Problem solving performance under variations in magnitude of differential reinforcement

Charles Henry Koski

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PROBLEM SOLVING PERFORMANCE UNDER VARIATIONS IN
MAGNITUDE OF DIFFERENTIAL REINFORCEMENT

by

Charles H. Koski

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Approved by:

[Signatures]

Chairman, Board of Examiners

Dean, Graduate School

Date

[Signature]

[Signature]

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CHK
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About 20 years ago, Crespi (1942, 1944) reported on the effects of variations in incentive magnitude that were introduced into certain learning situations involving the white rat. Crespi measured speed of running a straight-alley maze to different amounts of a food-reward, and his data showed that reinforcement did not affect the rate at which the rats approach a final level of performance but was significant in determining the final level they achieved. A few years later, Zeaman (1949) got essentially the same results, as measured by latency of the running response in the Graham-Gagne runway. These studies, which both showed that rats traverse a straight runway more rapidly for a large amount of food than for a smaller amount, leave little doubt that performance in the learning situation depends upon amount of reinforcement. Both Crespi's and Zeaman's procedure involved running one group of rats under a high amount of food-reward and the other group under a lower amount until both groups reached asymptotic performance levels. At this point, the amount of reward given the two groups was reversed for a number of trials; and on the second post-shift trial, an abrupt change in running speed occurred for both groups. The group originally getting low reward quickly shifted upward in performance upon receiving the amount of food originally given the high-reward group, and this latter group decreased running speed sharply after being shifted to the lower reward.

Hillman, Hunter, and Kimble (1953) demonstrated similar shifts in performance due to changes in drive level, using the same basic procedure that Crespi and Zeaman had employed. Thus, changes in amount of
reinforcement are shown to produce sudden increases or decreases in performance in a way which parallels that produced by changes in motivation—a result one could expect with a performance variable.

Hull (1951) gave special consideration to the role of incentive magnitude and derived a theorem to cover the effects of changes in this component (K). He says:

Other things constant, an abrupt shift in the incentive used during a maze-learning process will be followed first by a major shift in reaction potential and then by two or more progressively smaller shifts on successive trials, the series constituting a rapid learning process of the exponential variety, culminating in the course that the S/E would have followed had the new incentive been operating continuously from the beginning of learning.

Thus, Hull removed the incentive magnitude variable as a determiner of habit (H) and introduced a separate intervening incentive motivational variable (K) which is designed to handle both differences and shifts in incentive magnitude. Hull's notion does not, however, strictly account for the "elation effect"—superior acquisition in the animals shifted up, as compared to the group which had always received high reward, nor the "depression effect"—inferior acquisition in the animals shifted down. Both of these phenomena were demonstrated in the Crespi and Zeaman studies. A recent study by Marx and Piesper (1962), in which a design was employed whereby the animal receives a continuous contrast of incentive over an entire acquisition period, reports significant "elation" and "depression" phenomena. This finding points out that incentive variables may play an important role in determining the acquisition of a response, under certain conditions.

Specification of the exact way in which (K) combines with other intervening variables, particularly drive (D), has not been agreed upon. Hull originally proposed a multiplicative function as a working hypothesis;
and Spence, in his book *Behavior Theory and Conditioning* (1956), suggested that (D) and (K) add in linear fashion, for which there is some evidence (e.g., Ramond, 1954). Some recent work on the role of how (D) and (K) combine in the learning process has been done by Ehrenfreund and Badia (1961), who observed that "contrary to either a multiplicative or linear function, the picture supports the notion that the function describing how (D) and (K) combine is negatively accelerated, at least for the ranges employed." Their data, which support the notion that (D) and (K) add exponentially, are not in agreement with two other recent studies by Weiss (1960), and Reynolds and Pavlik (1960), which showed no interaction between drive and reward and, therefore, support the linear or simple additive function idea of Spence.

Several experiments where (D) has been held constant and the amount of reward (K) varied have yielded results which are in agreement on the point that performance increases in a negatively accelerated fashion with increases in (K). Among the studies leading to this conclusion are those of Maher and Wickens (1954), Crespi (1942), and Guttman (1953).

The above brief review of animal research on the effects of amount of reinforcement in the learning situation leads to three main points: (1) that greater amounts of reinforcement produce higher final levels of performance than lesser amounts; (2) upward and downward shifts in reward magnitude following asymptotic performance on a task produce abrupt changes in performance which level out at values somewhat higher or, under appropriate conditions, lower than the asymptotic values produced by these reward magnitudes operating continuously throughout the task; and (3) the function which relates performance to the amount of reinforcement is a negatively accelerated curve under conditions where drive is held constant.
Studies with humans involving amount of reward variables have led to essentially the same considerations as those from animal research, but the complexities have as yet retarded the development of major conclusions like those which have been stated above for animal behavior. In fact, no clearcut evidence of any general consistency can be found concerning the effects of incentive variations using human Ss; although on the whole, as pointed out by Ammons (1954), results have tended to corroborate the 1951 Hullian theory, which does not predict changes in performance level as great as those observed in the Crespi effect.

One recent study (O'Connor & Claridge, 1958) with male imbeciles tested the effects of change in incentive magnitude on the performance of a simple dexterity task. In this experiment, Hull's theorem of incentive change was not supported by the data. Rather, the performance of Ss, given a sudden increase in incentive, reached a significantly higher level than that of Ss, who had received this incentive from the outset of the task. This "elation" effect was not, however, accompanied by a corresponding "depression" effect on Ss receiving a reduced amount of incentive. The conclusion of these authors was that certain modifications rather than rejections in present concepts of habit, drive, and incentive are necessary to account for the inconsistencies noted between theoretical formulations and empirical evidence concerning the operation of these variables. This is particularly true concerning human behavior, for it surely must be agreed that, where incentive variables are combined with performance factors (e.g., task complexity, Ss' awareness of reward, etc.), their effect is more complex than Crespi outlined, not completely accounted for by Hullian concepts, and not supported by enough conclusive evidence to be explained adequately in the conditioning theories proposed by Spence.
The area of human problem solving suffers from a conspicuous
dearth of knowledge concerning the effects of incentive on learning and
performance. Lack of standardized tasks which are sensitive to the
major variables affecting problem solving has no doubt discouraged such
research, and the fact that pertinent data remain controversial in
studies involving relatively simple tasks has held back attempts to
manipulate incentive variables in the more complex task situations. It
is the contention of this writer that basic data need to be obtained on
the question of effects of incentive variation in a human problem-solving
situation if a comprehensive integration of theory and empirical fact
is to be maintained.

On a priori grounds, it can be stated that variations of
incentive magnitude in a human problem-solving situation lead to effects
which parallel in a fundamental way the effects shown in animal studies
on performance in less complex situations, given the appropriate con­
ditions in which to measure them. This speculation has merit for the
reason that it may curb oftentimes dead-end research toward a
progressively more parsimonious approach on a system of facts from which
to construct a theory of learning. If systematic variations in amount of
reward in a human problem-solving situation are shown to relate to
fundamental principles of learning established under simpler conditions,
involving either subhuman or human organisms, a significant step will
have been taken toward the above goal.

But a priori reasoning remains just that, and the need for
pertinent data is obvious. In view of this need, a program of experi­
mentation which seems most meaningful involves a very basic delineation
of the operation of systematically varied incentive on the performance
of a task which can be controlled under a variety of conditions. If such
an outline demonstrates that: (1) different amounts of reinforcement
produce different final levels of performance; (2) that the effect of
a change in reward late in a task produces abrupt shifts in performance;
and (3) the function relating amount of reinforcement and performance is
negatively accelerated, then further research would be encouraged on
motivational factors in the human problem-solving situation (e.g., drive,
anxiety) and on aspects of the human learning process in problem
situations such as "hypothesis testing" and "strategy behavior." These
latter aspects might be expected to conform to variations in amount of
reward in a manner similar, in principle, to the way in which amount of
reward determines performance in less complex situations.

Such a program is surely desirable in view of the lack of system
building in the human problem-solving area, for its ultimate value is in
providing the framework for a network of integrated predictions of complex
phenomena on the basis of known relations demonstrated with less complex
phenomena.

What task should be chosen to carry out such a program? One which
has been given sufficient experimental attention to warrant consideration
as a true "standard task" for research on human problem solving is
"anagrams," or the transposition of a basic letter combination into words.
A great deal of information has been gathered on problem solving using
anagrams as a medium of obtaining estimates of various factors on perfor-
mance. Although variations of the task have been used widely, different
investigators have employed a variety of procedures, and even the same
investigator has seldom used the same procedures twice. After an extensive
survey of this problem, Ammons and Ammons (1959a) worked out a standard
problem-solving task using anagrams and supported its utility by a rational evaluation of its use as a laboratory analogue of "real life" problem solving (Ammons & Ammons, 1959b). Among the practical features of this task which lend to its attractiveness for systematic experimental use, as noted by the above authors, are: (1) it can be used as a group or individual task; responses are essentially the same in both situations; (2) complete objective control of the similarity of "problems" is easy to achieve; (3) relative difficulty of "problems" can be determined easily by pretesting and can be precisely controlled; (4) when conditions are held constant, each S tends to be consistent in rate of production of solutions; and (5) transfer of solutions from one basic letter combination is small, relative to changes in performance produced by other important variables.

The validity of the Ammons Standard Anagram Task for the experimental study of problem solving involves the great number of similarities between some of its characteristics and those of "real-life" problem-solving situations. Among them are: (1) rearrangement of elements into new combinations is called for in the anagram task; (2) many solutions are possible, and differences among them are easily discriminable; (3) problems vary in difficulty; (4) greater flexibility of attack on a problem leads to more solutions; and (5) solutions have different habit strengths, forming a habit family hierarchy in principle determinable for a given S as well as for a reference group.

Problem

A series of investigations by the above author and associates (Kane, et al., 1959a), beginning with study of the relationship between
reward and frequency of solutions to anagrams, provide the background of the study carried out by this writer. In the first phase of this series, attempts were made to modify the solution frequencies by a form of verbal reinforcement. One class of responses was verbally rewarded (anagram solutions consisting of 4 or more letters) for one group while a control group was given no such reward. The Standard Anagram Task (Ammons & Ammons, 1959a) was used for a more objective control of administration and scoring procedures. Results showed that the ratio of longer to total solutions was not appreciably affected by verbal reinforcement. Although the Ss were not informed about the nature of the experiment, almost all of them reported some degree of awareness of the verbal reinforcement; that they were operating under some hypothesis was evident in these reports also. It was concluded that E reward could have simply reinforced these hypotheses held by Ss, and that self-reward possibly masked the effects of E reward. A replication study (Kane, et al., 1959b) designed to minimize Ss' awareness of the purpose of the experiment yielded data which were consistent with results in the original experiment, showing no significant differences between verbal reward and nonrewarded performance. It was apparent from a tabulation of Ss' replies to a question as to the purpose of the experiment that they formulated definite hypotheses about its nature, but that these were almost always incorrect. The authors concluded that, where conditions prevent the development of relevant hypotheses by Ss, E reward does not have a significant effect in changing what appear to be firmly established habits.

Subsequent to these findings, these authors stepped in the other direction and considered the possible effects of reward made very explicit to a S in a similar situation. To explore this possibility, an
alternative procedure was devised using points as a reward, with more points awarded for long (4 or more letters) solutions than for short (1, 2, or 3 letters) solutions. The differential employed was 1 point for short solutions and 5 for long solutions for the experimental group, and 1 and 1, respectively, for control Ss. The preliminary study using this procedure revealed a slight but not statistically significant increase in number of longer solutions \( (p > .10) \). Interviews with the Ss indicated a general feeling that the points had affected their performance.

Three further studies were then carried out, a portion of the data from the first being used to set up solution frequency norms for the Standard Anagram Task and to evaluate the stability of these norms (Robertson & Ammons, 1961). These solution frequency norms were found to be highly reliable (median \( r = .87, p \leq .01 \) for the six problems used) for two groups of 40 Ss.

The main purpose of the first study was to obtain more reliable data on reward effects in anagram problem-solving situations. Because only slight differences in performance between short-solution totals and long-solution totals had been shown when a 1-5 differential reward was used, the authors increased the differential to 1-10 and studied the effects of increasing the reward on both short and long solutions. This second study tested the hypothesis that frequency of solutions within a class would be affected (increased) as a function of differential point reward. The data supported this hypothesis and raised a new question. Does magnitude of reward affect production of total solutions, solutions within a class, or both? A further study similar to the above was completed with additional control groups in order to clarify the previous
findings (Ammons & Ammons, 1962). Analysis of the performance of these groups led to the following generalizations:

1. Total number of solutions produced increased slightly but steadily with practice on successive problems.

2. Level of performance is not appreciably affected by points given as reward, providing the point values are the same for all classes (in this case, two) of solutions.

3. Where there is differential reward for two classes of solutions, that class given the greater reward tends to show a relative increase in frequency, while the class given the smaller reward tends to show a relative decrease in frequency with practice on successive problems.

4. Where there is differential reward for two classes of solutions, that class which receives the higher reward shows an absolute increase in frequency with practice on successive problems, while the class receiving the lower reward shows an approximately constant absolute frequency.

The three studies described above show that a reward in the form of points can produce predictable changes in performance on the Standard Anagram Task. The reward value of "points" is hypothesized to be acquired through cultural means from the many social institutions in which individuals in our society tend to interact. Implicit in nearly all of these institutions is the influence that quantity indices have on their members. Prior to these more formal social groups, this "quantity" influence takes on many forms in a child's life—"having more" of any positive object is desirable and enhances the "value" of this object for the child and the group to which he belongs. This influence becomes more structured in classroom situations where "points" are substituted for objects and become highly rewarding in themselves over a considerable span in the life of an average member of this culture.
Many other human enterprises can be listed where the attainment of "points" is an end in itself, one which manifests considerably high motivation toward its attainment. Due to the similarities in the nature of the Standard Anagram Task to problem solving in a classroom situation, and the specific instructions to Ss concerning the scoring of this task, it is felt that the influence of points on behavior in this study is a rewarding one, in principle not greatly different in influence from points given for high performance in settings outside of the laboratory.

Accordingly, the writer proposed further research utilizing the Standard Anagram Task manipulating amount of points as an independent reward variable. It was the purpose of the present study to determine the effects of several levels of a differential point reward on two classes of anagram solutions. Previous research had indicated a general increase in the incidence of the more highly rewarded solution class, but had not resolved whether or not systematic differences in performance would occur corresponding to increases in the magnitude of a point differential reward.

Another question that might be answered in part by this experiment concerns (2) the quantitative description of the function relating performance on a problem-solving task to systematic increases in magnitude of reward. The question of performance shifts following reward reversals is obviously answered better by knowledge of reliable data on the two basic issues described above and, therefore, deemed a problem most efficiently considered only after such data were obtained.

Two null hypotheses were tested in this study. They are:

1. The relative frequency of a solution class receiving the upper magnitude of a differential point reward will be the same as
that of a solution class receiving the lower magnitude of reward.

2. Rates of the various groups will be ordered at random with respect to magnitude of point differential for that class of solutions receiving increasing reward.

A prediction was made that the function describing the relationship between performance on anagrams and increasing magnitude of a point-reward differential will be a negatively accelerated curve for that class of solutions receiving the higher reward.

Method

Materials. The six anagrams standardized by Robertson and Ammons (1961) were used in the study. This particular choice of letter combinations was made on the basis of a procedure for evaluating the relative frequency of total solutions by class (see Appendix A) and applied to 11 anagrams used in previous studies.

Text booklets were constructed out of 8½- by 11-inch paper and contained instructions for the anagram task on the first page and six problems, printed one to a page, at the tops of the next six pages (see Appendix B for sample booklets).

A stop-watch was used to control the time intervals during and between the problems.

Subjects. Because of the large number of Ss required for the study, the total sample could not be tested in one group session. Various smaller groups were drawn from undergraduate courses at the University of Montana and Montana State University. The U of M Ss numbered 370 and were taken from spring and summer classes of the 1962 school year in
groups which ranged from 11 to 34 Ss in size. An additional 75 Ss were tested at MSU during the summer of 1962, in groups ranging from 11 to 24 Ss each. The total sample was pared to 400 Ss. There were 39 rejects for failure to follow instructions in some manner and six random withdrawals to equate N's among the various groups.

One hundred and ninety-six of these final Ss were women with a median age of 22.33, and the remaining 204 were males with a median age of 23.65. All Ss were students who had been enrolled in the regular school year, a criterion imposed because it is not unusual to encounter many potential Ss over 30 years of age in attendance for summer classes only. Such Ss would not represent the normal age range of Ss drawn during the regular college year and were, therefore, excluded from the sample.

Since, however, some Ss were 30 years old, but more than 85 percent were 25 or under, the median was chosen as the most meaningful statistic for an estimate of the typical S's age.

All Ss were screened prior to the experiment by E as to any previous experience with an anagram task. This was done at each testing session by announcing the general nature of the Standard Anagram Task, E reading a few lines from the instruction page on each booklet (see Appendix B). The group was asked if anyone had been in an experiment using anagrams in the fashion described to them by E. Those indicating "yes" were asked further if the task involved "points" which they added during the course of the experiment. Any Ss also replying "yes" to this question were excluded from that session after being thanked for their cooperation. Names were cross-checked for the entire sample to determine
if any Ss were inadvertently tested more than once during the experiment. Only the first set of data was used where such duplications were noted.

Procedure. Solutions were divided into two classes: 1-, 2-, and 3-letter solutions and 4-or-more-letter solutions (4, 4*), which will be called "short" and "long" solutions, respectively, for the remainder of this thesis. A point differential system of varying the reward, which is comparable to the one in the Ammons studies mentioned above, was used. Their investigations had demonstrated that, when short and long solutions were given equal 1-point rewards, performance was very nearly equal to the performance exhibited by a group given an equal 10-point reward for both solution classes. But when solution classes were differentially rewarded, relative performance was effected for that class getting the higher point value. Accordingly, the lower magnitude for each of the differential reward factors was arbitrarily set at 1 point. That is, while varying the magnitude of reward for short solutions, long solutions were rewarded by 1 point, while varying the magnitude of reward for long solutions, short solutions were rewarded by 1 point.

Eight different point differentials were incorporated in the design, with two control differentials held constant at different magnitudes (see Table 1a). The N was 40 for each group. Four experimental groups received 5, 10, 20, or 50 points in reward for short solutions and 1 point for long solutions. The other four experimental groups were given 1 point for short solutions and 5, 10, 20, or 50 points for long solutions. Two control groups of Ss were rewarded equally for either short or long solutions, with one group given 1 point for both classes of solutions and the other group 50 points for both classes of solutions.
Solution frequencies for the 10 groups were obtained on the six anagrams shown in Appendix A. All groups were presented with GARDEN as the first basic letter combination, short and long solutions equally rewarded by 1 point. GARDEN was first in all conditions in order to provide an estimate of individual and group ability differences. The five remaining problems were assigned one of the 10-point differentials shown in Table la. For example, Ss in condition 1-5 worked on GARDEN first, with short and long solutions rewarded equally with 1 point, and completed problems 2 through 6 under 1-point reward for short solutions and 5-points reward for long solutions. The first number in each of the point differentials refers to the amount of reward given short solutions, and the last number refers to the amount of reward given long solutions.

The order of problems 2 through 6 was as shown in Appendix A for all conditions. These problems were rotated within each group, however, providing for control of problem difficulty by presenting them an equal number of times in each of the five positions. Table 1b shows the five-problem sequences that each group received, with eight Ss per sequence, a total of 40 Ss in each main group.

Subjects were tested in groups as available. Test booklets for the entire sample were prearranged as to reward groups and problem sequence in an order which distributed the 10 groups and five sequences as equally as possible throughout any particular testing session. To be specific, 600 booklets (includes an excess of 200 to allow for rejects and extra Ss as needed) were stacked in the following order: The top booklet of the stack was a 1-1 reward group booklet whose problems were arranged in sequence 1 of Table 1b; the second was a 1-5 reward booklet whose problems were arranged in sequence 2; the third a 1-10 booklet with
problem arrangement 3; the fourth a 1-20 booklet with sequence 4; the fifth a 1-50 booklet under sequence 5; the sixth a 5-1 booklet with problems in sequence 1; the seventh a 10-1 under sequence 2; the eighth a 20-1 under sequence 3; the ninth a 50-1 under sequence 4; and the tenth a 50-50 under sequence 5. This procedure was repeated 60 times, resulting in the final arrangement of the 600 test booklet with reward conditions and sequence of problems evenly distributed throughout the stack.

At each testing session, the number of Ss remaining in the group after the screening was done were counted and then asked to leave the room for a few minutes. The E then distributed the appropriate number of booklets face down at alternate desks, recalled the group, and instructed them to "fill in the seats only where booklets were found, leaving the booklets face down until instructed further."

Preliminary instructions followed the standard set suggested by Ammons and Ammons (1959). The first page acquainted the Ss with the Standard Anagram Task, stating that it was "a game in which you will construct words out of a basic letter combination which you will have in front of you while you work." The rules of the "game" were listed following this orientation, along with several examples of their operation.

A second set of instructions informed Ss that the author was "investigating the effects of variations in rules for playing this game." These instructions further indicated to Ss that they were to score their own solutions and also gave a complete set of scoring rules involving a hypothetical anagram, a "1-2 point differential," and an example of scored solutions to this anagram (see Appendix B). Complementary oral instructions
were used by E throughout each session to clarify questions and control the progress of the experiment closely (see Appendix C).

Subjects worked for 6 minutes on each problem and had a 1½-minute scoring interval after each problem. At the end of the scoring interval of the final problem, E instructed each particular group to describe, in a few sentences on the back of their test booklets, the effect, if any, that the points had on their behavior with the problems.

Immediately following each testing session, the booklets for that group were inspected and any Ss failing to meet the criteria for solution acceptability, as stated in Appendix D, or who did not apply the scoring rules properly, were rejected from the sample. New booklets matching the reward condition and sequence of problems for any such reject were then made up and inserted randomly into the set of booklets scheduled for use in the testing session to follow. This procedure was necessary in order to maintain approximately equal N's in terms of reward groups and problem sequences as the experiment progressed.

Results

Total frequency of solutions, frequency of 1-, 2-, 3-letter solutions and frequency of 4-, 4+ -letter solutions was determined for GARDEN (always Position 1) and for the five remaining practice positions. Respective means for each of these three measurements are listed separately in Tables 2, 3, and 4 on pages 72-74.

Four other measurements of solution production in the form of ratios were calculated utilizing the solution frequencies in the following manner:
1. Ratio of 1, 2, 3 to total solutions
2. Ratio of 4, 4+ to total solutions
3. Ratio of 1, 2, 3 to 4, 4+ solutions
4. Ratio of 4, 4+ to 1, 2, 3 solutions

Means representing the sum of the individual Ss' ratios divided by N (40) are presented for each of the four ratios listed above, separately in Tables 5, 6, 7, and 8, pages 75-78.

The means for each of the seven solution performance measurements listed above were plotted as a function of practice (positions). These figures (1-7) are given in Appendix E. From a consideration of the above figures, it was apparent that the relationships among reward groups were extremely complex during acquisition of this task for all seven indices used. Comparisons of performance under one particular reward level to performance under another at each position of practice, or differences between classes of solutions for separate conditions of reward, were considered not meaningful with the relatively small number of individuals in each group (N = 40). To determine these effects more precisely, the data were combined in two basic ways: (1) all reward groups for each of the two solution classes were pooled at each position of practice, e.g.:

\[
\left[ \frac{\sum (1-5) + (1-10) + (1-20) + (1-50)}{N} \right]
\]

for 1, 2, 3, and 4, 4+ solutions,

\[
\left[ \frac{\sum (5-1) + (10-1) + (20-1) + (50-1)}{N} \right]
\]

for 1, 2, 3, and 4, 4+ solutions,

\[
\left[ \frac{\sum (1-1) + (50-50)}{N} \right]
\]

for 1, 2, 3, and 4, 4+ solutions at all positions.

This procedure was applied to each of the measures and gives means for the high-reward and low-reward performance for each specific solution class.
combined over all appropriate reward groups. A summary measure was then obtained from these means by pooling the more highly rewarded solutions, regardless of class, over all differential reward conditions; e.g., the means of the $4^+, 4^+$ (long) solutions receiving higher reward, over all reward groups. For a graphic representation of these measures, see Figures 8, 9, 10, 12, 13, 15, and 16.

Similarly, the ratio measure (1) and (2) above, and (3) and (4) above were also combined, giving the curves plotted in Figures 11, 14, and 17 respectively.

The second basic combination of data was done as follows:

(2) pooling of all measurements in one particular reward group for each solution class over the last four positions of practice; e.g., the sum of the $(1-5)$ scores over $N$ for the short solutions for Positions 3 through 6; \( \frac{\varepsilon (1-5)}{N} \) for the long solutions for Positions 3 through 6; \( \frac{\varepsilon (1-5)}{N} \) for short solutions, Positions 3 through 6; \( \frac{\varepsilon (5-1)}{N} \) for long solutions, Positions 3 through 6, etc. All measures except total solutions were pooled in this fashion, yielding the curves plotted in Figures 18, 19, 21, 22, 24, and 25.

A summary performance measure corresponding to that related to (1) above was obtained by pooling the more highly rewarded solutions, regardless of class, over the last four positions of practice for each
condition of differential point reward. Thus the short and long solutions were pooled over Positions 3 through 6, the short-to-total and long-to-total solutions over Positions 3 through 6, and finally the short-to-long and the long-to-short solutions over Positions 3 through 6. Respective means for these are shown by the curves in Figures 20, 23, and 26.

Replies to the question asked Ss concerning their subjective estimate of any effect the points had on their behavior with the problems were judged and broken down into three basic categories: (1) no effect, (2) unspecified effect (i.e., "The points made me try harder on each problem"), and (3) definite effect (i.e., "I tried to get more short solutions because they were worth more points"). Results of this breakdown are shown in Table 9.

Since the variance within groups of Ss at each position of practice is not homogeneous due to the rotated problem sequence and distributions of ratio measurements typically show marked skewness, a parametric statistic could not be validly applied to these data. Accordingly, tests of Hypothesis 1 were performed utilizing a special case of the binomial test. An example to clarify the application of this test is explained below: Consider Hypothesis 1 as it pertains to Figure 9 (i.e., that solutions receiving the higher reward will show the same relative frequency as solutions receiving lower reward), and
its alternative (i.e., that high-rewarded solutions will show different frequencies from low-rewarded solutions). Conceived in another way, Hypothesis 1 can be restated in the following manner: (a) performance differences between high- and low-rewarded solutions at each trial position will distribute randomly, against its alternative; (b) that these differences will exhibit some systematic pattern of distribution over trial positions. It is clear, then, that a test which determines the probabilities associated with the occurrences of certain obtained patterns of differences is needed.

Assume, for purposes of illustration, that Hypothesis 1 is true for the differences between high- and low-rewarded groups at each trial position (Figure 9). Let \((Y)\), then, be the difference between groups at Position 2. Let \((P)\) be the probability of the event that \((Y)\) will be equalled or exceeded on \(x\) number of occasions, and \((Q)\) be the probability of the event that \((Y)\) will not be equalled or exceeded on \(x\) number of occasions. Hence, we have two distinct categories of events that the differences in performance between the top and bottom curves at each position in Figure 9 can logically fall into. Because of this imposed dichotomy, and the fact that the pooled scores represent independent groups of Ss for high- and low-reward performance, the major assumptions of the binomial test are met. If the \(H_0\) (no systematic pattern of differences) is true for the differences in the two curves at Positions 2 through 6 (Figure 9), we expect that \((Y)\) would be equalled or exceeded about as many times as it would not be equalled or exceeded. Thus \(P = Q = \frac{1}{2}\), or .50. If \(H_1\) (the alternate hypothesis) is true, then \(P\) and \(Q\) will deviate from .50 in certain degrees, depending upon the particular distribution of differences obtained.
To illustrate this in detail, consider the distribution of differences between the high and low curves of Figure 9. As a conservative test of Hypothesis 1, let (Y) be the performance difference at Position 2, represented by the ordinal distance between the high-reward groups (open circle) and the low-reward groups (closed circle). Utilizing this (Y) value, we can test the null hypothesis (Ho) that P = Q = .50 against its alternative (HI) that P < Q or Q < P. Since we have predicted in advance that (Y) will be equalled or exceeded significantly more often than not, a one-tailed binomial test is applied to the five differences. Note that all five equal or exceed (Y) in Figure 9, and to determine the significance of this value, we need only apply the formula for the sampling distribution of the binomial (Siegel, 1956) given by

\[ \sum_{i=0}^{X} \binom{N}{i} P^i Q^{N-i} \]  

In other words, we sum the probabilities of the observed value (x) with the values even more extreme. For the data above, P = .031, and we can reject the Ho at the .05 level of confidence and accept instead HI, which supports the prediction that more highly rewarded solutions will show greater relative frequencies than low-rewarded solutions over practice.

The binomial test was utilized in the manner described above for all tests of Hypothesis 1 as it relates to practice effects (Figures 9-17). Utilizing the same rationale, it is also possible to test Hypothesis 1 as it relates to differential reward effects (Figures 18-26). Consider Figure 22: Let Y be the difference between the ordinate values of the high-rewarded and low-rewarded groups at the 5-point abscissa value. If the Ho was true, P will equal Q at a value of .50
each, which in this case means that the differences between groups, relative to $Y$, will show no systematic variation with increasing magnitudes of point differential. Note that all four differences between the high and low groups are equal to or greater than $Y$, an event which could occur by chance about 6.2 times out of a hundred, or $P = .062$. Hypothesis 1 cannot, therefore, be rejected at the .05 level, although the trend of differences is in the predicted direction.

The likelihood of a type II error is obvious, and the fact that this test is limited by $(N)$ would require additional levels of point differential in order to obtain smaller probabilities. Notwithstanding this limitation, the above test was exacted as the most powerful available which would be sensitive to any systematic pattern of differences between points.

To summarize the results of the binomial test on Hypothesis 1, the reader is referred to Table 10, where probabilities ($P$) are listed for practice effects (Figures 9-17) and differential reward effects (Figures 18-26), measurements 1 through 9, inclusive. Inspection of this table reveals that, in general, Hypothesis 1 can be rejected at the .05 level for all differences between reward groups over practice and at the .07 level for differences between reward groups over increase in magnitude of point differential.

Testing of Hypothesis 2 was also restricted to a nonparametric technique, since it refers to an expected rank-order of magnitude of the effects of different experimental treatments and the customary
one-way analysis of variance does not satisfy this demand; the F-ratio is independent of the order in which the group means occur. Furthermore, the rotated problem sequences violate the assumption of homogeneity of variance among reward groups at each position of practice. A test which avoids these and other complications common to parametric statistics is a distribution-free k-sample test against ordered alternatives developed by Jonckheere (1954). The Jonckheere test supposes that, on each of \( n \) occasions, any one of \( k \) events may occur, and we can test the hypothesis that the \( k \) events occur randomly in the series of \( n \) occasions against the alternative that they occur in a particular ordered sequence.

Hypothesis 2 states that rates of the various reward groups will be ordered at random with respect to point differential. We need make only one major assumption in order to apply the Jonckheere test to Hypothesis 2: that the values representing mean performance for a particular solution class at various levels of a point differential reward be obtained from independent groups. This assumption is satisfied for the data corresponding to each of the 12 curves, considered separately, in Figures 18, 19, 21, 22, 24, and 25. A numerical example using the data from the \( 1-5, 1-10, 1-20, 1-50 \) reward groups for Positions 3, 4, 5, and 6 clarifies the application of this test; the upper curve in Figure 25 representing the means for these groups plotted over increasing magnitude of point differential.

Let the statistic

\[
S = 2 \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \frac{\pi_{ij} - \frac{k}{n}}{\pi_{i} \pi_{j}} \sum_{m=1}^{n} M_{i}^{m} M_{j}^{m}
\]

(2)
Reordering the data in the following manner will facilitate the computation of $S$:

<table>
<thead>
<tr>
<th>Positions of Practice</th>
<th>Reward Groups</th>
<th>1-5</th>
<th>1-10</th>
<th>1-20</th>
<th>1-50</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.387</td>
<td>2.244</td>
<td>1.880</td>
<td>1.685</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.958</td>
<td>1.962</td>
<td>2.103</td>
<td>1.587</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.440</td>
<td>1.615</td>
<td>2.362</td>
<td>1.984</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.814</td>
<td>1.822</td>
<td>2.524</td>
<td>2.104</td>
<td></td>
</tr>
</tbody>
</table>

Hypothesis 2 can be stated in the following form (Ho): the five samples have come from the same population, against the alternative (H1) that the populations are such that the values from the samples 1-5 through 1-50 (I-IV) are in an expected order of increasing magnitude. For the computation of $S$ we have $M_1=M_2=M_3=M_4=4$, $k=4$, $P_{12}=13$, $P_{13}=15$, $P_{14}=12$, $P_{23}=13$, $P_{24}=8$, and $P_{34}=3$. Using (2) above, $S=2 \times 64 - 96 = 32$.

To summarize the computation of $S$: the samples are ranged in the order implied by the alternative hypothesis (H1); and for each sample in turn, we determine for each value the number of items which are larger in all succeeding samples. This gives the sums of the values $p$, $i$, and $j$, which can then readily be applied to formula (2) above. The probability of the $S$ obtained for the data above is equal to or greater than .0751, and hence H1, that the samples came from populations which were stochastically ordered in the series I, II, II, and IV cannot be accepted at the .05 level, and the Ho must be retained.

The Jonckheere test was applied to the appropriate values of Tables 3 through 8 for the upper curves of Figures 18, 19, 21, 22, 24, and 25, respectively. The probabilities of the obtained $S$ values for these six tests are shown in Table 11.
None of the p values are small enough to warrant rejection of Hypothesis 2 at the .05 level, although the means for the four reward groups tend to increase as predicted for the ratio measures (Figures 21, 22, 24, and 25) in a fairly consistent manner, with the notable exception of the 50:1 level of point differential, where in Figures 22 and 25, the mean solution performance drops off to a value somewhat below that of groups at the 20:1 differential.

Discussion

The most salient features of the data presented here relate to Hypothesis 1: for practice effects, the binomial test yields probabilities which lead to rejections of the Ho for all of the relevant indices of solution production, at the .05 level of confidence, and it is concluded that solution performance varies markedly between classes as some function of differential point reward. The data in general exhibit some interesting characteristics concerning practice effects, from a consideration of performance differences between high- and low-reward groups, as measured by the various indices plotted in Figures 8 through 17. Figure 8 shows that total solution production between groups receiving high reward for short solutions (open circles) and groups receiving high reward for long solutions (closed circles) does not differ significantly (binomial test; $P = .188$) over practice, although the upper curve maintains a consistently higher ordinate value over the lower curve, a trend which is reversed at Position 6. While not statistically significant, this trend may be related to certain
performance differences noted between solution classes in Figures 8 through 17, in quite an unexpected way. To illustrate, consider Figure 9. The three curves display fairly obvious consistency in short-solution performance over practice: means for groups getting high reward (upper curve) increase steadily from a value of 7.308 at Position 1 to a high of 10.201 at Position 6; means for groups getting equal reward (middle curve) also increase steadily but less markedly, from a value of 7.212 at Position 1 to a high of 8.287 at Position 6. On the other hand, the lower curve shows that, where groups are receiving the low (1-point) value of the point differential for short solutions, performance actually decreases slightly from an initial value of 7.419 at Position 1 to a final value of 7.162 at Position 6.

A similar comparison of high, middle, and low curves of Figure 10 reveals an obvious contrast in the performance of groups on long solutions (lower curve) receiving the low (1-point) value of the point differential; whereas short-solution performance was shown to decrease slightly under the effect of low reward, long-solution performance increases steadily from a mean low of 4.987 solutions at Position 1 to a high of 7.214 at Position 6, under the same low reward.

This reward-effect contrast between classes of solutions where amount of reward is equal poses a considerable theoretical problem. The present writer proposes, however, that such an effect is an important feature of the human problem-solving process under these conditions, and that an analysis of the determinants of this effect may gain momentum in the light of the following interpretation: recall that total solutions performance tends to be higher for groups getting
the higher point reward for long solutions over the majority of practice intervals (Figure 8). This fact, along with the relations described above concerning contrasting reward effects between solution classes, pressed the writer into a closer look at Ss' replies to the question on the effects of points. A number of statements by Ss in certain categories, along with the relevant performance differences noted above, prompted a tentative explanation of how reward may affect problem-solving behavior for short solutions in a manner somewhat different from long solutions. Inspection of Table 9 reveals no clear trend in frequency of "no effect," "general effect," and "definite effect" replies among the various reward groups. In fact, where short solutions receive higher reward, the frequency of Ss who report that they strived for more short solutions (i.e., "definite effect") is slightly greater than (106 to 95) that for Ss reporting in the same fashion when rewarded more highly for long solutions. It can be concluded, then, that the decline in a short-solution performance over practice ("suppression effect") demonstrated by the data is something that Ss are probably not directly aware of, as the replies of Table 9 suggest. Replies which were deemed quite significant came from Ss in conditions where short solutions got higher reward, falling in the "general effect" category of Table 9. These statements were typified by one reaction--these Ss felt that their efforts to obtain long solutions rather than short solutions should have been rewarded more highly! Many Ss in the other two categories of Table 9 (high reward for short solutions) also gave supplementary statements about their feelings as to which class should have been rewarded more highly, oftentimes in contradiction to their initial statements as to what they actually tried to do on the task.
This general reaction may be conceptualized as evidence for an "implicit reward" factor inherent in the solving of a certain aspect of a problem requiring the use of more complex (long solutions), hence more satisfying and rewarding, strategies than used to solve simpler (short solutions) aspects of the problem. In terms of these data, the possibility exists that the solving of anagram problems under conditions where longer solutions are explicitly rewarded more highly than shorter ones is further enhanced for the longer solutions by reward inherent in the more satisfying (in the sense of greater felt accomplishment) strategies used to discover them. This enhancement is thought to be independent of the effects of point-differential reward. Attendant to the "implicit-reward" concept, the possibility also exists that the "suppression effect," described above, noted where short solutions receive a lower reward is due to the relative absence of "implicit reward" inherent in Ss' strategies for producing short solutions. Consequently, a differential-point reward enhances solving performance for highly rewarded short solutions and decreases it for lower rewarded short solutions because "implicit reward," although perhaps present in some degree, does not counteract the "suppression effect" of differential points enough to facilitate performance over practice. Long-solution strategies, on the other hand, possess greater degrees of "implicit reward" inherent in their successful application, and this is sufficient to overcome any "depression effects" contingent upon differential points. Hence, low-rewarded long-solution performance increases, whereas low-rewarded short-solution performance is unaffected, perhaps even drops somewhat, over practice with these problems.
It can be argued that the greatest percentage of increase in performance where long solutions receive the lower value of the point-differential reward occurs between Position 1 and 2 (see Figure 10) and, therefore, that the facilitation noted in these groups is due not to "implicit reward" inherent in strategies involving more complexity, but rather is due to the possibility that these strategies are not developed, because of their complexity, until the differential factor comes into effect at Position 2 (it will be remembered that at Position 1, the various groups are all working under equal 1-point reward for short and long solutions). This possible factor may surely have some relevance, probably in determining the magnitude of the "implicit reward" effect, but is not considered a completely satisfactory explanation in view of the fact that long-solution production rises quite steadily even after Position 2 (lower curve, Figure 10), whereas short-solution production decreases somewhat between Position 2 and 6.

Additional support of the "implicit reward" interpretation is offered by the data shown in Figures 12 and 13, and 15 and 16. These curves represent ratio measures which are somewhat more sensitive to the effect being considered because they involve relative frequencies between short, long, and total solutions. Again, Figure 12 shows that short solutions decrease markedly in frequency over practice relative to total solutions when receiving the lower reward, whereas long solutions, relative to total solutions, show an overall steady rate of production (Figure 13). Final support is offered in an alternative way by the rates of the upper curves in Figures 15 and 16. It can be seen by inspection of Figure 15 that, even where short solutions are receiving the high-point values, their
incidence relative to the frequency of long solutions in appropriate groups increases only slightly. Figure 16 presents a markedly different picture—performance for groups receiving high reward for long solutions increases steadily with far greater improvement over practice. This result can be expected with this particular ratio index of performance if the interpretation here is essentially correct, in view of the fact that a differential point reward, combining with implicitly rewarding factors inherent in complex strategies behavior with long solutions, enhances performance for these groups and may attenuate concentration on "simpler strategies," even though they are rewarded by more points.

Interpretation of Hypothesis 1, as it relates to increases in differential reward over groups can be generally rejected at the .07 level of confidence (see Table 10). The conclusion that can be drawn from the above is that, while tests were limited to the .07 level because $N$ in the binomial test has an upper limit of 4 for these data, the various groups receiving the higher point reward for a particular solution class perform at a higher level due to the effects of differential-of-point reward than groups getting the lower value.

Hence, Hypothesis 1 is generally rejected for both practice and differential reward effects (the latter being group effects), and the alternative hypothesis that performance on solutions receiving the high value of a point differential reward is greater than performance on these solutions when they receive the low value of the differential must be accepted instead.

Hypothesis 2 states that "rates of the various reward groups will be ordered at random with respect to point differential for that class
of solutions receiving increasing reward." A limitation to be noted at the outset in interpreting the data depicted by the upper curves of Figures 18, 19, 21, 22, 24, and 25, as they relate to this hypothesis, is that these curves show group functional relations and do not necessarily represent the changes in rate which might be observed if an individual S were successively administered the different levels of point differentials. With this limitation in mind, we may proceed to some tentative conclusions concerning the size of the point differential as an important determinant of solution performance.

Results of the Jonckheere test (Table 11) show that only measures 4 and 6 approach significant P values \( p > 0.08 \). For each of the four ratio indices (3-6), the trend of group means is in the direction predicted from the alternative to Hypothesis 2 (Figures 21, 22, 24, and 25), but rejection of Hypothesis 2 in general is not warranted. That the means of the groups are ordered in a pattern whose rate is determined by the size of the differential reward is not clear in the stochastic sense; but, indeed, it is of some significance that in Figures 22 and 25, the group means for the 50:1 ratio of reward groups show the only values which do not exceed all prior group means at smaller ratios. It is probably that the size of the point differential as a determinant of performance on these problems has some limiting value lying somewhere between the ratios of 20:1 and 50:1. Furthermore, the decrease for the 50:1 groups in Figures 21 and 24 may be due to the fact that a 50:1 differential exceeds this limiting value, causing Ss to significantly alter their strategies where this differential is in effect. One possible factor in such an alteration of strategies could
conceivably involve the "suppression effect" described earlier, with point reward and "implicit reward" inherent to the type of solution strategy interacting in some manner which attenuates performance at extreme point differentials, while having a facilitating effect at differentials within a certain optimum range.

To conclude the discussion of Hypothesis 1 and 2, the former can be generally rejected, with higher rewarded solutions more frequently produced than lower rewarded solutions over practice and among reward groups, while the latter must be accepted in general—some order effects being noted due to level of reward independent of practice but manifested in a way not predictable from the stochastic model referred to.

The prediction that the function describing performance as it relates to different sizes of a differential point reward will be negatively accelerated for that class of solutions receiving the increasing amount of reward, is not ideally borne out by the data for all appropriate curves. Figures 21, 22, 23, 25, and 26 show curves (upper) which fit the predicted function for a majority of points, but all show at least one reversal in rate at some point. It is concluded by this writer that the above prediction is by no means invalidated by the data, but is probably sound if utilized only with respect to the optimum range hypothesized earlier to lie somewhere between the 5:1 and the 50:1 reward differentials. It is obvious that further research will be necessary to determine this range.

How does all this relate to human problem solving in general? Of primary importance on this question is the successful demonstration
in this study of the application of a methodology which allows for rigorous control, predictions, and testing of hypotheses pertinent to performance on a standardized problem-solving task. This result has not, in the opinion of this author, been heretofore established in an adequately systematic manner. The present study seems to fill this initial gap in the literature with reasonable success. Equally important are the interpretations given these data. Human problem solving is obviously a complex process, but it seems evident from the results of this study that certain fundamental principles of learning do operate in problem solving of this kind in ways which are analogous to their role in simpler learning situations. Magnitude of reward is clearly shown to be a variable affecting behavior with anagrams in (quantifiable) ways similar to its effects on behavior in less complex learning tasks. Certainly, the magnitude of reward effect is prone to unique interaction with other elements of the task, as shown by the way in which performance varies, over practice, between two different solution classes treated alike in amount of reward, but the main effect remains unaltered upon comparison to consequences produced by different amounts of reward in simpler situations. Specifically, responses followed by higher reward will occur more often, in general, than responses given a lower reward, other things being equal. Such is the case in this study, as is systematically verified by the method described.

The particular comparative approach outlined above has explanatory limitations, however, and a more refined analysis of the problem is warranted in the light of some of the data of the present study. This analysis relates to the complex processes that no doubt occur which are relatively independent of the simpler effects of increases in amount of
reward. Strategies of Ss were shown to change systematically when operated on by a differential reward factor, but in ways that suggested the simultaneous operation of another unexpected reward variable—that of "implicit reward." The concept of "implicit reward" could be a valuable guide to further research on these changes. Aspects of problems which exhibit high degrees of "implicit reward" may be responded to in entirely different ways when rewarded explicitly from ways in which other aspects, relatively unaffected by this factor, are responded to under the same explicit reward. The research described here has possibly set up some useful criteria as a guide toward a more systematic and rigorous attack on the problem of the processes involved in human problem-solving behavior.

 Suggestions for further research. The purpose of this study seems adequately fulfilled by the data presented: the effects of four levels of a differential point reward were determined on two classes of anagram solutions, and the problem of the need for basic data concerning the general effects of amount of reward in a human problem-solving situation has been systematically attacked in a manner successful enough to warrant further application of the general method described. With this in mind, perhaps the most pressing problem elucidated by these findings is the optimum range of point differentials of reward which can be employed as a determinant of anagram performance. Thus, a study to determine this range is the next most logical step in a program of systematic investigation concerning the variable of differential reward. It should be added that this range can be expected to vary with respect to solution class, a prediction following from the interpretations given the data of the present study.
Assume that such an investigation determines quite accurately the maximum size of point differential which will produce an increase in performance over lower sizes for both short- and long-solution classes. Granting such an accomplishment, the present writer wishes to propose a design which could bring data to bear on the following problems: (1) strategy behavior with anagrams; (2) the effects of a shift in reward; (3) the "implicit reward" effect, as well as on other pertinent aspects of problem solving in general. The proposed experiment: Consider a factorial design employing four main groups of Ss, with two of the groups being differentially rewarded in one way and the other two groups in an opposite way. One of the first two groups (I) receives a maximum point reward for short solutions and a low reward for long solutions, practicing until asymptote. The other of the first two groups (II) receives the maximum number of points for long solutions and a low value for short solutions, also practicing until asymptote. At this point, the groups are subdivided, one portion of group I continuing under the same point differential (group Ia), and the other subgroup switching to the differential which group II received during practice (group Ib). Similarly, group II is divided, one portion continuing practice under the same differential (group IIa), and the other subgroup (group IIb) switching to the differential originally given group I. The performance measure for these four groups is frequency of short solutions, and the following predictions are made:

1. Strategies will show significant modifications after the shift in reward is introduced (i.e., successful strategies will tend to occur more frequently for solutions receiving the higher reward).
2. An abrupt change in performance will occur for both of the reward-shift subgroups; short-solution production increasing rapidly following an increase in reward and decreasing in like manner following a decrease in reward.

Groups III and IV receive the same treatments exactly. The reason for these two groups is to obtain an independent estimate of production of long solutions under these conditions. Predictions:

1. Strategies will show less modification following reward shift due to the effects of "implicit reward." Long-solution strategies, in other words, will probably be quite evident during preshift practice, even for the group receiving low reward for these solutions because of the inherent reward in their successful application. Hence, a shift to higher reward will not increase their frequency for that class as much as it is expected to for relatively "implicit reward-free" short-solution strategies.

2. The change in long-solution performance for the subgroup shifted from low to high reward (group IVb), although abrupt, will not cover as great an ordinate distance as the corresponding change occurring for group IIb, because the enhancing effects of "implicit reward" inherent to long-solution strategies should cause the difference between groups III and IV to be less during practice than that for groups I and II. Furthermore, the group shifted from high reward to low reward will show a relatively slow decline in long-solution performance due to the compounded effect of the point reward and "implicit reward" operating together with the continuing effects of the latter perseverating even after the shift in differential.
Summary

This experiment was conducted to study the effects of several levels of a differential point reward on two classes of solutions to the Ammons Standard Anagram Task. These effects were analyzed in relation to (1) practice and (2) magnitude of reward given one solution class as compared to a different magnitude given the other class. Two major hypotheses tested were:

1. The relative frequency of a solution class receiving the upper magnitude of a differential point reward will be the same as that of a solution class receiving the lower magnitude of reward.

2. Rates of the various reward groups will be ordered at random with respect to point differential for that class of solutions receiving increasing reward.

Eight experimental and two control groups practiced for 36 minutes on the Ammons Standard Anagram Task with the experimental groups receiving differential reward for two types of solutions and the control groups equal reward for these same solutions. The binomial test was applied to the performance differences of these groups relative to Hypothesis 1, resulting in its general rejection.

Hypothesis 2 could not be rejected, even though mean trends favored its alternative, while not being statistically significant, as determined by the Jonckheere k-sample test against ordered alternatives.

A prediction made that the relation between performance on anagrams and increasing magnitudes of a differential point reward would be a negatively accelerated curve was not ideally supported by the data, but
performance curves depicted the predicted function at an appreciable number of points to warrant further investigation of this relationship.

Interpretations of the findings relative to the general problem of the analysis of human problem-solving processes were given, and proposals for further research based on predictions utilizing the results of the present study were outlined.
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Weiss, R. F. Deprivation and reward magnitude effects on speed throughout the goal gradient. *J. exp. Psychol.*, 1960, 60, 384-390.

APPENDICES
In order to evaluate the possible bias of unequal availability, hence, the disproportionate effects of rewarding short and long solutions differentially, a frequency tabulation of total solutions by class was performed on 11 anagrams. Six had been used in the Robertson and Ammons study and five in the Ammons and Ammons study. It was found that, on the average, there is about the same incidence of long solutions as short solutions for the anagrams used in the first study mentioned above. Comparing the plots of total solutions (short plus long) by different groups revealed a great deal of variability between groups during acquisition but no indications that final level of performance is being affected by a contrast of reward in any consistent direction. The tabulation of the second group of anagrams revealed, however, that short-solution incidence was approximately twice that of long for each letter combination. The total solutions production was increased only slightly, however, for some groups, and was not greatly different from solutions production of equal reward groups.

To ensure at least an approximate proportionality in reward effect as a function of incidence of solutions within a class, the five anagrams with the most nearly equal totals of short and long solutions were chosen from the set of 11. They are as follows: GARDEN (control), YANERL, KERCHDAU, DWILBAEN, IPECVNRO, and UCENIDOR.
APPENDIX B

SAMPLE TEST BOOKLET FROM THE 1-10 REWARD CONDITION
WORD CONSTRUCTION GAME

This is a game in which you will construct words out of a basic letter combination which you will have in front of you while you work. After a few minutes with each letter combination, you will be given a short rest, then you will work on a different letter combination. As you work, you will turn over the pages so that the previous letter combination you have worked on will not be visible.

The rules you should follow are these:

1. Use any number of letters you wish out of the basic letter combination—from one to as many letters as there are in the letter combination.

2. Use each letter only once in a given word. Of course, you can construct many words using the same letter once each time as a part of each single word.

3. Construct only English words. Foreign words do not count. Neither do prefixes or suffixes (e.g., "pre-" or "-ing").

4. Construct no proper nouns, that is, no name whose first letter would be capitalized.

5. A basic word is counted only once; e.g., "bag" and "bags," or "cut" and "cuts" would count only once. An improperly spelled word is not counted, and neither are abbreviations and contractions.

Try the following letter combination: MDEA
Some of the words you could make would be: A, MAD, MA, DAM, DAME, and ME. DE would not be usable under the rules because it is a foreign word meaning "of" in several languages, and not an English word. MAE also would not count, since it is a proper noun—the name of a specific girl,
which name would always have the first letter capitalized. You could not use MADAM because that would mean that you were using the letters "m" and "a" twice in the same word. Remember, use each letter only once in each word you construct, use no proper nouns, use no foreign words, and use either singular or plural, but not both. These words would not count, and would just slow you down. Print the words you construct, starting under the word "Solutions" at the upper left of the page.

Do not turn a page until you are given the signal to do so.

Scoring Rules

We are investigating the effects of variation in rules for playing this game. It will help us a lot if you will score your own solutions. At the bottoms of the pages, you will find the scoring instructions. Also, you will be informed before you start on each letter combination what the rules are for scoring your solutions to that particular letter combination.

Here is an example. The person playing the game was asked to make words out of MDEA. His scoring instructions were to give 1 point for each acceptable solution 1 or 2 letters long, and 2 points for each solution 3 letters long or longer. He found the following solutions:

MDEA

  A
  MAD
  MA
  DE---
  DAM
  MAAE--
  DAME
  ME
  MADAM--

Scoring formula:

  1 or 2 letters = 1 point X (3) = 3
  3 and 3+ letters = 2 points X (3) = 6

TOTAL POINTS = 9
This is the way he scored these solutions. First he crossed out DE, which is a foreign word, then MAE, which is a proper noun, and finally MADAM, which uses M and A twice (i.e., more times than they appear in the basic letter combination MDEA). There were three 1- and 2-letter solutions left (A, MA, ME). He wrote this number 3 in the scoring formula and multiplied out "1 point X (3) = 3". His next step was to count the solutions which were 3 and 3+ letters long. He found 3 (MAD, DAM, DAME), and entered this in the scoring formula "2 points X (3) = 6". Finally, he found the total points for this basic letter combination by adding the 3 and 6 together, a total of 9.

Suppose we had the same solutions but a different scoring system. See if you can follow it this way:

Scoring formula:

1 or 2 letters = 2 points X (3) = 6
3 and 3+ letters = 1 point X (3) = 3

TOTAL POINTS = 9

We will use several different point systems, but all will be used in the same way. Please follow directions very carefully.

TRY TO MAKE A HIGH SCORE EACH TIME.
Basic letter combination:

GARDEN

Solutions:

Scoring formula:

1, 2, or 3 letters = 1 point \times ( ) =

4 and 4+ letters = 1 point \times ( ) =

TOTAL POINTS
Basic letter combination:

YANERL

Solutions:

Scoring formula:

1, 2, or 3 letters = 1 point \( \times () = \)

4 and 4+ letters = 10 points \( \times () = \)

TOTAL POINTS
Basic letter combination:
KERCHDAU

Solutions:

Scoring formula:

1, 2, or 3 letters = 1 point \( X ( ) = \)

4 and 4+ letters = 10 points \( X ( ) = \)

TOTAL POINTS
Basic letter combination:

DWILBAEN

Solutions:

Scoring formula:

1, 2, or 3 letters = 1 point \times ( ) =

4 and 4+ letters = 10 points \times ( ) =

TOTAL POINTS
Basic letter combination:

IPECVNRO

Solutions:

Scoring formula:

1, 2, or 3 letters = 1 point \( x ( \) =

4 and 4+ letters = 10 points \( x ( \) =

\[ \text{TOTAL POINTS} \]
Basic letter combination:
UCENIDOR

Solutions:

Scoring formula:

1, 2, or 3 letters = 1 point X ( ) =
4 and 4+ letters = 10 points X ( ) =

TOTAL POINTS
DIRECTIONS FOR ADMINISTERING THE STANDARD ANAGRAM TEST

1. Before Ss enter room, E lays out booklets in a systematic order, face down.

2. Ss are brought into testing room, and E takes people from "end" of booklet sequence and fills up all gaps earlier in the sequence of booklets, keeping them in order, and puts them on top of the remaining pile to be used with the next group.

3. When all Ss are seated, E instructs the group as follows:
   "Now turn your booklets over. Please put the following information at the top right of the first page, the instructions sheet—Name (print), Class identification (class from, section, instructor), Age to last birthday, Date, School term." Tester writes these items on board, and as soon as about two-thirds of the Ss are finished, tester says:

4. "Now, please read the instructions on the top page, headed WORD CONSTRUCTION GAME. When you are through, will you please look up, so I can tell when to go ahead with the next part. Do not turn any page until I give the signal." E waits for group to finish, and says:

5. "Now, listen carefully. We are studying the effects of various numbers of points on performance of this game. You will not be competing with each other, but each person only with himself. So that you can keep track of how you are doing, you will score your own problems. We don't expect you to do this perfectly, but you need to do it well enough so that you can judge your own learning. We will rescore all the papers later."

6. "Now turn the page, and read SCORING RULES. These are only samples, and what you want to find out is the method or procedure in
scoring. No one will have these exact point values. Please look up when you are through. Do not turn the page until you are asked to.

7. When the group is finished, E then says: "Let me summarize what you are going to be doing. You will take a set of letters and make as many English words as you can from them. Proper nouns won't count, and you can't use a letter more than once in each solution. When I say 'stop,' you will count up how many of each kind of solution you have, write these numbers in the little table at the bottom right of the page you have been working on, and figure out your score. Then you will be ready to start on the next problem when I give the signal. Any questions?"

8. "Write the solutions in columns, starting on the left on the page. As you are working, from time to time, I will call out a number. I want you to write down the number in the middle of the column, like this (E demonstrates). Sometimes you will not find any solutions between numbers, but write the next number called out anyway."

9. "Remember try to get as many points as you can. You are working against your own record each time. Now, turn the page, look at the scoring formula, and begin."

10. At the end of 1 minute, E says: "Write down a 1 and keep on going." At the end of each of the next 4 minutes, E calls out 2, 3, 4, and 5 respectively. At the end of the sixth minute, E says: "Stop, draw a line, and score. Work quickly, since you only have a short time."

11. After 1½ minutes of scoring on the first problem, E says: "Now, let's go to the next problem. Turn the page, look at the scoring formula, and begin. Try to get as many points as you can."
12. After the scoring interval for problem 6, E says: "Now, please go through the whole booklet and print your name in the upper right-hand corner on each page."

13. After almost all Ss have finished the above, E says:
"Now, we need just one more thing which will help us a lot in understanding the effects of the points. Please turn over the booklet and answer a question in two or three sentences, at most. The question is: 'In what ways, if any, did the points affect your behavior with the problems?'. Tester repeats the question.

14. After Ss have finished answering the above question, E says: "Thank you very much for helping us out. You may leave now; please leave the test booklets at the desks."
APPENDIX D
REJECT CRITERIA FOR THE AMMONS STANDARD ANAGRAM TASK

Any Ss having one or more of the following irregularities in his data, as agreed upon by at least two judges, were rejected from the sample:

1. Improper scoring of at least two anagrams. Indications of: (1) lack of understanding of point rewards; (2) emotional disturbance; (3) extreme uncooperativeness.

2. Consistent violations of instructions (using foreign words, one letter more than once, etc.) on (a) all six problems; (b) three or more on three or more problems, except with high performer who improves.

3. Obvious misinterpretation of procedure (copious display of nonsense syllables, production of solutions not related to basic letter combinations at top of each page, etc.).


Note: Webster's New Collegiate Dictionary was used as the reference for scoring all solutions.
APPENDIX E: FIGURES 1 - 7
Figure 1. Total solution production as a function of practice (positions) based on means of separate reward groups, e.g., $\frac{\sum_{i=1}^{5} x_i}{N}$ where $N = 40$. 
Figure 2. 1-, 2-, and 3-letter solution production as a function of practice (Positions) based on means of separate reward groups.
MEAN OF INDIVIDUALS 1, 2, 3 SOLUTIONS
Figure 3. 4-, 4+ -letter solution production as a function of practice (Positions), based on means of separate reward groups.
MEAN OF INDIVIDUALS' 4,4+ SOLUTIONS
Figure 4. Solution production, measured as a ratio of 1-, 2-, and 3-letter solutions to total solutions, as a function of practice (Positions). Based on means of separate reward groups.
Figure 5. Solution production, measured as a ratio of 4-, 4+ -letter solutions to total solutions, as a function of practice (Positions). Based on means of separate reward groups.
MEAN OF INDIVIDUALS' RATIOS OF 4, 4+ SOLUTIONS TO TOTAL
Figure 6. Solution production, measured as a ratio of 1-, 2-, and 3-letter to 4-, $4^+$-letter solutions.
Figure 7. Solution production, measured as a ratio of 4-, 4+-letter to total solutions, as a function of practice (Positions). Based on means of separate reward groups.
MEAN OF INDIVIDUAL RATIOS OF 4, 4+ TO 1, 2, 3 SOLUTIONS
Table 1

Differential Point Rewards for Eight Experimental and Two Control Groups

<table>
<thead>
<tr>
<th>Groups</th>
<th>Type of Response and Reward</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1-, 2-, &amp; 3- letter solutions</td>
</tr>
<tr>
<td>1-1</td>
<td>1 point</td>
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<tr>
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</tr>
<tr>
<td>50-50</td>
<td>50 points</td>
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Table 1b

The Five-Problem Sequences Presented to Each Group

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<th>No.</th>
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<tbody>
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<td>8</td>
</tr>
<tr>
<td>3</td>
<td>GARDEN DWILBAEN IPECVNRO UCENIDOR YANERL KERCHDAU</td>
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<td>GARDEN UCENIDOR YANERL KERCHDAU DWILBAEN IPECVNRO</td>
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</table>

Total = 40
Table 2

Mean of Total Solutions by Reward Groups (N = 40 at Each Point)

<table>
<thead>
<tr>
<th>Reward Condition</th>
<th>Positions</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>17.13</td>
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Table 3

Mean of 1-, 2-, & 3-Letter Solutions by Reward Groups (N = 40 at Each Point)

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Table 4

Mean of 4- , 4+ -Letter Solutions by Reward Groups (N = 40 at Each Point)

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Table 5
Mean of Ratios of 1-, 2-, & 3-Letter Solutions by Reward Groups (N = 40 at Each Point)

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Table 6

Mean of Ratios of 4-, 4+ -Letter Solutions by Reward Groups (N = 40 at Each Point)

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<th>Reward Condition</th>
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<td>.424</td>
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Table 7

Mean of Ratios of 1-, 2-, & 3-Letter to 4-, 4+-Letter Solutions by Reward Groups (N = 40 at Each Point)

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<th>Reward Condition</th>
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Table 8
Mean of Ratios of 4-, 4+ -Letter Solutions by Reward Groups (N = 40 at Each Point)

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<th>Reward Condition</th>
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<th>4</th>
<th>5</th>
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<td>1.196</td>
<td>1.365</td>
<td>1.159</td>
<td>1.635</td>
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Table 9

Ss' Replies to Questions on Effect of Points;
Summarized into Three Main Categories (N = 40 in Each Cond.)

<table>
<thead>
<tr>
<th>Category</th>
<th>Reward Conditions</th>
<th>Short Solutions Receiving Higher Reward</th>
<th></th>
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<tr>
<td></td>
<td>1-1</td>
<td>5-1</td>
<td>10-1</td>
<td>20-1</td>
<td>50-1</td>
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<tr>
<td>No Effect</td>
<td>16</td>
<td>12</td>
<td>7</td>
<td>10</td>
<td>9</td>
<td></td>
<td></td>
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<tr>
<td>General Effect</td>
<td>19</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Definite Effect</td>
<td>5</td>
<td>25</td>
<td>29</td>
<td>26</td>
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<table>
<thead>
<tr>
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<th>Reward Conditions</th>
<th>Long Solutions Receiving Higher Reward</th>
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<tr>
<td></td>
<td>50-50</td>
<td>1-5</td>
<td>1-10</td>
<td>1-20</td>
<td>1-50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Effect</td>
<td>25</td>
<td>15</td>
<td>16</td>
<td>10</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>General Effect</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Definite Effect</td>
<td>8</td>
<td>22</td>
<td>22</td>
<td>28</td>
<td>23</td>
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Totals 80 80 80 80 80 = 400
### Table 10

**Binomial Probabilities Associated with Tests of Practice**

**Effects (1a) and Differential Reward Effects (1b) Under Hypothesis 1**

<table>
<thead>
<tr>
<th>Measurement Index</th>
<th>(1a)</th>
<th>(1b)</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Fig.</td>
<td>P</td>
</tr>
<tr>
<td>1. $1+2+3$</td>
<td>9</td>
<td>.031</td>
</tr>
<tr>
<td>2. $4+4^+$</td>
<td>10</td>
<td>.031</td>
</tr>
<tr>
<td>3. $1+2+3$</td>
<td>12</td>
<td>.031</td>
</tr>
<tr>
<td>4. $4+4^+$</td>
<td>13</td>
<td>.031</td>
</tr>
<tr>
<td>5. $1+2+3$</td>
<td>15</td>
<td>.031</td>
</tr>
<tr>
<td>6. $4+4^+$</td>
<td>16</td>
<td>.031</td>
</tr>
<tr>
<td>7. $\frac{\Xi(1+2+3)+\Xi(4+4^+)}{N}$</td>
<td>11</td>
<td>.031</td>
</tr>
<tr>
<td>8. $\frac{\Xi(1+2+3)+\Xi(4+4^+)}{(Total)}$</td>
<td>14</td>
<td>.031</td>
</tr>
<tr>
<td>9. $\frac{\Xi(1+2+3)+\Xi(4+4^+)}{(4+4^+)/(1+2+3)}$</td>
<td>17</td>
<td>.031</td>
</tr>
<tr>
<td>10. $\frac{\Xi(1+2+3)+\Xi(4+4^+)}{N}$</td>
<td>8</td>
<td>.188</td>
</tr>
</tbody>
</table>
Table 11

Probabilities Associated with Jonckheere Test of Hypothesis 2

<table>
<thead>
<tr>
<th>Measurement Index</th>
<th>Figure</th>
<th>P</th>
<th>Table</th>
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</thead>
<tbody>
<tr>
<td>1. 1+2+3</td>
<td>18</td>
<td>.5183</td>
<td>3</td>
</tr>
<tr>
<td>2. 4+4+</td>
<td>19</td>
<td>.5183</td>
<td>4</td>
</tr>
<tr>
<td>3. (\frac{1+2+3}{\text{Total}})</td>
<td>21</td>
<td>.1241</td>
<td>5</td>
</tr>
<tr>
<td>4. (\frac{4+4+}{\text{Total}})</td>
<td>22</td>
<td>.0751</td>
<td>6</td>
</tr>
<tr>
<td>5. (\frac{1+2+3}{4+4+})</td>
<td>24</td>
<td>.1058</td>
<td>7</td>
</tr>
<tr>
<td>6. (\frac{4+4+}{1+2+3})</td>
<td>25</td>
<td>.0751</td>
<td>8</td>
</tr>
</tbody>
</table>
Figure 8. Total solution production as a function of practice (Positions) for individuals combined by common reward groups; e.g., $\sum \frac{(1-5) + (1-10) + (1-20) + (1-50)}{N}$
M E A N  O F  I N D I V I D U A L S'  T O T A L  S O L U T I O N S

Q — Q G R O U P S  W I T H  1 ,  2 ,  3  S O L U T I O N S  R E C E I V I N G  H I G H  R E W A R D
£ — A G R O U P S  W I T H  4 ,  4 +  S O L U T I O N S  R E C E I V I N G  H I G H  R E W A R D
C O N T R O L  G R O U P S  R E C E I V I N G  E Q U A L  R E W A R D

POSITIONS

MEAN O F INDIVIDUALS' TOTAL SOLUTIONS

GROUPS WITH 1, 2, 3 SOLUTIONS RECEIVING HIGH REWARD
GROUPS WITH 4, 4+ SOLUTIONS RECEIVING HIGH REWARD
CONTROL GROUPS RECEIVING EQUAL REWARD

POSITIONS

1 2 3 4 5 6
Figure 9. 1-, 2-, and 3-letter solution production as a function of practice (Positions) for individuals combined by common reward groups.
1, 2, 3 solutions receiving high reward

1, 2, 3 solutions receiving reward equal to 4, 4+

1, 2, 3 solutions receiving low reward
Figure 10. 4-, 4+ -letter solution production as a function of practice (Positions) for individuals combined by common reward groups.
SOLUTIONS RECEIVING HIGH REWARD

SOLUTIONS RECEIVING REWARD EQUAL TO 1, 2, 3

SOLUTIONS RECEIVING LOW REWARD

POSITIONS

MEAN OF INDIVIDUALS, 4, 4+ SOLUTIONS
Figure 11. Solution production, disregarding class, as a function of practice (Positions) for individuals combined by groups receiving common point values for 1-, 2-, and 3-letter and 4-, 4+-letter solutions.
COMBINED MEANS OF INDIVIDUALS, 1, 2, 3 AND 4,4 SOLUTIONS

HIGH-REWARD CLASS OF SOLUTIONS
(ALL SOLUTIONS RECEIVING 5, 10, 20, OR 50 POINTS)

EQUAL REWARD CLASS OF SOLUTIONS
(GROUPS 1-1 AND 50-50)

LOW REWARD CLASS OF SOLUTIONS
(ALL SOLUTIONS RECEIVING 1 POINT)

POSITIONS

COMBINED MEANS OF INDIVIDUALS, 1, 2, 3 AND 4,4 SOLUTIONS

HIGH-REWARD CLASS OF SOLUTIONS
(ALL SOLUTIONS RECEIVING 5, 10, 20, OR 50 POINTS)

EQUAL REWARD CLASS OF SOLUTIONS
(GROUPS 1-1 AND 50-50)

LOW REWARD CLASS OF SOLUTIONS
(ALL SOLUTIONS RECEIVING 1 POINT)

POSITIONS
Figure 12. Solution production, measured as a ratio of 1-, 2-, and 3-letter to total solutions, as a function of practice (Positions) for individuals combined by common reward groups.
Mean of Individual Ratios of 1, 2, 3 to Total Solutions

1, 2, 3 Solutions Receiving Higher Reward (5, 10, 20 or 50 Points)

1, 2, 3 Solutions Receiving Reward Equal to 4, 4 (Groups 1-1 and 50-50)

1, 2, 3 Solutions Receiving Low Reward (1 Point)
Figure 13. Solution production measured as a ratio of 4-, 4+ -letter to total solutions as a function of practice (Positions) for individuals combined by common reward groups.
4,4⁺ solutions receiving higher reward (5, 10, 20 or 50 points)

4,4⁺ solutions receiving reward equal to 1, 2, 3 (Groups 1-1 and 50-50)

4,4⁺ solutions receiving low reward (1 point)
Figure 14. Solution production, disregarding class, as a function of practice (Positions) for individual ratios of $\frac{1}{\text{Total}}, \frac{2}{\text{Total}}, \frac{3}{\text{Total}}$ and $\frac{4}{\text{Total}}, \frac{4^+}{\text{Total}}$ combined by groups receiving common point values for $1^-$, $2^-$ and $3$-letter and $4^-$, $4^+$-letter solutions.
COMBINED MEANS OF INDIVIDUAL RATIOS

HIGH-REWARD CLASS OF SOLUTIONS IN NUMERATION
OF RATIO (ALL SOLUTIONS RECEIVING 5, 10, 20 OR
50 POINTS)

EQUAL REWARD CLASS OF SOLUTIONS
(GROUPS 1-1 AND 50-50)

LOW-REWARD CLASS OF SOLUTIONS IN NUMERATION
OF RATIO (ALL SOLUTIONS RECEIVING 1 POINT)
Figure 15. Solution production, measured as a ratio of 1-, 2-, and 3-letter to 4-, 4+ -letter solutions, as a function of practice (Positions) for individuals combined by common reward groups.
MEAN OF INDIVIDUAL RATIOS OF 1,2,3 TO 4,4 SOLUTIONS

1,2,3 SOLUTIONS RECEIVING HIGHER REWARD (5, 10, 20, 50 POINTS)

1,2,3 SOLUTIONS RECEIVING REWARD EQUAL TO 4,4 (GROUPS 1-1 AND 50-50)

1,2,3 SOLUTIONS RECEIVING LOW REWARD (1 POINT)
Figure 16. Solution production, measured as a ratio of 4-, 4+-letter
to 1-, 2-, and 3-letter solutions, as a function of practice (Positions)
for individuals combined by common reward groups.
Mean of individual ratios of 4, 4+ to 1, 2, 3 solutions

- 4, 4+ solutions receiving higher reward
- 4, 4+ solutions receiving reward equal to 1, 2, 3
- 4, 4+ solutions receiving low reward

Positions: 1, 2, 3, 4, 5, 6
Figure 17. Solution production, disregarding class, as a function of practice, (Positions) for individual ratios of $\frac{1, 2, 3}{4^+}$ and $\frac{4, 4^+}{1, 2, 3}$ combined by groups receiving common point values for 1-, 2-, and 3-letter and 4, 4+—letter solutions.
HIGH-REWARD CLASS OF SOLUTIONS IN NUMERATION OF RATIO (ALL SOLUTIONS RECEIVING 5, 10, 20 OR 50 POINTS)

EQUAL-REWARD CLASS OF SOLUTIONS (GROUPS 1-1 AND 50-50)

LOW-REWARD CLASS OF SOLUTION IN NUMERATOR OF RATIO (ALL SOLUTIONS RECEIVING 1 POINT)

COMBINED MEANS OF INDIVIDUAL RATIOS OF (1/2+6/7+4/6) SOLUTIONS

POSITIONS

1 2 3 4 5 6
Figure 18. 1-, 2-, and 3-letter solution production as a function of increments in differential reward for individuals combined by common reward differential over last four positions of practice; e.g., \( \frac{6}{3} \frac{(1 \cdot 2 \cdot 3)}{N} \) for (1-5), (5-1); \( \frac{6}{3} \frac{(1 \cdot 2 \cdot 3)}{N} \) for (1-10), (10-1); etc.
MEAN OF INDIVIDUALS: 1, 2, 3 SOLUTIONS FOR LAST FOUR POSITIONS (3-6)

1, 2, 3 SOLUTIONS RECEIVING HIGH REWARD

1, 2, 3 SOLUTIONS RECEIVING LOW REWARD

POINT REWARD FOR MORE HIGHLY REWARDED CLASS OF SOLUTIONS
Figure 19. $4^-$, $4^+$-letter solution production as a function of increments in differential reward for individuals combined by common reward differential over last four positions of practice; e.g.,

$\sum_{i=1}^{6} (4, 4)$ for (1-5), (5-1); $\sum_{i=3}^{6} (4, 4)$ for (1-10), (10-1); etc.
Means of individuals' $4, 4^+$ solutions for last four positions (3-6)

- Solutions receiving high reward
- Solutions receiving low reward

Point reward for more highly rewarded class of solutions
Figure 20. Solution production, disregarding solution class, as a function of increments in differential reward for individuals combined by groups receiving common point values for 1-, 2-, and 3-letter and 4-, 4+ -letter solutions; e.g., \[ \sum_{i=1}^{6} \left( \frac{1 2 3}{3} \right) \text{ for } 5-1 \text{ group } + \sum_{i=1}^{3} \left( \frac{4 4}{3} \right) \text{ for } 1-5 \text{ group } ; \text{ etc.} \]
COMBINED MEANS OF INDIVIDUALS' 1, 2, 3 AND 4, 4+ SOLUTIONS FOR LAST FOUR POSITIONS (3-6)

- HIGH REWARD CLASS OF SOLUTIONS
- EQUAL REWARD CLASS OF SOLUTIONS
- LOW REWARD CLASS OF SOLUTIONS

POINT REWARD FOR MORE HIGHLY REWARDED CLASS OF SOLUTIONS
Figure 21. 1-, 2-, and 3-letter solution production, measured as a ratio of 1-, 2-, and 3-letter to total solutions, as a function of increments in differential point reward for individuals combined by groups receiving common reward differential over last four positions of practice.
1,2,3 SOLUTIONS RECEIVING HIGH REWARD

1,2,3 SOLUTIONS RECEIVING LOW REWARD

POINT REWARD FOR MORE HIGHLY REWARDED CLASS OF SOLUTIONS
Figure 22. 4-, 4⁺-letter solution production, measured as a ratio of 4-, 4⁺-letter to total solutions as a function of increments in differential point reward for individuals combined by groups receiving common reward differential over last four positions of practice.
Means of Individual 4,4 Solutions to Total for Last Four Positions (3-6)

4,4 Solutions Receiving High Reward

4,4 Solutions Receiving Low Reward

Point Reward for More Highly Rewarded Class of Solutions
Figure 23. Solution production, disregarding solution class, as a function of increments in differential reward for individuals combined by groups receiving common point reward values for 1-, 2-, and 3-letter and 4-, 4+ -letter solutions over last four positions of practice; e.g., the same as Figure 13 using ratios rather than frequencies.
HIGH-REWARD CLASS OF SOLUTIONS
IN NUMERATOR OF RATIO

LOW-REWARD CLASS OF SOLUTIONS
IN DENOMINATOR OF RATIO

POINT REWARD FOR MORE HIGHLY REWARDED CLASS OF SOLUTIONS
Figure 24. 1-, 2-, and 3-letter solution production, measured as a ratio of 1-, 2-, and 3-letter to 4-, 4+ -letter solutions, as a function of increments in differential reward for individuals combined by groups receiving common reward differential over last four positions of practice.
M E A N  O F  I N D I V I D U A L  R A T I O S  O F  1 , 2 , 3  T O  4 , 4 *  S O L U T I O N S  P O R  L A S T  F O U R  P O S I T I O N S  ( 3 - 6 )

1 , 2 , 3 S O L U T I O N S  R E C E I V I N G  H I G H  R E W A R D

1 , 2 , 3 S O L U T I O N S  R E C E I V I N G  L O W  R E W A R D

Figure 25. $4^-$, $4^+$-letter solution production, measured as a ratio of $4^-$, $4^+$-letter to $1^-$, $2^-$, and $3^-$-letter solutions, as a function of increments in differential reward for individuals combined by groups receiving common reward differential over last four positions of practice.
MEAN OF INDIVIDUAL RATIOS OF $4,4^+$ TO 1,2,3 SOLUTIONS FOR LAST FOUR POSITIONS (3-6).

4,4$^+$ SOLUTIONS RECEIVING HIGHER REWARD

4,4$^+$ SOLUTIONS RECEIVING LOW REWARD

POINT REWARD FOR MORE HIGHLY REWARDED CLASS OF SOLUTIONS
Figure 26. Solution production, disregarding class, as a function of increments in differential reward for individuals combined by groups receiving common point values for 1-, 2-, and 3-letter and 4-, 4+ -letter solutions over last four positions of practice; e.g.,

$$\sum_{3}^{6} \left( \frac{(1+2+3)}{(4+4^+)} \right) \text{ for 5-1 group}; \quad \sum_{3}^{6} \left( \frac{(4+4^+)}{(1+2+3)} \right) \text{ for 1-5 group, etc.}$$
HIGH-REWARD CLASS OF SOLUTIONS
IN NUMERATOR OF RATIO

LOW-REWARD CLASS OF SOLUTIONS
IN NUMERATOR OF RATIO

COMBINED MEANS OF INDIVIDUAL RATIOS
OF 1/2, 2/3, AND 4/3 SOLUTIONS

POINT REWARD FOR MORE HIGHLY REWARDED CLASS OF SOLUTIONS