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MATH 414.01: Deterministic Models - A Course in Mathematical Biology

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Math 414
A Course in Mathematical Biology
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Abstract

Mathematical Biology is a fast growing area of mathematics that began in the late 1920's with the work of Lotka & Volterra in population models and Kermack & McKendrick in epidemiology. We have developed a senior-level interdisciplinary course that serves the needs of applied mathematics as well as the modelling needs of students in the Wildlife Biology and Chemistry programs.

Much of Mathematical Biology involves the analysis of nonlinear difference and differential equations (and systems). Most topics can be approached using graphical techniques easily mastered by upper- division non-majors. These tools provide genuine insight into the application of mathematics in the world around us. The models show how certain processes work in biology and predict outcomes that give insight into biological mechanisms.

Much of the course involves the use of eigenvalues in analyzing local behavior, a task within reach of interdisciplinary students. We focus on 2-dimensional systems, but also mention behaviors in higher dimensions.

A third aspect of the course is the use of technology: we use some simple TRUE BASIC programs to introduce certain models of population growth (Leslie model), and to discuss the chaotic behavior of logistic models. Students are then taught to use PHASEPLANE, as a tool for finding numerical solutions of differential and discrete systems.

This one-semester course includes aspects of chaotic motions, phase-plane analysis, and bifurcation theory, but virtually all of the theoretical material requires only a basic understanding of sophomore calculus.

Outline of Course
(45 days)

1. Difference Equations: (7 days)
   (a) Linear: Compound Interest, Fibonacci, Bird Populations
   (b) Nonlinear: Logistic Equation, Steady States, Stability
   (c) Period Doubling and Chaos
2. Difference Systems: (5 days)
   (a) Linear: Leslie Model (survivability and fecundity)
   (b) Nonlinear: Nicholson-Bailey Model

Exam #1

3. Differential Equations: (6 days)
   (a) Linear: Growth, Decay, Oscillations (periodic, damped)
   (b) Scaling (applied to Logistic, harvesting)
   (c) Nonlinear: Ludwig's Budworm Model (hysterisis)
4. Differential Systems: (10 days) (all autonomous)
   (a) Linear: 2 x 2 and 3 x 3, elimination, eigenvalues
   (b) Nonlinear: Phase Planes, Nullclines, SS and Stability
   (c) Dimensional Analysis (applied to Predator/Prey model)
   (d) Limit Cycles
   (e) Lotka-Volterra Competition Models

Exam #2

5. Biological Oscillators: (5 days)
   (a) BZ Reaction (Field-Noyes model)
6. Reaction Diffusion Models: (10 days)
   (a) Traveling Waves: Diffusion Equation, Slime Molds
   (b) Showalter automata for BZ waves
   (c) Young's Automata, Wave Equation, Turing Patterns

Final Exam

∗ All linear equations are solved via eigenvalue/eigenfunction approach

∗ Nonlinear models all studied using graphical methods