the AO3 method. While this approach is viable, the simpler
approach (classifying each plant that distance measurements were made to) was used in this
investigation to reduce sampling time due to the limited amount of time available.

**Relative Efficiency**

Jordan, et al. (2004) present a useful approach to compare the relative efficiencies of
various sampling methods. Their unique approach evaluates the efficiencies of a sampling
method based on the amount of sampling time required to attain a certain level of confidence
relative to another sampling method with the understanding that there is a trade-off between the
cost (time) of sampling and the quality (confidence interval) of results. Evaluating relative
efficiency using this method provides a more descriptive measure of sampling method efficiency
as opposed to measuring efficiency by solely comparing the amount of time required to sample a
certain area land (e.g. Rempel, et al., 2012).

**Quaking Aspen Restoration**

The Distance Methods were evaluated in quaking aspen following restoration treatments
because monitoring aspen regeneration tends to be a relatively tedious task using a fixed-radius
plot sampling approach. Quaking aspen (*Populus tremuloides*) stands have been in decline in
many areas across western North America for many reasons including fire suppression (Bartos
D. L., 2001; Bartos & Campbell, 1998), high browse pressure from ungulates (Bartos &
Campbell, 1998), and even climate change (Worrall, et al., 2010; Worrall, et al., 2013). As a
result many aspen stands have been subject to a variety of restoration treatments to help reverse the decline in targeted aspen stands. These treatments usually focus on reinvigorating aspen stands by increasing reproduction through root suckering and keeping wildlife and/or livestock browse pressure below a threshold level. Successful aspen restoration can be achieved by implementing specific treatment techniques including felling of aspen or encroaching conifer species (Shepperd, 2004; Jones, et al., 2005), mechanical separation of clonal root system (Shepperd W. D., 2001), prescribed burning (Bartos, et al., 1991), and/or temporary wildlife/livestock exclusion fencing of treated aspen stands (Kota & Bartos, 2010). Monitoring data is collected from treated aspen stands periodically in the years following treatment completion to determine if implemented treatments are effectively increasing aspen regeneration and vigor or if additional treatments are necessary.
Methods

Study Sites

This study was conducted in four treated aspen stands on four livestock allotments owned and managed by the Bureau of Land Management on the eastern front of the Wyoming Range in Sublette County, Wyoming (Figure 1). These four sampled aspen stands are hereafter referred to by the name of the allotment in which they are located: Maki Creek (42.8425679° N, -110.4336866° W); Camp Creek (42.7423976° N, -110.4403171° W); Upper Billies Canyon (42.7201823° N, -110.4332376° W); and Red Canyon (42.6814175° N, -110.4563661° W).

Average temperatures in the area range from 12.9° F in January to 59.6° F in July (Pinedale, WY TAPS Station, 1971-2000 average). Mean annual precipitation is 11.29 in. (Pinedale, WY TAPS Station, 1971-2000 average). Soils in these areas consist of Typic argicryolls (mollisols) and Calcic argicryolls (mollisols) derived from sedimentary and metamorphic slope alluvium in the Maki Creek and Red Canyon stands, as well as Typic haplocryalfs (alfisols), Ustic haplocryalfs (alfisols), and Typic argicryolls (mollisols) derived from sedimentary and metamorphic slope alluvium/colluvium in the Camp Creek and Upper Billies Canyon stands (USDA Natural Resources Conservation Service, Web Soil Survey, 2013). The forest canopy generally consists of dead and some live quaking aspen (*Populus tremuloides*) with some subalpine fir (*Abies lasiocarpa*) snags. The understory mainly consists of thick and/or patchy quaking aspen regeneration with some grass and forb species. The four aspen stands have all been treated as part of the ongoing Wyoming Front Aspen Restoration Project (WYFARP). The general objective of these aspen restoration treatments was to improve forest health and wildlife habitat by eliminating encroaching subalpine fir while stimulating the growth of and protecting a new cohort of quaking aspen (Bureau of Land Management - Pinedale Field Office, 2006). These
Figure 1: Treated stands within the WYFARP including sample stands and the relative position of the Wyoming Front Aspen Restoration Project treatment area within the state.
treatments involved felling and slashing of most subalpine fir and some quaking aspen followed by prescribed burning two years later. Prescribed burning was completed in Maki Creek in 2009, Red Canyon in 2010, Camp Creek in 2011, and Upper Billies in 2012. In addition to these treatments, some of the stands received special treatments to prevent excess browsing pressure from livestock. Maki Creek had a temporary electric fence installed for 2 years after it was determined that yearling steers were over-browsing the treatment area. A range rider was employed for 2 years starting after treatment at Red Canyon to prevent livestock from entering the treated stands. Upper Billies has not received any additional treatments (E. Maichak, personal communication, March 13, 2012).

**Sampling Design**

One treated stand from each of the four livestock allotments was randomly selected for sampling. To determine sample plot locations within the stands, a slightly modified version of the procedure most commonly used to monitor aspen regeneration and browse pressure in this area was followed (Abendroth, et al., 2006). The only modification to the original ACP protocol was sample size, which will be addressed below. A starting point at the edge of the treated stand was selected. A random transect was established by spinning the dial on a compass and stopping at a random azimuth direction. If this direction happened to not lead into the stand, another random azimuth would be selected until the transect direction would lead into the stand. If at any time a plot would be established near the edge of the stand a new random transect would be established that would lead back into the stand as well as away from previously sampled plots. Under this protocol, fixed-radius plots containing no aspen regeneration (zero plots) are commonly excluded from the sample during data collection. This convention was maintained during data collection so data was not collected at zero plots. Transects would be allowed to
cross only in the case that a previous transect line had to be intersected in order for the current transect to lead back into the stand. Random pacing intervals between Aspen Circular Plots were chosen using a random number table. If this interval was short enough that sample plots would potentially overlap (less than 5 paces), another random interval would be chosen. The same pacing interval was used between all plots within the individual stand being sampled. Aspen circular plots and Distance Method plots were established at each interval along the transect line. Additionally, another Distance Method plot was established between each interval resulting in twice the number of Distance Method plots relative to circular plots. Figure 2 below visually displays this sampling procedure.

**Figure 2:** Visual display of sampling design within an aspen stand/clone.

**Aspen Circular Plots**

Aspen Circular Plots (ACP) are fixed-radius plots that can vary in size between 1/50 acre and 1/500 acre. Fixed-radius plot sizes are arbitrarily selected using the data collectors’ best judgement and visual evidence of stand heterogeneity and sucker density. If the sampled stands appear to be extremely patchy or sucker density does not appear to be very high, then larger
fixed-radius plots are suggested (Abendroth, et al., 2006). For this study, 1/500 acre circular plots were selected because the sampled stands all appeared to have high aspen sucker density. A string cut to the length of the radius of a 1/500 acre plot (5.7 feet) was used to define the boundary of the circular plots. During data collection, either one person or a stake would hold one end of the string at plot center, while another person would hold the other end of the string and count all aspen stems within the string distance of plot center. Within each circular plot, all suckers are counted and categorized based on 5 height classes (0-1’, 1-3’, 3-6’, 6-10’, and >10’). Additionally, each aspen stem within the circular plot is classified as “browsed” or “un-browsed” based on whether or not the tallest terminal leader of the aspen stem has been eaten off, or browsed (Figure 3).

![Figure 3: Illustrations depicting examples on how to classify aspen regeneration as browsed or un-browsed. This diagram was taken from “Aspen Circles Monitoring Protocol” (Abendroth, 2011).](image)

The amount of time elapsed in seconds to collect data from each circular plot was also recorded. Under the Aspen Circular Plot monitoring procedure, sampling continues until a desired reliability/error for mean aspen sucker density is achieved (commonly 80% reliability
with 20% error) or 30 circular plots are collected, whichever occurs first. Optimal sample size was calculated using the equation below.

\[ N = (t_\alpha)^2 \cdot \frac{(s)^2}{(P \cdot x)^2} \]

Where:

\( N = \) required sample size

\( t_\alpha = \) t table value for desired reliability (80% C.I. was used)

\( s = \) standard deviation

\( P = \) percent error (0.20 was used)

\( x = \) mean # stems/plot

This calculation was done after each circular plot was tallied until the sample size collected is greater than the calculated number of plots, \( N \). Unfortunately, these conditions could not be met in all of the sampled stands due to the highly variable nature of the aspen regeneration in the stands and the limited amount of time available for data collection.

**Distance Methods**

As mentioned before, Distance Method plots were located at every ACP sample point and at the midpoint between ACP sample points. It is important to note that this sampling procedure differs from what is recommended for some of the Distance Methods used. Other authors have recommended using a semi-systematic sampling procedure by establishing a grid of potential sampling points and systematically selecting actual sample points from them (Clark & Evans,
Since this investigation will be comparing estimates of aspen regeneration density between multiple Distance Methods and Aspen Circular Plots, it was determined that it would be best to collect these data from the same locations to limit the potential error introduced if the data were collected from different locations within the stand.

All measurements for the Distance Methods were taken to the nearest \( \frac{1}{4} \) inch. Time elapsed in seconds to implement each of the Distance Methods was recorded. In addition to these measurements, browse pressure and stem height class data were collected only in the Maki Creek and Upper Billies Canyon stands. Within these stands, each stem that distance measurements were made to were classified based on the five height classes used in the Aspen Circular Plot methodology (0-1’, 1-3’, 3-6’, 6-10’ and >10’) as well as if the terminal leader was browsed or un-browsed using the same classification criteria that is used in the ACP sampling method. Distance measurement procedures for each of the Distance Methods tested in this paper are described below.

**Closest Individual (CI)**

The distance from plot center to the center of the nearest aspen stem \((r_i)\) was measured (Figure 4).

**Nearest Neighbor (NN)**

The closest aspen stem to plot center is identified. The distance from the closest aspen stem to its nearest neighbor \((r_i)\) is measured (Figure 5).
Figure 4: Closest Individual (CI) method. The dashed line represents the transect. The red point represents plot center and the black points represent individual stems. The arrowed line labeled “r_i” represents the distance to be measured.

Figure 5: Nearest Neighbor (NN) method. The arrowed line labeled “r_i” represents the distance to be measured.
**T-Square (TS)**

The closest aspen stem to plot center was identified. A line was established using a narrow pole which bisects the closest aspen stem and is perpendicular to the line going through plot center and the closest aspen stem. This will create a “T” shape, giving the method its name. The distances between plot center and closest aspen stem ($r_i$) and between the closest aspen stem and the nearest neighbor aspen stem on the side opposite side of the perpendicular line from plot center ($z_i$) were measured (Figure 6).

![T-Square (TS) method](image)

**Figure 6: T-Square (TS) method.** The arrowed lines labeled “$r_i$” and “$z_i$” represent the distances to be measured. The red dashed line represents the line perpendicular to “$r_i$” forming the top of the “T”.

**Ordered Distance (OD)**

The distance between plot center and the second (OD2) and third (OD3) closest aspen (R$_i$) was measured. Figure 7 depicts field measurements for both OD2 and OD3.
Corrected Point Distance (CPD)

Three measurements are taken using this method: the distance from plot center to the closest aspen stem ($r_p$), from the closest aspen to its nearest neighbor ($r_n$), and from the nearest neighbor to its nearest neighbor excluding the closest aspen stem that has already been measured ($r_m$) (Figure 8).

Point-Centered Quarter

The area around plot center was divided into four even quadrants with the plot center at the vertex. The quadrants were oriented so that one of the dividing lines follows the transect line and the other is perpendicular to the transect line. PVC poles were used to mark the location of these lines at each sample point. In each of the four quadrants, the distance from plot center to the nearest aspen stem is measured (Figure 9).
Figure 8: Corrected Point Distance (CPD) method. The arrowed lines labeled “r_p”, “r_n”, and “r_m” represent the distances to be measured.

Figure 9: Point Centered quarter (PCQ) method. The perpendicular solid line represents the line which divides the plot into quadrants along with the transect line. The arrowed lines labeled “r_{i1}”, “r_{i2}”, “r_{i3}”, and “r_{i4}” represent the distances to be measured.
Quartered Neighbor

The area around the stopping point was divided into the same four quadrants as used in with the Point-Centered Quarter method with the stopping point at the vertex. In each of the four quadrants, the distance from the closest plant to its nearest neighbor was measured in each of the four quadrants (Figure 10).

![Figure 10: Quartered neighbor (QN) method. The arrowed lines labeled “r_{i1}”, “r_{i2}”, “r_{i3}”, and “r_{i4}” represent the distances to be measured.](image)

Angle Order

The area around the stopping point was divided into the same four quadrants used in the previous two methods described. There are many possible measurements using this method. The original procedure suggests measuring the distance from the sample location to the third closest plant in each quadrant (AO3) because this has been shown to reduce the standard error while maintaining ease and efficiency of the method (Morisita, 1960). Additionally, the distance between plot
center and the second closest aspen stem was measured (AO2). Figure 11 illustrates data collection for both AO2 and AO3.

![Figure 11: Angle Order (AO) method. The left pane depicts AO2 measurements and the right pane depicts AO3 measurements. The arrowed lines labeled “ri1”, “ri2”, “ri3”, and “ri4” represent the distances to be measured.](image)

**Data Analysis**

Mean, variance, standard error and 95% confidence intervals of stand aspen sucker density were calculated from each of the methods being compared using the method-specific estimators. In addition to the standard 95% confidence intervals for the Distance Methods, BCa 95% confidence intervals were also estimated using a bootstrap resampling approach with 10,000 bootstrap resamples from each of the four allotment datasets. Some of the Distance Methods have had multiple estimators developed and used to estimate stem density. This study will test many of these other estimators in addition to the most widely accepted and used
estimators. As a note, all density estimators produce an estimate of aspen suckers per inch$^2$ since measurements were taken in inches. To convert these estimates to aspen suckers per acre, density estimates are simply multiplied by a correction factor, 6,272,640 inches$^2$ per acre. The estimators used for each of the Distance Methods are summarized in Table 1 below.

Since it is well known that the accuracy of many of the Distance Methods is affected by spatial pattern, two indices were used to quantify and categorize the spatial pattern exhibited by the aspen suckers in each of the stands; Index of Dispersion (I) (as presented in Krebs C. J., 1999) and Standardized Morisitas Index (SMI) (Smith-Gill, 1975). Conveniently, both of these indices utilize data that was collected in this study from circular plots. Aspen regeneration has been shown to exhibit a spatially aggregated pattern likely because multiple aspen suckers are known to sprout from the same root node. Additionally, sprouting root nodes have been shown to be aggregated (Shepperd, 1993). The aspen regeneration in these sampled stands is expected to exhibit a similar pattern. An SMI value of -1 indicates spatial uniformity and 1 indicates spatial aggregation. A chi-square test is conducted using I to determine if the pattern deviates from Poisson.

To determine if any one of the alternative methods could be used in place of the ACP method, each of the distance methods will be compared to ACP based on their ability to accurately estimate sucker density per acre, browse pressure, and height class distribution as well as on their relative efficiency compared to ACP. Estimated sucker density per acre and 95% confidence intervals for each of the alternative methods are compared to density estimates and 95% confidence intervals derived from Aspen Circular Plots. The efficiencies of the Distance Methods are compared to the Aspen Circular Plot method using a relative efficiency formula.
Table 1: Summary of density estimators analyzed in this study. Superscripted asterisks on variance estimates indicate that confidence intervals were calculated using an estimator different from what is typically used.

<table>
<thead>
<tr>
<th>Distance Method Category</th>
<th>Estimator</th>
<th>Notation</th>
<th>Density Estimate (in²)</th>
<th>Variance (in²)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closest Individual</td>
<td>Closest Individual</td>
<td>OCI</td>
<td>(\frac{N}{2 + \sum p(i:j)}^2)</td>
<td>NA</td>
<td>Cottam et al., 1953; Cottam &amp; Curtis, 1956</td>
</tr>
<tr>
<td></td>
<td>Pollard Closest Individual</td>
<td>PCI</td>
<td>(\frac{N - 1}{\pi \sum (p(i:j)^2)})</td>
<td>(\frac{PCI}{\pi N})</td>
<td>Pollard, 1971</td>
</tr>
<tr>
<td>Nearest Neighbor</td>
<td>Nearest Neighbor</td>
<td>ONN</td>
<td>(\frac{N}{1.67 + \sum n_i}^2)</td>
<td>NA</td>
<td>Cottam &amp; Curtis, 1956</td>
</tr>
<tr>
<td></td>
<td>Pollard Nearest Neighbor</td>
<td>PNN</td>
<td>(\frac{N - 1}{\pi \sum (n_i)^2})</td>
<td>(\frac{PNN}{\pi N})</td>
<td>Pollard, 1971</td>
</tr>
<tr>
<td>T-Square</td>
<td>Besag and Gleaves T-Square</td>
<td>BGTS</td>
<td>(\frac{2N}{\pi \sum (x_i^2)})</td>
<td>NA</td>
<td>Besag &amp; Gleaves, 1973</td>
</tr>
<tr>
<td>Ordered Distance</td>
<td>Ordered Distance (2nd closest individual)</td>
<td>OD2</td>
<td>(\frac{2N - 1}{\pi \sum (p(z_i)^2)})</td>
<td>(\frac{OD2^2}{2N - 2})</td>
<td>Morisita, 1954</td>
</tr>
<tr>
<td></td>
<td>Ordered Distance (3rd closest individual)</td>
<td>OD3</td>
<td>(\frac{3N - 1}{\pi \sum (p(z_i)^2)})</td>
<td>(\frac{OD3^2}{3N - 2})</td>
<td>Morisita, 1954</td>
</tr>
<tr>
<td>Point-Centered Quarter</td>
<td>Point-Centered Quarter</td>
<td>OPQ</td>
<td>(\frac{4N}{\sum \sum p(i:j)}^2)</td>
<td>NA</td>
<td>Cottam et al., 1953; Cottam &amp; Curtis, 1956</td>
</tr>
<tr>
<td></td>
<td>Pollard Point-Centered Quarter</td>
<td>PPQ</td>
<td>(\frac{4(4N - 1)}{\sum \sum p(i:j)}^2)</td>
<td>(\frac{PPQ^2}{4N - 2})</td>
<td>Pollard, 1971</td>
</tr>
<tr>
<td>Quartered Neighbor</td>
<td>Quartered Neighbor</td>
<td>OQN</td>
<td>(\frac{4N}{\sum \sum n_{ij}})</td>
<td>NA</td>
<td>Zhu &amp; Zhang, 2009</td>
</tr>
<tr>
<td></td>
<td>Pollard Quartered Neighbor</td>
<td>PQN</td>
<td>(\frac{4(4N - 1)}{\sum \sum (n_{ij})^2}^2)</td>
<td>(\frac{PQN^2}{4N - 2})</td>
<td>Pollard, 1971</td>
</tr>
<tr>
<td>Compound Methods</td>
<td>Diggle Compound</td>
<td>DCINN</td>
<td>(\sqrt{\frac{\frac{N}{\pi \sum p(i:j)^2}}{\sum \sum (p(i:j)^2)}} + \left(\frac{N}{\pi \sum (n_i)^2}\right) \frac{DCINN^2}{N})</td>
<td></td>
<td>Diggle, 1975</td>
</tr>
<tr>
<td></td>
<td>Engeman, et al. Compound</td>
<td>BDAV3</td>
<td>(\sqrt{\frac{OCI + ONN + \left(\frac{N}{1.67 + \sum m_i}\right)^2}{\pi \sum (n_i)^2}})</td>
<td>NA</td>
<td>Engeman et al., 1994</td>
</tr>
<tr>
<td></td>
<td>Diggle Compound Point-Centered Quarter/Quartered Neighbor</td>
<td>DPOQN</td>
<td>(\sqrt{PPQ + PQN})</td>
<td>NA</td>
<td>Diggle, 1975; Leavitt &amp; St. Clair, 2011</td>
</tr>
<tr>
<td></td>
<td>Diggle T-Square Compound</td>
<td>DCITS</td>
<td>(\sqrt{\left(\frac{N}{\pi \sum (p(i:j)^2)}\right) + \frac{2N}{\pi \sum (x_i^2)}})</td>
<td>NA</td>
<td>Diggle, 1975</td>
</tr>
<tr>
<td></td>
<td>Byth T-Square Compound</td>
<td>BTS</td>
<td>(\frac{N^2}{2 \sum p(i:j) \sum \sqrt{\sum z_i}})</td>
<td></td>
<td>Byth, 1982</td>
</tr>
<tr>
<td>Corrected Point Distance</td>
<td>Corrected Point Distance – R not limited (m, excluded)</td>
<td>CPDL1</td>
<td>(\frac{d_{PL}}{3.473})</td>
<td>(3.717(A_1))</td>
<td>Batchelor, 1973; Batchelor, 1975</td>
</tr>
<tr>
<td></td>
<td>Corrected Point Distance – R not limited (m, included)</td>
<td>CPDL2</td>
<td>(\frac{d_{PL}}{6.946})</td>
<td>(3.717(A_1) + 3.717(A_2))</td>
<td>Batchelor, 1973; Batchelor, 1975</td>
</tr>
<tr>
<td></td>
<td>Corrected Point Distance – R limited (m, excluded)</td>
<td>CPDL1</td>
<td>(\frac{d_{PL}}{3.473f})</td>
<td>(1 + 2.473f)(A_1))</td>
<td>Batchelor, 1973; Batchelor, 1975</td>
</tr>
<tr>
<td></td>
<td>Corrected Point Distance – R limited (m, included)</td>
<td>CPDL2</td>
<td>(\frac{d_{PL}}{2(4 + 2.473f)^{1/2}})</td>
<td>(1 + 2.717f)(A_1) + (1 + 2.717f)(A_2))</td>
<td>Batchelor, 1973; Batchelor, 1975</td>
</tr>
<tr>
<td>Angle Order</td>
<td>Angle Order (2nd closest individual)</td>
<td>AO2</td>
<td>(\frac{2 - 1}{N} \sum \sum \left(\frac{1}{p(i:j)}\right))</td>
<td>NA</td>
<td>Morisita, 1957</td>
</tr>
<tr>
<td></td>
<td>Angle Order (3rd closest individual)</td>
<td>AO3</td>
<td>(\frac{3 - 1}{N} \sum \sum \left(\frac{1}{p(i:j)}\right))</td>
<td>NA</td>
<td>Morisita, 1957</td>
</tr>
</tbody>
</table>
$A_1 = \text{index of dispersion derived from } n_i \text{ distances}, A_2 = \text{index of dispersion derived from } m_i \text{ distances}, d_c = \text{density estimate calculated using distance-restricted } p_{1i} \text{ distances}, d_{c1} = \text{density estimate calculated using unrestricted } p_{1i} \text{ distances}, f = \text{proportion of non-excluded sample points to total sample points}, m = \text{distance measurements to the nearest neighbor to } n_i \text{ excluding } p_{1i}, N = \text{number of sample points}, n = \text{distance measurements to the nearest neighbor of } p_{1i}, n_1 = \text{distance measured to the nearest neighbor of } p_{1i} \text{ in four quadrants around a sample point}, p_{1j} = \text{distance measurements to the closest individual to a sample point}, p_{2j} = \text{distance measurements to the 2}\text{nd closest individual to a sample point}, p_{3j} = \text{distance measurements to the 3}\text{rd closest individual to a sample point}, p_{1jk} = \text{distance measurements to the closest individual in four quadrants around a sample point, } p_{2jk} = \text{distance measurements to the 2}\text{nd closest individual in four quadrants around a sample point, } p_{3jk} = \text{distance measurements to the 3}\text{rd closest individual in four quadrants around a sample point, } z_i = T\text{-square distances to the nearest neighbor to } p_{1i}.$

(Jordan, et al., 2004) which takes into account time required to implement each method in the field as well as the estimated variance of sucker density from each method. Variance of estimated sucker density is used in comparing method efficiencies because sucker density is a primary determining factor in whether aspen restoration treatments are successful or not. A good sampling method will at least estimate aspen sucker density efficiently. Relative efficiencies of each of the distance methods relative to Aspen Circular Plots are calculated using the equation below.

$$RE = \frac{(2 \ast \hat{t}_x) \ast s^2_x}{\hat{t}_{ACP} \ast s^2_{ACP}}$$

where $RE = \text{relative efficiency}$

$\hat{t}_x = \text{mean sampling time in seconds for one of the distance methods}$

$s^2_x = \text{calculated variance of the selected distance method associated with } \hat{t}_x$

$\hat{t}_{ACP} = \text{mean sampling time in seconds for ACP}$

$s^2_{ACP} = \text{calculated variance for ACP}$

Distance Method mean sample times are multiplied by two since there were twice as many Distance Method plots in each stand. Calculated relative efficiency values less than 1 indicate
that the Distance Method is more efficient relative to ACP while calculated relative efficiencies greater than 1 indicate that the distance method is less efficient relative to ACP.

Due to the underlying assumption of spatial randomness for many of the Distance Methods, these particular methods (CI, NN, OD3, TS, PCQ, QN, and compound estimators) are not expected to accurately estimate aspen sucker density. As for the methods that were developed to work across a wider range of spatial patterns (compound estimators, CPD, and AO), it is predicted that at least one of these will accurately estimate aspen sucker density. As for browse pressure and height class relative density, I predict that all of the Distance Methods will accurately estimate these parameters. All of the Distance Methods are expected to be more efficient relative to ACP (calculated relative efficiencies less than 1).

All data analysis and calculations were conducted using custom scripts written in R (R Development Core Team, 2011). Bootstrap analysis was conducted using the ‘bootstrap’ package for R (S original, from StatLib and by Rob Tibshirani & R port by Friedrich Leisch, 2012). Results figures were generated using the ‘ggplot2’ package for R (Wickam, 2009).
Results

ACP estimates indicate fairly high stem densities in the four sampled stands. Estimated aspen regeneration density ranged from 22,031 (±10,435) stems per acre in the Maki Creek stand to 34,750 (±10,341) stems per acre in Camp Creek stand (Table 2). Percent margin of error achieved with ACP ranged from 29.7% in Camp Creek to 47.4% at a 95% confidence level in Maki Creek. Percent browse pressure and height class distribution data were collected in the oldest treated stand (Maki Creek) and the youngest treated stand (Upper Billies Canyon). Percent browse pressure was relatively high in the Upper Billies stand at 35.85% (±8.78). Estimated browse pressure in Maki Creek was 16.90% (±13.00). Aspen stems were primarily concentrated in the 0 to 1 foot height class while very few occupied the 1 to 3 foot and 3 to 6 foot height classes in the Upper Billies stand. Aspen regeneration in Maki Creek mainly fell in the 1 to 3 foot and 3 to 6 foot height classes (Table 3). ACP results are summarized in Table 2 and 3 below.

Regarding aspen stem spatial distribution patterns, both spatial pattern indices used in this study indicate that the stands are significantly non-random. In addition, the indices also suggest that the aspen stems exhibit a spatially aggregated or patchy pattern (Table 4). The implications of aspen sucker spatial pattern in regards to the methods and estimators tested in this paper will be discussed further in the ‘Discussion’.

Density estimates and 95% confidence intervals (where applicable) and 95% BCa bootstrap confidence intervals for Distance Method estimates relative to ACP density estimates are summarized in the following tables and figures: Maki Creek (Table 5-6 and Figure 12-13),
Table 2: Summary of Aspen Circular Plot (ACP) stem density and browse pressure results for each of the stands, including estimated stem density (stems per acre) with 95% confidence intervals, percent margin of error based on 95% confidence level for the estimates, browse pressure with 95% confidence intervals, and sample size. Browse pressure values are expressed as a percentage of aspen stems that were classified as browsed.

<table>
<thead>
<tr>
<th>Stand</th>
<th>Stem Density</th>
<th>Margin of Error (%)</th>
<th>Browse Pressure (%)</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maki Creek</td>
<td>22,031 (±10,435)</td>
<td>47.4%</td>
<td>35.85 (±8.78)</td>
<td>16</td>
</tr>
<tr>
<td>Upper Billies Canyon</td>
<td>23,550 (±10,303)</td>
<td>43.8%</td>
<td>16.90 (±13.00)</td>
<td>30</td>
</tr>
<tr>
<td>Red Canyon</td>
<td>27,053 (±8,781)</td>
<td>32.5%</td>
<td>NA</td>
<td>19</td>
</tr>
<tr>
<td>Camp Creek</td>
<td>34,750 (±10,341)</td>
<td>29.8%</td>
<td>NA</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 3: ACP aspen regeneration height class distribution results for Upper Billies Canyon and Maki Creek. Values are expressed as percent of estimated aspen regeneration density.

<table>
<thead>
<tr>
<th>Stand</th>
<th>Height Class (feet)</th>
<th>Height Class Distribution (%) (± 95% C.I.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Billies Canyon</td>
<td>0-1’</td>
<td>80.24 (±6.76)</td>
</tr>
<tr>
<td></td>
<td>1-3’</td>
<td>19.19 (±6.52)</td>
</tr>
<tr>
<td></td>
<td>3-6’</td>
<td>0.56 (±1.05)</td>
</tr>
<tr>
<td>Maki Creek</td>
<td>0-1’</td>
<td>1.60 (±1.85)</td>
</tr>
<tr>
<td></td>
<td>1-3’</td>
<td>39.55 (±11.58)</td>
</tr>
<tr>
<td></td>
<td>3-6’</td>
<td>51.48 (±10.07)</td>
</tr>
<tr>
<td></td>
<td>6-10’</td>
<td>7.36 (±5.64)</td>
</tr>
</tbody>
</table>

Table 4: Spatial distribution indices of aspen regeneration for each of the sampled allotments derived from fixed-area plot counts. In general, I values greater than 1 and SMI values greater than 0 indicate spatially clumped patterns. Asterisk superscripts denote statistically significant non-random patterns indicated by the corresponding index (p < 0.025).

<table>
<thead>
<tr>
<th>Stand</th>
<th>Index of Dispersion (I)</th>
<th>Std. Morisita Index (SMI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maki Creek</td>
<td>34.81*</td>
<td>0.52*</td>
</tr>
<tr>
<td>Upper Billies Canyon</td>
<td>64.66*</td>
<td>0.52*</td>
</tr>
<tr>
<td>Red Canyon</td>
<td>24.54*</td>
<td>0.51*</td>
</tr>
<tr>
<td>Camp Creek</td>
<td>24.89*</td>
<td>0.51*</td>
</tr>
</tbody>
</table>
Upper Billies (Table 7-8 and Figure 14-15), Red Canyon (Table 9-10 and Figure 16-17), and Camp Creek (Table 11-12 and Figure 18-19).

In general, the majority of the Distance Method estimators do not provide reliable estimates of aspen sucker density. Only a few of the alternative estimators produced density estimates that were not significantly different than ACP density estimates based on 95% confidence intervals (Figure 14-19). Only one of the tested methods (AO3) estimated height class distributions within Aspen Circular Plot 95% confidence intervals (Table 15). All of the tested methods estimated browse pressure within Aspen Circular Plot 95% confidence intervals except for the PCQ methods (Table 16). In terms of efficiency, the majority of the methods were consistently more efficient than ACP. The only exceptions were BTS, BDAV3, OQN, CPDL2, CPDNL3, CPDL3, and AO3 (Table 13-14). More detailed results corresponding to each estimator are provided in the sections below.

**Closest Individual and Nearest Neighbor Estimators**

Both the original Closest Individual (OCI) estimator and Pollard’s (PCI) estimator yielded density estimates significantly different than ACP density estimates. These estimators all produced density estimates much lower than ACP estimates. All CI estimators were significantly different than ACP estimates except for the OCI estimate with bootstrap confidence intervals. Relative to the other estimators, the CI estimators were very precise despite their inaccuracy (Figures 20-21). Both of the CI estimators were much more efficient relative to ACP (Table 13). Neither of the CI methods consistently estimated height class distribution within ACP confidence intervals (Table 15). Each of the CI methods did provide consistently similar browse pressure estimates (Table 16).
Table 5: Maki Creek aspen regeneration density (stems per acre) results for spatial distribution biased estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
<thead>
<tr>
<th></th>
<th>ACP</th>
<th>OCI</th>
<th>PCI</th>
<th>ONN²</th>
<th>PNN¹</th>
<th>BGTS¹</th>
<th>OD2</th>
<th>OD3</th>
<th>OPQ</th>
<th>PPQ</th>
<th>OQN¹</th>
<th>PQN²</th>
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</thead>
<tbody>
<tr>
<td>Density</td>
<td>32,466</td>
<td>22,031</td>
<td>11,596</td>
<td>8,418</td>
<td>4,888</td>
<td>132,700</td>
<td>45,781</td>
<td>54,973</td>
<td>4,573</td>
<td>5,479</td>
<td>3,867</td>
<td>1,232</td>
</tr>
<tr>
<td>95% UCL</td>
<td>32,466</td>
<td>22,031</td>
<td>11,596</td>
<td>8,418</td>
<td>4,888</td>
<td>132,700</td>
<td>45,781</td>
<td>54,973</td>
<td>4,573</td>
<td>5,479</td>
<td>3,867</td>
<td>1,232</td>
</tr>
<tr>
<td>Bootstrap</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>95% LCL</td>
<td>32,466</td>
<td>22,031</td>
<td>11,596</td>
<td>8,418</td>
<td>4,888</td>
<td>132,700</td>
<td>45,781</td>
<td>54,973</td>
<td>4,573</td>
<td>5,479</td>
<td>3,867</td>
<td>1,232</td>
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</tbody>
</table>

Table 6: Maki Creek aspen regeneration density (stems per acre) results for adjusted bias estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
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<tr>
<th></th>
<th>ACP</th>
<th>DCINN²</th>
<th>BDAV3</th>
<th>DCTTS²</th>
<th>BTS¹</th>
<th>DPQQN</th>
<th>CPDNL2²</th>
<th>CPDNL3</th>
<th>CPDL2²</th>
<th>CPDL3</th>
<th>AO2²</th>
<th>AO3²</th>
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<tbody>
<tr>
<td>Density</td>
<td>32,466</td>
<td>22,031</td>
<td>11,596</td>
<td>8,418</td>
<td>4,888</td>
<td>132,700</td>
<td>45,781</td>
<td>54,973</td>
<td>4,573</td>
<td>5,479</td>
<td>3,867</td>
<td>1,232</td>
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<tr>
<td>95% UCL</td>
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<td>22,031</td>
<td>11,596</td>
<td>8,418</td>
<td>4,888</td>
<td>132,700</td>
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<td>4,573</td>
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<td>1,232</td>
</tr>
<tr>
<td>Bootstrap</td>
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<td>22,031</td>
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<td>3,867</td>
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<table>
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<th></th>
<th>ACP</th>
<th>DCINN²</th>
<th>BDAV3</th>
<th>DCTTS²</th>
<th>BTS¹</th>
<th>DPQQN</th>
<th>CPDNL2²</th>
<th>CPDNL3</th>
<th>CPDL2²</th>
<th>CPDL3</th>
<th>AO2²</th>
<th>AO3²</th>
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<td>22,031</td>
<td>11,596</td>
<td>8,418</td>
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<td>45,781</td>
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<td>4,573</td>
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<td>95% UCL</td>
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<td>11,596</td>
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<td>4,888</td>
<td>132,700</td>
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<td>54,973</td>
<td>4,573</td>
<td>5,479</td>
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<td>1,232</td>
</tr>
</tbody>
</table>

38
Figure 12: Maki Creek aspen regeneration density results with standard 95% confidence intervals where applicable. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Figure 13: Maki Creek aspen regeneration density results with bootstrap BCa 95% confidence interval. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDNL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Table 7: Upper Billies Canyon aspen regeneration density (stems per acre) results for spatial distribution biased estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>ACP</th>
<th>OCI</th>
<th>PCI</th>
<th>ONN(^1)</th>
<th>PNN</th>
<th>BGTS</th>
<th>OD2</th>
<th>OD3</th>
<th>OPQ</th>
<th>PPQ</th>
<th>OQN(^2)</th>
<th>PQN(^2)</th>
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<tr>
<td><strong>Density</strong></td>
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</tr>
<tr>
<td><strong>95% UCL</strong></td>
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<td>682</td>
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<td>1,302</td>
<td>249</td>
<td>3,486</td>
<td>3,390</td>
<td>675</td>
<td>812</td>
<td>1,582</td>
<td>602</td>
<td>36,756</td>
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<tr>
<td><strong>95% LCL</strong></td>
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</tr>
</tbody>
</table>

Table 8: Upper Billies Canyon aspen regeneration density (stems per acre) results for adjusted bias estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
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<th>Estimator</th>
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<th>DCINN</th>
<th>BDAV(^2)</th>
<th>DCITS</th>
<th>BTS(^2)</th>
<th>DPQNN</th>
<th>CPDNL(^1)</th>
<th>CPDNL3</th>
<th>CPDL(^1)</th>
<th>CPDL3</th>
<th>AO(^1)</th>
<th>AO(^3)</th>
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<tbody>
<tr>
<td><strong>Density</strong></td>
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<tr>
<td><strong>95% UCL</strong></td>
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<td>62,491</td>
<td>1,889</td>
<td>28,806</td>
<td>352,433</td>
<td>34,941</td>
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<tr>
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<td>947</td>
<td>63,882</td>
<td>926</td>
<td>4,198</td>
<td>1,826</td>
<td>20,002</td>
<td>244,723</td>
<td>22,191</td>
<td>227,213</td>
<td>20,527</td>
<td>32,444</td>
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</tr>
<tr>
<td><strong>Bootstrap</strong></td>
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<tr>
<td><strong>95% LCL</strong></td>
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</tbody>
</table>

Figure 14: Upper Billies Canyon aspen regeneration density results with standard 95% confidence intervals where applicable. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDNL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Figure 15: Upper Billies Canyon aspen regeneration density results with bootstrap BCa 95% confidence interval. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDNL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Table 9: Red Canyon aspen regeneration density (stems per acre) results for spatial distribution biased estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
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<th>PCI</th>
<th>QNN²</th>
<th>PNN²</th>
<th>BGTS²</th>
<th>OD2</th>
<th>OD3</th>
<th>OPQ</th>
<th>PPQ</th>
<th>OQN²</th>
<th>PQN²</th>
</tr>
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<tbody>
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<td></td>
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<td>5,409</td>
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<td>13,031</td>
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<td>9,225</td>
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<td>631</td>
<td>--</td>
<td>9,473</td>
</tr>
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<td><strong>95% LCL</strong></td>
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<td>881</td>
<td>389</td>
<td>24,931</td>
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</table>

Table 10: Red Canyon aspen regeneration density (stems per acre) results for adjusted bias and unbiased estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

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<th>DCITS²</th>
<th>BTS²</th>
<th>DPQQN</th>
<th>CPDLN2¹</th>
<th>CPDLN3</th>
<th>CPDL2²</th>
<th>CPDL3²</th>
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<th>AO3²</th>
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<td>7,184</td>
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<td>80,360</td>
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</tr>
<tr>
<td><strong>95% LCL Density</strong></td>
<td>27,053</td>
<td>4,012</td>
<td>171,573</td>
<td>5,280</td>
<td>11,727</td>
<td>2,891</td>
<td>26,310</td>
<td>277,783</td>
<td>41,862</td>
<td>286,768</td>
<td>37,132</td>
<td>48,027</td>
</tr>
<tr>
<td><strong>Bootstrap 95% LCL</strong></td>
<td>18,271</td>
<td>3,809</td>
<td>--</td>
<td>--</td>
<td>6,996</td>
<td>2,745</td>
<td>9,590</td>
<td>101,250</td>
<td>3,364</td>
<td>23,048</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td><strong>95% LCL</strong></td>
<td>--</td>
<td>2,065</td>
<td>33,357</td>
<td>5,409</td>
<td>12,879</td>
<td>999</td>
<td>9,470</td>
<td>114,181</td>
<td>17,769</td>
<td>132,991</td>
<td>21,567</td>
<td>24,641</td>
</tr>
</tbody>
</table>
Figure 16: Red Canyon aspen regeneration density results with standard 95% confidence intervals where applicable. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Figure 17: Red Canyon aspen regeneration density results with bootstrap BCa 95% confidence interval. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDNL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Table 11: Camp Creek aspen regeneration density (stems per acre) results for spatial distribution biased estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
<thead>
<tr>
<th>Density</th>
<th>ACP</th>
<th>OCI</th>
<th>PCI</th>
<th>ONN&lt;sup&gt;1&lt;/sup&gt;</th>
<th>PNN&lt;sup&gt;2&lt;/sup&gt;</th>
<th>BGTS&lt;sup&gt;2&lt;/sup&gt;</th>
<th>OD2</th>
<th>OD3</th>
<th>OPQ</th>
<th>PPQ</th>
<th>OQN&lt;sup&gt;2&lt;/sup&gt;</th>
<th>PQN&lt;sup&gt;1&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>95% UCL</td>
<td>45,091</td>
<td>12,639</td>
<td>8,885</td>
<td>85,503</td>
<td>30,448</td>
<td>46,718</td>
<td>12,695</td>
<td>14,099</td>
<td>11,606</td>
<td>7,271</td>
<td>119,223</td>
<td>48,934</td>
</tr>
<tr>
<td>95% LCL</td>
<td>34,750</td>
<td>7,194</td>
<td>4,468</td>
<td>33,913</td>
<td>6,736</td>
<td>12,791</td>
<td>6,867</td>
<td>7,630</td>
<td>6,996</td>
<td>3,633</td>
<td>70,194</td>
<td>30,242</td>
</tr>
<tr>
<td>Bootstrap 95% UCL</td>
<td>24,409</td>
<td>3,480</td>
<td>2,195</td>
<td>6,974</td>
<td>1,486</td>
<td>3,084</td>
<td>3,510</td>
<td>3,644</td>
<td>3,458</td>
<td>1,585</td>
<td>38,786</td>
<td>15,643</td>
</tr>
<tr>
<td>95% LCL</td>
<td>--</td>
<td>3,480</td>
<td>2,195</td>
<td>6,974</td>
<td>1,486</td>
<td>3,084</td>
<td>3,510</td>
<td>3,644</td>
<td>3,458</td>
<td>1,585</td>
<td>38,786</td>
<td>15,643</td>
</tr>
</tbody>
</table>

Table 12: Camp Creek aspen regeneration density (stems per acre) results for adjusted bias and unbiased estimators and their standard and bootstrap 95% lower (LCL) and upper (UCL) confidence limits. A “1” superscripted on the estimator name indicates that the estimated density falls within the fixed-area plot confidence interval. A “2” superscripted on the estimator name indicates that the estimated confidence interval overlaps the fixed-area plot confidence interval.

<table>
<thead>
<tr>
<th>Density</th>
<th>ACP</th>
<th>DCINN</th>
<th>BDAV3</th>
<th>DCITS&lt;sup&gt;2&lt;/sup&gt;</th>
<th>BTS&lt;sup&gt;2&lt;/sup&gt;</th>
<th>DPQNN</th>
<th>CPDNL&lt;sup&gt;2&lt;/sup&gt;</th>
<th>CPDNL3</th>
<th>CPDL&lt;sup&gt;2&lt;/sup&gt;</th>
<th>CPDL3</th>
<th>AO&lt;sup&gt;2&lt;/sup&gt;</th>
<th>AO&lt;sup&gt;3&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>95% UCL</td>
<td>45,091</td>
<td>14,389</td>
<td>210,127</td>
<td>35,854</td>
<td>59,930</td>
<td>17,403</td>
<td>45,947</td>
<td>496,543</td>
<td>50,941</td>
<td>395,762</td>
<td>55,052</td>
<td>179,247</td>
</tr>
<tr>
<td>95% LCL</td>
<td>34,750</td>
<td>117,374</td>
<td>7,667</td>
<td>16,276</td>
<td>10,555</td>
<td>27,261</td>
<td>303,676</td>
<td>27,661</td>
<td>222,365</td>
<td>35,538</td>
<td>74,773</td>
<td></td>
</tr>
<tr>
<td>Bootstrap 95% UCL</td>
<td>24,409</td>
<td>5,464</td>
<td>117,374</td>
<td>7,667</td>
<td>16,276</td>
<td>10,555</td>
<td>27,261</td>
<td>303,676</td>
<td>27,661</td>
<td>222,365</td>
<td>35,538</td>
<td>74,773</td>
</tr>
<tr>
<td>95% LCL</td>
<td>--</td>
<td>5,341</td>
<td>--</td>
<td>9,180</td>
<td>9,992</td>
<td>16,161</td>
<td>180,034</td>
<td>11,501</td>
<td>92,458</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Bootstrap 95% LCL</td>
<td>--</td>
<td>1,875</td>
<td>51,362</td>
<td>4,225</td>
<td>8,225</td>
<td>5,688</td>
<td>16,921</td>
<td>189,818</td>
<td>16,294</td>
<td>135,694</td>
<td>25,395</td>
<td>40,868</td>
</tr>
</tbody>
</table>
Figure 18: Camp Creek aspen regeneration density results with standard 95% confidence intervals where applicable. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Figure 19: Camp Creek aspen regeneration density results with bootstrap BCa 95% confidence interval. Asterisks denote adjusted bias or unbiased estimators. BDAV3, CPDNL3, and CPDL3 are not displayed due to density estimates falling outside the range of the graph.
Table 13: Relative efficiency values of the biased estimators relative to Aspen Circular Plots (ACP). Values less than 1 indicate that the estimator is more efficient than ACP. An asterisk superscript above the estimator name indicates the estimator is not consistently more efficient than ACP.

<table>
<thead>
<tr>
<th></th>
<th>OCI</th>
<th>PCI</th>
<th>ONN</th>
<th>PNN</th>
<th>BGTS</th>
<th>OD2</th>
<th>OD3</th>
<th>OPQ</th>
<th>PPQ</th>
<th>OQN*</th>
<th>PQN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Billies Canyon</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.03</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.27</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Camp Creek</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.09</td>
<td>0.02</td>
<td>0.09</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.01</td>
<td>&lt;0.00</td>
<td>0.61</td>
<td>0.12</td>
</tr>
<tr>
<td>Red Canyon</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.21</td>
<td>0.01</td>
<td>0.11</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>1.64</td>
<td>0.30</td>
</tr>
<tr>
<td>Maki Creek</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.36</td>
<td>0.06</td>
<td>0.26</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>&lt;0.00</td>
<td>0.81</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 14: Relative efficiency values of the adjusted-bias estimators relative to Aspen Circular Plots (ACP). Values less than 1 indicate that the estimator is more efficient than ACP. An asterisk superscript above the estimator name indicates the method is not consistently more efficient than ACP.

<table>
<thead>
<tr>
<th></th>
<th>DCINN</th>
<th>BDAV3*</th>
<th>DCTTS</th>
<th>BTS*</th>
<th>DPQQN</th>
<th>CPDNL2</th>
<th>CPDL2*</th>
<th>CPDL3*</th>
<th>AO2</th>
<th>AO3*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Billies Canyon</td>
<td>&lt;0.00</td>
<td>1.34</td>
<td>0.01</td>
<td>0.05</td>
<td>&lt;0.00</td>
<td>0.02</td>
<td>0.03</td>
<td>2.44</td>
<td>3.66</td>
<td>0.03</td>
</tr>
<tr>
<td>Camp Creek</td>
<td>0.01</td>
<td>1.13</td>
<td>0.18</td>
<td>0.18</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
<td>2.99</td>
<td>1.91</td>
<td>0.10</td>
</tr>
<tr>
<td>Red Canyon</td>
<td>&lt;0.00</td>
<td>33.83</td>
<td>0.30</td>
<td>1.04</td>
<td>0.01</td>
<td>0.41</td>
<td>1.65</td>
<td>24.04</td>
<td>39.95</td>
<td>0.49</td>
</tr>
<tr>
<td>Maki Creek</td>
<td>&lt;0.00</td>
<td>1.27</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
<td>0.26</td>
<td>0.23</td>
<td>25.78</td>
<td>11.84</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Table 15: Height class distribution results for all estimators. Values represent percent of aspen stems falling in each height class. An asterisk superscript above the estimator name indicates the estimator(s) that consistently estimated height class relative densities within fixed area plot 95% confidence intervals.

<table>
<thead>
<tr>
<th>Height Class</th>
<th>ACP</th>
<th>CI</th>
<th>NN</th>
<th>BGTS</th>
<th>TS</th>
<th>OD2</th>
<th>OD3</th>
<th>DCINN</th>
<th>BDAV3</th>
<th>PQ</th>
<th>QN</th>
<th>DPQQN</th>
<th>CPD</th>
<th>AO2</th>
<th>AO3*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upper Billies Canyon</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-1’</td>
<td>80.24 (±6.76)</td>
<td>88.33</td>
<td>91.67</td>
<td>86.67</td>
<td>87.5</td>
<td>86.67</td>
<td>88.33</td>
<td>90</td>
<td>87.22</td>
<td>87.5</td>
<td>82.08</td>
<td>84.79</td>
<td>86.67</td>
<td>80.83</td>
<td>76.67</td>
</tr>
<tr>
<td>1-3’</td>
<td>19.19 (±6.52)</td>
<td>11.67</td>
<td>8.33</td>
<td>10</td>
<td>10.83</td>
<td>13.33</td>
<td>11.67</td>
<td>10</td>
<td>12.78</td>
<td>12.5</td>
<td>17.5</td>
<td>15</td>
<td>13.33</td>
<td>18.75</td>
<td>22.92</td>
</tr>
<tr>
<td>3-6’</td>
<td>0.56 (±1.05)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.42</td>
<td>0.21</td>
<td>0</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td><strong>Maki Creek</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0-1’</td>
<td>1.60 (±1.85)</td>
<td>3.13</td>
<td>15.63</td>
<td>6.25</td>
<td>4.69</td>
<td>3.23</td>
<td>3.13</td>
<td>9.38</td>
<td>10.42</td>
<td>3.91</td>
<td>3.91</td>
<td>3.91</td>
<td>4.17</td>
<td>4.69</td>
<td>0.78</td>
</tr>
<tr>
<td>1-3’</td>
<td>39.55 (±11.58)</td>
<td>50</td>
<td>25</td>
<td>37.5</td>
<td>43.75</td>
<td>51.61</td>
<td>28.12</td>
<td>37.5</td>
<td>37.5</td>
<td>46.09</td>
<td>44.53</td>
<td>45.31</td>
<td>41.67</td>
<td>42.97</td>
<td>45.31</td>
</tr>
<tr>
<td>3-6’</td>
<td>51.48 (±10.07)</td>
<td>40.62</td>
<td>53.12</td>
<td>56.25</td>
<td>48.44</td>
<td>41.93</td>
<td>65.63</td>
<td>46.87</td>
<td>46.87</td>
<td>44.53</td>
<td>46.09</td>
<td>45.31</td>
<td>51.04</td>
<td>46.88</td>
<td>45.31</td>
</tr>
<tr>
<td>6-10’</td>
<td>7.36 (±5.64)</td>
<td>6.25</td>
<td>6.25</td>
<td>0</td>
<td>3.12</td>
<td>3.23</td>
<td>3.12</td>
<td>6.25</td>
<td>5.21</td>
<td>4.69</td>
<td>5.47</td>
<td>5.08</td>
<td>3.12</td>
<td>4.69</td>
<td>7.81</td>
</tr>
</tbody>
</table>

Table 16: Browse pressure results for all estimators. Values represent percent of aspen stems that were browsed. An asterisk superscript above the method name indicates the method(s) that do not consistently estimate browse pressure within fixed area plot 95% confidence intervals.

<table>
<thead>
<tr>
<th>ACP</th>
<th>CI</th>
<th>NN</th>
<th>BGTS</th>
<th>TS</th>
<th>OD2</th>
<th>OD3</th>
<th>DCINN</th>
<th>BDAV3</th>
<th>PQ</th>
<th>QN</th>
<th>DPQQN</th>
<th>CPD</th>
<th>AO2</th>
<th>AO3*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upper Billies Canyon</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.85 (±8.78)</td>
<td>33.33</td>
<td>35</td>
<td>39.66</td>
<td>36.49</td>
<td>36.67</td>
<td>28.33</td>
<td>34.17</td>
<td>36.67</td>
<td>26.25</td>
<td>32.92</td>
<td>29.58</td>
<td>36.11</td>
<td>32.5</td>
<td>31.25</td>
</tr>
<tr>
<td><strong>Maki Creek</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Figure 20: Closest Individual regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 21: Closest Individual regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
The Nearest Neighbor (NN) estimates performed less consistently across allotments. Original NN (ONN) density estimates never differed significantly from ACP estimates based on bootstrap BCa confidence intervals. However, ONN estimates were not very precise. Pollard NN (PNN) density estimates were generally lower than ACP estimates with the exception of Maki Creek (Figure 22-23). The PNN density estimate from Upper Billies was the only one to differ significantly from ACP estimates when using 95% bootstrap BCa confidence intervals (Figure 23). When using standard 95% confidence intervals for PNN, Red Canyon, Camp Creek, and Upper Billies density estimates were all significantly different from ACP estimates (Figure 23). There also might be a connection between the accuracy of these two estimators and stem density. It appears that as stem density increased, PNN became more accurate while ONN shifted from overestimating density to becoming more accurate (Figure 22). Relative efficiencies of these estimators were more varied but the NN methods were still more efficient relative to ACP (Table 13). The NN estimators performed similarly to CI estimators in regards to height class distribution and browse pressure estimates in that they only consistently estimated browse pressure accurately (Table 15-16).

T-Square Estimators

This group of estimators also produced mixed results. Besag and Gleaves’ T-square (BGTS) estimates were significantly different from ACP in the Upper Billies Canyon and Camp Creek allotments. Generally, BGTS density estimates were lower than circular plot estimates except in Maki Creek. Diggle’s T-Square (DCITS) density estimates were not significantly different from ACP except in Upper Billies Canyon. DCITS density estimates were lower than ACP estimates in every one of the allotments. Byth’s T-Square (BTS) density estimates were
Figure 22: Nearest Neighbor regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 23: Nearest Neighbor regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
generally lower than ACP estimates but none of the estimates were significantly different than ACP density estimates (Figure 24-25).

None of these methods produced accurate estimates for relative density by stem height class but all of them accurately estimated browse pressure (Table 15-16). All of these methods were more efficient relative to ACP except BTS (Table 13-14).

*Ordered Distance Estimators*

The Ordered Distance (OD) group of estimators generally produced consistently inaccurate, but relatively precise density estimates. OD2 and OD3 stem density estimates were all significantly different from ACP estimates. OD estimates were also all much lower than ACP density estimates (Figures 26-27). OD2 and OD3 did not consistently estimate relative density by stem height class accurately but did consistently estimate browse pressure accurately (Table 15-16). Both of these methods were much more efficient relative to ACP since calculated relative efficiencies were close to 0 (Table 13).

*Point-Centered Quarter Estimators*

The Point-Centered Quarter estimators performed similarly to the CI estimators. Both the original (OPQ) and Pollard’s (PPQ) estimator produced inaccurate estimates much lower than ACP estimates. OPQ and PPQ estimates were much more precise than ACP estimates. This combined with the relatively short sampling time of PCQ relative to ACP resulted in very low relative efficiency values for both PCQ estimators indicating that these methods were much more efficient relative to ACP (Figure 28-29).
Figure 24: T-Square regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 25: T-Square regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Figure 26: Ordered Distance regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 27: Ordered Distance regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Figure 28: Point Centered Quarter regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 29: Point Centered Quarter regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Similar to many of the other distance methods, OPQ and PPQ did not accurately estimate height class relative densities (Table 15-16). Unlike many of the other distance methods OPQ and PPQ did not accurately estimate browse pressure consistently (Table 13).

*Quartered Neighbor Estimators*

The Quartered Neighbor estimators produced mixed results. Original Quartered Neighbor (OQN) generally overestimated aspen sucker density. In the case of Red Canyon and Camp Creek, OQN greatly overestimated aspen sucker density (Figure 30-31). However, OQN density estimates were not significantly different than ACP estimates due to relatively large OQN 95% bootstrap BCa confidence intervals (Figure 31).

Pollard’s estimator with the Quartered Neighbor method (PQN) generally underestimated aspen sucker density. Standard 95% confidence intervals indicate that PQN density estimates were significantly different than ACP estimates in all but Camp Creek (Figure 30). Bootstrap BCa 95% confidence intervals indicate that none of the PQN density estimates were significantly different than ACP density estimates (Figure 31).

OQN was more efficient at estimating density relative to ACP in all but Red Canyon, again due to relatively large confidence intervals. PQN was much more efficient relative to ACP at estimating aspen sucker density in all cases (Table 13). Neither of the Quartered Neighbor estimators accurately estimated height class relative density (Table 15). Both Quartered Neighbor estimators accurately estimated browse pressure (Table 16).
Figure 30: Quartered Neighbor regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 31: Quartered Neighbor regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Compound Estimators

The two Diggle compound estimators (DCINN and DPQQN) performed similar to one another. Each underestimated aspen sucker density in all four of the stands. DCINN and DPQQN density estimates were significantly different than ACP estimates based on 95% confidence intervals (Figure 32). DCINN estimates are significantly different than ACP estimates in all but Maki Creek. DPQQN aspen sucker density estimates were significantly different than ACP estimates in all stands based on 95% bootstrap BCa confidence intervals (Figure 33). All of the Diggle compound estimators were much more efficient relative to ACP at estimating aspen sucker density (Table 14).

The Engemann, et al. (1999) compound estimator (BDAV3) dramatically overestimated aspen sucker density in all of the stands. Due to their gross inaccuracy, the resulting BDAV3 estimates had to be displayed separately from the other compound estimators. BDAV3 density estimates were extremely imprecise as illustrated by the relatively large bootstrap BCa confidence intervals (Figure 34). As a result, BDAV3 estimates were significantly different than ACP density estimates in only two of the stands: Maki Creek and Camp Creek (Figure 35). Given the extremely low precision of BDAV3, it was less efficient than ACP in all stands (Table 14).

Again, all of these compound estimators did not accurately estimate height class relative density consistently, but they did accurately estimate browse pressure (Table 15-16).

Corrected-Point Distance Estimators

The four types of Corrected-Point Distance estimators used for this sampling method varied quite a bit in their performance. The two estimators that used only two distance
Figure 32: Regeneration density estimates from two of the compound estimators compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 33: Regeneration density estimates from two of the compound estimators compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Figure 34: BDAV3 regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
measurements (CPDNL2 and CPDL2) were much more accurate and precise relative to their other CPD counterparts. None of their density estimates were significantly different than ACP estimates (Figure 35-36). CPDNL2 was always more efficient relative to ACP. CPDL2 was more efficient than ACP in all but Red Canyon (Table 14).

The two CPD estimators that used all three distance measurements (CPDNL3 and CPDL3) both dramatically overestimated aspen sucker density and were extremely imprecise (Figure 37-38). Additionally, CPDNL3 and CPDL3 were less efficient relative to ACP in all cases (Table 14).

The CPD estimators did not accurately estimate height class relative densities but they did accurately estimate browse pressure (Table 15-16).

*Angle-Order Estimators*

The two Angle-Order estimators produced somewhat mixed results. AO2 generally overestimated sucker density in all stands. None of the AO2 sucker density estimates were significantly different than ACP estimates based on bootstrap BCa 95% confidence intervals. AO3 generally overestimated aspen sucker density but AO3 density estimates were never significantly different than ACP estimates. Despite this, both AO2 and especially AO3 were generally less precise than ACP based on their relatively large 95% confidence intervals (Figure 39).

AO2 was always more efficient relative ACP at estimating sucker density (Table 12). The relatively large amount of error in the case of the AO3 estimate in Camp Creek and Red Canyon caused AO3 to be less efficient than ACP (Figure 39).
Figure 35: Corrected Point Distance – \( m \), excluded regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 36: Corrected Point Distance – m excluded regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Figure 37: Corrected Point Distance – mi included regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with 95% confidence intervals where applicable.
Figure 38: Corrected Point Distance – mi included regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
Figure 39: Angle Order regeneration density estimates compared to Aspen Circular Plot regeneration density estimates in all four stands with bootstrap BCa 95% confidence intervals.
AO2 did not consistently estimate height class relative densities accurately. AO3 was the only distance method estimator to accurately estimate this parameter in both stands (Table 15). Both AO2 and AO3 consistently estimated browse pressure accurately (Table 16).
Discussion

Accuracy

As seen in the ‘Results’ section, the majority of the Distance Methods did not accurately estimate aspen sucker density consistently in all cases and therefore did not meet the first expectation. This was not surprising with some of the methods tested because of the inherent bias of many of their estimators associated with the spatial pattern exhibited by the individuals within the population. Some of the density estimates displayed in the ‘Results’ provide clear visual evidence for the known inherent bias associated with spatial pattern of many of the Distance Method estimators (e.g. CI, NN, PCQ, QN, and OD estimators). Since both spatial pattern indices used in the analysis indicate that the aspen regeneration is arranged in an aggregated spatial pattern, apparent patterns in estimator error for many of the Distance Methods can be attributed to the spatial arrangement of the aspen regeneration. Further discussion on the accuracy of each of the Distance Methods and the implications of spatial pattern as it applies to each estimator will follow.

Browse pressure and height class distribution are two parameters that are also included when monitoring aspen restoration. These parameters are unbiasedly estimated through the ACP sampling strategy. There are two basic strategies to estimate these parameters using Distance Methods. One involves collecting distance measurements to stems representing each height and browse category at each sample location. The other involves classifying stems to which measurements are made to. The latter strategy was chosen for this study and is generally the more efficient choice of the two since it requires fewer measurements per sample point. While estimates for these parameters were very similar between the Distance Methods and ACP,
confidence intervals cannot be calculated using this approach for the Distance Methods. Consequently, estimates could not be compared in the standard way. This puts the Distance Methods at a disadvantage relative to ACP. Estimates for these parameters were considered accurate if they fell within the ACP 95% confidence intervals. As seen in the results, this did not always happen. In the few cases where estimates fell outside of the ACP bounds, they were generally off by just a few percentage points so these estimates were not grossly inaccurate. Using the alternative approach to estimating these parameters would likely result in some of the Distance Methods being less efficient than ACP since up to 6 sets of measurements would have to be taken at each sample point; one set of measurements to stems representing each stem height class.

**Relative Efficiency**

The efficiency of each of the Distance Methods was evaluated using both sample time and estimator precision relative to ACP. Many of the Distance Methods used in this comparison study were much more efficient than ACP estimates, as originally expected. This is not surprising since it takes substantially less time to collect quick measurements at each plot compared to counting and categorizing stems within a circular plot. This is the main advantage to Distance Methods that makes them such an attractive choice in scenarios where sampling using fixed area plots is more difficult. Estimator precision appeared to have the most effect on the relative efficiency of the individual Distance Methods. Some of the Distance Method estimators were also highly precise (e.g. CI and OD estimators), further reducing their relative efficiency values and therefore increasing their efficiency compared to ACP. Some of them were less precise, sometimes having substantially wider 95% confidence intervals than ACP density estimates (e.g. BTS and ONN). Their relatively short sample times acted as a buffer in
calculating their relative efficiencies. However, if these methods were to be used, a higher level of precision would be expected requiring additional sample plots and more sample time. This additional sample time could be enough in some cases to cause these methods to be less efficient than ACP. A few estimators were so imprecise that they ended up being consistently much less efficient than ACP (e.g. CPDNL3, CPDL3, and BDAV3). These results on their own would be enough to rule them out as potential sampling alternatives if their inaccuracy had not already been a factor.

A brief summary of the performance of each of the Distance Methods in regards to their accuracy, relative efficiency, and whether or not they can be considered for use is summarized in Table 17. The following sections provide a more in depth discussion of the performance of each of the Distance Methods.

**Closest Individual and Nearest Neighbor**

As seen in the results, both the original (OCI) and Pollard’s (PCI) estimators for this method dramatically underestimated stem density in all stands and therefore performed poorly. Since all of the sampled stands are spatially aggregated and these estimators are known to be biased when used in non-random spatial patterns, it is likely that the poor performance of these estimators is due to the spatial pattern of the aspen regeneration. Both OCI and PCI have been shown to underestimate density in spatially clumped patterns by other studies (Engeman, et al., 1994; Oldemeyer & Regelin, 1980) which was also the case in this assessment.

The Nearest Neighbor estimators were more sporadic in their performance. Other studies have shown that Nearest Neighbor estimators tend to overestimate density, and therefore have a
Table 17: Summary of the performance of each of the Distance Methods. See Table 1 for specific estimators and references.

<table>
<thead>
<tr>
<th>Category</th>
<th>Distance Method/Estimator</th>
<th>Aspen Regeneration Density Accurate</th>
<th>Browse Pressure Accurate</th>
<th>Height Class Distribution Accurate</th>
<th>Relative Efficiency &lt;1</th>
<th>Consider For Use</th>
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<td>No</td>
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<tr>
<td></td>
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<td>Yes</td>
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<tr>
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<td>BGTS</td>
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<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
positive bias in aggregated spatial patterns (Engeman, et al., 1994; Oldemeyer & Regelin, 1980; Hijbeek, et al., 2013). This was not always the case for the Nearest Neighbor estimators used in this study. Both ONN and PNN accurately estimated sucker density in a few instances. When estimates were not accurate, ONN overestimated sucker density, as would be expected, and PNN underestimated density, which was not expected.

To understand why PNN estimates were not biased in their typical direction and why ONN and PNN estimates were not consistently biased, the distance measurements from the field must be considered. CI distance measurements should be relatively large when compared to NN distance measurements in a spatially aggregated pattern for their corresponding density estimators to be similarly biased in opposing directions. This was similarly stated by Diggle (1975). The Pollard estimators utilize CI and NN field measurements in exactly the same way to estimate density. Comparing the PCI and PNN density estimates for each of the stands, PNN estimates were always larger than PCI estimates which implies that NN distances are smaller on average than CI distances. However, none of the PNN density estimates appear to be positively biased relative to ACP density estimates which indicates that NN distances were not nearly as small as would be expected in aggregated patterns in the sampled stands.

Upon examining the raw data, NN distances were fairly small to begin with (average NN distances ranged between 6.02 and 10.71 inches in the four stands). It is difficult to imagine many field situations where aspen interstem distances would be substantially smaller than what was observed in the four sampled stands. Following that line of thought, the high stem density may have an effect on NN estimator accuracy. Engeman, et al. (1994) suggested that density has an effect on estimator quality. Hijbeek, et al. (2013) noted in supplementary material that as density increased in aggregated spatial patterns, bias of Nearest Neighbor density estimates
tended to decrease. There may also be too much variation in the NN distances to yield a small enough mean NN distance.

All of the CI and NN estimators were consistently more efficient than ACP. This is not surprising since CI and NN regeneration density estimates were extremely precise relative to ACP. Despite this, the inconsistent accuracy and bias by design of these CI and NN estimators makes them inappropriate choices for aspen restoration monitoring methods.

*Compound Estimators*

None of the compound estimators accurately estimated aspen regeneration density. This study found somewhat similar estimator error relative to spatial pattern as the study conducted by Engeman, et al. (1994) for these two estimators. However, DCINN had a negative bias only in populations with the lowest level of spatial clumpiness within their study. The direction of error of BDAV3 in this study is consistent with the direction of bias noted by Engeman et al. (1994) although the level of estimator error in this study was much higher. White, et al. (2008) found that BDAV3 worked well estimating density in a variety of natural populations with varying spatial patterns but it is clear from this study that BDAV3 does not work well estimating aspen regeneration density.

These estimators were developed to work well in a variety of spatial patterns by balancing out the opposing bias directions of the original CI and NN estimators, as described previously (Diggle P. J., 1975). However, since the PNN estimators were not consistent in their expected positive bias in aggregated spatial patterns as discussed above, the resulting DCINN and especially BDAV3 estimates in the four stands were innaccurate. As a result, neither of these sampling methods appear to be good choices for monitoring aspen regeneration.
The DPQQN compound estimator also performed poorly. This is not surprising considering that this estimator is the geometric mean of PPQ and PQN, which utilize very similar measurements as NN and CI estimators, and both tended to underestimate aspen regeneration density. This compound estimator was suggested as a way to eliminate bias from the Point-Centered Quarter and Quartered Neighbor density estimates (Leavitt & St. Clair, 2011) but as with DCINN, bias from PPQ and PQN estimates must be in opposite directions in order for DPQQN to balance out the bias. Since this did not appear to hold true in this case, DPQQN is not a good estimator to consider for use in monitoring aspen regeneration.

DCINN and DPQQN were consistently more efficient than ACP, which is not surprising since each of these estimates were shown to be extremely precise relative to ACP. On the other hand, BDAV3 was never more efficient than ACP. The extreme inprecision of this estimator clearly had an impact on the efficiency of this estimator. Improving the precision of this estimator would require more time to sample which would further impact the efficiency of this method. The poor relative efficiency of BDAV3 further suggests that it should not be considered as an alternative sampling method for monitoring aspen regeneration.

**T-Square**

Of the three estimators in this group, only BTS consistently estimated aspen sucker density accurately. The two compound estimators BTS and DCITS suggested by Byth (1982) and Diggle (1975), respectively were developed to create a more robust estimator using T-Square distance measurements that would perform better in a variety of spatial patterns relative to more simple estimators utilizing only one distance measurement, like BGTS. Based on this study, only BTS was consistently more accurate than BGTS. As seen in the results, DCITS generally
underestimated aspen regeneration density suggesting that aggregated patterns caused this estimator to be negatively biased. This is consistent with Engeman, et al. (1994). The only exception was the BGTS estimate in Maki Creek which fell within the ACP 95% confidence intervals. PNN also estimated density accurately in the Maki Creek stand. Since T-Square measurements are essentially Nearest Neighbor measurements modified only by restricting the direction in which measurements are made, it is not surprising that T-Square estimates were also accurate in some cases as well. These estimators should not be considered for use in monitoring aspen regeneration due to their inconsistent accuracy and general bias in aggregated spatial patterns. Although BTS met the accuracy requirement, it was relatively imprecise in some cases and was not always more efficient than ACP and therefore it should not be used.

Ordered Distance

This group of estimators consistently underestimated aspen sucker density in all four of the sampled stands. These results reflect what other studies have noted that they tend to underestimate density in spatially aggregated patterns and that they are biased in non random spatial patterns (Engeman, et al., 1994; White, et al., 2008; Hijbeek, et al., 2013). Morisita (1954) and Pollard (1971) suggest that increasing the k value for these estimators, and therefore measuring to the next farthest stem from the sample point decreases the variance. By comparing 95% confidence intervals from these two estimators, it seems that this is not always the case. In the cases that OD3 estimates had smaller confidence intervals than OD2, the differences were relatively small. Due to the inaccuracy and bias of these estimators in nonrandom spatial distributions, OD2 and OD3 are not suggested for use in monitoring aspen regeneration.

Point-Centered Quarter
The two estimators for this sampling method performed as expected in the aggregated populations sampled in this study. OPQ and PPQ consistently underestimated aspen sucker density. This is similar to the findings of other methods comparison studies regarding the performance of these estimators in aggregated populations (Engeman, et al., 1994; Hijbeek, et al., 2013). This group of sampling methods cannot be relied upon to estimate density without bias regardless of spatial pattern and therefore it should not be used to monitor aspen regeneration.

*Quartered Neighbor*

Relative to one another, these two estimators performed somewhat different. Since the two estimators used for this sampling method are essentially the same as the Point-Centered Quarter estimators, it seems reasonable to expect that they should be biased in opposite directions given the known relationship between closest individual and nearest neighbor distances and density estimates in aggregated distributions. While this relationship was expressed as expected when OQN and OPQ density estimates are compared, this was not true with PQN and PPQ estimates. However, PQN density estimates were still greater than PPQ estimates. The lack of expected bias might also be explained by the potential effect of density on estimator accuracy as explained in the Nearest Neighbor methods section.

Both of these estimators met the accuracy requirement based on overlapping 95% confidence intervals, however the estimates themselves did appear to have a consistent direction of error relative to ACP estimates. Based on this consistency of each of the Quartered Neighbor estimators and since the aspen regeneration in all of the sampled stands exhibits a spatially aggregated pattern, spatial pattern may be a contributing factor causing these estimators to be
biased. This is not surprising since the estimators used are the same as those used for Point-Centered Quarter methods which are known to be biased by non-random spatial pattern (Engeman, et al., 1994). Other factors, such as stem density, may also be contributing to the bias of these estimators, but this cannot be concluded from the limited results of this investigation. The relatively wide OQN bootstrap BCa 95% confidence intervals also show that this estimator can be very imprecise and potentially less efficient than ACP. As a result of these findings and the known bias of the estimators, OQN and PQN are not recommended for use in monitoring aspen regeneration.

Corrected Point Distance

Two of the estimators in this group performed as originally hypothesized. The two CPD estimators which utilized only closest individual and nearest neighbor distance measurements (CPDL2 and CPDNL2) performed the best of the four tested. Batcheler & Bell (1970) suggest using the second nearest neighbor measurement (m_i) and the corresponding estimators (CPDL3 and CPDNL3) to correct for potential clumping within clumps, or “second order of contagion” in their words, within the population. Based on the significant overestimation of stem density, it appears that CPDL3 and CPDNL3 over-correct for this factor. Rempel, et al. (2012) noted this behavior as well both in simulated artificial point populations and natural shrub populations. They also found that the CPD estimator utilizing only closest individual and nearest neighbor measurements was much less biased than when second nearest neighbor measurements are used as well. This was also found to be true in this investigation. Estimates were noticeably more precise when no limits were placed on distance measurements (CPDNL2 and CPDNL3).
The inprecision of the CPDL2, CPDL3, and CPDNL3 estimates substantially increased their relative efficiency values and caused them to be much less efficient relative to ACP. However, CPDNL2 was much more precise and therefore tended to have lower relative efficiency values. The efficiency of CPDNL2 combined with its accuracy made this the best performing estimator of the four CPD estimators tested and probably the best performing Distance Method overall.

Despite its generally superior performance relative to the other Distance Methods and its high efficiency, this method should still be viewed with caution. Even with the empirical correction factors, these estimators may not be consistently reliable seeing as they did not perform as originally intended. Batcheler states that both $A_1$ and $A_2$ should be used as correction factors (equivalent to CPDNL3 and CPDL3) in cases where $A_1$ is greater than 0.5 (Batcheler, 1973). This was the case in all of the sampled stands yet these two estimators performed poorly compared to their other CPD counterparts in which $A_{\text{max}}$ is used (CPDNL2 and CPDL2). Additionally, these estimators are all based on empirical models and may not be reliable if the spatial pattern of the sampled population falls outside the range in which these estimators were developed.

**Angle Order**

AO2 and AO3 performed fairly well relative to many of the other Distance Methods examined in this study. However, these estimators still appear to have some flaws. Both AO estimators performed as originally hypothesized, however these estimators (especially AO3) were relatively imprecise in some cases. As noted in the results, AO3 tended to overestimate density in the sampled aspen stands suggesting that spatial pattern may still have an effect on
these estimators. The direction and level of estimator error for AO2 and AO3 appear to be consistent with those noted by other investigations in spatially aggregated patterns (e.g. Laycock & Batcheler, 1975; Engeman, et al., 1994; White, et al., 2008).

Of the two, AO2 appears to be the best performing estimator based on its consistently lower estimator error and narrower bootstrap BCa confidence intervals. AO3 did not consistently have relative efficiency values below 1 indicating that it was not always more efficient than ACP. Since AO2 was almost always the more precise estimator of the two, AO2 had the lowest relative efficiency values and was always more efficient than ACP. With its accuracy and efficiency, AO2 was the better performing estimator of the two, and it was also one of the best performing estimators in this study. This is somewhat surprising since other studies have found that AO3 often outperforms AO2 in regards to stem density estimation, but generally not by much (Engeman, et al. 1994; White, et al., 2008). Given the limited scope of this study, it is difficult to determine exactly why this is. AO3 estimator accuracy seemed to decrease slightly in conjunction with increasing density between the four stands suggesting that stem density might be affecting its accuracy (Figure 39). Based on the results, spatial pattern and perhaps stem density may have some effect on the accuracy of these estimators, so these estimators should be considered with caution. An additional note of caution relates to the assumptions of these estimators. Even though the AO estimators were developed to work in spatially non-random populations, they still assume spatial randomness at the sample plot scale (Morisita, 1957).

Based on the findings of this study and those conducted by other researchers, many of these Distance Methods cannot be used to consistently produce unbiased estimates of stem density regardless of the spatial pattern or any other underlying characteristic of the population. To understand why these estimators are biased, they can be compared to other sampling methods.
and estimators commonly used in natural resource sampling. It is important to focus on the estimator portion of the sampling strategy rather than the sampling method when making this comparison. The field sampling method might be completely different between strategies (e.g. sampling fixed area plots vs. sampling individual stems) however many of the commonly used sampling strategies (e.g. fixed area plots and point-relascope sampling) utilize a form of the Horvitz-Thompson estimators (Horvitz & Thompson, 1952). No matter which population parameter is being estimated, Horvitz-Thompson estimators always account for the probability of the measured population unit being included in the sample. This is otherwise known as the inclusion probability of an individual. This is what makes Horvitz-Thompson estimators superior because using the inclusion probability in the estimator allows for unbiased estimation of many standard characteristics, like total basal area or average volume of merchantable timber, without making assumptions regarding the population’s spatial pattern, density, or any other underlying attribute.

Although it might not be initially clear, fixed area plot sample designs like ACP use inclusion probabilities of population individuals to unbiasedly estimate a variety of population parameters. The inclusion probability of any individual is expressed as the proximity to an individual stem that a fixed-area plot center must fall within, which is equivalent to the size of the fixed-area plot being used. Most of the Distance Method sampling strategies investigated in this paper do not use inclusion probabilities but instead rely on a model which has assumptions that must be met for them to be accurate. Delineating stem inclusion areas and calculating inclusion probabilities for individual stems sampled using Distance Methods would be an incredibly arduous task, especially in aspen regeneration, and would cause these sampling methods to be drastically less efficient than fixed area plot sampling strategies. Some of the
Distance Methods in this study try to overcome this flaw using adjustment factors (CPD) or other assumptions (AO), and some of them performed well in this investigation. However, these methods were only tested in four aspen regeneration stands which may not reflect the full range of conditions that the method would be expected to perform in.

Additional research should focus more specifically on those methods which met the conditions of this investigation: CPDNL2 and AO2. These methods should be further tested in treated aspen stands to get a better idea of how consistent their accuracy and efficiency is across a wider range of sampling conditions. Natural resource professionals are encouraged to test the better performing Distance Methods themselves to determine if their performance remains consistent. In addition, other potentially more efficient fixed area plot sampling designs and estimators, such as double sampling, should be investigated. A basic double sampling simulation was conducted using the ACP data that was collected. The results of this simulation were promising and suggest that double sampling may be as accurate and more efficient relative to ACP.
Conclusion

A variety of alternative sampling methods were tested to determine if one or more of them would be a better alternative to the Aspen Circular Plot procedure currently used for monitoring aspen restoration treatments in the Wyoming Front Aspen Restoration Project in western Wyoming. The alternative methods, commonly known as Distance Methods, were assessed based on their ability to accurately and efficiently estimate three common parameters used in monitoring aspen regeneration after restoration treatments are implemented. These parameters include aspen sucker density per acre, browse pressure (percent of aspen suckers whose terminal leaders were bitten off), and height class distribution (percent of aspen suckers categorized by height class). Aspen regeneration density estimates were compared using traditional 95% confidence intervals and BCa bootstrap 95% confidence intervals, where applicable. Distance Method browse pressure and height class distribution estimates were deemed accurate if they fell within ACP 95% confidence intervals. Distance method efficiency was analyzed using a mathematical combination of the two times the mean Distance Method sample plot completion time and estimated variance of aspen stem density relative to the mean sample time and estimated variance from the ACP method.

Prior to analysis, it was hypothesized that the design-biased Distance Methods (CI, NN, OD, TS, PCQ, QN, and compound estimators) would not accurately estimate aspen sucker density while the remaining Distance Methods (CPD and AO) would accurately estimate aspen sucker density. All of the Distance Methods were predicted to be more efficient compared to ACP based on calculated relative efficiencies for each method. Finally, they were all expected to accurately estimate browse pressure and height class distribution. Eight of the Distance Method estimators consistently estimated aspen regeneration density accurately; ONN, BTS, OQN, PQN,
CPDNL2, CPDL2, AO2, and AO3. All but PPQ estimated browse pressure accurately and only AO3 estimated height class distribution accurately. All of the Distance Methods were consistently more efficient than ACP except for OQN, BDAV3, BTS, CPDL2, CPDNL3, CPDL3 and AO3.

Since none of the Distance Methods met all of the expectations, none of them can be strongly recommended as an accurate and more efficient alternative sampling strategy for monitoring aspen regeneration over ACP. However, some of them could potentially be considered further as an alternative sampling strategy for estimating aspen regeneration density and browse pressure. CPDNL2 and AO2 performed the best of all the Distance Methods that were tested. They accurately estimated stem density and browse pressure. These methods were also consistently more efficient than ACP so using them would require less sampling time. Based on the performance of these two Distance Methods, they are worth considering for further field trials. ONN, BTS, and OQN also performed well, but their density estimates were generally less precise than the leading two estimators. The other Distance Methods were shown to perform poorly in this spatially aggregated sampling situation. Therefore, these methods and their respective estimators cannot and should not be relied upon to accurately estimate density unless their specific assumptions related to spatial pattern are met.

Even with the somewhat promising results from the two best Distance Methods, these methods should still be considered with caution. This project assessed these alternative methods in only four treated aspen stands in western Wyoming, which do not encompass a very wide range of sampling conditions that may be encountered. Also, the Distance Methods rely on models to estimate stem density and these models have a limited range of application. The best performing Distance Methods along with other design-unbiased and potentially more efficient
fixed area plot sampling designs should be tested further before any strong recommendations can be made for their use in monitoring aspen regeneration.
References


