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EFFECTS OF VARIOUS ASYMPTOTIC RESTRICTIONS
ON HUMAN TRIAL-AND-ERROR LEARNING

by

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INTRODUCTION

A research program in human learning recently begun by Noble (Noble, 1954) is attempting to extend Hull's S-R reinforcement theory (Hull, 1952) from the special case of two-choice simple trial-and-error learning to the general case of multiple-choice compound trial-and-error learning. On the basis of Hull's assumptions, the following rational equation was proposed (Noble, 1957c) to describe the acquisition curves:

$$R_p = a(1)r^N \quad [1]$$

where R_p = probability of correct choice based on the pooled responses of a group of S_s ; a = asymptote or limit of R_p ; i = initial probability at the outset of learning (given by the reciprocal of the number of available, independent and mutually-exclusive responses); r = rate parameter (theoretically related to stimulus conditions, task factors, work distribution, reward contingencies, and individual differences); and where N = number of practice trials.

Previous experiments have investigated the generality of the equation under variations of response availability (Noble, 1955, 1957a), task length (Noble, 1957b), locus of reward (C. E. Noble & J. L. Noble, 1958), delay of reward (Noble & Alcock, 1958), reward schedule (Taylor & Noble, 1960), specificity of instructions (Noble, Alcock, & Farese, 1958), type of choice procedure (C. E. Noble, Alcock, & J. L. Noble, 1958), individual differences (C. E. Noble, J. L. Noble, & Alcock, 1958), and work distribution (Noble & Taylor, 1959). The percentage

of variance in R_p accounted for by Equation 1 in these studies has ranged in 24 independent samples from 91.8% to 99.9% with the higher values predominating (mean = 98.7%).

The present experiment is a more stringent test of the applicability of Equation 1 in which the limit of acquisition (a) was varied for different groups, all other variables being kept constant. Various asymptotic restrictions were introduced by manipulating the percentage of reinforceable responses available to Ss in a standard 10-link problem. The empirical question, in other words, was to determine the influence of different "degrees of solubility" on the basic form of human trial-and-error acquisition curves.

Most of the earlier investigations discovered a high degree of predictability for Equation 1 when i was varied and a held constant. This experiment was designed to assess the predictability of the formula under exactly the opposite conditions. Our expectation was that Equation 1 would continue to predict human behavior with approximately the same accuracy as before, and that r would be some function of a . Due to the nature of the equation, it was also deduced that initial proficiency would vary directly with these asymptotic restrictions.

Method

Apparatus. The Selective Mathometer, described in detail elsewhere (Noble & Farese, 1955), is shown in Fig. 1. It is an automatic 19-key pushbutton device, designed for the study of complex learning in human Ss . Pacing stimuli are presented serially by an electronically-controlled slide projector, to which S responds by selecting keys on a semicircular panel. The slides consisted of an invariant

sequence of 10 electrical circuit symbols, and the correct solution for all Ss was to press the odd-numbered keys in the order: 3, 13, 17, 9, 19, 5, 1, 11, 7, 15. Scores were recorded by polygraph and counters. Time constants for stimulus duration (2-sec.), intraserial interval (1.75-sec.), interserial interval (7.50-sec.), reward delay (0-sec.) and reward duration (.75-sec.) were the same as in most previous experiments. All data were collected in an air-conditioned laboratory, shown in Fig. 2; E and S were isolated in adjoining sound-treated chambers connected by a one-way vision mirror and an inter-com system.

Subjects. The total of 220 Ss sampled were students enrolled at Montana State University during the spring and summer quarters of 1959, of whom 70 were rejected due to machine failure, E error, S error, and for the purpose of sub-group balancing. The distribution of rejections were as follows:

Reason for Rejection	Groups				
	10	9	8	6	4
Machine failure	8	4	12	13	4
<u>E</u> error	1		2	3	2
<u>S</u> error	2	1			
Sub-group balancing		7	2	6	3

The remaining 150 Ss were composed of 74 men and 76 women, whose ages

ranged from 17 to 59 years (mean = 25.18 yr.). None had previous experience with the Mathometer. All received standard instructions (Noble, Alcock, & Farese, 1958, p. 255) which emphasized accuracy, quick responding, and the dependency of the green light on correct choices. The nature of the apparatus and the instructions was such that Ss received no indication of differential treatment in the various conditions.

Procedure. The Ss were divided into five experimental groups of 30 each, and assigned to the different conditions by a simple forward-reverse counterbalancing plan. The treatments differed in the fact that the number of reinforceable keys varied from 4 to 10. The reference or control group (Group 10) practiced under standard conditions where 10 keys were available and all rewardable, thus making it possible for Ss in this group to find the appropriate key for each stimulus. Group 10 thus worked on a soluble problem, where $\underline{a} = 1.00$. Group 9 was presented with the same stimuli and had the same keys available as Group 10, but one of the keys was non-reinforceable; i.e. one key always failed to produce reward or feedback, making it possible for Ss in this group to find an appropriate key for only 9 of the 10 stimuli. Group 9 therefore worked on an insoluble problem, where $\underline{a} = .90$. For Group 8 the situation was the same as above except that two keys were non-reinforceable; hence $\underline{a} = .80$. Group 6 had four keys non-reinforceable, giving $\underline{a} = .60$. Finally, Group 4 practiced with six non-reinforceable keys, so $\underline{a} = .40$.

The four groups whose asymptotic values were less than 1.00 were further sub-divided into three equal subgroups ($\underline{n} = 10$) which were dif-

ferentiated from each other by the pattern of non-reinforceable keys. In all cases the keys were eliminated by random selection as shown below:

Groups	Keys Eliminated		
	Subgroup 1	Subgroup 2	Subgroup 3
9	9	17	3
8	9,5	9,15	1,7
6	3,13,17,9	3,13,9,15	3,9,11,15
4	3,19,1,11,7,15	9,19,5,1,11,15	3,13,9,5,11,15

A non-correction procedure (C. E. Noble, Alcock, & J. L. Noble, 1958) was used throughout, with S being allowed only one choice for each slide. All Ss were instructed to use the forefinger of the preferred hand only, to return the hand to the fixation plate after each response, and not to press any buttons during the inter-stimulus interval. Each S practiced for 50 trials or mastery, whichever came first, the latter being defined as five successive trials without error, a criterion which had been established in a previous study (Noble, 1957b).

Results and Discussion

The acquisition curves are presented in Fig. 3, where the probabilities of correct (reinforceable) responses (R_p) are plotted as a function of trials (N) for the five values of the asymptote parameter, a. It should be emphasized that each curve contains relative frequency measurements of only those acts which are both correct (R_+) and rewardable (G), and that the denominator (number of possible responses) for

each trial score is 10. As predicted from theoretical considerations (Hull, 1952; Noble, 1957c), all five curves are asymmetrical and sigmoidal. Moreover, they are in excellent agreement with Equation 1. The best-fitting theoretical equations, obtained by minimizing the sums of the squares of the residuals to determine the rate parameter $\underline{r}$, are as follows:

$$\text{Group 10: } R_p = 1.00 (.10)^{.916N} \quad [2]$$

$$\text{Group 9: } R_p = .90 (.10)^{.922N} \quad [3]$$

$$\text{Group 8: } R_p = .80 (.10)^{.922N} \quad [4]$$

$$\text{Group 6: } R_p = .60 (.10)^{.913N} \quad [5]$$

$$\text{Group 4: } R_p = .40 (.10)^{.902N} \quad [6]$$

The computational procedure for finding $\underline{r}$ has been fully described elsewhere (Noble, 1957b). A preliminary estimate of the inflection point of the curve (N_1) was made by dropping a perpendicular from a point on the ordinate equal to the ratio $\underline{a}/\underline{e}$ (where $\underline{e} = 2.718$) to the abscissa. This approximate N_1 value is then substituted in the formula,

$$\log_e \underline{r} = \frac{-\log_e (-\log_e i)}{N_1} \quad [7]$$

to obtain an estimate of $\underline{r}$. The exact value of $\underline{r}$ is secured when the quantity $\Sigma(R_p - \tilde{R}_p)^2$ reaches a minimum. In all previous studies, the ordinate at which acceleration of the R_p curve passed to zero was a constant; namely, $.368 = 1/\underline{e}$, the reciprocal of the base of the natural system of logarithms. A salient feature of the present experiment is the demonstration that this interesting property of Equation 1 is merely the special case of a soluble problem where the asymptotic value is 1.00. In the general case of problems ranging in degree of solubility

(probability of mastery = $\underline{a}$) from zero to unity, the inflection point should occur at $\underline{a}/\underline{e}$. Empirical confirmation of this prediction can be seen in Fig. 3.

Coefficients of determination for Equations 2 through 6 were computed directly from the variances of the residuals and the $\underline{R}_p$ values, as recommended by Noble (Noble, 1957b). Predictability is over 99% in every case. As a test of our hypothesis that $\underline{r}$ would be a function of $\underline{a}$, the rate parameter of best fit for Group 10 is compared with each of the others in Table 1. Judging by the relatively greater amounts of variance in $\underline{R}_p$ scores accounted for by Equations 3, 4, 5, and 6 when their least-squares $\underline{r}$ values are employed rather than an $\underline{r}$ of .916, it is clear that the rates of change are systematically, although not monotonically, affected by manipulation of the asymptotes.

Zero accelerations occur at the following trials ($\underline{N}_i$) for the five fitted acquisition curves: Group 10: 9.5; Group 9: 10.27; Group 8: 10.27; Group 6: 9.1; Group 4: 7.58. Examination of the empirical points in Fig. 3 indicates that the $\underline{R}_p$ values for Group 10 deviated markedly from its theoretical curve. A $\underline{z}$ test of proportions taken at the point of maximal deviation (Trial 5) indicates that a deviation ($\underline{R}_p - \hat{\underline{R}}_p$) this large or larger would occur 38.98% of the time by chance. The Trial 5 discrepancy, therefore, is not significant.

As a further test of the differential-rate hypothesis, a 10x5 mixed-factorial analysis of variance was performed on the $\underline{R}_+$ scores, arranged in 10 blocks of five trials ($\underline{N}$) for the five experimental groups ($\underline{a}$). The summary is shown in Table 2. Main effects of $\underline{N}$ and $\underline{a}$ and their interaction are all significant. As statistical evidence of

non-parallel trends, the $N \times a$ interaction supports our expectation that $\bar{r}$ would vary as a function of a .

It is of some interest to evaluate the effects of degree of solubility on the difficulty (Noble, 1957b) of acquiring comparable amounts of "knowledge" (number of R_+ 's) about the task or problem. To this end a trials-to-criterion analysis was made of the five groups in which the criterion was defined as that trial on which the same four correct and rewarded responses were made for the fifth trial in succession. Figure 4 shows the median number of trials required to reach this criterion (R_N) as a function of the percentage of reinforceable responses ($100a$). The relationship is an inverse proportionality whose best-fitting, least-squares equation is $Y = -.176X + 35.924$. Calculation of the coefficient of determination indicates that 96% of the variance in Y is attributable to X . From this we conclude that difficulty is a negative linear function of increasing solubility over the present range.

Further examination of Fig. 3 indicates that the initial proficiency (R_p on Trial 1) of the different groups varies directly with the asymptotic restrictions. By reference to Equations 2 through 6 it is evident that the theoretical origin of each group's curve (R_p on Trial 0) is given by the product of a and i . This important characteristic of Equation 1 may be rationalized by applying the multiplication theorem of the probability calculus, as follows. When 10 keys are available, the probability of getting an R_+ by chance alone (not considering reward contingencies) is .10 ($= i$). When the number of reinforceable responses is restricted, however, we are

dealing with the combined probability of independent events, each less than unity. For this reason, we must calculate the joint probability that a particular key will be selected ($\underline{R+}$) and that this response will be followed by reinforcement ($\underline{G}$). In abstract terms: $p(\underline{R+ \cap G}) = p(\underline{R+}) \cdot p(\underline{G})$. In the case of Group 4, for example, $p(\underline{R+}) = \underline{i} = .10$ and $p(\underline{G}) = \underline{a} = .40$; hence $\underline{R_p}$ on Trial 0 = $\underline{a} \cdot \underline{i} = .04$. The joint probabilities of the other groups are computed in similar fashion. Thus is the rationale of Equation 1 directly coordinated with the relative frequency theory of probability.

In conclusion, it appears from these results that Equation 1 has again proved itself to be an excellent predictor of human acquisition data in complex learning situations. The versatility of the equation is indicated not only by its ability to describe the empirical results with an error of less than 1%, but also by the extent to which it lends itself to systematic formulation in probability terms.

Summary

This experiment investigated the effects of asymptotic restrictions on the forms of acquisition curves in a complex trial-and-error learning problem in man. One hundred fifty college students, divided into five experimental groups of 30 $\underline{Ss}$ each, practiced a series of 10 pushbutton responses on the Selective Mathometer for 50 trials. The experimental variable consisted of manipulating the number of reinforceable keys in the various groups to produce 40%, 60%, 80%, 90%, and 100% limits on the attainable levels of mastery.

The response probability ($\underline{R_p}$) curves were asymmetrical and sigmoidal when plotted as a function of amount of practice ($\underline{N}$). Each group's

curve originated at a hypothetical R_p value given by the combined probability of making a correct choice (i) and of receiving reinforcement (a). Noble's rational equation $R_p = a(i)r^N$ accounted for over 99% of the variance in the dependent variable scores (R_p) in all five instances. The rate parameters (r) were systematically, although not monotonically, affected by variation of the asymptotes. It was shown that in general the inflection points of the theoretical curves occurred at ordinate values of a/e , where $e = 2.718$.

An analysis of the influence of degree of solubility on difficulty agreed with the effects of asymptotic restrictions on initial proficiency. By measuring the median trials required to master four correct responses in all groups, it was found that difficulty was an inverse linear function of solubility.

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APPENDIX

An informal interview was held with 50 of the Ss following the practice session on the Mathometer. The object of the interview was to gain some understanding of the problem as seen by S. Questioning dealt with S's approach to the solution, what S thought he was doing while trying to find the solution, and if S did not find the solution, how he accounted for his failure.

Initial "random" selection was reported by 80% of the Ss, while 20% reported that they started with some system. Of the 80% reporting initial random selection, 40% reported that they developed a system within 10 trials. Of those starting with a system and those developing a system, 77% used the same system, that of pressing a button continuously through a trial until they received a green light, then pressing the next button until they received a green light, remembering to press the correct button for the first green light, etc. It was observed that in the majority of instances of this behavior, S could do this successfully until approximately 4 solutions had been found, after which considerable temporary "loss" of correct responses occurred. No other system was used by more than one S.

Questioning revealed that 30% were consciously aware that they were attempting to associate buttons and symbols through memorization. The balance were unable to describe how they learned to solve the problem.

In those groups where solution was impossible, 60% felt that given sufficient time, they could have solved the problem, while 40% felt that something was wrong with the machine. Frustration and anger inspired by the task were acknowledged by 80% of the sample, the balance denying that they had had any emotional reaction. In the latter case it was suspected by E that this was largely a defense mechanism since in most instances ample evidence of irritation had been observed in the form of frowns, biting of lips, beating on the response panel, and excessive vigor of button pressing.

In general the most apparent behavior characteristic noted by E during questioning was the unawareness of the means by which learning took place. E felt that many of the answers on which the preceding tabulation was based were purely post hoc rationalizations by S.

Fig. 1. Floor plan of the Selective
Learning Research Laboratory

Fig. 2. The Selective Mathometer

Fig. 3. Compound trial-and-error learning (R_p)
as a function of practice (N) with different
asymptotes (a) as the parameter. The origins
are rational.

Fig. 4. Difficulty in learning four choices (R_N) as a function of degree of solubility (100 $\underline{a}$) of the 10-link problem. The fitted line is empirical.

Table 1
Rate Parameter of Group 10 Compared with the Rate Parameters
of Best Fit for the Other Experimental Groups

Group	$\underline{r}$	Goodness-of fit Index (%)	$\underline{r} = .916$
10	.916	99.06	
9	.922	99.57	98.59
8	.922	99.41	98.58
6	.913	99.31	98.46
4	.902	99.15	95.17

Table 2
 Analysis of Variance of Correct Responses (R+)
 During Successive 5-Trial Blocks for the
 Various Experimental Groups

Source	df	Mean Square	F
Between <u>Ss</u>	149		
Asymptote (<u>a</u>)	4	18008.34	80.36*
Error (<u>b</u>)	145	224.10	
Within <u>Ss</u>	1350		
Trial Blocks (<u>N</u>)	9	15466.55	92.90*
Interaction (<u>Nxa</u>)	36	296.34	1.78*
Error (<u>w</u>)	1305	166.48	
Total	1499		

* $p < .01$